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Abstract

Economic rationality demands the decision maker (DM)'s degrees of beliefs to be coherent, that is to obey the rules of probability calculus. This view is usually referred to as Probabilism. Among the various justifications of Probabilism, the Dutch Book Argument (DBA) occupies a prominent place. DBA purports to show that DM's aversion to sure financial losses is sufficent to ensure that her beliefs are coherent. A tacit assumption of DBA is that DM is capable to implement a heuristic error-correction process, ECC, that yields rational beliefs. The main aim of this paper is to challenge this assumption. In order for DBA to be convincing, ECC must empower DM to detect *each and every* Dutch Book that may be made against her, no matter how complex this Book turns out to be. A complex Dutch book is one that requires very sophisticated calculations before its financial consequences are deduced. In the presence of complex Dutch Books, the only point that DBA makes clear is that DM has to be "computational omnipotent" on pain of incoherence.

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1 Introduction

The notion of the "rationality of beliefs" is fundamental to several disciplines, such as economics, philosophy of science and psychology. Beliefs refer to propositions (or events) and come in degrees. A decision maker (DM)'s subjective probability, $P(A)$, of a proposition A is usually interpreted as the degree to which DM is confident in the truth of A. It is usually assumed that A belongs to a Boolean algebra, $\mathcal{F}$, of propositions, describing events that are relevant for the decision problem at hand. A DM is rational if her subjective probability function, P, defined on $\mathcal{F}$ is coherent, that is if P obeys the rules of probability calculus. This view is usually referred to as "Probabilism". This means that what is normatively appealing (the normative standard) is dictated by what is formally admissible. At this point the following question arises: Why is Probabilism equivalent to rationality? In other words, what are the arguments (if any) that support the view that unless P is coherent in the formal sense, DM is irrational?

There are two main arguments for Probabilism, hereafter referred to as AP1 and AP2.[1] AP1 utilizes the so-called "representation theorems", according to which Probabilism follows logically from a set of plausible rationality constraints (e.g. transitivity) on DM's preference relation $\succeq$: P is coherent iff $\succeq$ satisfies (for example) Savage's (1974) axioms. In this type of argument, "the burden of proof" has moved from beliefs to preferences. But then the question that was originally raised for beliefs now re-emerges for preferences: What are the arguments (if any) supporting the view that rationality of preferences amounts to $\succeq$ satisfying the axioms of Savage? The main such argument is the so-called "Money Pump Argument" (MPA) which is structurally similar to AP2, to which we now turn.

AP2 is the (in)famous Dutch Book Argument (DBA). DBA is based on the so-called Dutch Book theorem, which shows that a DM who does not have coherent beliefs is susceptible to a Dutch Book. The latter is a set of bets, each of which appears to be fair to DM (by her own standards) but all together assure that DM will incur a net financial loss, whatever the outcome turns out to be. Since a rational DM does not exhibit this type of susceptibility (so the argument goes) it follows logically (by a modus tollens argument) that she entertains coherent beliefs[2].

It is worth emphasizing that in the context of both DBA and MPA, DM's aversion to certain financial losses forces her to obey a calculus (namely, Kolmogorov's or Savage's calculus) that she may not know or might have never heard of. On this view, what DBA and MPA have (allegedly) managed to ac-

[1] Joyce (1998, 2009) offers a third argument for Probabilism, namely the "Accuracy Domination Argument". This argument (being "epistemic rather than pragmatic) is based on the idea that a rational DM prefers to form beliefs that are as close to the true ones as possible. Joyce's s main result is that under a set of axioms, if DM's probability function P does not obey the rules of probability calculus, then there exists another probability function P' which obeys these rules and is closer to the truth in every possible world.

[2] DBAs were introduced by Ramsey (1926) and de Finetti (1937). For a recent survey of these arguments see Hajek (200?).

complish is to translate a "practical" or "pragmatic" notion of rationality into a "formal" one. In other words, DM's attempts to avoid being money-pumped or Dutch-booked make her develop a system of "thought police" that "clubs her into line when she violates certain principles of right reasoning" (Garber 1983, p. 101). This means that both DBA and MPA are based on the fundamental assumption that DM is capable to implement a heuristic error-correction process, ECC, that yields rational beliefs and rational preferences. ECC may be outlined as follows: DM begins with a probability function, P, and examines whether, under P, she is susceptible to a Dutch Book. If she is, then she denies that Book and adjusts her initial P (corrects her initial error) until her susceptibility disappears. This in turn implies that ECC may be thought of as a rational adaptation processes for achieving some specific goals which, by its very nature, applies to any decision maker, regardless of DM's level of expertise in the relevant subject matter. In other words, is ECC is a purely a-priori process, and as such it does not depend on the presence of any empirical information and DM's ability to process this information. Besides, the very conception of DBA was motivated by the ambition to make formal rationality accessible to the ordinary decision maker. To sum up, the only properties that an ordinary DM is required to have in order to reach rationality are, first, an aversion to suffering a sure monetary loss and, second, the analytical skills to implement ECC.

Does an ordinary DM have the skills to carry out the necessary calculations that are involved in ECC? In other words, does such a DM have the logical sophistication to detect any (initial) incoherence of her beliefs? The answer to this question depends on the degree of complexity of the Dutch Book that DM is faced with. For simple Dutch Books, DM is likely to have the computational capacity to detect the aforementioned incoherence. However, in order for DM to be deemed as rational, she must be able to repel *any conceivable* Dutch Book made against her, simple or complex. A complex Dutch book may be thought of as one that requires very complex calculations before its financial consequences are deduced. In the present paper, we define the degree of complexity of a Dutch Book as the number of rows in the relevant pay-off matrix. The number of rows increases with the number of propositions (bets), on which the Dutch Book is based, at an exponential rate. As a result, relatively simple Dutch Books (that is, those based on a small number of propositions) exhibit a high degree of complexity, which in turn renders ECC infeasible. Put differently, Dutch Books that appear to be "simple" in terms of the number of propositions involved may be "complex" in terms of computational tractability, "given the severe time and memory limitations of a 'fast and frugal' cognitive system." (Oaksford and Chater, 2007 p. 16). It seems that as we move up in the scale of complexity, DBAs run out of steam. In order to execute ECC for every Dutch book, DM must possess "computational omnipotence".

In Section 2, we first analyze the logical structure of a typical DBA, thus locating within this structure the role of ECC. Second, we derive the rate at which the degree of complexity of a Dutch Book increases with the number of hypotheses, n, on which the Dutch Book is based. This rate depends, apart

from n, on whether these hypotheses are mutually exclusive and/or exhaustive. Section 3 concludes the paper.

2 The Degree of Complexity of Dutch Books

A typical DBA has the following form (adapted from Hajek 200?):

P1: DM's subjective probability of A, $A \in \mathcal{F}$, is given by DM's betting price on the truth of the proposition A.

P2: (Dutch Book Theorem - DBT): If DM's system of betting prices on the propositions of $\mathcal{F}$ violate the rules of probability calculus, then there is a Dutch Book consisting of bets at those prices.

P3: If there is a Dutch Book consisting of bets at DM's betting prices, then DM is susceptible to financial losses.

P4: If DM is so susceptible, then she is irrational

C(onclusion): If DM's subjective probabilities violate the rules of probability calculus, then DM is irrational

This argument is valid. Whether it is sound is another matter. Let us for the moment assume that the premises **P1-P4** are true (the argument is sound) and clarify what DBA has established and what has not. The basic result of DBA is proposition **C**. From this it immediately follows (modus tollens) that the proposition **Ca**: "If DM is rational then her subjective probabilities do not violate the rules of probability calculus" is true. Similarly, the propositions **Cb**: "If DM is not susceptible to the financial losses (as described above), then her subjective probabilities do not violate the rules of probability calculus" and **Cc**: "If there is no Dutch Book consisting of bets at DM's betting prices, then DM's system of betting prices (subjective probabilities) do not violate the rules of probability calculus" are also true. However, DBA is silent on the truth of the proposition **P2a**: "If DM's system of betting prices (subjective probabilities) do not violate the rules of probability calculus, then there is no Dutch Book consisting of bets at those prices". The truth of this proposition, usually referred to as the Converse Dutch Book Theorem (CDBT) was proved by Lehman 1955 and Kemeny 1955. DBT and CDBT may be joined together as follows: "DM's system of betting prices on the propositions of $\mathcal{F}$ does not violate the rules of probability calculus, *if and only if* there is no Dutch Book consisting of bets at those prices.

Let us now discuss the soundness of DBA: P2, being a mathematical theorem is uncontroversial. **P1** and **P3** are plausible (for counter-arguments see Hajek 200?). **P4** is the key premise which relates susceptibilty to financial loss with irrationality. Why should this type of susceptibility imply irrationality? The standard answer to this question is that a rational DM should have detected her own weakness, exclusively by means of a-priori reasoning, thus changing her system of betting prices accordingly. This in turn implies that apart from an aversion towards being Dutch-booked, DM has also the cognitive ability to detect that she is about to be Dutch-booked. This means that a hidden assumption on which **P4** is based is the following, **P4a**: "DM is able to detect

each and every Dutch book that can be made against her". In particular, **P4a** assumes that DM is capable to implement the aforementioned ECC heuristic procedure, that yields coherence.

Is DM capable of implementing ECC? Assume that a clever betting opponent (the Dutch bookie) has detected a certain type of initial incoherence in DM's credences with respect to the truth/falsity of the proposition H and designs a simple Dutch Book, $db1$, accordingly. For example, DM has initially assigned probabilities equal to p_1 and p_2 to the truth and falsity of H, respectively with $p_1 + p_2 < 1$. Before accepting $db1$, DM performs "the necessary calculations" and realizes that she is about to be Dutch booked. As a result, she corrects her initial error, in the sense that she now forms new subjective probabilities p_1' and p_2' such that $p_1' + p_2' = 1$, thus restoring coherence. In this case, DM has implemented successfully ECC. Is DM's success to avoid $db1$ sufficient to establish her rationality? Definitely not. As already mentioned, DM must be able to diagnose *any* Dutch Book made against her, no matter how complex it may be. The specific Dutch Books that are usually employed in the literature (in order to motivate their supportive role for rationality) are extremely simple. Specifically, they are based on the truth or falsity of a very small number of (mutually exclusive and/or exhaustive) propositions, which render the number, n, of the rows of the corresponding pay-off matrix relatively small. For example, consider a Dutch Book that is based on the truth or falsity of two propositions, H_1 and H_2. If these propositions are both mutually exclusive and exchaustive (for example, "Heads" and "Tails") then, $n = 2$. On the other hand, if H_1 and H_2 are mutually exclusive but not exhaustive, then n increases to 3 (see, also the entry Dutch Book Arguments, 2022, in Stanford Encyclopedia of Philosophy). Such simple books imply a quite feasible ECC. However, when DB is faced with a slightly more complex DB, for example one with three propositions that are neither mutually exclusive nor exhaustive, her ability to implement ECC becomes questionable. Such a DB is the following:

A Slightly More Complex Dutch Book

Assume that DM is faced with three propositions, H_1, H_2 and H_3. DM does not know whether these propositions are mutually exclusive and/or exhaustive. Assume that DM's subjective probabilities are such that

$$P(H_1 \vee H_2 \vee H_3) \neq P(H_1)+P(H_2)+P(H_3)-P(H_1 \wedge H_2)-P(H_2 \wedge H_3)-P(H_1 \wedge H_3)+P(H_1 \wedge H_2 \wedge H_3), \tag{1}$$

which means that DM's probability function P is not additive. Next, assume that

$$P(H_1 \vee H_2 \vee H_3) < P(H_1)+P(H_2)+P(H_3)-P(H_1 \wedge H_2)-P(H_2 \wedge H_3)-P(H_1 \wedge H_3)+P(H_1 \wedge H_2 \wedge H_3). \tag{2}$$

A crucial assumption (on which **P3** depends) is that DM's opponent (the bookie) is aware of (2). On the other hand, it is rather implausible that the ordinary DM knows that in order to avoid susceptibility, she has to set $P(H_1 \vee H_2 \vee H_3)$ equal to the right-hand side of (2). The bookie may exploit this epistemic asymmetry by offering DM the following bets:

the bet that pays \$1 if H_1 and 0 otherwise for the price of $P(H_1)$,

the bet that pays \$1 if H_2 and 0 otherwise for the price of $P(H_2)$,

the bet that pays \$1 if H_3 and 0 otherwise for the price of $P(H_3)$ and

the bet that pays \$1 if $H_1 \wedge H_2 \wedge H_3$ is true and 0 otherwise for the price of $P(H_1 \wedge H_2 \wedge H_3)$.

The bookie then buys (since DM is willing to sell) the following bets:

the bet that will pay him \$1, if $H_1 \vee H_2 \vee H_3$ is true and 0 otherwise, for the price of $P(H_1 \vee H_2 \vee H_3)$,

the bet that will pay him \$1, if $H_1 \wedge H_2$ is true and 0 otherwise, for the price of $P(H_1 \wedge H_2)$,

the bet that will pay him \$1, if $H_1 \wedge H_3$ is true and 0 otherwise, for the price of $P(H_1 \wedge H_3)$, and

the bet that will pay him \$1, if $H_2 \wedge H_3$ is true and 0 otherwise, for the price of $P(H_2 \wedge H_3)$.

The possible payoffs to DM are summarized in the following table:

H_1	H_2	H_3	Net Payoff
T	F	T	$1 - P(H_1) - P(H_2) + 1 - P(H_3) + P(H_1 \wedge H_2) + P(H_1 \wedge H_3) - 1 + P(H_2 \wedge H_3)$
T	T	T	$1 - P(H_1) + 1 - P(H_2) + 1 - P(H_3) + P(H_1 \wedge H_2) - 1 + P(H_1 \wedge H_3) - 1 + P(H_2 \wedge H_3)$
F	T	F	$-P(H_1) + 1 - P(H_2) - P(H_3) + P(H_1 \wedge H_2) + P(H_1 \wedge H_3) + P(H_2 \wedge H_3) - 1$
F	F	F	$-P(H_1) - P(H_2) - P(H_3) + P(H_1 \wedge H_2) + P(H_1 \wedge H_3) + P(H_2 \wedge H_3) - 1$
T	F	F	$1 - P(H_1) - P(H_2) - P(H_3) + P(H_1 \wedge H_2) + P(H_1 \wedge H_3) + P(H_2 \wedge H_3) - P$
T	T	F	$1 - P(H_1) + 1 - P(H_2) - P(H_3) + P(H_1 \wedge H_2) - 1 + P(H_1 \wedge H_3) + P(H_2 \wedge H_3)$
F	T	T	$-P(H_1) + 1 - P(H_2) + 1 - P(H_3) + P(H_1 \wedge H_2) + P(H_1 \wedge H_3) + P(H_2 \wedge H_3) - 1$
F	F	T	$-P(H_1) - P(H_2) + 1 - P(H_3) + P(H_1 \wedge H_2) + P(H_1 \wedge H_3) + P(H_2 \wedge H_3) - 1$

It is easy to see that for each row of this table, that is for each possible outcome, DM suffers a monetary loss, which means that all the bets together entail a certain loss for DM. Of course, if

$$P(H_1 \vee H_2 \vee H_3) > P(H_1) + P(H_2) + P(H_3) - P(H_1 \wedge H_2) - P(H_2 \wedge H_3) - P(H_1 \wedge H_3) + P(H_1 \wedge H_2 \wedge H_3),$$

then the bookie can easily construct another Dutch Book by simply reversing the direction of the relevant bets.

3 Conclusions

Is it possible for a DM to form a system of beliefs that satisfies the requirements of a formal calculus, without DM having any conscious knowledge of this calculus? The Dutch Book Argument is an attempt to answer this question in the affirmative, in which case DBA can be used as a pragmatic justification of rationality. The main point of this paper is that this type of justification is unconvincing. In order for this argument to have some force, DM must be able to detect (by means of a heuristic trial-and-error process, referred to as ECC in the paper) *every* Dutch Book that can be made against her, either simple or complex. It is worth emphasizing that the existence of even a single undetected Book sufficies for characterizing DM as irrational. This in turn leaves DM vulnerable to a betting opponent (the bookie) who can design Dutch Books

so complex that the ordinary DM will almost surely fail to detect. In such cases, ECC breaks down, which in turn implies that DM's aversion to monetary losses by itself is not enough to ensure the coherence of her beliefs. Hence, the main aim of the Dutch Book Argument, namely to justify a formal concept of rationality on the basis of a pragmatic one, fails. If we insist on identifying rationality with Probabilism, then a justification of the latter should be pursued along a less practical path. To that end, one may defend Probabilism on grounds similar to those that render Propositional Logic normatively appealing. This view was expounded by Ramsey (1927) who argues that "the laws of probability are laws of consistency, an extension to partial beliefs of formal logic, the logic of consistency." (1927, p. ??). Hence, a probabilistically incoherent DM is not irrational just because she is vulnerable to sure monetary losses, but rather because her system of beliefs exhibits a structural flaw that is unacceptable on cognitive (rather than pragmatic) grounds.

The foregoing discussion raises the following queston: In view of the extreme analytical/computational demands that Probabilism places on the ordinary DM, is probabilistic coherence an unreasonable demand for rationality? In other words, should we lower the bar on our demands for rationality? If yes, then the question is how much should we lower the bar? One suggestion that stems from the foregoing discussion is to acknowledge as rational those decision makers that are capable of detecting "simple" Dutch Books. Then of course the question is how to define "simplicity", that is where to draw the line between "simple" and "complex" Books. Hacking (1967) makes a suggestion along these lines. Specifically, he considers Dutch Books designed by bookies who are on a par to DM in terms of their computational and analytical skills. If DM succeeds in detecting this type of Books then she is considered rational. This "more realistic" conception of rationality is criticized by Eells (1990), who argues that Hacking's proposal entails the rather paradoxical result that DM is rational (in the sense of not being Dutch booked) as long as she is not aware of her incoherence.

In a nutshell, Dutch Book Arguments do not serve their intended purposes of being justifications of rationality, with the latter being tantamount to Probabilism. On the contrary, if they are properly interpreted, they show the implausibility of Probabilism itself as a plausible demand for rationality. In spite of this implausibility (and because of the inadequacies of the proposed alternatives), Probabilism continues to be considered as the main tenet of Economic rationality.