Sustainability and the Economics of the Environment: Cost-Benefit Analysis and the Dynamics of the Long-Run Discount Rate

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Abstract. This paper reviews the arguments for and the implications of employing Declining Discount Rates (DDRs) in CBA and in the analysis of economic growth and sustainability. We show that there exist several growth models in which a relationship has been found between the long-run equilibrium under DDRs and that in which a zero discount rate is employed. This can have the effect of pushing the optimum under DDRs away from the conventional utilitarian outcome towards the Green Golden Rule (GGR) level of capita or environmental stocks. Furthermore, in response to worries that the GGR places weight on the future at too great an expense to the present, we highlight the result of Li and Lofgren (2000): DDRs can evoke a solution to resource management problems in which the objective function explicitly takes into account the preferences of present and future generations, such as those posited by. Either zero or conventional discounting does not achieve this solution. It is in these senses that DDRs can be seen to encourage a more equal treatment of generations and promote sustainable outcomes. We also discuss different methodologies for the estimation of a working schedule of DDRs assuming that future discount rates and the past provides information about the future. The policy implications of this are that a correctly specified model of discount rates provides a schedule of DDRs, which values atmospheric carbon reduction 150% higher than conventional exponential discounting, and almost 90% higher than incorrectly specified models. In this sense sustainable outcomes are more likely to emerge from project appraisal with DDRs, but given that the theory of DDRs for CBA reviewed relates to the socially efficient discount rate, such outcomes can also be thought of as efficient.

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1. Introduction

Discounting is an issue that continues to receive much attention in the analysis of economic growth and sustainability, Cost Benefit Analysis (CBA) and in the study if micro-economic behaviour. With the advent of a distinct long-term policy arena however, in which long-term decisions must be made concerning various environmental, natural resources and other issues, such as climate change, biodiversity loss and nuclear build, attention has necessarily turned towards alternative methods of determining inter-temporal values than exponential discounting. In particular, the use of discount rates that decline with the time horizon, Declining Discount Rates (DDRs) has received much attention as a useful alternative and the reasons for this attention are numerous.

Firstly, the use of conventional constant exponential discounting over long-time horizons ensures that the welfare of generations in the distant future is discounted back to peanuts. As Weitzman (1998) states, 'to think about the distant future in terms of standard discounting is to have an uneasy intuitive feeling that something is wrong, somewhere'. Chichilnisky (1996) referred to this as the 'tyranny' of exponential discounting, in that it makes the current generation a dictator over future generations. Such unequal treatment of generations caused Ramsey (1928) to label discounting of future utilities as 'ethically indefensible'. Secondly, not only does this trouble our intuition and sense of fairness, it is also clearly contrary to the widely supported goal of sustainable development. Sustainable development requires that policies and investments now have due regard for the need to secure sustained increases in per capital welfare over longer time horizons than might normally be considered in policy-making (Atkinson et al. 1997). In this regard, the use of DDRs has been found to offer solutions to resource management problems that adhere to desirable axioms of intergenerational choice, such that neither the present nor the future generation holds a dictatorship over the other in determining optimal management. That is, in many cases DDRs can ensure intergenerational equity and sustainability. Lastly, there is a wide body of experimental and empirical evidence associated with the 'hyperbolic' discounting literature (for example Lowenstein and Elster 1992, Frederick, Lowenstein and O'Donoghue 2002) suggesting that individuals actually employ discount rates that decline over time in evaluating projects or scenarios. For this reason it seems sensible to incorporate such preferences into CBA and the analysis of economic growth and sustainability.

So, on the one hand the use of DDRs is often seen as a resolution of what Pigou called the 'defective telescopic faculty' of conventional exponential discounting (Pigou 1932) in that greater weight is placed upon the consequences of projects that occur in the far distant future and the preferences of future generations are more clearly registered. On the other hand, it appears on occasion to reflect how people actually behave. However, despite these arguments questions remain for the practitioner of CBA: what formal justifications exist for using a DDR in CBA? And, if we accept the theoretical arguments for DDRs, what is the optimal trajectory of the decline? In this paper we discuss the implications of DDRs for sustainability and intergenerational equity and review the various arguments for the use of DDRs in CBA.

2. A Review of Discount Rates

Project Appraisal or Cost Benefit Analysis (CBA) and the Net Present Value (NPV) criterion with which it is associated is rooted in the tradition of discounted utilitarianism. The utilitarian objective is to maximise the sum of net welfare changes for generations within the prescribed time horizon. CBA can be thought of as consisting of two stages in determining the NPV. Firstly the impacts and the costs and benefits of particular projects or policy interventions must be assessed in terms of their incidence in time and their economic value. Secondly, a judgement must be made concerning the relative value of costs and benefits that accrue in different time periods, i.e. a discount function needs to be selected. The discount function employed reflects the manner in which the numeraire changes in value depending upon its incidence in time and hence the discount rate will usually depend upon the numeraire. However whatever the numeraire a decision must also be made concerning the behaviour of the discount rate over time. Koopmans (1960), for example, provides an axiomatic approach to the selection of the discount function which provides a rationale for conventional constant rate exponential discounting. Following the tradition of e.g. Little and Mirlees (1974) and Lind (1982) it is usual in CBA to evaluate all costs and benefits using consumption as the numeraire and employing exponential discounting. In this sense the NPV of a public project with time horizon *T* can be evaluated as follows:

(1) $$\int_0^T (b_t - c_t)\exp(-at)dt$$

where b_t and c_t represent the costs and benefits at time *t* and *a* represents the chosen social discount rate (SDR). Where consumption is the numeraire the social rate of discount is commonly called the consumption rate of interest/discount or the social rate of time preference. We denote this by δ. This discount rate reflects how the contribution of increments of consumption to the underlying utilitarian welfare function changes over time. It also reflects the economic arguments for discounting in CBA.

Firstly individuals discount consumption in the future because they are impatient. This is reflected by pure rate of time preference or utility discount rate, ρ [1]. Secondly, utility maximising individuals discount the future in accordance with how they expect their wealth to change in the future. There are two important effects here, the *wealth effect* and the *prudence effect* (see for example, Gollier 2002a). Put simply, if individuals expect their wealth to increase in the future they value current consumption more highly and as a result discount the future more heavily. Inversely, if individuals are ‘prudent’, i.e. they increase savings in response to greater uncertainty about future consumption, then they will value consumption in the future more, and hence discount the future at a lower rate[2]. These effects and the consumption decisions of utility maximising individuals are commonly represented by the Ramsey equation (Ramsey 1928):

(2) $$r = \delta = \rho + \mu g$$

where r is the risk free rate of return o marginal opportunity cost of capital, here μ represents the elasticity of marginal utility of consumption, a measure of the curvature of the utility function and hence the desire to smooth consumption over time, and *g* represents the growth rate of consumption[3]. Equation (2) shows, with reasonable assumptions concerning preferences ($\mu > 0$), positive growth will increase δ [4]. The equivalent of (2) when growth is uncertain is (Gollier 2002a):

(3) $$r = \delta \approx \rho + \mu E[\tilde{g}_{t+1}] - \frac{\mu}{2}\mathrm{var}(\tilde{g}_{t+1})P(C)$$

where $P(C)$ is a measure of individual's relative prudence as a function of consumption *C*, $E[\tilde{g}_{t+1}]$ is today’s expectation of growth in period $t+1$ and $\mathrm{var}(\tilde{g}_{t+1})$ is the variance. Kimball (1990) shows that if individuals are prudent then $P(C) > 0$ and hence the associated value of δ decreases with variance[5]. Equation (3) shows that the overall effect on δ depends upon the balance between the prudence effect (the third element) and the wealth effect (the second element). Under uncertainty the term μ represents another element of individuals’ preferences for risk: it is the coefficient of relative risk aversion, and together with the measure of prudence $P(C)$, equation (3) shows how the discount rate is dependent upon such preferences.

In a competitive economy δ will be equal to the social (risk free) rate of return on capital, *r*, which, in the absence of distortions such as taxes and externalities, will equal the private rate of return on capital, *i*. However, under the (realistic) assumption imperfect markets these rates are unlikely to be equal and thus the appropriate discount rate is not immediately obvious. (Lind, 1982). For this reason economists and others have argued over which of these several discount rates should be used as the SDR, *r*, *i*, or δ. In a competitive economy these rates are equal, reflecting the interaction of utility maximising consumption decisions and profit maximising production decisions. Nevertheless, a consensus in recent literature appears to have been reached that the SDR should equal the opportunity cost of capital, *r* (Portney and Weyant 1999).

A number of additional arguments have been posited for once and for all adjustments to the level of the discount rate in particular circumstances. For example, Krutilla and Fisher (1975) suggested that the discount rate should be reduced for projects that have a significant environmental component since if environmental goods are increasing in scarcity and incomes are growing, future generations will harbour a greater willingness to pay for such goods. Gravelle and Rees (2000) used a similar argument for the case of health benefits. Such an approach implies a composite discount rate for the evaluation of these particular benefits and costs, which is reduced by the inclusion of the growth rate of willingness to pay[6]. However, Horowitz (2002) rightly highlights the importance of separating out contemporaneous and intertemporal valuation issues from the discounting issues. Weitzman (1994) also called for a reduction in the level of the discount rate applied for CBA in order to account for the increased diversion of consumption required in order to meet environmental standards in the face of greater output. He showed that consumption externalities lead to such 'environmental drag' and can cause a divergence between the social and private rates of return to capital, particularly where environmental damage is not easily reversed. A number of other arguments exist for this once and for all reduction in the level of the discount rate[7].

In the analysis of economic growth and sustainability the tradition of discounted utilitarianism has also received much attention. The objective function in such models is frequently concerned with the maximisation of welfare over time by a representative social planner, i.e. it is utility rather than consumption that is the important value. The objective function in such cases is commonly:

$$(4) \qquad \max_{\{C_t\}} W = \int_0^T u(C_t)\exp(-\rho t)dt$$

subject to the constraints of the particular model in hand, where $u(C_t)$ represents the utility at time *t*. The appropriate discount rate in this case, and for all cases where utility is the numeraire, is the utility discount rate ρ.

Clearly, there is a correspondence between the two discount rates described thus far: ρ and δ, and both of these concepts arise in the discounted utilitarian framework, respectively for valuing changes in utility and consumption that occur at different points in time. However, the correspondence between the two will depend upon the assumptions contained in the underlying welfare function. For example, Equation (2) reflects the assumptions contained in the Ramsey model, that is that utility depends solely on consumption. The two rates will differ in general and we should be aware that it is quite possible to have positive discounting of consumption and zero discounting of utility, or vice versa, occurring simultaneously[8].

Economic growth theorists differ in their opinions with regard to the discounting of utility in this way. For example, Chichilnisky (1996) framed the discussion in the language of social choice in her analysis of sustainable growth by noting that the utilitarian objective function for which $\rho > 0$ places an effective *dictatorship* of the present over the future: positive discounting reduces to zero the importance of future generation's welfare in the calculus of economic growth. Indeed, due to this unequal treatment of generations many of the early growth theorists were strongly opposed to discounting utility. For example, Ramsey (1928) stated that such a practice is 'ethically indefensible' while Harrod (1948) stated that it represented a 'triumph of reason over passion'. As a result there are many examples of growth models in which the objective function in Equation (4) has been evaluated using a zero utility discount rate.

The implications of using zero discount rates are numerous and of great interest in the analysis of growth and sustainability starting with the analysis of Ramsey (1928) and culminating more recently with the analysis of, among others, Li and Löfgren (2001). Of particular importance is the analysis of alternative growth paths or interventions in which benefits or costs occur over an infinite horizon since when the welfare effects are positive over such a horizon the integral in (4) is unbounded, making comparisons between different alternatives on this basis impossible. This is coupled with problems in the analysis of the long-run equilibrium (Barro 1999). However, since there is general agreement that the essence of sustainability and the analysis thereof is generally thought to lie in a 'treatment of the present and the future that places a positive value on the very long-run' (Heal 1998) the choice of discount rate and/or the use of zero discount rates has remained a matter of great importance. As a result this choice has received much attention in the literature.

This chapter is concerned with an alternative approach to discounting which is relevant to and has been extensively studied with regard to both CBA and models of optimal growth and sustainability. In addition to calls for once and for all reductions in or zero discount rates for the sake of intergenerational equity, environmental or other reasons, discount rates that decline with time, or Declining Discount Rates (DDRs) have arisen as an alternative way in which to incorporate these efficiency and equity goals. We now turn to these issues.

3. Declining Discount Rates

In this section we review the use of DDRs in the analysis of economic growth and sustainability and show how current work views the role of DDRs in considering intergenerational equity. This discussion concerns the utility discount rate ρ. We then go on to review the theoretical justifications that have emerged for the use of declining consumption rate of interest, δ, for CBA.

3.1 Growth, Sustainable Resource Management, Intergenerational Equity and Declining Utility Discount Rates, ρ

3.1.1 DDRs, Growth and Environment

A number of authors have discussed the implications of DDRs for optimal and sustainable growth in the context of economic growth models. Important contributions in this area include Heal (1998), Barro (1999), Chichilnisky (1996) and Li and Löfgren (2000, 2001). In many of these cases the analysis is undertaken in the context of the stylised discounted utilitarian in which the objective is to maximise the inter-temporal sum of discounted utility. In this sense where DDRs are employed they refer to the pure rate of time preference, i.e. ρ, which in general will differ from the consumption rate of interest commonly used in CBA, δ, as described in Section 2. The motivation for the use of such DDRs comes from the empirical and experimental evidence that has been discussed at length in the 'hyperbolic' discounting literature[9]. This is true of Li and Löfgren (2000, 2001) and Barro (1999) for example. The hyperbolic discounting literature provides empirical evidence suggesting that the discount rates that individuals actually apply vary and decline with the time horizon involved. For example, there is evidence to show that individuals discount the short run at rates up to 15% whilst the discount rate applied to the long-run falls to close to 2% for horizons beyond 100 years (Lowenstein and Elster 1992).

The implications for the utilitarian social planner's decisions of employing DDRs have been addressed by several authors in different contexts. Perhaps the simplest analysis of these implications is the analysis of an economy dependent upon an exhaustible resource by Heal (1998, Ch 2). He develops a model of resource exploitation á la Hotelling (1931) and shows that if the social planner employs DDRs the path of consumption declines far slower than in the presence of conventional exponential discounting. Naturally, although consumption eventually falls to zero, the decline is much slower and so certain future generations enjoy higher levels of consumption. This illustrates one way in which intergenerational equity is partially addressed by DDRs: inequality increases at a slower rate here (Heal 1998).

With regard to renewable resources it is a common result that, where utility depends upon the amenity value of environmental stocks, the optimal stationary solution as the discount rate goes to zero coincides with that of the so-called Green Golden Rule (GGR), a variant of Phelps' golden rule in the context of environmental resources (Phelps 1961, Heal 1998, Li and Löfgren 2000). The GGR is an important concept in the analysis of sustainability and is characterised by the highest sustainable or long run level of utility. In this sense the GGR equilibrium treats each generation more equally and leads to a level of the resource stock that is higher than that under conventional utilitarianism[10]. Interestingly, Heal (1998) shows that a solution to the renewable resource problem involving the use of DDRs that are asymptotic to zero as $t \to \infty$ is asymptotically equivalent to the solution in the presence of zero discount rates, since the dynamical equations are asymptotically equivalent. That is, when DDRs are used the long run stationary solution tends towards the GGR. This represents a more concrete example of the relation between the use of DDRs and the concepts of sustainability and intergenerational equity.

Barro (1999) looked at the implications for the Ramsey model of using DDRs. Motivated by the work of Laibson he analysed what is widely recognised as a thorny problem in the application of DDRs, namely time inconsistency[11]. He showed that where there is non-commitment, such that time inconsistent policies can be implemented, the optimal path might mimic that observed under conventional discounting. He concludes that the 'introduction of variable rates of time preference leaves the basic properties of the Ramsey model intact'. However, Barro (1999) assumed that the discount rate declined asymptotically to some positive constant and was not interested in environmental sustainability.

Li and Löfgren (2001) address this issue in the context of the Ramsey and Brock (Brock 1977) growth models, the latter incorporates environmental quality into the Ramsey model. They also assume that the DDR declines asymptotically to zero. In comparing optimal growth paths for the discounted utilitarian for whom $\rho > 0$, zero discounting ($\rho = 0$) and for DDRs, they find that in the Ramsey model the stable arm of the saddle growth path of consumption under DDRs is bounded by those arising under utilitarianism and zero discounting. Specifically, the consumption path starts off in the region of the discounted path and converges in the long run to that of the zero discounting case. The stationary solution in the zero discounting Ramsey case is equivalent to Phelps (1961) golden rule where the capital stock is held at the maximum sustainable yield (MSY) stock. Just as in the renewable resource case with amenity value described above, the introduction of DDRs leads to a steady state at the golden rule level of utility and takes more consideration of long run sustainability[12]. However, where stock pollution is introduced in the Brock model, these results do not hold in general, due mainly to the interaction of capital and pollution stocks, and it becomes unclear whether or not environmental quality increases or decreases when DDRs are employed.

These are just some of the impacts that DDRs have upon the traditional analysis of optimal economic growth, environmental resource management and sustainability. In many cases the use of DDRs leads to optimal long run steady states which mimic the sustainable outcomes that arise under zero discounting: the GGR. However, it is widely thought that such outcomes place too much weight upon the far-distant future generations, at some considerable cost to the present or near future. Further contributions have attempted to move away from the pure utilitarian or sustainability maximands towards a more general formulation balancing the objectives of the present and the future more satisfactorily. In the examples that follow this balance is defined in terms of axioms of social/intergenerational choice.

3.1.2 Intergenerational Equity vs Dictatorship

Perhaps the most interesting contributions in this area are those which endeavour to tackle the issue of intergenerational equity axiomatically. Chichilnisky (1996) sets out a series of desirable axioms of sustainability and derives objective functions that adhere to them. Beyond this Heal (1995) and Li and Löfgren (2000) show the importance of using DDRs to solve renewable resource allocation problems that also adhere to these axioms. Perhaps the most important of these from the perspective of intergenerational equity is the axiom of non-dictatorship, which states that there should neither be a dictatorship of the present over the future nor vice versa in evaluating long run economic growth. Chichilnisky (1996) notes that a utilitarian social planner who employs conventional discounting implies a dictatorship of the present over the future. That is, in such a representation there always exists a point in time at which the costs and benefits that accrue to the future generations do not enter into the calculus of the current utilitarian. In order to overcome the dictatorship of either generation over the other Chilchilnisky proposes an augmented objective function that explicitly incorporates, or is 'sensitive' to, the welfare of current and future generations. Chichilnisky's criterion is:

$$\max_{c,s} \alpha \int_0^\infty u(c_t, q_t)\Delta(t)dt + (1-\alpha)\lim_{t\to\infty} u(c_t, q_t) \quad (5)$$

where utility (u) is a time invariant function of consumption (c) and the resource stock (q) at each time period (t) and $\Delta(t)$ is the discount factor which could be the conventional exponential factor. Intuitively, the *lim* term reflects the sustainable utility level attained by a particular policy decision regarding c_t and q_t. This can be interpreted as the well being of generations in the far distant future and is the term that if maximized alone is associated with the GGR. Chichilnisky's approach is therefore a mixture of the discounted utilitarian approach, allowing for DDRs or constant exponential discounting, and an approach that ranks paths of consumption and natural resource use according to their long-run characteristics, or sustainable utility levels. Notice that $\alpha \in (0,1)$, represents the weights the social planner applies to each of the components of the objective function, respectively current and future generations. Chichilnisky shows that the maximisation of (5) avoids the dictatorship of one generation over another. However, while Heal (1995) shows that the maximization of (5) in the presence of non-renewables does not exhaust the resource stock, leading to a positive long-run level of utility, the solution in the presence of renewable resources requires the use of DDRs. In the latter case however, the dictatorship axiom is violated: the present is implicitly a dictator over the future [13].

In response to these issues, Li and Löfgren (2000) treat the future slightly differently. They assume society consists of two individuals, a utilitarian and a conservationist, each of which makes decisions over the inter-temporal allocation of resources. However, the former discounts the future at the constant rate $\rho_U > 0$ and the latter discounts at the rate $\rho_C = 0$. The utility functions of these two individuals are identical, and again have consumption (*c*), and the resource stock (*q*), as their arguments:

$$\text{(6)} \qquad \max\ U = \alpha U_1 + (\alpha - 1)U_2 = \int_0^\infty u(c_t, q_t)\Delta(t)\,dt$$

$$U_1 = \int_0^\infty u(c_t, q_t)\exp(-\rho_U t)\,dt$$

$$U_2 = \lim_{\delta \to 0} \int_0^\infty u(c_t, q_t)\exp(-\rho_C t)\,dt$$

where $\Delta(t)$ is the effective discount factor. The overall societal objective is to maximise a weighted sum of well-being for both members of the society, given their different respective weights upon future generations and subject to a renewable resource constraint. As in the case of Heal (1995), Li and Löfgren show that the use of a DDR which declines asymptotically to zero generates a solution to this problem where the DDR in this case is:

$$\text{(7)} \qquad a(t) = -\frac{1}{t}\left[\ln\left\{(1-\alpha)\exp(-\rho_C t) + \alpha\exp(-\rho_U t)\right\}\right]$$

For $\rho_C = 0$ this gives a discount factor equal to $\Delta(t) = (1-\alpha) + \alpha\exp(-\rho_U t)$, which has a minimum value of $(1-\alpha)$, the weight attached to the conservationist or future generations. This ensuring that the effective discount *rate* declines to zero. Thus, unlike the utilitarian discount function, which tends to zero as time reaches towards infinity, the weighted discount function of Li and Löfgren's model results in a positive welfare weight for the conservationist. For this reason there is no dictatorship of present over future generations. As the utilitarian's welfare level is explicitly considered, neither will there be a dictatorship of the future over the present. Thus, the axiom of non-dictatorship is adhered to.

Both Chichilnisky and Li and Löfgren show that a declining *utility* discount rate is consistent with non-dictatorship of one generation over another. In this way the 'tyranny of the present over the future' associated with constant rate discounting is overcome. However, whereas Chichilnisky allows the use of DDRs, the axioms of sustainability employed say nothing about the need for DDRs to generate sustainable and equitable solutions to resource allocation problems. Heal and Li and Löfgren show the importance of employing DDRs for this purpose, but only Li and Löfgren's formulation achieves intergenerational equity in the sense of avoiding dictatorship. Perhaps equally important here is the fact that the dual objective function also provides clearer guidance as to the best path towards the sustainable solution, something that is absent from the definition of the GGR. One interpretation of this is that the absence of dictatorship also represents a reasonable efficiency equity trade-off.

3.2 Cost Benefit Analysis (CBA) and Declining Social Rate of Time Preference, δ

3.2.1 Discount Rates Determined

In our discussion of the determination of the correct discount rate for CBA above we have reviewed several arguments for once and for all reductions in the *level* of the discount rate in particular circumstances. Both Fisher and Krutilla (1975) and Weitzman (1994) provide separate rationales for a lower 'environmental discount rate' and although the arguments are not rooted in consideration of intergenerational equity per se, this lower discount rate would naturally place greater weight upon the far-distant future. However, as we have described above, such a reduction would still afford a dictatorship of the present over the future if the discount rate remained positive. Perhaps for this reason the issue of DDRs for CBA has emerged, motivated less by the experimental evidence that has given rise to the notion of hyperbolic discounting, but more by the analysis of the socially efficient discount rate captured by the Ramsey equation, versions of which are seen in Equation (2) and (3).

Two important contributions in this area are those of Weitzman (1998) and Gollier (2002a, 2002b) each of which analyses the impact of uncertainty upon the determination of the social rate of time preference, δ, in a competitive economy. This is not to say that the issue of DDRs in a deterministic world has not been the subject of discussion. Weitzman (1994), for example, showed that the divergence between the social and private rates of return on capital, *r* and *i* respectively, is captured by the following equation:

(8) $$r = i\left[1 - Z\left(1 + \frac{1}{E}\right)\right]$$

where Z represents the proportion of national income spent on environmental goods and projects (e.g. cleanups), whilst E represents the elasticity of environmental improvement with respect to expenditure on environmental goods (e.g. preservation, mitigation), and $E > 0$. This reflects the tension in his model between investments in environmental protection and the production of consumption goods and the associated 'environmental drag': the fraction of extra consumption arising from a marginal investment that would have to be diverted to maintain the environmental standard. Notice that the social rate of discount, *r*, is lower than the private rate, *i*, for all positive levels of *Z* and *E*[14]. The important implication here is that the socially efficient discount rate will be *declining over time* if the proportion of income spent on environmental goods, *E*, is increasing over time. With positive growth this is guaranteed if environmental resources are luxury goods. A similar result holds if the elasticity of environmental improvement is declining over time. This analysis, summarised by equation (8), shows that even in a deterministic world consideration of preferences for the environment alone can provide an argument for DDRs[15].

3.2.1 Declining Discount Rates and Uncertainty

Clearly, the one thing that can be said with certainty about the far-distant future is that future states of the world are uncertain. Recent work by Weitzman (1998, 2001) and Gollier (2002a, 2002b, 2002c) has investigated the impact of uncertainty upon the determination of the social discount rate for CBA and found that the arguments for DDRs are compelling. Their analysis of uncertainty concerning future states of the world, have focussed respectively upon the opportunity cost of capital, *r*, and growth, *g*. Furthermore, just as Weitzman (1994) introduced preferences for environmental goods as a determinant of the SDR in a structural model, Gollier (2002a, 2002b) shows that in an uncertain world it is preference for risk that becomes important.

Uncertain Marginal Productivity of Capital (*r*) *and DDRs*

In an interesting paper, Weitzman (1998) develops ideas first formalised by Dybvig et al (1996) and shows how uncertainty regarding the marginal productivity of capital, *r*, leads to a DDR. He argues that there are good reasons to expect that in the long-run *r* is uncertain: there is uncertainty concerning capital accumulation, the degree of diminishing returns, the state of the environment, the state of international relations, and the level and pace of technological progress etc.

Weitzman (1998) shows the relationship between the socially efficient discount rates and the time horizon when it is assumed that *r* is uncertain and agents are risk-neutral[16]. He shows that, when these agents wish to maximise the NPV of investment either at an uncertain per-period risk free interest rate, $\tilde{R}$, or in a project that yields a sure benefit in period *T*, the socially efficient discount rate (before the realisation of the uncertain risk free rate) is declining with time. In other words, the yield curve is declining. This result comes from the observation that we should average over discount factors rather than rates and discounted values are a convex function of the discount rate. In discrete time, recall that the discount factor for a time period *t*, (A_t) is given by:

(9) $$A_t = \frac{1}{1+r_1} * \frac{1}{1+r_2} * \frac{1}{1+r_3} * ...... * \frac{1}{1+r_t}$$

With conventional discounting $r_t = r$. When the social rate of return is uncertain however, there are several potential states of the world, each with an associated discount rate and probability of realisation. For simplicity, imagine there are two potential future states of the world, state 1 and 2, each with an associated interest rate, R_1 and R_2, and probability of being realised, p_1 and p_2, where $p_1 + p_2 = 1$. Assuming that R_1 and R_2 are constant across time in each scenario the associated discount factors for each scenario are:

$$A_{1t} = \frac{1}{(1+R_1)^t} \text{ and, } A_{2t} = \frac{1}{(1+R_2)^t}$$

In the face of uncertain *r*, agents are unsure as to how to evaluate the opportunity cost of the project, and hence which discount factor to employ in determining the NPV. Agents must make some judgement of the discount factor and will use the expected, or *certainty equivalent discount factor*. Weitzman defines the certainty equivalent discount factor for risk neutral agents as the expected value:

(10) $$E\left[(1+\tilde{R}_t)^{-t}\right] = p_1.(1+R_1)^{-t} + p_2(1+R_2)^{-t}$$

Gollier (2002a) notes that, given the assumption of risk neutrality, there would be arbitrage were it not the case that:

(11) $$E\left[(1+\tilde{R}_t)^{-t}\right]^{-\frac{1}{t}} = 1+r_t$$

Where r_t is the equilibrium rate of interest for risk-neutral agents prior to the realisation of $\tilde{R}$, and is defined by the point at which the expected cost of purchasing the claim of \$1 at time *t* is equal to the present value of the benefit. Equation (11) shows that r_t is the appropriate socially efficient discount *rate* for use in CBA, and this is the *certainty equivalent discount rate* (CER)[17]. It is easy to show that the CER is a declining function of time and a formal proof of this result can be sketched by noting that Equation (8) is simply a re-statement of Jenson's inequality: (1 + r_t) is an harmonic mean of $(1+\tilde{R})$ over time, which is less than the arithmetic mean and tends to its lowest possible value, $R_{\min}$, as $t \to \infty$ [18]. This is a well-known result which can be derived from Pratt's theorem (Gollier 2002c)[19].

A numerical example is useful to see how these results are borne out. Table 1 assumes there are 2 potential scenarios $(j = 2)$, the probabilities of which are distributed uniformly ($p_1 = p_2 = 0.5$).

Table 1. Numerical Example of Weitzman's Declining Certainty Equivalent Discount Rate

Interest Rate Scenarios	**Discount Factors in Period t**				
	10	**50**	**100**	**200**	**500**
2% ($p_1 = 0.5$)	0.82	0.37	0.14	0.02	0.00
5% ($p_2 = 0.5$)	0.61	0.09	0.01	0.00	0.00
Certainty Equivalent Discount Factor, $E\left[\left(1+\tilde{R}_t\right)^{-t}\right]$ (7)	0.72	0.23	0.07	0.01	0.00
Average CER, r_t (8)	**3.38%**	**2.99%**	**2.65%**	**2.35%**	**2.14%**
Marginal CER, $\tilde{r}_t$	**3.28%**	**2.57%**	**2.16%**	**2.01%**	**2.00%**

The intuition behind this result is that calculating the CER rate requires taking a weighted average of several discount rate scenarios, where the weights are the discount *factors*. The discount factors in each scenario decrease exponentially over time in the way we observe when using conventional constant discount rates. In scenarios with higher discount rates, the discount factors decline more rapidly to zero. As such, the weight placed on scenarios with high discount rates itself declines with time, until the only relevant scenario is that with the lowest conceivable interest rate. In effect, the power of exponential discounting reduces the importance of future scenarios with high discount rates to zero, since the discount factor in these scenarios goes to zero. Since in the *ex ante* equilibrium the certainty equivalent rate of discount must equal the socially efficient discount rate in all periods of time, this results in a SDR which declines over time. This behaviour is exhibited in Table: the CER approaches the lowest discount rate of the 2 scenarios considered, i.e. 2%. In year 200 the marginal CER has fallen to 2.01%, and by year 500 this rate has fallen 2.0%.

Weitzman's argument seems very convincing: uncertainty in the discount rate itself leads to an arbitrage in which the socially efficient discount rate is a declining function of time. In addition, the apparent ease of application renders it appealing to the practitioner. However, Gollier (2002c) argues that Weitzman's logic relies critically upon a tacit assumption that we are maximising the Expected Net Present Value (ENPV) of a project such that the current generation that bears the risk of variation in the SDR. He illustrates this point by analysing the socially optimal discount rate that arises when we use an alternative criterion for project appraisal, the Expected Net Future Value (ENFV).

ENPV vs ENFV

In order to find the ENPV of a project that costs \$1 today and yields \$Z at time *T* when the discount rate, $\tilde{R}$, is uncertain the planner will evaluate:

$$\text{ENPV}: -1 + ZEe^{-\tilde{R}t} \geq 0$$

If this condition holds, then the agent should proceed with the project. The certainty equivalent per period discount rate in this environment, $r_w(t)$, is that which satisfies $Ee^{-\tilde{R}t} = e^{-r_w(t)t}$, and this is declining over time as described above.

Alternatively, Gollier asks us to imagine that we want to maximise the ENFV, i.e. we wish to rank our projects on the basis of maximising the value of assets that accumulate to future generations. The ENFV rule can be thought of as:

$$\text{ENFV} : -Ee^{\tilde{x}t} + Z \geq 0$$

in which case the certainty equivalent per period *interest* rate, $r(t)$ is that which satisfies $Ee^{\tilde{x}t} = e^{r(t)t}$. Noting that $r_w(t) \neq r(t)$, Gollier suggests that, when we rank projects by ENFV the socially efficient discount rate, $r(t)$ is in fact *increasing* over time, and converges to the highest possible value of *r* as $t \to \infty$, the precise mirror image of Weitzman's (1998) result. Gollier argues that both of these criteria cannot be correct and that since the two only differ in the location of the residual risk: when we use ENPV, agents in the present are bearing the risk and under the ENFV it is the future generations that are bearing the risk, we need some method of choosing how to allocate risk in order to choose between them[20].

In many ways this seems like a bizarre result: the location of risk affects the decision of risk neutral agents. Indeed Hepburn and Groom (2004) show that this particular conundrum has an altogether different interpretation which has nothing to do with the location in time of risk. They show that ENPV and ENFV are special cases of a more general Expected Net Value (ENV) criterion which is dependent upon the base-year chosen for project evaluation. In this light they show that the certainty equivalent discount rate is increasing in the base-year chosen for CBA (the temporal numeraire) but decreasing with the passage of time in the manner of Weitzman (1998). This aside, as shown by equation (3), when we are considering uncertainty it is eminently sensible to understand the role of risk preferences; the extent of risk aversion, the level of prudence, in determining the discount rate. This is the approach taken by Gollier (2002a, 2002b).

The Effect of Uncertain Growth, g, on the Social Time Preference Rate, δ

In the absence of currently existing financial markets which extend to the far-distant future, Gollier analyses the economic arguments for discounting the long run contained in δ. As described in (2) above there are two underlying characteristics of individual preferences which determine δ, i) pure impatience, represented by the utility discount rate, ρ, and ii) the desire to smooth growing wealth over time reflected by the term $\mu.g$. Under certainty μ reflects the degree of aversion to fluctuations in consumption, however in the environment of uncertain growth that is the focus of Gollier, this term represents the coefficient of relative risk aversion. This captures individuals' preferences for risk and how these preferences vary with income. The effect of individual preferences for risk upon the *level* of the discount rate has already been described: the wealth effect increases the discount rate and prudent individuals facing uncertain growth reduce the discount rate. These effects can be understood with reference to Equation (2) and (3) above. What is also clear from equation (3) is that changes over time in individuals' preferences for risk

Gollier analyses the yield curve in the context of a Lucas (1977) tree economy[21]. Simply put, Gollier (2002a, 2002b) shows that where growth is uncertain but definitely positive, that is there is no prospect of recession, and individuals exhibit Decreasing Relative Risk Aversion (DRRA) (μ decreases with income), the socially efficient discount rate will also be decreasing over time. In other words as incomes grow over time the prudence effect outweighs the wealth effect and the yield curve is downward sloping. The corollary of this is that under the same conditions when individuals display Constant Relative Risk Aversion

(CRRA) μ remains constant and the socially efficient discount rate remains flat: that is the prudence and wealth effects exactly compensate one another and the yield curve is flat.
The complexity of the analysis is dependent upon the assumptions concerning the probability distribution of growth[22]. When the prospect of recession is introduced the conditions for a declining yield curve become highly specialised. For example, if there is a risk of recession in the long run, the yield curve is declining only if individuals display both DRRA and Increasing Absolute Prudence (IAP): $P'(c) > 0$ [23]. This represents a distinct class of utility functions with restrictions upon 4th derivatives. Furthermore, if the risk of recession is extended to all future periods, short-run and long run, a declining yield curve requires restrictions the 5th derivatives of the utility function. As Gollier himself states, there is little hope that such conditions can be tested in the near future.

So, despite the apparent resolution that Gollier's provides in response to the conundrum arising from Weitzman's analysis regarding the intergenerational allocation of risk, the necessary conditions for DDRs to be theoretically justified become highly restrictive. This is particularly so when one makes realistic assumptions concerning the probability distribution of growth (that is, there is a positive probability of negative growth). Nevertheless, these conditions are testable in theory.

3.2.3 Summary

So far we have reviewed the current rationales for the use of DDRs in CBA and the effect that the use of DDRs will have upon models of economic growth and sustainability. We have found with regard to the second that the use of a utility discount rate (ρ) that declines over time is frequently justified by reference to the hyperbolic discount rate literature and can also result from objective functions that combine traditional utilitarian and objectives with those of a conservationist interested in long-term sustainability. We have also noted the correspondence of the steady state of some of these models with those employing zero discount rates. This has made clearer the relation between the justification for hyperbolic discounting from the perspective of experimental evidence and calls for the use of DDRs in order to address issues of sustainability and intergenerational equity.

The use of DDRs in CBA has been advocated for similar reasons: the consideration of intergenerational equity and sustainability. However, in the case of CBA, where the discount rate employed is commonly the consumption rate of interest, δ, the theoretical justification for the use of DDRs has recently emerged from the analysis of economic behaviour under uncertainty. The theoretical contributions of Weitzman and Gollier appear to be compelling in this sense.

For the practitioner however, one important question emerges from the theoretical literature: how are we to generate a schedule of workable DDRs for day to day use in the long-term policy arena? In the following section we provide a brief review of some of the approaches taken in this area.

4. Determining a Schedule of Declining Discount Rates for CBA

One of the practical steps involved in undertaking CBA is to determine the appropriate level of the discount rate. As described above, CBA\ usually uses units of consumption as the numeraire and thus the appropriate discount rate is the so-called consumption rate of interest or Social Rate of Time Preference, δ, which in a competitive economy is equal to the marginal opportunity cost of capital or risk free rate, *r*. For example, the UK Government uses δ = 3.5% as the discount rate for CBA where the decomposition is ρ = 1%, μ = 1% and g = 2.5% (see, for example, Pearce and Ulph, 1999 for a discussion). Similarly, if the practitioner wishes to implement DDRs for CBA a methodology is required to determine the appropriate schedule over time based upon the theoretical contributions outlined above.

The two most compelling arguments for declining δ come from Weitzman and Gollier. The rationale for declining discount rates provided by Gollier (2002a, 2002b) is perhaps the most theoretically rigorous of these contributions. But determination of the trajectory requires very specific information concerning the preferences of current generations at the very least, and, in the long run, the preferences of future generations. (With the infinitely lived representative agent approach there is effectively only one agent, and thus one generation. The reference to current and future generations is therefore an intuitive interpretation of the long run). These parameters include the aversion to consumption fluctuations over time, the pure time preference rate, and the degree of relative risk aversion. For the case with zero recession, restrictions on the fourth and fifth derivatives of the utility function become necessary. In addition, the probability distribution of growth needs to be characterised in some way. Clearly, the informational requirements of the Gollier approach could be daunting.

In order to implement the approach suggested by Weitzman (1998), it is necessary to characterise the uncertainty of the interest rate. In general terms this characterization amounts to defining a probability distribution for the future discount rate, and its behaviour over time. In this sense there are 2 ways in which we can interpret the example in Table 1 (Table on Numerical Example of Weitzman's Declining Certainty Equivalent Discount Rate). Firstly, it could represent the thought experiment of Weitzman (1998), in which we are currently uncertain about interest rates, and yet the interest rates will persist indefinitely ex post realisation. In this sense we have a probability distribution for the current uncertainty, which assumes that interest rates of 2 and 5% are equally likely, and we employ this distribution for all future periods. Uncertainty is therefore regarded as existing from day one, and all that is required is the current probability distribution of the discount rate.

In a further article, Weitzman (2001) takes precisely this approach. In order to establish the probability distribution for the socially optimal discount rate he undertakes a survey of over 2000 academic economists, and a so-called `blue ribbon' selection of 50, as to their opinion on the constant rate of discount to use for CBA. The responses were distributed with a gamma distribution with mean 4%, and standard deviation 3%, providing an ad hoc working assumption to determine the schedule of DDRs. The assumption implicit in the use of the gamma distribution is that there is uncertainty in the present about the interest rate in the future and that when uncertainty is resolved the realised interest rate will persist forever.

Newell and Pizer (2003) (N&P) take an alternative view. Rather than assuming uncertainty in the present, they state that we are currently fairly certain about the discount rate but uncertainty increases in the future. From this standpoint they characterise the uncertainty of the discount rate by econometric modelling of the time series process of interest rates. The estimated model is used to forecast future rates based upon their behaviour in the past. From these forecasts they derive numerical solutions for the CER. In doing so they are also able to provide a (weak) test of another assumption important to the Weitzman (1998) result, namely the presence of persistence of discount rates over time. They compare the discount rates modelled as a mean reversion process to a random walk model, and find support for the latter. The practical implications of implementing the declining discount rates that result are significant. When applied to global warming damages, the present value of damages from carbon emissions increases by 82%, compared with the same damages evaluated at the constant treasury rate of 4%. In monetary terms this translates into an increase in the benefits of carbon mitigation from \$5.7/ton of carbon, to \$10.4/ton of carbon.

Groom, Koundouri, Panopoulou and Pantelidis (2004) use US and UK interest rate data and by building on N&P's approach in determining DDRs, they make the following points concerning model selection and the use of DDRs in general. Firstly, N&Ps approach is predicated upon the assumption that the past is informative about the future and therefore characterizing uncertainty in the past can assist in forecasting the future and determining the path of CERs. If one subscribes to this view it is important to characterize the past as well as possible by correctly specifying the model of the time series process. This is particularly so when dealing with lengthy time horizons where the accuracy of forecasts is important. Indeed the selection of the econometric model is of considerable moment in operationalising a theory of DDRs that depends upon uncertainty, because econometric models contain different assumptions concerning the probability distribution of the object of interest. Groom, Koundouri, Panopoulou and Pantelidis shown for US and UK interest rate data that the econometric specification should allow the data generating process to change over time, and that State Space and Regime Switching econometric models (see Groom, Koundouri, Panopoulou and Pantelidis (2004) for the detailed description of the models) are likely to be appropriate. Secondly, selection between well specified models can and should be undertaken by reference to measures of efficiency such as coefficients of variation, confidence bounds and out-of-sample forecast MSEs.

In conclusion, one could assert the path of the CER differs considerably from one model to another and therefore each places a different weight upon the future. The policy implications of these estimates is revealed in the estimation of the value of carbon emissions reduction, with values which are up to 150% higher than when using constant discount rates, and up to 88% higher than the Random Walk model employed by N&P.

5. Conclusion

This paper has reviewed the arguments for and the implications of employing Declining Discount Rates in CBA and in the analysis of economic growth and sustainability. The review shows that there are several growth models in which a relationship has been found between the long-run equilibrium under DDRs and that in which a zero discount rate is employed. This can have the effect of pushing the optimum under DDRs away from the conventional utilitarian outcome towards the Green Golden Rule level (GGR) of capital or environmental stocks. Furthermore, in response to worries that the GGR places weight on the future at too great an expense to the present, we highlight the result of Li and Lofgren (2000): DDRs can evoke a solution to resource management problems in which the objective function explicitly takes into account the preferences of present and future generations, such as those posited by. That is, the use of DDRs can balance the preferences of current and future generations such that neither are dictators over the other. This solution is not achieved by either zero or conventional discounting. It is in these senses that DDRs can be seen to encourage a more equal treatment of generations and promote sustainable outcomes.

In the application of CBA, in which consumption is generally the numeraire, we have shown that there exists a body of theoretical work justifying the use of DDRs based upon the analysis of decisions under uncertainty. We have also discussed the implications a correctly specified model of discount rates, which provides a schedule of DDRs that values atmospheric carbon reduction 150% higher than conventional exponential discounting, and almost 90% higher than incorrectly specified models. In this sense sustainable outcomes are more likely to emerge from project appraisal with DDRs, but given that the theory of DDRs for CBA reviewed relates to the socially efficient discount rate, such outcomes can also be thought of as efficient.

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[1] Some authors have suggested that ρ reflects impatience arising from the instantaneous risk, or hazard rate of death at a particular point in time. See for example Pearce and Ulph (1992)

[2] Following the definition of Kimball (1990) individuals are called

[3] $\mu = -\frac{u''(C)}{u'(C)}C$, where $u(C)$ is the individual's utility function and $u'(.)$ is the first derivative of the utility function and $u''(.)$ is the second and so on.

[4] With certain knowledge of each of the parameters on the RHS of (2) the level of the SDR is known with certainty. For example the UK Government employs δ as the test rate for project and policy appraisal. They assume that $\rho = 1\%$, $\mu = 1$, and $g = 2.5\%$, making the social time preference rate equal to 3.5% (HM Treasury, 2003).

[5] Precisely, individuals are defined as prudent where $u''' > 0$. Noting that relative prudence is defined as $P(C) = -\frac{u'''}{u''}C$, and that $u'' < 0$, where individuals are prudent $P(C) > 0$.

[6] If willingness to pay (WTP) for environment evolves at some pre-determined rate, say α, the rationale for this increase in WTP being that preferences for environmental resources are changing over time due to income growth or increased scarcity (Fisher and Krutilla, 1975), then WTP for a unit of environmental goods at time *t* can be written as: $WTP_t = WTP_0(1+\alpha)^t$, where WTP_0 is willingness to pay at *t* = 0. In a deterministic world this means that we can derive an 'environmental' discount rate, w, such that the present value of benefits (costs) that accrue at time *t* can be written as: $WTP_0 / (1+w)^t$, where $w = (r-\alpha)/(1+\alpha)$, where *r* is the conventional discount rate and $r > w$.

[7] See Pearce et al 2003, for a review.

[8] For example, if utility depends upon the amenity value of and environmental stock, *q*, as well as consumption then the relation between δ and ρ will reflect the changes in these stocks. The relation under certainty then becomes $\delta = \rho + \mu_{C,C}g_C + \mu_{C,S}g_S$, where the μ terms represent the elasticity of marginal utility with respect to consumption and *q*, and the *g* terms represent the growth of the consumption and *q* respectively. Note that in steady states $g_C = g_S = 0$ and the two concepts coincide.

[9] Useful references in this area include Laibson (1997) Loewenstein (2000), Loewenstein and Prelec (1992).

[10] Where both consumption and environmental stocks (amenity value) enter into the utility function this is achieved where the marginal rate of substitution between consumption and the stock are equal to the marginal rate of transformation of the stock of renewable resources.

[11] For more on the issue of time inconsistency see Heal (1998, CH7) and Pearce et al (2003)

[12] The MSY level of the capital stock reflects the point at which the marginal productivity of capital equals zero. Li and Löfgren (2001) assume that the production function is increasing up to this stock level and decreasing thereafter.

[13] However, Dasgupta (2001) highlights a criticism (attributed to Kenneth Judd) of this approach on the basis that there is a way in which all generations can have their cake and eat it too. Suppose the current generation devises a plan that maximizes only the integral part of the maximand in equation (12). It simultaneously announces its intention to abandon that plan at some date in the distant future, at which point it will switch to a plan that then maximizes only the asymptotic part of the maximand. The farther this switching date, the more nearly the integral part will be maximized. But there will always be an infinite number of dates after the currently planned switching date, and hence it will always be possible to increase welfare by postponing the switching date.
[14] For a given level of *Z*, when the elasticity is low, and environmental expenditures are ineffective at cleaning up environmental damage, this divergence is increased. Weitzman's interpretation, from the perspective of optimal growth, is that this is a signal that the economy is finding prior environmental damage difficult to undo, and one solution is to reduce growth (if this is a feasible policy option). Alternatively, where the elasticity is high, a better solution might be to increase environmental expenditures (Weitzman 1994).
[15] Other more ad hoc proposals for DDRs exist. Rabl (1996) for example suggests that utility should not be discounted in the long term, and hence not included in the calculation of the SDR for CBA, not because of the 'ethical indefensibility' suggested by Ramsey, but rather upon the inadequacy of financial markets in performing long-term redistribution of resources. The implication here is that ρ represents the desire of the current generation to redistribute wealth, which is constrained by the time horizon covered by current financial markets, usually about 30 years. His proposal implies a stepped schedule of discount rates for CBA, i.e. with ρ set to zero after a period of 30 years or so.
[16] This is not crucial for this particular result to hold but is important for ease of exposition. The certainty equivalent could be defined to incorporate higher moments of the distribution of discount rates to reflect risk aversion, with a loss of tractability.
[17] It is important to note here that equation (8) reflects the discount rate that should be used to discount costs and benefits that occur at time *t* back to the present. However, Weitzman (1998) defines the CER as the rate of change of the certainty equivalent discount factor over time, thus his CER represents the period to period discount rate. The former can be thought of as the average CER, whilst the latter can be though of as the marginal CER. Weitzman shows that the marginal CER declines over time, whilst Gollier (2002a) shows how the average CER declines over time.
[18] It is worth noting once more the distinction between the *average* CER and the *marginal* CER. The discount rate discussed above, following Gollier (2002a), is the *average* CER. It is the per- period discount rate that would need to apply over the entire time horizon under consideration to ensure there are no opportunities for costless arbitrage. In contrast, Weitzman (1998) discusses the marginal CER, defined in continuous time by $\tilde{r}_t = -\frac{d\tilde{A}_t / dt}{\tilde{A}_t}$ rather than the solution to (6) above. Both the certainty equivalent average and marginal discount rates are declining over time in equilibrium; the marginal discount rate declines more rapidly. However, Weitzman notes that at the limit, as $t \to \infty$, both are the same.
[19] A rough sketch of the proof is as follows: r_t can be thought of as the certainty equivalent of a random pay-off, $\tilde{x}$, for an agent with a constant degree of absolute risk aversion *t*. As risk aversion increases, i.e. *t* increases, it is well known that the certainty equivalent r_t will decrease (Pratt 1964). Furthermore, as $t \to \infty$, r_t will tend to the lower bound of $\tilde{x}$.
[20] Under ENPV after the realisation of the uncertain discount rate the NPV may or may not be positive, and since the payoff in the future is certain, any residual losses are borne by the present generation. However, when we use the ENFV criterion it is future generations that are bearing the risk. The present generation makes a certain contribution to the project, but the rate at which the fund accumulates, and hence the outcome in the future, is uncertain before the realisation of $\tilde{R}$.
[21] The Yield curve describes the term structure of financial assets.
[22] It is also dependent upon the inter-temporal relationships. For the purpose of the analysis Gollier (2002a) assumes that the growth shocks are independently and identically distributed. Although this is unrealistic, it avoids the complications associated with the analysis of serially correlated shocks.
[23] There are a number of additional necessary conditions for this to hold - for details see Gollier (2002b)).