DEPARTMENT OF INTERNATIONAL AND EUROPEAN ECONOMIC STUDIES

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Fiscal Tightening and Skills Mismatch

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Working Paper Series

23-13

May 2023

Fiscal Tightening and Skills Mismatch*

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Abstract

The paper documents a new link between fiscal tightening and the vertical skills mismatch rate (share of over-qualified workers). Using data for Greece, where this rate exceeds one-third, and a variety of structural Bayesian vector auto regressions and identification strategies, we show that fiscal tightening increases mismatch. We then introduce the latter in a DSGE model with heterogeneous households and labor frictions. In line with the data, a fiscal tightening shock raises the mismatch rate in the model. This result holds for production function specifications both with and without capital-skill complementarity (CSC).

Keywords: skills mismatch, fiscal shocks, capital-skill complementarity, SVAR, DSGE model.

JEL Classification: J24, F41, J63, E62, O41.

* The research project was supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the "2nd Call for H.F.R.I. Research Projects to support Post-Doctoral Researchers" (Project Number: 201). We thank Tryfonas Christou for early input. We are grateful to Nicolò Maffei-Faccioli, Vasia Panousi and Filippos Petroulakis for comments and suggestions.

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1 Introduction

This paper. The public debt to GDP ratio soared across the world during COVID-19, posing a challenge for policymakers. While the literature has studied the so-called unintended consequences of fiscal consolidations, little is known about their effects on skills mismatch, which in many European economies has risen significantly. Notably, Greece has experienced a more than doubling of its vertical mismatch rate (i.e., share of over-qualified workers) between 1995 (15%) and 2019 (32%), currently ranking third in the Eurozone after Spain and Cyprus. Mismatch implies that resources are allocated inefficiently, leading to lower productivity.[1] While the literature on resource misallocation is rich and growing, the effects of fiscal policy have so far been understated.[2] This paper documents a new link between fiscal tightening (i.e., spending cuts or income tax hikes) and vertical skills mismatch.[3] Our empirical and theoretical findings suggest that one of the unintended consequences of fiscal retrenchment in the labor market may be a rise in the mismatch rate.

Empirical evidence. Using Greek data from 1995 to 2019 and Bayesian techniques, we run a variety of SVAR models, with four endogenous variables: (i) a cyclically adjusted fiscal variable (primary balance, primary expenses or tax revenues), (ii) per capita GDP or the unemployment rate, (iii) gross debt, and (iv) the mismatch rate. Cyclically adjusted variables help to minimize the simultaneous correlation with the GDP. We use both a Cholesky decomposition and various schemes with sign restrictions to identify fiscal adjustment shocks. First, we assume that an expenditure-based (tax-based or cyclically adjusted primary balance (CAPB)-based) adjustment should decrease government expenditure (increase tax revenue or the CAPB) for three consecutive years. Additionally, all adjustments should decrease, with a lag, the debt-to-GDP ratio for three years (see Pappa et al., 2015). Alternatively, we leave the debt ratio unrestricted following the evidence that fiscal consolidations, especially in the absence of growth-enhancing structural reforms and strong institutional frameworks, may be unsuccessful or even self-defeating (e.g., IMF (2023), Alesina et al. (2015), Pappa et al. (2015)). Finally, given the potentially contractionary effects of tax hikes documented in the literature and supported by the impulse responses from our DSGE model (see below), we also identify tax shocks with a third set of sign restrictions. We assume that, on top of increasing revenues, such adjustments reduce GDP with a lag, for three periods, while debt is again unrestricted.

Our findings are as follows. In the case of a positive shock to the CAPB, the mismatch response is statistically significant and positive under all identification strategies used. The response of GDP, when statistically significant, is positive in the Cholesky decomposition and when we restrict the debt-to-GDP ratio to fall, while

[1] Skill-mismatch employment can be beneficial by allowing high-skilled individuals to transition out of unemployment faster, but at the cost of a lower wage compared to their non-mismatched counterparts. Moreover, high-skilled workers do not take into account the effect that their search for low-skill positions has on low-skilled searchers.

[2] For example, Restuccia and Rogerson (2017) state that *"misallocation may reflect discretionary provisions made by the government or other entities (such as banks) that favor or penalize specific firms. Such provisions are often referred to as "crony capitalism" or even "government corruption." Examples are subsidies, tax breaks, or low interest rate loans granted to specific firms, along with unfair bidding practices for government contracts, preferential market access, or selective enforcement of taxes and regulations."* Ramey and Shapiro (1998) analyze the effects of sector-specific changes in government spending in a two-sector dynamic general equilibrium model in which the reallocation of capital across sectors is costly.

[3] In this paper, we focus on over-qualification while abstracting from under-qualification.

it is negative if debt is left unrestricted. Following a positive shock to government revenues, our Cholesky findings again indicate that the mismatch rate significantly increases. The same conclusion holds with sign restrictions if debt is left unrestricted or if we restrict the tax shock to be contractionary. Finally, in the case of a negative shock to the cyclically adjusted primary expenses, we obtain statistically significant and positive mismatch responses, especially when using the Cholesky decomposition or sign restrictions where public debt is unrestricted.

Theoretical model. Motivated by the empirical evidence suggesting that fiscal tightening can lead to an increase in skills mismatch, we then introduce the latter in a DSGE model with skill and wealth heterogeneous households, capital-skill complementarity (CSC), and search and matching (S&M) frictions. For simplicity, high-skilled households make investment decisions, while low-skilled households are hand-to-mouth consumers. Involuntary unemployment explains the existence of skills mismatch (i.e. employment of a high-skilled worker in a job requiring only low skills) in the model, which arises endogenously from an interplay of households' and firms' decisions. High-skilled households decide the share of their members who search for a low-skill job and firms posting low-skill vacancies decide on a share allocated to high-skilled workers. Mismatched workers continue searching on-the-job to find an upgraded position. In the event of a mismatch, a trade-off arises: the worker is more productive than the non-mismatched counterpart, but receives a higher wage and also the mismatch maybe terminated if she quits to take up a high-skill job via on-the-job search. The presence of CSC in the production function is motivated by its empirical plausibility, but we also conduct sensitivity analysis with respect to this specification.[4] S&M frictions are instrumental to model the on-the-job search of mismatched workers and have been extensively used in the mismatch literature (see the literature paragraph below).

In line with the data, we find that a negative shock to wasteful government spending or a positive income tax rate shock in the model raises the mismatch rate. The recessionary effects of the shocks lead to a reduction in hirings and quits from mismatch jobs, as well as a shift towards mismatches. The rise in the mismatch rate after fiscal tightening in our DSGE model holds for production function specifications both with and without CSC. Interestingly, for tax shocks, we find that the rise is magnified in the absence of CSC. The intuition is as follows. When capital and high-skilled labor are complements in production, it is more difficult for firms to substitute high-skilled labor with capital, which affects their decision about the creation of high-skill jobs. Therefore, the demand loss from fiscal tightening causes a weaker decrease in high-skill vacancies and a weaker rise in mismatch employment. Furthermore, the decrease in high-skilled employment is mitigated making complementary capital more productive and thus mitigating the fall in investment demand and output. This seems to suggest a dynamic demand-mitigation mechanism arising in the presence of CSC, which goes in the opposite direction from the dynamic-amplification mechanism found for monetary shocks in Dolado et al. (2021). The main policy implication of our findings is that fiscal consolidation policy makers should take into account the potential implications for skills mismatch in the labor market.

[4] The importance of the CSC relationship has been showcased by many empirical studies (starting, e.g., with Griliches (1994), Machin and Van Reenen (1998) and the seminal paper by Krusell et al. (2000)).

Literature. The paper relates to several strands of the literature. Firstly, we add to the literature on skills mismatch by documenting the effect of fiscal tightening. The S&M approach, pioneered by Mortensen and Pissarides (1994), has provided a structural approach for the study of mismatch in the labor market. Along this line, Albrecht and Vroman (2002) build a model of endogenous skill mismatch for the study of wage inequality. On-the-job search in Dolado et al. (2009) introduces an additional source of between and within-group wage inequality. Chassamboulli (2011) and Barnichon and Zylberberg (2019) also use S&M models to study the cyclical dimension of under-employment that arises from high-skilled workers who temporary accept lower-ranked jobs, but continue to search for an upgraded job. The S&M approach has been also used to study skills mismatch along with the effects of immigration flows (see, e.g. Iftikhar and Zaharieva (2019) and Liu et al. (2017)). Şahin et al. (2014) conceptualize the notion of mismatch unemployment and measure how much of the rise in the US unemployment rate during the Great Recession is attributable to mismatch across sectors. Baley et al. (2022) study the cyclical dynamics of skill mismatch (both in terms of under-qualification and over-qualification) and its impact on labor productivity in a quantitative business cycle model with labor market and information frictions. Estimated to the U.S., the model replicates salient business cycle properties of mismatch. Our findings are in line with Brunello et al. (2019) who conclude that skills mismatch is counter-cyclical in Europe as skilled individuals are more willing to take up lower skill-demanded jobs when facing the unemployment threat during a recession, and this effect tends to prevail on the cleansing effect of recessions.[5]

By focusing on skills mismatch, the paper is also related also to the large literature on resource misallocation (see for a survey Restuccia and Rogerson (2017)), exploring the causes and consequences of capital, labor and land misallocation. This literature shows that low-income countries are not as effective in allocating their factors of production to their most efficient use. The notion that the allocation of inputs across establishments matters substantially for aggregate productivity is supported by studies in the United States and elsewhere. We add to this research by exploring the effects of fiscal policy on skills mismatch and thus labor misallocation.

In addition, the paper relates to the literature on the role of fiscal policy in labor market dynamics (see, e.g., Pappa (2009), Monacelli et al. (2010), Ramey (2011), Brückner and Pappa (2012), Rendahl (2016)). Specifically, we extend the fiscal consolidation literature, both theoretically (see, e.g., Erceg and Lindé (2012), Erceg and Lindé (2013), Pappa et al. (2015), Bandeira et al. (2018), House et al. (2020), Bandeira et al. (2022)) and empirically (see, e.g., Guajardo et al. (2014), Alesina et al. (2015), Fotiou (2022), Lambertini and Proebsting (2023)) by revealing new unintended consequences for skills mismatch and thus labor misallocation. This literature shows that spending cuts are less recessionary and a more potent means for improving the fiscal position than tax increases. Our novel finding is that fiscal tightening, both through tax increases and spending cuts, can raise the skills mismatch rate. To the best of our knowledge, the macroeconomics literature using DSGE and SVAR models has not considered skills mismatch up to now.

Finally, a connection can also be established with the recent literature studying the role of CSC in D(S)GE models. Dolado et al. (2021) embed the CSC framework in a New Keynesian model with S&M frictions to study the distributional effects of monetary policy shocks. Oikonomou (2023) follows a similar modelling

[5] Matching models with endogenous separations suggest that mismatch is *procyclical* due to a cleansing of unproductive matches (e.g., Mortensen and Pissarides (1994)). Others have argued that mismatch is *countercyclical* due to various sullying forces (e.g., Barnichon and Zylberberg (2019)).

approach but in an open economy model that features changes in the skill composition of the domestic workforce due to heterogeneous migration outflows. Sakkas and Varthalitis (2021) build a dynamic general equilibrium model with a CSC structure in the production function and a frictionless labor market to study aggregate and distributional implications of fiscal consolidation policies. We focus instead on the mismatch response to fiscal tightening shocks.

Layout. The paper is organized as follows. Section 2 offers new empirical evidence about the effect of fiscal tightening on the skills mismatch rate. This evidence motivates our theoretical model, which follows in Section 3. Section 4 discusses the calibration strategy and Section 5 presents impulse responses from the DSGE model. Section 7 concludes.

2 Empirical motivation

In this section, we present new empirical evidence about the effect of fiscal tightening on the vertical skills mismatch (over-education) rate in the Greek labor market. This evidence motivates our theoretical model, which follows in the next section.

2.1 Data

We use annual data from 1995 to 2019. The annual frequency and the time span of our sample are dictated by the availability of mismatch data and fiscal data, respectively, for Greece. The country is a unique case of a decade-long crisis after the Great Recession and implementation of several fiscal consolidation packages, and currently has one in three workers employed in jobs for which they are over-qualified.

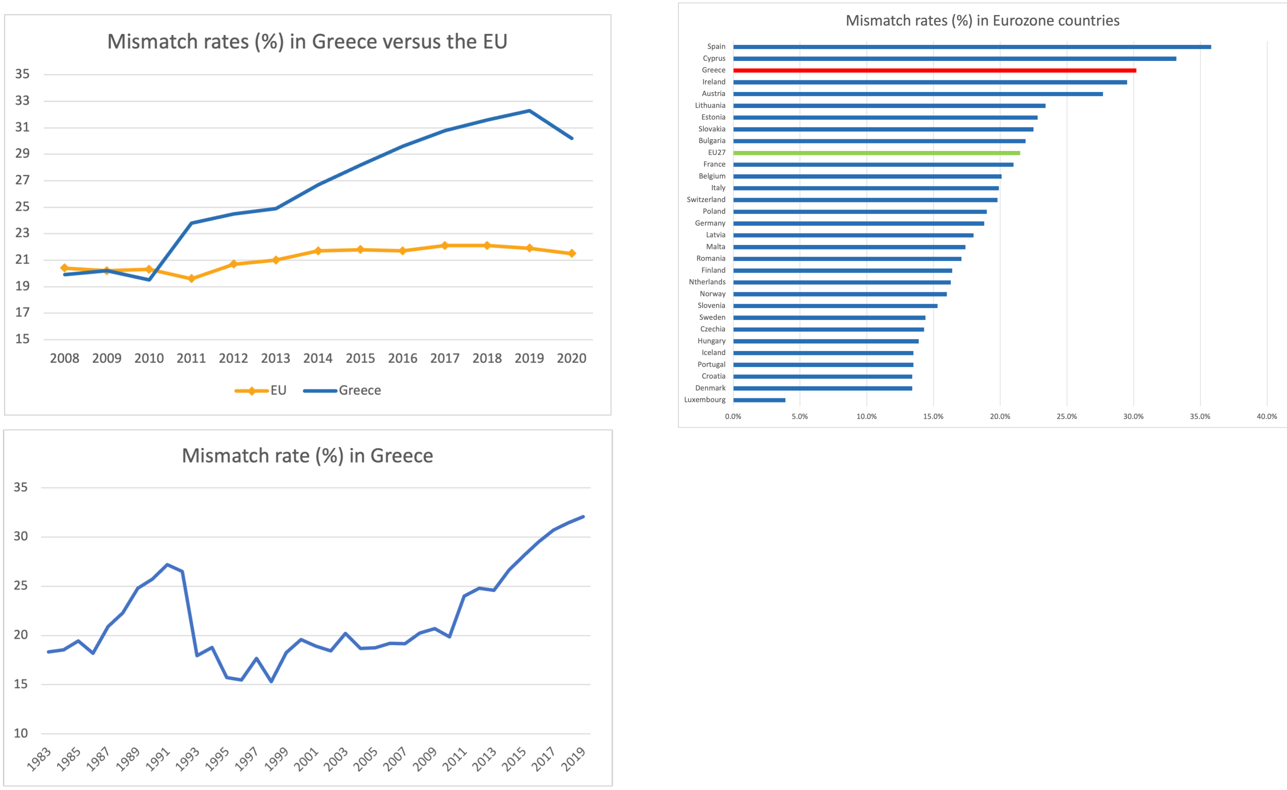
2.1.1 Mismatch data

We construct our (vertical) mismatch employment series based on micro data from the annual Labor Force Survey conducted by the Hellenic Statistical Authority (Elstat). The survey classifies employees based on their educational level and occupation type. An employee is classified as highly skilled if she holds a university degree. A highly skilled individual is then classified as matched if she works as a manager, professional or technician/associate professional. The mismatch employment rate is defined as the share of highly skilled individuals who are not employed in any of the above categories in the total number of highly skilled employees. Following the definition provided by Eurostat, which provides a similar mismatch series starting from 2008, we exclude from our mismatch measure employees whose occupation type is unclassified.

Figure 1 shows the mismatch rates, defined as the share (%) of those aged 20-64 with tertiary education and working in ISCO 4-9 in Greece from 1983 to 2019 (bottom graph), in Greece versus the EU from 2008 to 2020 (top left graph) and in Eurozone countries in 2020 (top right graph). In other words, we plot the percentage of employees with tertiary education working in occupations in which their skills are not necessary. We can see that the vertical mismatch rate in the Greek labor market roughly doubled between 1995 (15%) and 2019 (32%).[6] During the first years of the crisis, i.e. from 2010 to 2013, the rise amounted to more than 31%, while

[6]Figure A.1 in the Online Appendix shows that during that period the total number of high-skilled employees (denominator

Figure 1: Share (%) of those aged 20-64 with tertiary education and working in ISCO 4-9



Source: Eurostat (top graphs) and authors' own calculations based on micro data from the Labor Force Survey of Elstat (bottom graph)

it dropped to 28% from 2013 to 2017. Regarding the recent evolution of the mismatch rates in Greece and the EU, we can see that even though in 2008 the two rates were similar (close to 20%), Greece has had a sharp increase after 2010 which has widened the gap with the EU average. Among Eurozone countries, Greece ranks third in 2020 after Spain and Cyprus, well above the EU27 average of 21.5%.[7]

2.1.2 Fiscal data

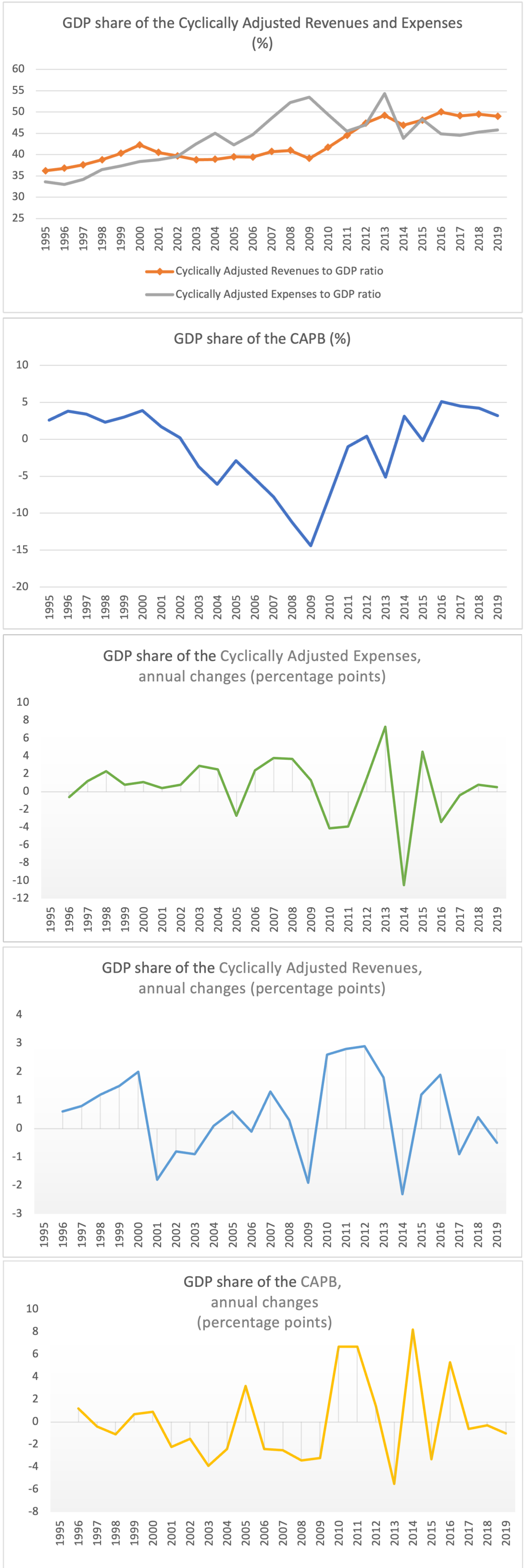
All fiscal (and other macro) data is expressed as a share of GDP and is taken from the AMECO database of the European Commission. We both use the cyclically adjusted primary balance (CAPB) and also distinguish between expenditure-based and revenue-based fiscal adjustments by using CA primary expenses and CA government revenues.[8] CA variables are estimated based on the Hodrick-Prescott filter and help to minimize the simultaneous correlation with the GDP.

of the mismatch rate) tripled, while the total number of mismatched high-skilled employees (nominator of the mismatch rate) shows an almost seven-fold increase.

[7]Regarding horizontal mismatch, according to OECD data for 2019, Greece tops the list of Eurozone countries with 41.7% of its employees working in occupations that are irrelevant to their education. At the same time, the EU27 and OECD averages are 32.2% and 31.7%, respectively.

[8]Primary expenses equal the general government's total expenses minus the interest rate payments.

Figure 2: Fiscal series and their volatility



Source: AMECO database and authors' own calculations

Table II: Sign restrictions of Pappa et al. (2015)

	Shock Variable		
	Gov Expenses / GDP	Tax Revenue (or CAPB) / GDP	Public Debt / GDP
Adjustment	t=0,1,2	t=0,1,2	t=1,2,3
Expenditure-based	-	n/a	-
Tax-based (or CAPB)	n/a	+	-

To take a view of the variation in our fiscal series, we plot them both in levels and in annual changes in Figure 2. Until 2001, Greece recorded higher CA government revenues than expenditures as a share of GDP, with a CA primary surplus between 2.5 and 4.5% of GDP. From 2001 until 2011, this trend was reversed. As a result, the country registered a CA primary deficit, which peaked at 15% in 2009, even though it was officially projected to be a fiscal deficit of only 6%. The CAPB turned into a surplus again from 2013 onward with a peak at 5% of GDP in 2015.

Let us see in more detail how the events unfolded during the debt crisis by following the description in Pagoulatos (2018). Greece experienced a debt-driven growth, fueled by external capital inflows, resulting from inadequate fiscal management, loose credit following euro-accession, and credibility granted by Eurozone membership. However, these private flows came to a "sudden stop" in 2010, which forced a bailout. The first adjustment program (2010-2013) led to the implementation of a wide array of heavily front-loaded austerity measures, which exacerbated the recession and caused complete target miss. The second program (2012-2014) included debt restructuring and focused on decreasing labor costs to improve competitiveness. The third program (2015-2018) was the only one completed and contained much of what had been left undone. Its size increased due to the 2015 economic downturn.

The effects of the three programs (see also Table A.1 in the Online Appendix) are clearly illustrated in the bottom graph of Figure 2, which depicts the evolution of the CAPB as a share of GDP in annual changes. The largest change in the GDP share of CA government expenses is recorded in 2014 with a decrease of almost 11 percentage points. Other reductions are observed in 2005 (nearly 3 percentage points), in 2010 and 2011 (4 percentage points) and in 2016 (3 percentage points). Note that Greece came under the Excessive Deficit Procedure, for the second time since 2004, in February 2009. In terms of the GDP share of CA government revenues, the largest change is recorded in 2014 with an increase of around 8 percentage points, followed by other increases in 2010 and 2011 (6 percentage points), 2016 (5 percentage points) and 2005 (3 percentage points).

2.2 Methodology

We implement a VAR methodology to identify the effect of fiscal shocks on the vertical mismatch employment rate in the Greek labor market and we resort to Bayesian techniques due to our small sample size. In the VAR, there are always four endogenous variables; the fiscal variable, per capita GDP (or the unemployment rate),

Table III: Same sign restrictions as in Pappa et al. (2015) but with unrestricted debt ratio

	Shock Variable		
	Gov Expenses / GDP	Tax (or CAPB) Revenue / GDP	Public Debt / GDP
Adjustment	t=0,1,2	t=0,1,2	t=1,2,3
Expenditure-based	-	n/a	n/a
Tax-based (or CAPB)	n/a	+	n/a

Table IV: Alternative sign restrictions for tax-based adjustments

	Shock Variable		
	Tax Revenue / GDP	GDP (or Unempl Rate)	Public Debt / GDP
	t=0,1,2	t=1,2,3	t=1,2,3
Tax-based adjustment	+	- (+)	n/a

gross debt, and the mismatch employment rate. We include one lag and a set of control variables, including the long-term interest rate, a linear or a combination of a linear and quadratic time trend, and a dummy controlling for the economic adjustment programs during the last decade in Greece. When we examine a shock to either CA revenues or expenditures, the other CA fiscal variable enters the model as control. Furthermore, when the fiscal shock is identified from the CAPB, there is no fiscal variable as control.

Turning to the identification of the shocks, we initially use a simple Cholesky decomposition where the fiscal variable is ordered first, GDP (or unemployment rate) second, gross debt third, and the mismatch employment rate last. Since the zero restrictions imposed by this procedure can be contested with annual data, we also use sign restrictions to identify fiscal adjustment shocks (see also Pappa et al., 2015). We assume that an expenditure-based adjustment should decrease government expenditure for three consecutive years, while a tax-based adjustment should increase tax revenue for three consecutive years. Accordingly, a fiscal adjustment must increase the CAPB for the same period. Additionally, all adjustments should decrease, with a lag, the debt-to-GDP ratio for three years. Table II summarizes the sign restrictions.

In addition, we follow an alternative sign restrictions scheme (see Table III) where we leave the ratio of public debt to GDP unrestricted. We base this strategy on the literature pointing out that fiscal consolidations, especially tax-based ones and in economies with widespread tax evasion, may be self-defeating (e.g., Alesina et al. (2015), Pappa et al. (2015), Erceg and Lindé (2013)). As stated in IMF (2023): "The debt-reducing effects of fiscal adjustments are reinforced when accompanied by growth-enhancing structural reforms and strong institutional frameworks. At the same time, because these conditions and accompanying policies may not always be present, and partly because fiscal consolidation tends to slow GDP growth, consolidations on average have negligible effects on debt ratios." Finally, given the substantial number of studies highlighting the contractionary effects of tax hikes, we use for the case of tax-based adjustments a third scheme of sign restrictions, which is displayed in Table IV. We assume that, on top of increasing tax revenues (as a share of GDP) for three periods, such adjustments reduce GDP or increase the unemployment rate, with a lag, for three periods. Note that these restrictions are supported by the sign of the corresponding impulse responses from our DSGE model in Section 3. Debt is again left unrestricted like in Table III.

2.3 Empirical responses

Let us now examine the responses of the endogenous variables to a one standard deviation shock in the fiscal variable of interest each time. In the case of government expenses, the shock is negative, while it is positive when we consider government revenues or the CAPB. In the figures that follow, we show 68% confidence bands.

CAPB and skills mismatch. In this exercise, we do not separate yet the effects of expenditure-based and revenue-based adjustments. Instead, we use the Greek government's CAPB. In Figure 3, we identify the shock based on the Cholesky decomposition as well as the sign restrictions displayed in Tables II and III. The main takeaway is that the mismatch response is statistically significant and positive under all these identification strategies. The response of GDP, when statistically significant, is positive in the Cholesky decomposition and when we restrict the debt-to-GDP ratio to fall, while it is negative if debt is left unrestricted. The debt response is positive, when left unrestricted, and statistically significant when we use a combination of a linear and a quadratic time trend.

CA primary expenses and skills mismatch. Figures 4 depicts the responses to a cut in the CA primary expenses under a Cholesky decomposition as well as the sign restrictions of Table II and Table III. We generally obtain statistically significant and positive mismatch responses in all these figures, especially when using the Cholesky decomposition and sign restrictions where public debt is left unrestricted. The response of GDP is positive except when we leave debt unrestricted while using a linear trend. Public debt falls in the Cholesky case, while when left unrestricted in a sign restrictions scheme it moves either way, depending on whether a linear or a combination of a linear and a quadratic trend is used.

CA tax revenues and skills mismatch. We now turn to the effects of a revenue-based adjustment on skills mismatch. Figure 5 depicts the responses to a rise in the CA primary revenues under a Cholesky decomposition as well as the sign restrictions of Table II, Table III, and Table IV. In the Cholesky decomposition, we assume that the elasticity of revenues to GDP is almost one. Thus, the cycle should not significantly affect the revenues-to-GDP ratio. When we use the Cholesky decomposition, our findings again indicate that revenue-based consolidations increase significantly the skills mismatch rate. When we use the sign restrictions in Pappa et al. (2015), the response remains positive but looses statistical significance. Note also that these restrictions are not satisfied when we use both a linear and quadratic time trend. When debt is left unrestricted and we use both these time trends, the response of mismatch is positive and statistically significant. Finally, if we restrict the tax shock to be contractionary, we obtain again positive and statistically significant responses of the mismatch rate. Note that the shock appears to be contractionary also in the Cholesky case. When we force the model to keep only shocks that decrease the debt to GDP ratio, we get that a tax-focused fiscal tightening can have expansionary effects.

Robustness. Even though we have already shown results for different specifications in terms of the time trend and different identification strategies, we also include in the Online Appendix all the previous responses when we replace GDP in the VAR regressions with the unemployment rate. Our main finding that fiscal tightening can lead to a rise in skills mismatch in the labor market continues to hold in all cases.

In the case of the positive shock to the CAPB, the effect on unemployment is negative, when statistically significant (see Figure A.3). In the case of the negative spending shock, the response of unemployment is negative (see Figure A.4), which may be reflecting a reduction in labor participation (see Brückner and Pappa (2012)). Finally, if we restrict the tax shock to be contractionary, we obtain again positive and statistically significant responses of the mismatch rate as was the case also in the VAR with GDP. When we force the model to keep only shocks that decrease the debt to GDP ratio, we get that the response of unemployment is not statistically significant (see Figure A.5). The response of the latter is positive and statistically significant under the Cholesky decomposition.

In sum, the findings of our empirical exercises, using Greek data and a variety of specifications and identification methods, seem to suggest that fiscal tightening may lead to a rise in skills mismatch in the labor market. Next, we turn to our theoretical investigation by incorporating skills mismatch in a DSGE model.

3 Theoretical model

3.1 Description of the model

We build an heterogeneous agents DSGE model of a small open economy (SOE) with domestic and imported goods. Households differ in two aspects: (i) access to capital and financial markets and (ii) type of labor they supply, i.e. high or low-skilled. In particular, households that have access to domestic and international capital and financial markets, own firms, and supply skilled labor services to firms are labelled as "high-skilled" (h). The second household type, labelled as "low-skilled" (l), does not receive any profits from firms to which supplies unskilled labor services. The labor market is governed by a standard search and matching (S&M) mechanism. Vacancies posted by firms require different skill types. High-skilled workers are characterized by a lower separation rate and a lower elasticity of substitution with capital than low-skilled workers.

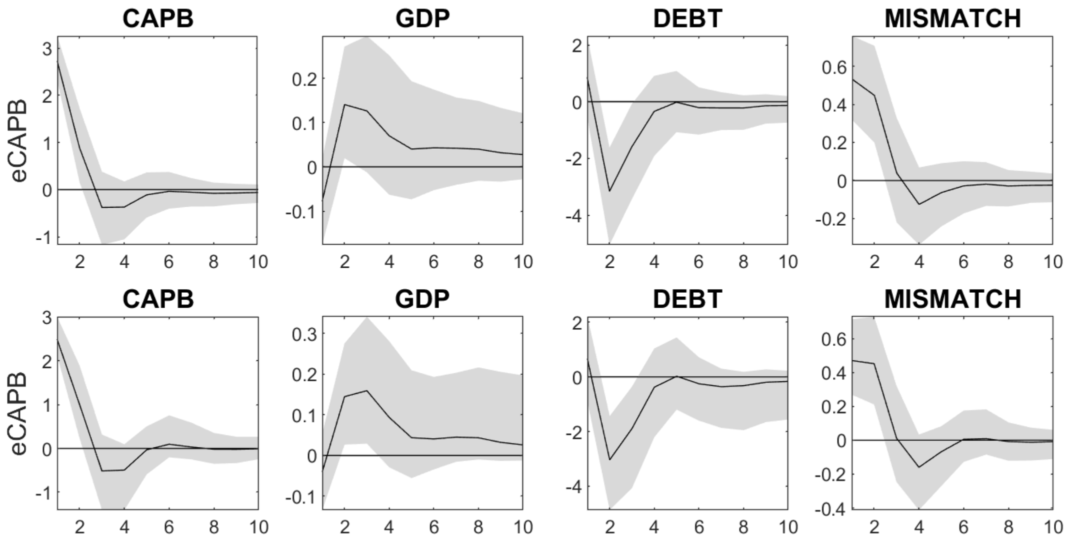
3.2 Population and skill mismatch

The economy is comprised of low-skilled households, $l = 1, ..., N_t^l$, and high-skilled households, $h = 1, ..., N_t^h$. Total population is given by $N_t = N_t^l + N_t^h$. We denote by $t^l \equiv N_t^l/N_t$ and $t^h \equiv N_t^h/N_t$ the non-equal population shares of low-skilled and high-skilled households, respectively.

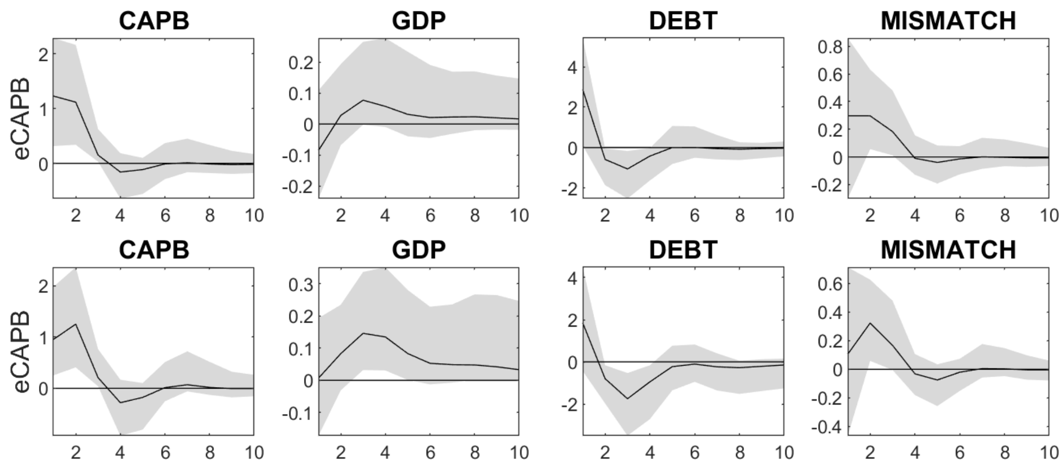
Each low-skilled household l consists of members that are employed in a low-skill position, $n_t^{l,l}$, members that are unemployed, u_t^l, and members that are out of the labor force and enjoy leisure, l_t^l, so that:

$$n_t^{l,l} + u_t^l + l_t^l = 1 \tag{1}$$

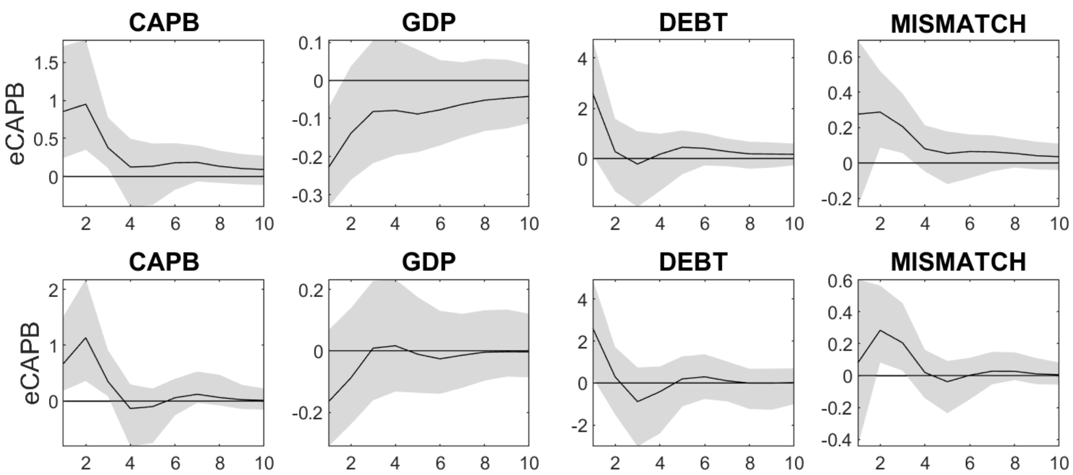
Figure 3: Responses to a one standard deviation shock in CAPB



(a) Cholesky (row 1: linear trend, row 2: linear and quadratic trend)



(b) Sign restrictions of Pappa et al. (2015) (row 1: linear trend, row 2: linear and quadratic trend)

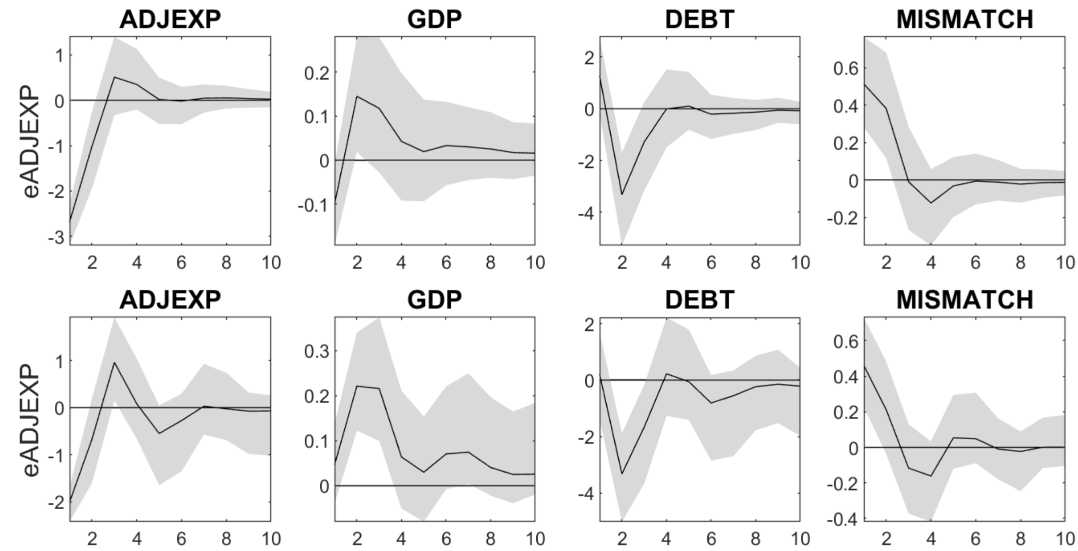


(c) Sign restrictions of Pappa et al. (2015) but with unrestricted debt ratio

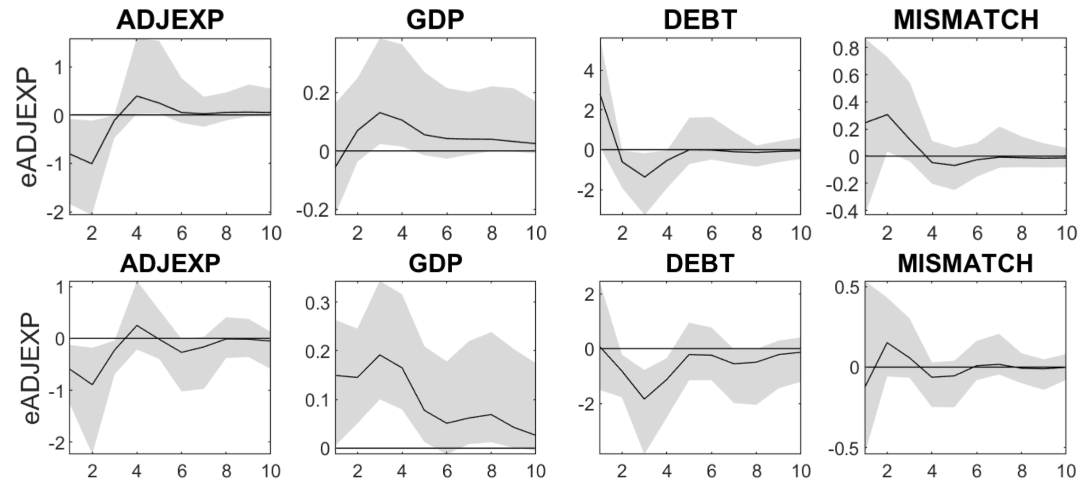
(row 1: linear trend, row 2: linear and quadratic trend)

Note: We use Bayesian methods and present 68% posterior confidence bands. The horizontal axis refers to years.

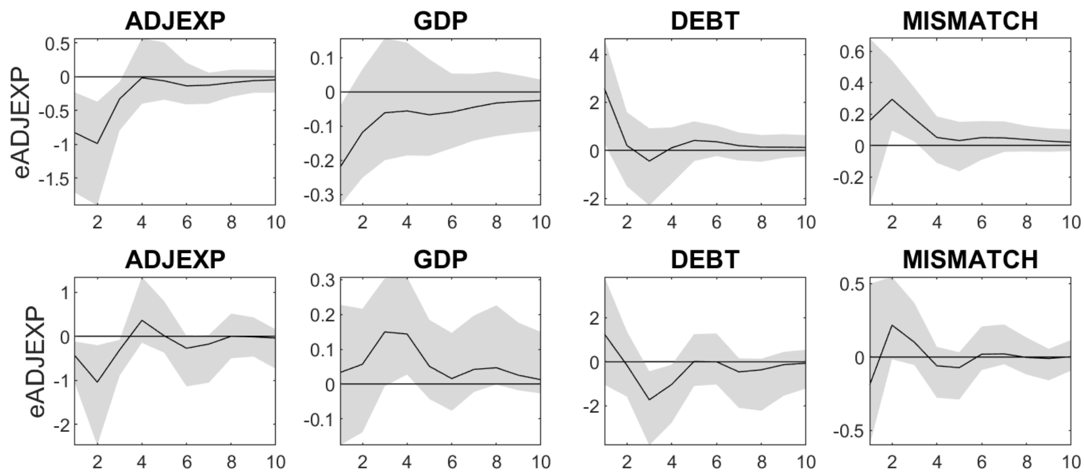
Figure 4: Responses to a one standard deviation negative shock in cyclically adjusted primary expenses



(a) Cholesky (row 1: linear trend, row 2: linear and quadratic trend)



(b) Sign restrictions of Pappa et al. (2015) (row 1: linear trend, row 2: linear and quadratic trend)

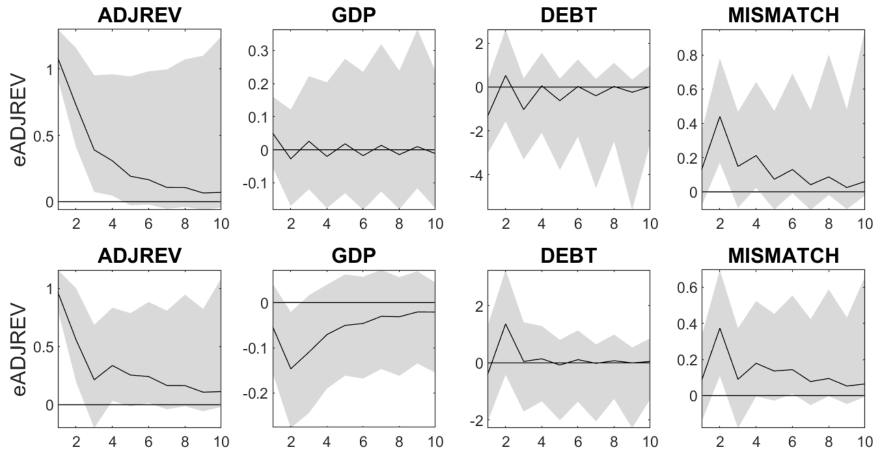


(c) Sign restrictions of Pappa et al. (2015) but with unrestricted debt ratio

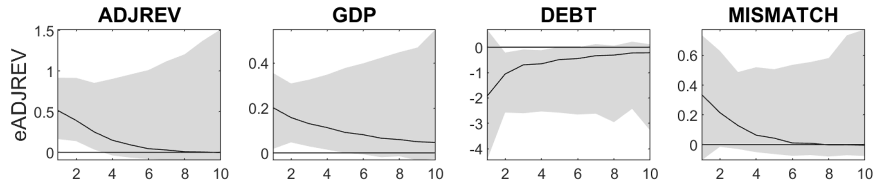
(row 1: linear trend, row 2: linear and quadratic trend)

Note: See Figure 3.

Figure 5: Responses to a one standard deviation shock in the cyclically adjusted revenues

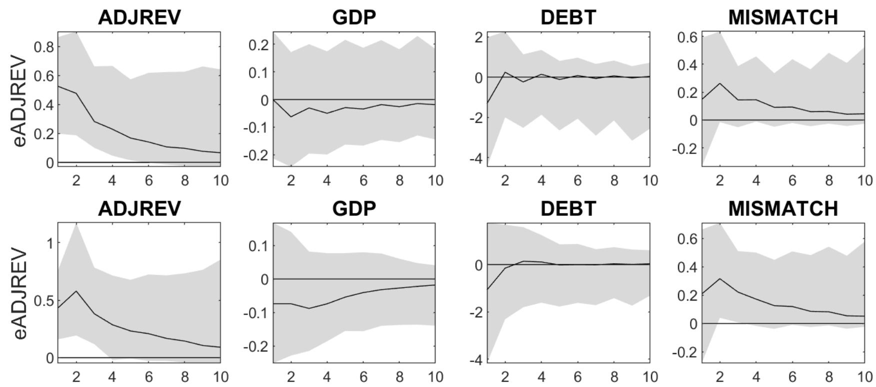


(a) Cholesky (row 1: linear trend, row 2: linear and quadratic trend)



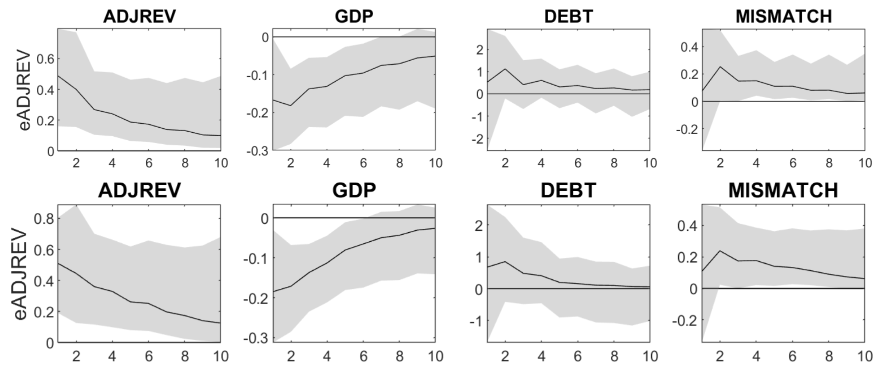
(b) Sign restrictions of Pappa et al. (2015)

(linear trend, sign restrictions not satisfied with both linear and quadratic trend)



(c) Sign restrictions of Pappa et al. (2015) but with unrestricted debt ratio

(row 1: linear trend, row 2: linear and quadratic trend)



(d) Sign restrictions on GDP with unrestricted debt ratio

(row 1: linear trend, row 2: linear and quadratic trend)

Note: See Figure 3.

Note that the presence of endogenous participation is important here. The external margin allows households to adapt their labor supply in the face of shocks and helps to ensure that the model does not overstate the change in skills mismatch induced by a shock. Moreover, we can capture the reduction in participation after a negative government spending shock due to the standard wealth effect.

Each high-skilled household h consists of members that are employed n_t^h, members that are unemployed, u_t^h, and members that are out of the labor force and enjoy leisure, l_t^h. We assume that high-skilled employees, n_t^h, are either employed in a high-skill position, $n_t^{h,h}$, or a low-skill position (hereafter, called mismatched (overqualified) employees), $n_t^{h,l}$, so that:

$$n_t^h = n_t^{h,h} + n_t^{h,l} \tag{2}$$

To keep the model tractable, we refrain from considering horizontal mismatch within the same skill group. As previously, the members of a high-skilled household are normalized to unity:

$$n_t^{h,l} + n_t^{h,h} + u_t^h + l_t^h = 1 \tag{3}$$

The household chooses the fraction of the high-skilled unemployed who look for high-skill positions s_t, while the remaining $(1 - s_t)$ search for mismatch (low-skill) positions:

$$u_t^h = s_t u_t^h + (1 - s_t) u_t^h = u_t^{h,h} + u_t^{h,l} \tag{4}$$

where $u_t^{h,h} \equiv s_t u_t^h$ and $u_t^{h,l} \equiv (1 - s_t) u_t^h$. Following the literature, we assume that mismatched workers continue searching on-the-job and applying to vacancies with a high-skill requirement. The efficacy of this search, denoted by $\phi(z_t)$, depends positively on the endogenous effort they exert, z_t, while the cost of searching is denoted by $b(z_t)$, with $d'b(z_t)/dz_t > 0$. If the search is successful, they quit the mismatch position in favor of the high-skill position (see below).[9]

3.3 Labor market

The model considers three labor sub-markets depending both on the workers' skill type and on the position's skill requirements. In each sub-market, jobs are created through a matching function. We denote with $M_t^{l,l}$ and $M_t^{h,h}$ the aggregate matches in the low-skill and high-skill labor markets respectively, whereas aggregate mismatches are denoted by $M_t^{h,l}$. The respective matching functions are given by:

$$M_t^{l,l} = \mu_1((1 - x_t)V_t^l)^{\mu_2}(N_t^l u_t^l)^{1-\mu_2} \tag{5}$$

$$M_t^{h,l} = \mu_1(x_t V_t^l)^{\mu_2}((1 - s_t)N_t^h u_t^h)^{1-\mu_2} \tag{6}$$

$$M_t^{h,h} = \mu_1(V_t^h)^{\mu_2}(s_t N_t^h u_t^h + \phi(z_t)N_t^h n_t^{h,l})^{1-\mu_2} \tag{7}$$

where μ_1 denotes the efficiency of the matching process, μ_2 denotes the elasticity of matches with respect to vacancies, V_t^j denotes the aggregate vacancies posted by firms for skill type $j = l, h$, x_t is the fraction of low-skill vacancies that are allocated by firms to high-skilled applicants, thus generating a mismatch. In equation (7), total searchers for a high-skill position comprise both the high-skilled unemployed job seekers, $s_t N_t^h u_t^h$, and the mismatched employees who perform on-the-job search, $\phi(z_t)N_t^h n_t^{h,l}$.

[9] All unemployed members search with intensity one and there is no pecuniary cost associated with their search.

Probabilities and labor market tightness. We define the hiring probabilities as follows:

$$\psi_{H,t}^{l,l} = \frac{M_t^{l,l}}{N_t^l u_t^l} \qquad \psi_{H,t}^{h,l} = \frac{M_t^{h,l}}{(1-s_t)N_t^h u_t^h} \qquad \psi_{H,t}^{h,h} = \frac{M_t^{h,h}}{s_t N_t^h u_t^h + \phi(z_t) N_t^h n_t^{h,l}} \tag{8}$$

We also define the vacancy-filling probabilities:

$$\psi_{F,t}^{l,l} = \frac{M_t^{l,l}}{(1-x_t)V_t^l} \qquad \psi_{F,t}^{h,l} = \frac{M_t^{h,l}}{x_t V_t^l} \qquad \psi_{F,t}^{h,h} = \frac{M_t^{h,h}}{V_t^h} \tag{9}$$

Finally, labor market tightness in each sub-market is given by:

$$\vartheta_t^{l,l} = \frac{(1-x_t)V_t^l}{N_t^l u_t^l} \qquad \vartheta_t^{h,l} = \frac{x_t V_t^l}{(1-s_t)N_t^h u_t^h} \qquad \vartheta_t^{h,h} = \frac{V_t^h}{s_t N_t^h u_t^h + \phi(z_t) N_t^h n_t^{h,l}} \tag{10}$$

Employment laws of motion. The law of motion for aggregate mismatched employment is given by:

$$N_t^h n_{t+1}^{h,l} = \left(1 - \sigma^l - \phi(z_t)\psi_{H,t}^{h,h}\right) N_t^h n_t^{h,l} + \psi_{H,t}^{h,l}(1-s_t)N_t^h u_t^h \tag{11}$$

where new mismatches, $(1-s_t)N_t^h u_t^h$, occur with probability $\psi_{H,t}^{h,l}$, σ^l is the exogenous destruction rate of low-skill positions (taken to be the same both if there is mismatch and if not), and $\phi(z_t)\psi_{H_t}^{h,h}$ is the endogenous destruction rate due to on-the-job search leading to quits to take up a non-mismatch job.

The laws of motion of aggregate non-mismatched employment are as follows:

$$N_t^l n_{t+1}^{l,l} = (1-\sigma^l) N_t^l n_t^{l,l} + \psi_{H,t}^{l,l} N_t^l u_t^l \tag{12}$$

$$N_t^h n_{t+1}^{h,h} = (1-\sigma^h) N_t^h n_t^{h,h} + \psi_{H,t}^{h,h}\Big(s_t N_t^h u_t^h + \phi(z_t) N_t^h n_t^{h,l}\Big) \tag{13}$$

where low-skilled employment and high-skilled employment increase with probability $\psi_{H,t}^{l,l}$ and $\psi_{H,t}^{h,h}$ respectively and decrease with positions being destroyed at rate σ^j, with $j = l, h$.

3.4 Households

The lifetime utility of each household with skill type j, $j = 1, 2, ..., N_t^j$, is:

$$E_t \sum_{t=0}^{\infty} {\beta^*}^t u(c_t^j, l_t^j) \tag{14}$$

where E_0 denotes rational expectations conditional on the information set available at time zero, the time discount factor is $\beta^* \in (0,1)$, c_t^j and l_t^j is household $j's$ consumption and leisure time at time t. We assume that the instantaneous utility function takes the form:

$$u(c_t^j, l_t^j) = \frac{(c_t^j)^{1-\eta}}{1-\eta} + \Phi^j \frac{{l_t^j}^{1-\phi}}{1-\phi} \tag{15}$$

where $\eta > 0$ is the inverse of intertemporal elasticity of substitution, $\Phi^j > 0$ is the relative preference for leisure and ϕ is the inverse of the Frisch elasticity of labor supply.

3.4.1 The representative low-skilled household

We assume that the household l is a hand-to-mouth consumer, with the following budget constraint:

$$c_t^l = (1-\tau_t) w_t^{l,l} n_t^{l,l} + \bar{\omega} u_t^l + \bar{g}_t^{t,l} \tag{16}$$

where $0 \leq \tau_t < 1$ is the income tax rate, $w_t^{l,l}$ is the wage rate in a low-skill job, $\bar{\omega}$ is the unemployment benefit and $\bar{g}_t^{t,l}$ are transfers given to low-skilled households.

Each household l acts competitively by taking prices as given and chooses consumption, c_t^l, leisure, l_t^l, members employed in a low-skill position, $n_{t+1}^{l,l}$, and members searching for employment, u_t^l. The optimization problem, which is standard, is presented in the Online Appendix.

3.4.2 The representative high-skilled household

Differently from low-skilled households, each high-skilled household receives an interest income $r_t^k k_t^h$ and $r_t^d d_t^h$ from capital and net foreign assets where r_t^k and r_t^d denote the respective returns, as well as a (tax-free) share of firms' profits $\pi_{d,t}$. Based on the above, the household $h's$ budget constraint is:

$$c_t^h + i_t^h + e_t r_t^d d_t^h + b(z_t) n_t^{h,l} = (1-\tau_t)(w_t^{h,h} n_t^{h,h} + w_t^{h,l} n_t^{h,l} + r_t^k k_t^h) + \pi_{d,t} + e_t d_{t+1}^h + \bar{\omega} u_t^h + \bar{g}_t^{t,h} \tag{17}$$

where e_t is the real exchange rate, d_t^h is the debt position of the household from trading an international non-state contingent bond, $0 \leq \tau_t < 1$ is a total income tax rate, $w_t^{h,h}$ is the high-skill wage rate, $w_t^{h,l}$ is the mismatch wage rate, $\bar{\omega}$ is the unemployment benefit, and $\bar{g}_t^{t,h}$ are lump-sum transfers. The on-the-job search of mismatched workers for high-skill positions entails a cost $b(z_t)$ per mismatched worker.
The capital law of motion evolves according to:

$$k_{t+1}^h = (1-\delta) k_t^h + i_t^h - \frac{\Xi}{2} \left(\frac{k_{t+1}^h}{k_t^h} - 1 \right)^2 k_t^h \tag{18}$$

where δ is the depreciation rate and parameter Ξ controls the capital adjustment costs, which help to obtain smooth impulse responses.

Each high-skilled household decides to invest in capital, k_{t+1}^h, and in an international non-state contingent bond, d_{t+1}^h; chooses consumption c_t^h, leisure l_t^h, members employed in a high-skill position, $n_{t+1}^{h,h}$, members employed in a mismatch position, $n_{t+1}^{h,l}$, members searching for a job, u_t^h, the fraction of unemployed members that search for a high-skill position versus a mismatch (low-skill) position, s_t^h, and the intensity of the on-the-job search, z_t^h. The optimization problem is presented in the Online Appendix. Below we show the first order conditions for s_t and z_t, which are relevant for skills mismatch.
$[s_t]$

$$\lambda_{n_t^{h,h}} \psi_{H,t}^{h,h} = \lambda_{n_t^{h,l}} \psi_{H,t}^{h,l} \tag{19}$$

$[z_t]$

$$\lambda_{c_t^h} \frac{b'(z_t)}{\phi'(z_t)} = \psi_{H,t}^{h,h} \Big(\lambda_{n_t^{h,h}} - \lambda_{n_t^{h,l}} \Big) \tag{20}$$

where $\lambda_{n_t^{h,h}}$, $\lambda_{n_t^{h,l}}$, and $\lambda_{c_t^h}$ are the Lagrange multipliers on equations (13), (11), and (17). Equation (19) states that the values of job seeking for a non-mismatch and a mismatch job should be equal, subject to the respective hiring probabilities. Condition (20) states that the marginal cost of on-the-job search intensity, in units of consumption, must be equal to the excess relative value of working in a non-mismatch job subject to the job-finding probability.[10]

3.5 Firms

Each intermediate good firm $f = 1, 2, ..., N_t^f$ uses capital k_t^f, low-skilled employment $n_t^{l,f}$, and high-skilled employment $n_t^{h,f}$ to produce the domestic intermediate good $y_{i,t}^f$ with a CES technology:

$$y_{i,t}^f = A_t \Big[\alpha (n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}} \Big]^{\frac{\epsilon}{\epsilon-1}} \tag{21}$$

$$x_{i,t}^f = \Big[\zeta (k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}} \Big]^{\frac{\rho}{\rho-1}} \tag{22}$$

where A_t denotes the exogenous technology process, $0 \leq \alpha$ and $0 \leq \zeta$ control the income shares, $0 \leq \epsilon$ is the elasticity of substitution both between low-skilled labor and capital and between low-skilled labor and high-skilled labor, and $0 \leq \rho$ is the elasticity of substitution between capital and high-skilled labor. For capital-skill complementarity, we need $\rho < \epsilon$ (see Krusell et al. (2000)).

As in e.g. Iftikhar and Zaharieva (2019), we assume different productivity between the two types of workers employed in low-skill positions. Following Acemoglu (2001), we assume that sub-markets with different skill requirements produce different intermediate goods, which are used in the production. Thus, for every firm f in the intermediate goods sector, the input of low-skill positions is equal to:

$$n_t^{l,f} = n_t^{l,l,f} + q^h n_t^{h,l,f} \tag{23}$$

where $n_t^{l,l,f}$ denotes demand for low-skilled workers, $q^h n_t^{h,l,f}$ denotes demand for high-skilled workers in a low-skill (mismatch) position, and $q^h \geq 1$ reflects the effective productivity of a high-skilled worker in a low-skill position.

The intermediate good $y_{i,t}^f$ is sold domestically, $y_{d,t}^f$, and abroad, $y_{d,t}^*$. In aggregate terms, this is:[11]

$$Y_{i,t} = Y_{d,t} + Y_{d,t}^* \tag{24}$$

Defining the real exchange rate as $e_t = \frac{P_t^*}{P_t}$, foreign aggregate demand $Y_{d,t}^*$ is given exogenously by:

$$Y_{d,t}^* = (1-\omega^*)(\frac{p_{d,t}}{e_t})^{-\gamma^*} Y_t^* \tag{25}$$

where the parameters ω^* and γ^* to be the foreign counterparts for the home bias and elasticity of substitution and Y_t^* is the foreign GDP.[12]

The representative final good firm aggregates the domestic intermediate good $Y_{d,t}$ and imported aggregate goods $Y_{f,t}$ to produce the economy-wide final good Y_t using a CES technology. Details for this part, which is standard, are presented in the Online Appendix.

[10] We only consider cases where $\lambda_{n_t^{h,h}} > \lambda_{n_t^{h,l}}$ is true in the steady state.

[11] $Y_{i,t} = \sum_{f=1}^{N_t^f} y_{i,t}^f$, $Y_{d,t} = \sum_{f=1}^{N_t^f} y_{d,t}^f$ and $Y_{d,t}^* = \sum_{f=1}^{N_t^f} y_{d,t}^{f\ *}$.

[12] The structure of the foreign economy is similar to the home economy but, due to the small size of the latter, domestic developments have a negligible effect on foreign economy dynamics.

Firm's profit maximization problem

Firms post positions requiring high skills $v_t^{h,f}$ and positions requiring low skills $v_t^{l,f}$. Since a fraction of high-skilled searchers apply for low-skill positions, firms consider all applications for such positions and decide also the fraction of low-skill vacancies targeting high-skilled applicants, x_t. Therefore, skills mismatch arises endogenously from an interplay of the households and firms decisions in the model. Recall that, in the event of a mismatch, the following trade-off arises for the firm: the worker is more productive than the non-mismatched worker (see eq. (23)), but the mismatch maybe terminated if she quits to take up a non-mismatched job via on-the-job search (see eq. (11)). Additionally, the firm chooses the amount of capital to demand.

$$Q(n_t^{l,l,f}, n_t^{h,f}, n_t^{h,l,f}) = \max_{v_t^{l,f}, v_t^{h,f}, k_t^f, x_t} \Big\{ p_{d,t} y_{i,t}^f - w_t^{l,l} n_t^{l,l,f} - w_t^{h,l} n_t^{h,l,f} - w_t^{h,h} n_t^{h,f} - r_t^k k_t^f - \kappa^l v_t^{l,f} - \kappa^h v_t^{h,f} \\ + E_t \left[\Lambda_{t,t+1} Q(n_{t+1}^{l,l,f}, n_{t+1}^{h,f}, n_{t+1}^{h,l,f}) \right] \Big\} \quad (26)$$

where κ^l and κ^h denote the marginal cost of posting a vacancy requiring low and high skills, respectively. As the household owns the firm, the term $\Lambda_{t,t+1} \equiv \beta \frac{\partial u_{c_{t+1}}}{\partial u_{c_t}}$ refers to the household's stochastic discount factor in which β is the household's discount factor. The maximization is subject to the employment laws of motion, eqs. (11)-(13), using the vacancy-filling probabilities.
Denoting the marginal productivity of k_t^f, $n_t^{l,l,f}$, $n_t^{h,f}$ and $n_t^{h,l,f}$ as:

$$y_{i,t}^{k,f} \equiv \frac{\partial y_{i,t}^f}{\partial k_t^f} \qquad y_t^{l,l} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{l,l,f}}, \qquad y_t^h \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{h,f}}, \qquad y_t^{h,l} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{h,l,f}} \quad (27)$$

The first order conditions are given by:

$[n_{t+1}^{l,l,f}]$

$$\frac{\kappa^l}{\psi_{F,t}^{l,l}} = E_t \Lambda_{t,t+1} \Big\{ p_{d,t+1} y_{i,t+1}^{l,f} - w_{t+1}^{l,l} + \frac{(1-\sigma^l)\kappa^l}{\psi_{F,t+1}^{l,l}} \Big\} \quad (28)$$

$[n_{t+1}^{h,f}]$

$$\frac{\kappa^h}{\psi_{F,t}^{h,h}} = E_t \Lambda_{t,t+1} \Big\{ p_{d,t+1} y_{i,t+1}^{h,f} - w_{t+1}^{h,h} + \frac{(1-\sigma^h)\kappa^h}{\psi_{F,t+1}^{h,h}} \Big\} \quad (29)$$

$[n_{t+1}^{h,l,f}]$

$$\frac{\kappa^l}{\psi_{F,t}^{h,l}} = E_t \Lambda_{t,t+1} \Big\{ p_{d,t+1} y_{i,t+1}^{h,l,f} - w_{t+1}^{h,l} + \frac{(1-\sigma^l - psi_{H,t+1}^{h,h} \phi(z_{t+1}))\kappa^l}{\psi_{F,t+1}^{h,l}} \Big\} \quad (30)$$

$[k_t^f]$:

$$r_t^k = p_{d,t} y_{i,t}^{k,f} \quad (31)$$

where x_t is given by:

$$x_t = \frac{n_{t+1}^{h,l,f} - (1-\sigma^l - \psi_{H,t}^{h,h}\phi(z_t)) n_t^{h,l,f}}{v_t^{l,f} \psi_{F,t}^{h,l}} \quad (32)$$

Eqs. (28)-(30) state that the marginal cost of hiring should equal the expected marginal benefit, given by the marginal productivity of labor minus the wage cost plus the continuation value. The termination of the match occurs exogenously with probability σ^j where $j = h, l$ and, in the case of mismatch employment, also endogenously due to quits in order to take up an upgraded position, $\psi_{H,t+1}^{h,h}\phi(z_{t+1})$. In eq. (32), we see that the endogenous quits from mismatch jobs, $\phi(z_t)\psi_{H,t}^{h,h}n_t^{h,l,f}$, positively affect the share of low-skill positions that a firm allocates to high-skilled (thus, mismatched) applicants so as to fill the gaps created. Details about derivations as well as the expressions for the values to the firm of an additional unit of high-skilled, low-skilled and mismatch employment are included in the Online Appendix.

3.6 Wage bargaining

The Nash bargaining problem is to maximize the weighted sum of log surpluses for each employment status. The wages for non-mismatch and mismatch employment are thus given as the optimal solution of the following problems:

$$\max_{w_t^{j,j}} \left\{ \left(1-\theta^{j,j}\right) \ln V_t^{n_t^{j,j}} + \theta^{j,j} \ln V_t^{n_t^{j,j,f}} \right\}, \quad j = l, h \tag{33}$$

$$\max_{w_t^{h,l}} \left\{ \left(1-\theta^{h,l}\right) \ln V_t^{n_t^{h,l}} + \theta^{h,l} \ln V_t^{n_t^{h,l,f}} \right\} \tag{34}$$

where θ denotes the bargaining power of firms, $V_t^{n_t^{h,l,f}}$ and $V_t^{n_t^{j,j,f}}$ are respective value functions of an additional unit of mismatch and non-mismatch employment to each firm, and $V_t^{n_t^{j,j}}$ and $V_t^{n_t^{l,l}}$ are the marginal values of having an additional household member employed in a non-mismatch and mismatch position, respectively. Delegating all derivations in the Online Appendix since this part is quite standard, the equilibrium wages are as follows:

$$w_t^{j,j} = (1-\theta^{j,j})\left(p_{d,t}y_{i,t}^{j,j,f} + \frac{(1-\sigma^j)\kappa^j}{\psi_{F,t}^{j,j}}\right) - \frac{\theta^{j,j}}{\lambda_{c_t^j}(1-\tau_t)}\left(-\Phi^j(l_t^j)^{-\phi} + \lambda_{n_t^{j,j}}(1-\sigma^j)\right), \quad j = l, h \tag{35}$$

$$\begin{aligned} w_t^{h,l} = (1-\theta^{h,l})&\left(p_{d,t}y_{i,t}^{h,l,f} + \frac{(1-\sigma^l-\phi(z_t)\psi_{H,t}^{h,h})\kappa^l}{\psi_{F,t}^{h,l}}\right) \\ &- \frac{\theta^{h,l}}{\lambda_{c_t^h}(1-\tau_t)}\left(-\Phi^h(l_t^h)^{-\phi} + \lambda_{n_t^{h,l}}(1-\sigma^l) - \lambda_{c_t^h}b(z_t) + (\lambda_{n_t^{h,h}} - \lambda_{n_t^{h,l}})\psi_{H,t}^{h,h}\phi(z_t)\right) \end{aligned} \tag{36}$$

In both equations above, the term weighted by the workers' bargaining power $(1-\theta)$ includes the value of the marginal product of labor and the continuation value to the firm. This continuation value accounts, in the case of the mismatch wage (see eq. (36)), for the fact that there is an increased likelihood of a match termination due to on-the-job search, which pushes down on the wage. The term weighted by the firms' bargaining power θ includes the utility cost of labor for workers and the continuation value to the household. Additionally, eq. (36) implies that the likelihood of quitting because of on-the-job search, $\psi_{H,t}^{h,h}\phi(z_t)$, enables firms to bargain a lower mismatch wage. On the other hand, the on-the-job search cost $b(z_t)$ increases the mismatch wage that firms need to pay if workers accept a mismatch job.

3.7 Government

For simplicity, we abstract from public debt in the model opting for a parsimonious theoretical framework that can rationalize our empirical evidence. The government taxes the household income at the rate $0 \leq \tau_t < 1$

and uses these revenue to finance aggregate unemployment benefits to low-skilled and high-skilled households, $\bar{\omega}U_t^l$ and $\bar{\omega}U_t^h$ respectively, lump-sum transfers to both types of households, denoted as G_t^l and G_t^h, as well as government consumption, G_t^c. The latter is modelled as a waste in the economy. The government budget constraint is the following:[13]

$$\bar{\omega}U_t^l + \bar{\omega}U_t^h + G_t^{t,l} + G_t^{t,h} + G_t^c = \tau_t\Big(w_t^{l,l}\sum_{l=1}^{N_t^l} n_t^{l,l} + w_t^{h,l}\sum_{h=1}^{N_t^h} n_t^{h,l} + w_t^{h,h}\sum_{h=1}^{N_t^h} n_t^{h,h} + r_t^k\sum_{h=1}^{N_t^h} k_t^h\Big) \tag{37}$$

3.8 Closing the model

We address the known issue of non-stationarity that arises in the small open economy models by assuming the following debt-elastic interest rate:

$$r_t^d = r_t^* + rp_t \tag{38}$$

where r_t^* is the foreign interest rate taken as given by the small open economy and rp_t is the risk-premium, which takes the following form:

$$rp_t = \psi^{rp}\left(exp(\frac{e_t d_{t+1}}{gdp_t} - \frac{ed}{gdp}) - 1\right) + \epsilon_t^{rp} \tag{39}$$

where ϵ_t^{rp} denotes the risk premium shock and variables without a time subscript take steady-state values. Aggregating the household's budget constraint using the market clearing conditions, the government's budget constraint, and aggregate profits, we obtain the law of motion for net foreign assets:

$$e_t(r_t d_t - d_{t+1}) = nx_t \tag{40}$$

where nx_t are total net exports defined as:

$$nx_t = p_{d,t}y_{d,t}^* - p_{f,t}y_{f,t} \tag{41}$$

In turn, real GDP is defined as:

$$gdp_t = y_t + nx_t \tag{42}$$

3.9 Market clearing conditions

Final good

$$y_t = t^l c_t^l + t^h c_t^h + s_t^c y_t + i_t + \kappa^l v_t^l + \kappa^h v_t^h + b(z_t)t^h n_t^{h,l} \tag{43}$$

where s_t^c is the output share of government consumption G_t^c. We define t^l and t^h to be the shares of low and high-skilled households in the population, respectively, i.e. $t^l \equiv \frac{N_t^l}{N_t^l+N_t^h}$ and $t^h \equiv \frac{N_t^h}{N_t^l+N_t^h}$.

Intermediate domestic good

$$y_{i,t} = \frac{1}{N_t}\sum_{f=1}^{N_t^f} y_{i,t}^f = \frac{N_t^f}{N_t} y_{i,t}^f = t^h y_{i,t}^f \tag{44}$$

[13] $U_t^l = \sum_{l=1}^{N_t^l} u_t^l, U_t^h = \sum_{h=1}^{N_t^h} u_t^h, G_t^{t,l} = \sum_{l=1}^{N_t^l} \bar{g}_t^{t,l}, G_t^{t,h} = \sum_{h=1}^{N_t^h} \bar{g}_t^{t,h}$.

Capital, investment and foreign assets (in per capita terms):

$$\sum_{h=1}^{N_t^h} k_t^h = \sum_{f=1}^{N_t^f} k_t^f \quad , \quad k_t = \frac{N_t^h}{N_t} k_t^h = t^h k_t^h \tag{45}$$

$$i_t = \frac{1}{N_t} \sum_{h=1}^{N_t^h} i_t^h = \frac{N_t^h}{N_t} i_t^h = t^h i_t^h \tag{46}$$

$$d_t = \frac{1}{N_t} \sum_{h=1}^{N_t^h} d_t^h = \frac{N_t^h}{N_t} d_t^h = t^h d_t^h \tag{47}$$

Skill-specific unemployed ($j = l, h$)

$$\frac{U_t^j}{N_t} = \frac{1}{N_t} \sum_{j=1}^{N_t^j} u_t^j = t^j u_t^j \tag{48}$$

Skill-specific vacancies ($j = l, h$)

$$v_t^j = \frac{1}{N_t} \sum_{f=1}^{N_t^f} v_t^{j,f} = \frac{1}{N_t} V_t^j = \frac{N_t^f}{N_t} v_t^{j,f} \tag{49}$$

4 Calibration

In this section, we discuss our parameterization. We calibrate the model at an annual frequency (in line with Section 2) to match salient features of the Greek economy at the onset of the Global Financial Crisis around 2008. We present the key parameters of our model in Table V and selected targeted steady-state values in Table VI. Online Appendices M and N report the set of equations in the Decentralized Competitive Equilibrium and the Steady State Equilibrium, respectively. Online Appendix O provides all the details about the calibration strategy.

For conventional parameters, we follow the literature (see, e.g., Oikonomou (2023)). For less conventional parameters, we target related moments of the Greek economy. To match the model to the data, we define output in our model y as the difference between real Gross Domestic Product and net exports (see eq. (D.48)). Following usual practice (e.g., Kehoe and Prescott (2002); Conesa et al. (2007)), we define investment in the model as total investment (gross fixed capital formation) in the data. We match government consumption in the model with the series of government consumption (final consumption expenditure of general government) in the data. Private consumption is then defined residually as the difference between total and government consumption.

Households. Using data on population by educational attainment from Eurostat (2021), we set the population weights of the two households, t^l and t^h, to be 0.69 and 0.31, respectively. Household-specific unemployment rates are calibrated using Eurostat (2021) data on the unemployment rates by educational attainment level, namely tertiary and non-tertiary education levels. Thus, we set u^l and u^h equal to 0.12 and 0.07 for high and low-skilled households, respectively. Using data on employment by educational attainment, we solve for the household-specific employment rates in the economy, $n^l = 0.49$ and $n^h = 0.62$. As expected, employment rates are higher for the high-skilled than the low-skilled households. Through the household composition equations (eqs. (S.11) and (S.24)), we can then pin down the fractions of the non-active members as $l^l = 0.39$ and $l^h = 0.12$. This suggests that labor market non-participants represent a higher fraction for the low-skilled household than the high-skilled household. For the subjective discount factor, we use eq. (S.14) and calibrate $\beta = 0.96$ to match a 3.72% real interest rate. Furthermore, we calibrate the inverse of the Frisch elasticity of labor supply for high and low-skilled workers to 1.5. For the inverse elasticity of the intertemporal substitution η, much of the literature uses econometric estimates between 0 and 2 (see, e.g., Hansen and Singleton (1983)); we thus set it equal to 2. We calibrate the inverse elasticity of the intertemporal substitution of low-skilled households Φ^l to 0.44 using eq. (S.9). The respective inverse elasticity for the high-skilled household Φ^h is calibrated to 0.01 using eq. (S.17) indicating that low-skilled workers receive higher utility from leisure in comparison to their high-skilled counterparts. Finally, we calibrate the value of the depreciation rate equal to 0.05, using eq. (S.21), and the ratios of aggregate investment to output and of aggregate capital stock to output based on the data as follows, i.e. $(i/y) = 0.19$ and $(k/y) = 3.95$.

Labor market. We set σ^h and σ^l to 0.06 and 0.10, respectively, which suggests that jobs are destructed more easily for low-skilled workers than high-skilled workers. The latter is close to values found for total employment destruction rates in Hobijn and Şahin (2009). We set the filling rates of low-skill vacancies to $\psi_F^{l,l} = 0.60$ and the hiring probabilities $\psi_H^{h,h}$ and $\psi_H^{h,l}$ to 0.14 and 0.88 respectively, indicating that a high-skilled searcher is more likely to find a low-skill, rather than a high-skill, position. We use eqs. (S.22) and (S.23) to calibrate $\phi(z)$ and the share of searchers for a high-skill position s. Then, using eq. (S.10), we calibrate $\psi_H^{l,l}$ to 0.41. We find matches $m^{l,l}$, $m^{h,l}$ and $m^{h,h}$ from eqs. (S.1)-(S.3). We calculate $\psi_F^{h,l}$ and the product xv^l by solving a system of two eqs. (S.5) and (S.30). A solution to v^l is given by eq. (S.4), so that $v^l = 0.09$, and thus we calculate x from $x = xv^l/v^l$. We use eqs. (S.25) and (S.26) to solve for the matching efficiency parameter μ_1 and matching elasticity parameter μ_2, values commonly set in the literature (e.g., Petrongolo and Pissarides (2001), Oikonomou (2023) for Greece). We solve for $\psi_F^{h,h}$ and v^h using eqs. (S.6) and (S.27). Using the same approach as for v^l yields the per capita vacancies requiring low skills, $v^h = 0.01$.

Using the resource constraint (eq. (S.44)), data on the private consumption to output ratio ($c^p/y = 0.59$), the public consumption to output ratio ($s^c = c^g/y = 0.2$) as derived endogenously from the steady-state system of equations, and the aggregate investment to output ratio ($i/y = 0.19$), and by setting $\kappa^h = 0.1$ the resulting value for total vacancy costs to output ratio is 1%.[14] Using eq. (S.31), we calibrate the efficiency of mismatched workers q^h to 1.08, indicating that mismatched workers are more productive than their low-skilled counterparts in low-skill type occupations, by 8%. Using data on the average annual compensation per employee and the per educational attainment level from the "Survey on the structure and distribution of wages in firms (2006)", we obtain the wage premia of high-skilled versus low-skilled workers, $w^{h,h}/w^l = 1.47$ and of mismatched versus low-skilled workers, $w^{h,l}/w^l = 1.05$. These premia are broadly in line with Figure A.2, panel b, in the Online Appendix. We then use these wage ratios and eq. (S.35) to find the three wages $w^{h,h}, w^{h,l}, w^l$ and to calibrate $\kappa^l = 0.13$. The latter value indicates that it is more costly to post a vacancy for a low-skill position than a high-skill position, which is similar in Oikonomou (2023). We calibrate the firms' bargaining power parameters, $\theta^{h,h}$,

[14]The value is close to the range reported in 1997 National Employer Survey, which shows that 2% – 3% of GDP is dedicated to recruiting (https://census.gov/econ/overview/mu2400.html).

$\theta^{h,l}$ and θ^{l}, to 0.03, 0.26 and 0.26 to satisfy eqs. (S.28), (S.29) and (S.30), respectively. Finally, for the cost and efficacy of the on-the-job search we adopt the following functional forms: $b(z_t) = b_1(z_t)^{b_2}$ and $\phi(z_t) = \phi_1(z_t)^{\phi_2}$, respectively. We normalize the efficacy parameter ϕ_1 to 1 and we set the cost parameter b_2 to 2 for a simple quadratic specification. Then, by using equations (S.19), (S.46) and (S.47), we solve for $\phi_2 = 1.12$, $b_1 = 0.16$ and for the on-the-job search effort to end mismatch, $z = 1.33$.

Production. We set the elasticity of substitution between h-labor and capital ρ to 0.67 as in Krusell et al. (2000). We set the weight attached to low-skilled labor $\alpha = 0.36$, close to Oikonomou (2023), and the elasticity of substitution between low-skilled labor, capital and high-skilled labor $\epsilon = 1.12$. Capital-skill complementarity (CSC) requires that $\rho < \epsilon$, which holds here. We also compare impulse responses for an alternative calibration with $\rho = \epsilon = 1.12$ (no CSC case) in Section 5.3. By targeting y^{d^*}/y, y^{f^*}/y and d/y, we calibrate the home bias parameter $\omega = 0.86$, and the elasticity of substitution between home-produced and imported goods $\gamma = 3.91$.

Using eq. (S.42) and data on the imports to output ratio (y^f/y=0.25), we calibrate the price of imported goods $p^f = 0.86$. We normalize the price level P to 1 and use eq. (S.40) to calibrate the price of domestic goods $p^d = 1.03$. We normalize total factor productivity in eq. (S.31) to one, i.e. $A = 1$. Furthermore, using the production function, eq. (S.39), we pin down the ratio of the intermediate good distributed domestically to output, $y^d/y = 0.75$. Then, using eq. (S.41) and data on the exports to output ratio, $y^{d^*}/y = 0.21$, we pin down the ratio between the intermediate good firm output and the economy wide output, $y^i/y = 0.97$. Using eq. (S.45), we find the exchange rate to be $e = 0.86$. Then, using the definition of $e \equiv P^*/P$, we find the foreign general price level equal to $P^* = 0.86$. Finally, using the production function, we solve for output, $y = 2.08$, which pins down y^* in eq. (S.41) equal to 6.78.

Government. We set the labor income tax rate τ equal to the effective tax rate of total income in Greece, which is 0.3 (Christou et al. (2021)). We set the unemployment benefit to 0.55 of the low-skill income. Using eqs (S.17) and (S.18), we solve a system for two unknowns, Φ^h and s^c. Finally, using the government budget constraint (eq. (S.43)), we pin down government transfers.

Shocks. Finally, we set the parameter for the persistence of the shocks equal to 0.95 and the corresponding standard errors equal to 0.01, targeting a 1% deviation from the steady state.

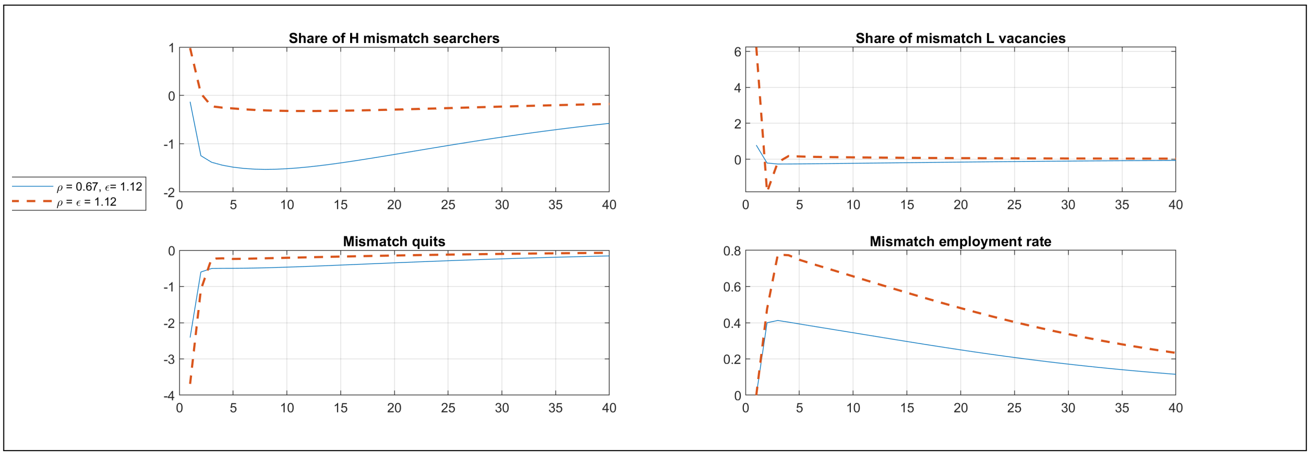
Table V: Parameterization

A. Set from data		Value	Source
t^l, t^h	Population weights of households	0.69, 0.31	Eurostat (2021)
ϵ	EoS (l-labor, capital and h-labor)	1.12	$k/y = 3.95$, $y^f/y = 0.25$, $y^{d^\star}/y = 0.21$, $d/y = 0.1$
B. Derived from steady-state equations		Value	Rationale
β	Discount factor	0.96	3.72% annual interest rate
δ	Depreciation rate	0.05	$i/y = 0.19$, $k/y = 3.95$
$\omega, \omega^\star$	Home Bias	0,86	eq. (S.45), 3.72% annual interest rate
$\gamma, \gamma^\star$	EoS (Home-produced, imported goods)	3.91	$k/y = 3.95$, $y^f/y = 0.25$, $y^{d^\star}/y = 0.21$, $d/y = 0.1$
ζ	Weight attached to capital	0.94	$k/y = 3.95$, $y^f/y = 0.25$, $y^{d^\star}/y = 0.21$, $d/y = 0.1$
s^c	Government spending share in output	0.20	(S.15),(S.16),(S.17),(S.18),(S.20), (S.43), $c^p/y = 0.59$
q^h	Efficiency of mismatched workers	1.08	eq. (S.35)
b_1	On-the-job search cost	0.16	eqs. (S.19) & (S.46)
ϕ_2	Efficiency of on-the-job search	1.12	(S.47)
κ^l, κ^h	Vacancy costs	0.13, 0.1	$\frac{w^h}{w^l}$=1.25 and $(\kappa^l v^l + \kappa^h v^h)/y = 0.01$
μ_1	Matching efficiency	0.56	eqs. (S.25) & (S.26)
μ_2	Matching elasticity	0.80	eqs. (S.25) & (S.26)
Φ^l	Relative disutility for labor (l)	0.44	eq.(S.9)
Φ^h	Relative disutility for labor (h)	0.01	(S.15),(S.16),(S.17),(S.18),(S.20), (S.43), $c^p/y = 0.59$
$\theta^{h,h}, \theta^{h,l}, \theta^{l,l}$	Firms' bargaining power	0.03, 0.26, 0.26	eqs. (S.28),(S.29),(S.30)
C. Set according to the literature		Value	Source
ρ	EoS (h-labor and capital)	0.67	Krusell et al. (2000)
α	Weight attached to l-labor	0.36	Common value in the literature
ϕ	Inverse Frisch elasticity	1.5	Common value in the literature
η	Inverse elasticity of intertemporal substitution	2	Hansen and Singleton (1983)
$\bar{\omega}$	Unemployment benefits	0.85	55% of low-skill income
σ^l, σ^h	Job destruction rates	0.10, 0.06	Close to Hobijn and Şahin (2009)
τ	General tax rate	0.30	Christou et al. (2021)
r^*	World interest rate	0.04	Common value in the literature
Ξ	Capital adjustment costs	4.00	Dolado et al. (2021)
D. Other		Value	Rationale
ϕ_1	Efficacy of on-the-job search	1	Normalization
b_2	On-the-job search cost	2	Quadratic cost

Table VI: Steady-state variables

Variable	**Description**	**Value**
y	Output	2.08
y_d^*/y	Exports to output ratio	0.21
y_d	Domestic demand	1.58
y_f/y	Imports to output ratio	0.25
c^l, c^h	Consumption: low- and high-skilled	0.75, 2.29
i/y	Investment to output ratio	0.19
k/y	Capital to output ratio	3.95
d/y	Net foreign assets to output ratio	0.10
$n^{l,l}$, $n^{h,h}$	Employment rates: low-skill and high-skill jobs	0.49, 0.62
$n^{h,l}$	Employment rate: mismatch jobs	0.19
l^l, l^h	Non-participants: low- and high-skilled	0.39, 0.12
u^l, u^h	Unemployment rates: low- and high-skilled	0.12, 0.07
r_k	Return on capital	0.12
$w^{l,l}$, $w^{h,h}$	Low- and high-skill wages	1.55, 2.28
$w^{h,l}$	Mismatch wage	1.63
$m^{l,l}$, $m^{h,h}$	Low- and high-skill matches	0.034, 0.011
$m^{h,l}$	Mismatches	0.017
v^l, v^h	Low- and high-skill vacancies	0.09, 0.01
$\psi_H^{l,l}$, $\psi_H^{h,h}$	Low- and high-skill hiring probabilities	0.41, 0.14
$\psi_H^{h,l}$	Mismatch hiring probability	0.88
$\psi_F^{l,l}$, $\psi_F^{h,h}$	Low- and high-skill vacancy-filling probabilities	0.60, 0.78
$\psi_F^{h,l}$	Mismatch vacancy-filling probability	0.50
p_d	Domestic good price	1.03
r_d	Rate for foreign assets	0.037
s^t	Output share of government transfers	0.05
$exch$	Exchange rate	0.86
z	On the-job search effort to end mismatch	1.33
$b(z)$	Cost function of on the-job search	0.28
$\phi(z)$	Efficacy function of on the-job search	1.37
x	Fraction of low-skill positions given to high-skilled	0.37
$1-s$	Fraction of high-skilled searching for mismatch	0.89

Figure 6: Responses of mismatch variables to a 1% positive income tax shock



Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. The horizontal axis depicts years. The mismatch employment rate refers to the share of mismatch employees in the total number of the high-skilled household's employed members, $n_t^{h,l}/(n_t^{h,l}+n_t^{h,h})$*. H and L refer to high and low skills, respectively.*

5 Fiscal shocks and skills mismatch

In this section, we investigate how skills mismatch reacts to fiscal tightening shocks, namely a positive shock to the income tax rate and a negative shock to government spending. We present impulse responses from our DSGE model for selected variables under our baseline calibration where the production function exhibits CSC (see the continued lines in the Figures that follow). Online Appendix P includes responses for additional variables.

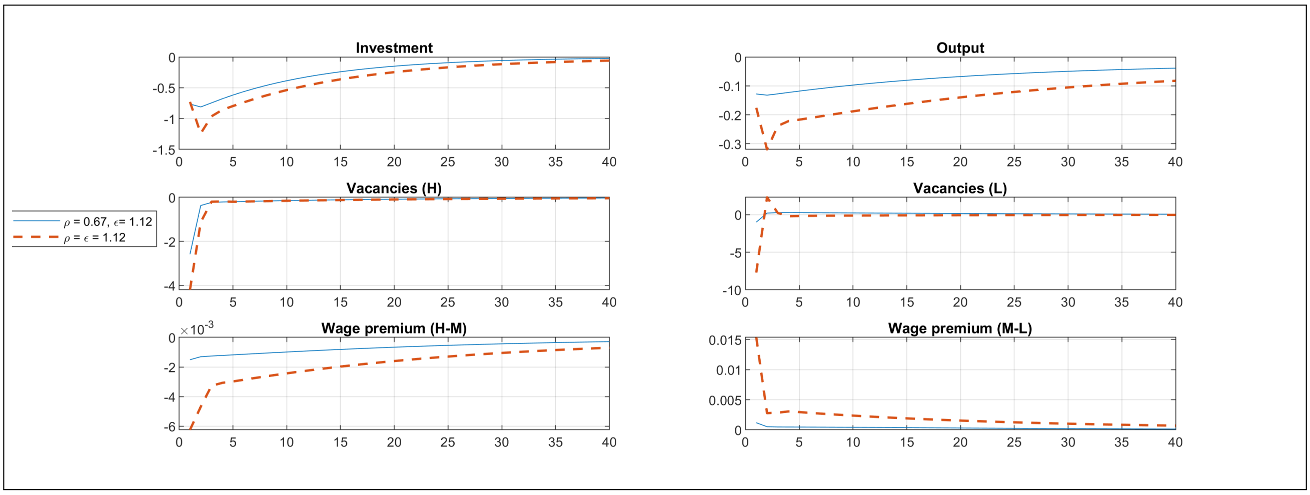
5.1 Tax shock

In line with the empirical evidence in Section 2, Figure 6 shows that the mismatch employment rate rises persistently following an unexpected increase in the income tax rate. This happens because (a) firms increase on impact the share of low-skill vacancies going to high-skilled workers and (b) quits from mismatch jobs to take up an upgraded position decrease sharply, following the decline in high-skill vacancies (see Figure 7). Notice also in Figure 6 that the high-skilled household decreases the share of its unemployed members searching for a mismatch job. However, this effect is not strong enough to counteract (a) and (b) above, and therefore we see a persistent increase in the mismatch rate.[15]

The tax hike leads to a persistent fall in investment and output (see Figure 7). The recession induced by the distortionary effects of the tax shock causes a drop in job openings, with the effect being stronger for vacancies requiring high skills. In turn, high-skilled employment declines persistently, while the fall in low-skilled employment is more short-lived (see Figure A.7 in the Online Appendix). Accordingly, the wage premium of the high-skilled non-mismatched employees versus their mismatched counterparts declines persistently, while

[15]Mismatch employment and the associated wage also increase (see Figure A.7 in the Online Appendix).

Figure 7: Responses of investment, output and vacancies to a 1% positive income tax shock



Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively. M refers to mismatch.

the wage premium of mismatched workers versus their low-skilled counterparts rises.

Figure 8 shows that consumption rises and labor market participation falls for the low-skilled household as a result of the negative incentive to work and the higher lump-sum transfers under a balanced government budget. For the high-skilled household, who are also affected by the drop in their investment income, consumption shows the opposite response and persistently declines. Participation of high-skilled households declines as well, but the effect is substantially smaller compared to low-skilled households.

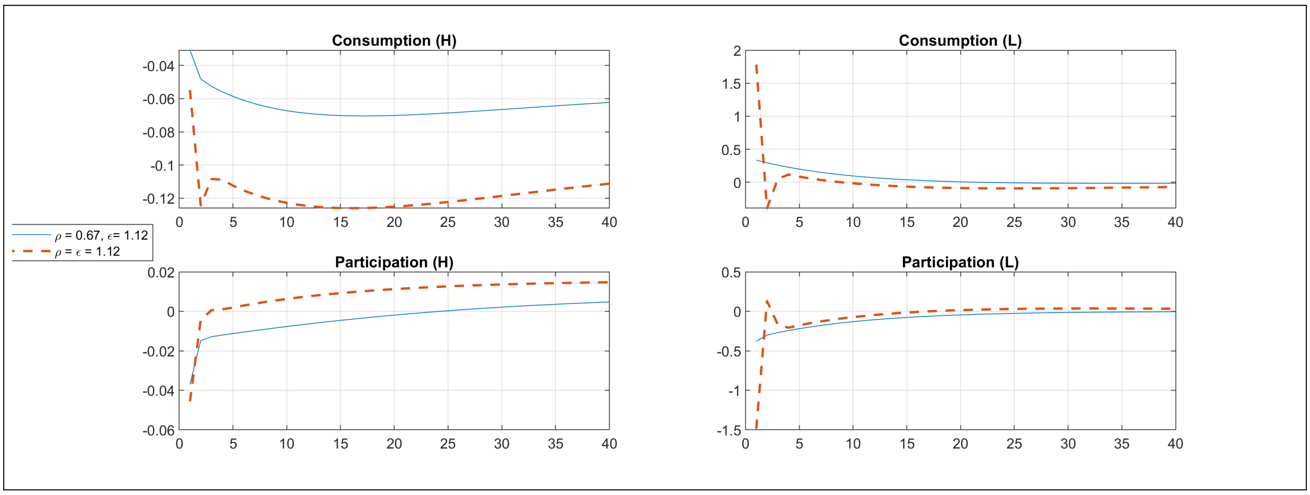
5.2 Government spending shock (negative)

Let us start again with the results from the baseline calibration with CSC in the production function. In line with the data, Figure 9 shows that the mismatch employment rate rises persistently following a cut in wasteful government spending.[16] This result is driven by the following: (a) quits from mismatch jobs decrease sharply, (b) firms increase the share of low-skill vacancies that target high-skilled workers, and (c) the high-skilled household increases the share of unemployed searching for a low-skill (mismatch) job.

The fiscal contraction leads to a crowding-out of investment, a persistent decrease in output, and a reduction in hirings due to the demand loss (see the resource constraint in eq.(43)). To understand (a), notice in Figure 10 that high-skill vacancies decrease, thereby reducing the chances for mismatch employees of getting an upgraded job via on-the-job search. Firms reduce even more strongly vacancies requiring low skills, which leads to a substantial exit of low-skilled workers from the labor force (see Figure 11). To understand (b) and (c), notice in Figure 10 that the wage premium of high-skilled non-mismatched employees versus their mismatched counterparts declines persistently, while the wage premium of mismatched workers versus their low-skilled

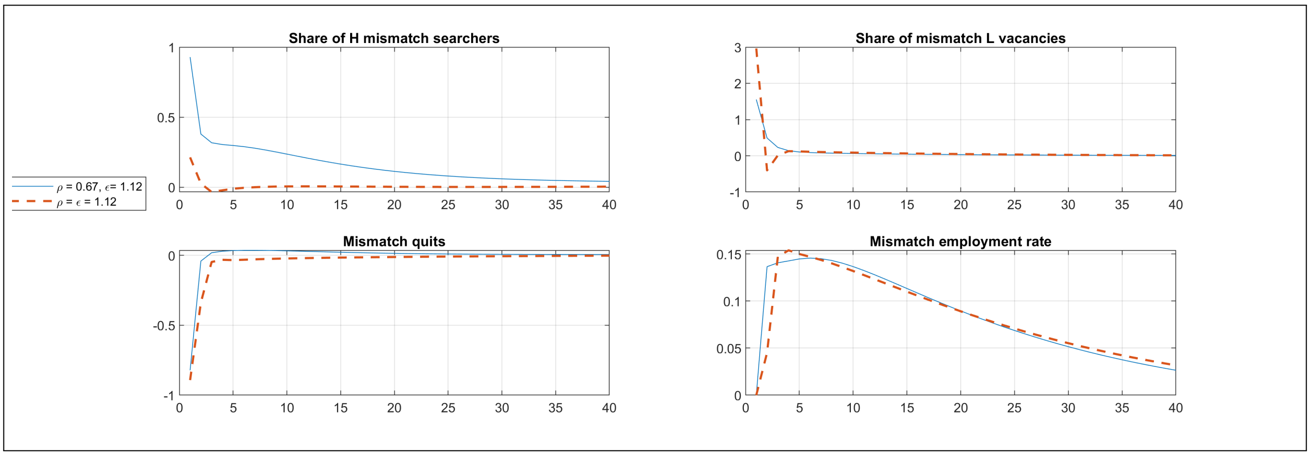
[16] Mismatch employment and the associated wage increase (see Figure A.7 in the Online Appendix).

Figure 8: Responses of consumption and participation to a 1% positive income tax shock



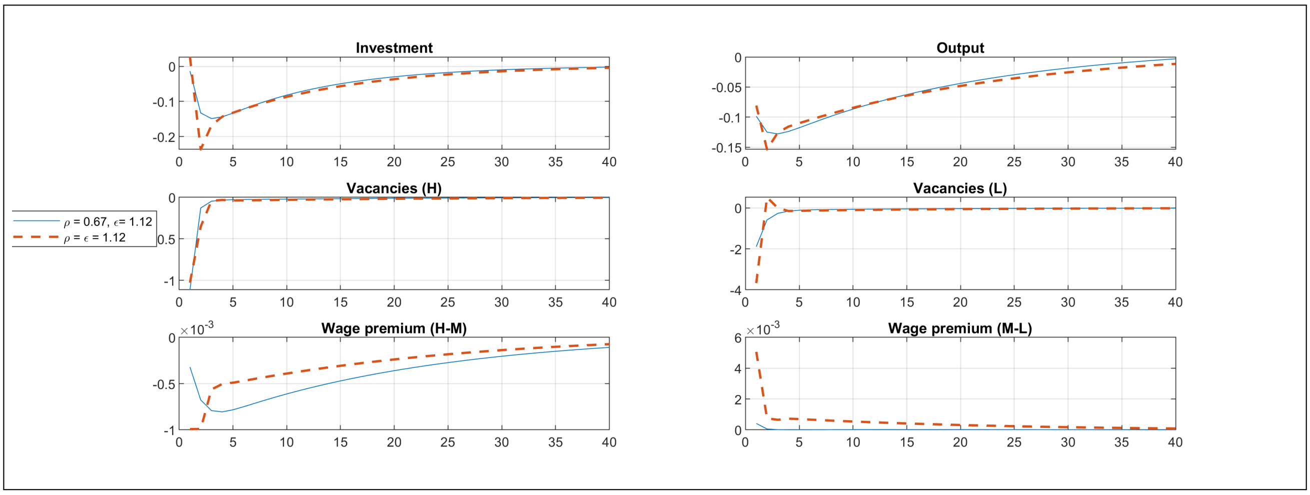
Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively.

Figure 9: Responses of mismatch variables to a 1% negative government spending shock



Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively. The mismatch employment rate refers to the share of mismatch employees in the total number of the high-skilled household's employed members, $n_t^{h,l}/(n_t^{h,l} + n_t^{h,h})$.

Figure 10: Responses of investment, output and vacancies to a 1% negative government spending shock



Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively. M refers to mismatch.

counterparts rises.

For both household types, Figure 11 shows that consumption rises and labor market participation falls due to the positive wealth effect from the spending cut, i.e., households (expect to) receive increased lump-sum transfers. The effects are much more pronounced for the low-skilled household (hand-to-mouth agent) since the high-skilled household faces also a decrease in income from capital and net foreign assets (see Figure A.6 in the Online Appendix). The drop of both labor demand (vacancies) and labor supply (participation) induces the negative responses of high-skilled and low-skilled employment, with the effect again being stronger for the latter (see Figure A.7 in the Online Appendix).

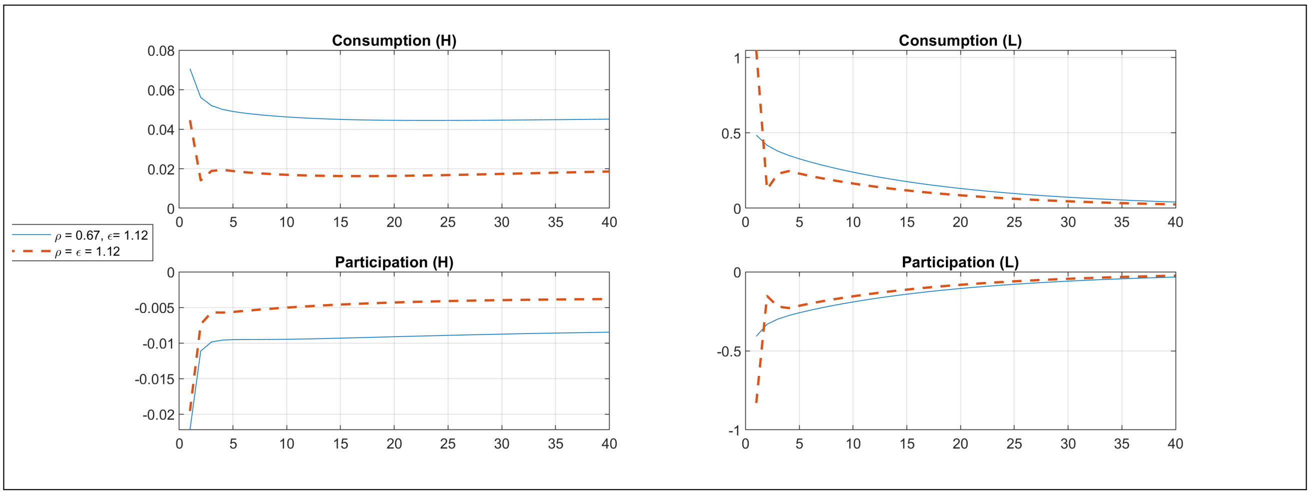
5.3 Sensitivity analysis

Next, we compare impulse responses from our DSGE model for selected variables under our baseline calibration against an alternative calibration (see the dashed lines in the Figures that follow) without CSC in the production process ($\rho = \epsilon$).The goal is to test the sensitivity of our main finding, which is that the mismatch employment rate increases both after a tax hike and a spending cut, to an alternative specification of the production function.

Tax shock. Without CSC, the output contraction is stronger due to the reinforced fall in investment compared to the baseline calibration (see Figure 7). Accordingly, the drop in vacancies is magnified for both skill types. As Figure 6 shows, in this case the mismatch employment rate still rises and significantly more than in the baseline calibration. This happens because the high-skilled household experiencing now a stronger drop in investment income, responds by increasing, rather than decreasing, the share of searchers for a mismatch job. In turn, firms allocate on impact an even higher share of low-skill vacancies to mismatched applicants. Moreover, the fall in quits from mismatch jobs is magnified on impact.

The intuition is as follows. When capital and high-skilled labor are complements in production, as in the

Figure 11: Responses of consumption and participation to a 1% negative government spending shock



Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively.

baseline calibration, it is more difficult for firms to substitute high-skilled labor with capital, which affects their decision about the creation of high-skill jobs. Therefore, the demand loss from fiscal tightening causes a weaker decrease in high-skill vacancies and thus a weaker rise in mismatch employment. Furthermore, the decrease in high-skilled employment is mitigated making complementary capital more productive, which in turn mitigates the fall in investment demand and output. This seems to suggest a dynamic demand-mitigation mechanism induced by the presence of CSC in the production function, which goes in opposite direction from the dynamic demand-amplification mechanism found for monetary shocks in Dolado et al. (2021).

Returning to Figure 7, we see that both the fall in the wage premium of high-skilled non-mismatched workers versus their mismatched counterparts and the rise in the wage premium of mismatched workers versus their low-skilled counterparts are reinforced relative to the baseline calibration. Finally, Figure 8 shows that, without CSC, the fall in consumption for high-skilled households is magnified persistently, while their participation now slightly rises after the third period due to the stronger investment income loss. For low-skilled households, the impact responses of consumption and participation are also magnified.

Spending shock. Without CSC, we observe in Figure 10 a sharper decrease in low-skill vacancies on impact and also in investment and output at peak (two periods after the shock). Accordingly, both the decrease in the participation of the low-skilled household and the increase in its consumption are reinforced on impact (see Figure 11). The high-skilled household experiences a smaller rise in consumption and a smaller decrease in participation, as it is affected adversely by the reinforced fall in investment. In reaction to the fact that low-skill vacancies fall more strongly, the high-skilled household increases by less the share of members searching for a mismatch job, despite the fact that firms raise more strongly on impact the share of those vacancies going to mismatch applicants. Importantly, the positive response of the mismatch rate continues to hold. As we can see in Figure A.7 in the Online Appendix, mismatch employment still rises but with a slight delay. When we take into account also the total number of the high-skilled household's employed members, which is the denominator in the mismatch employment rate, we see that the resulting variation between the baseline and the alternative calibration is smaller. Therefore, the presence of CSC in the model affects the response of the mismatch employment rate little when the government slashes wasteful spending.

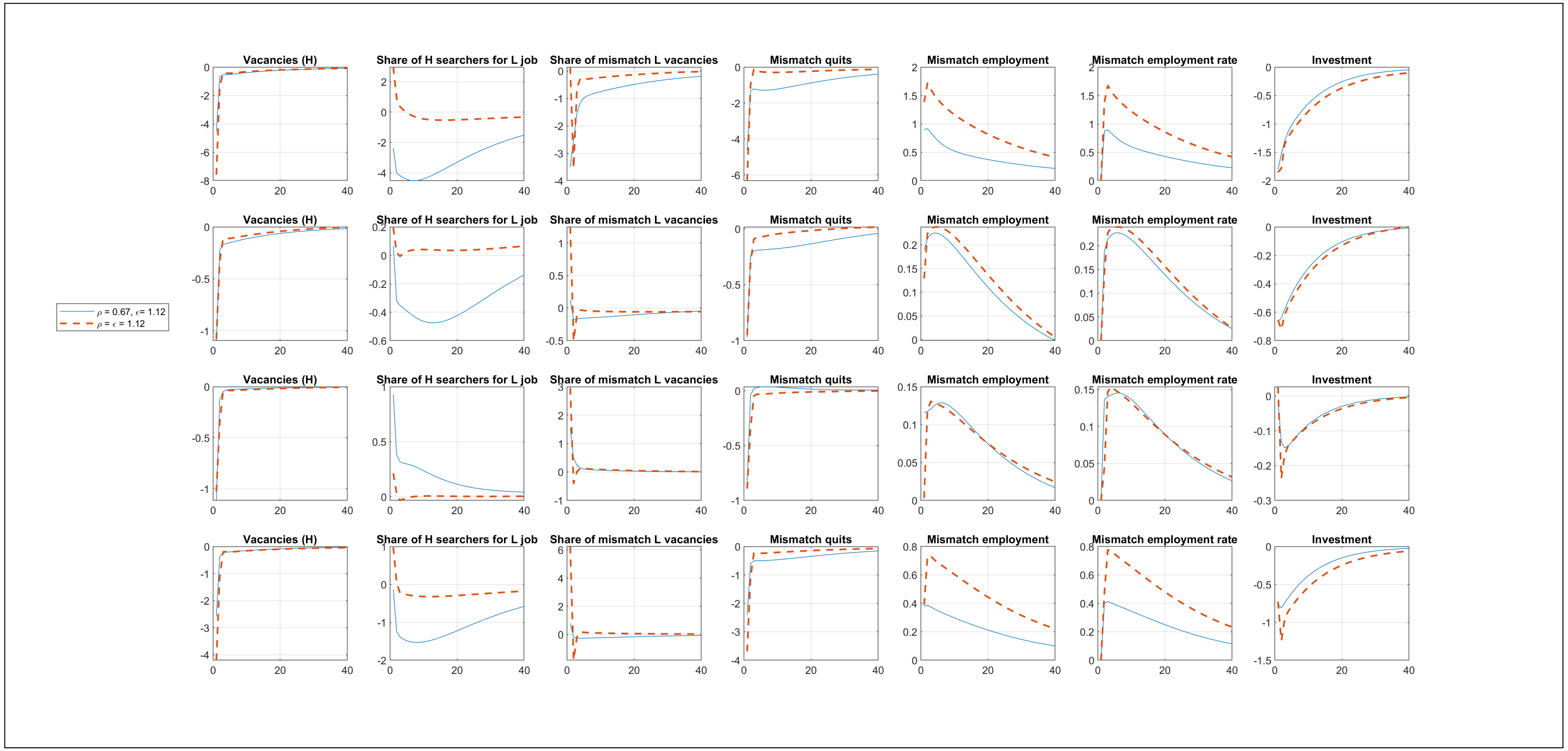
In sum, we have shown that our main finding on the positive response of the mismatch rate to a fiscal tightening shock, whether spending-based or tax-based, is robust to an alternative specification of the production function without CSC.

6 Comparison and other macro shocks

Before we explore the responses from our DSGE model to other macroeconomic shocks, let us sum up the main differences in the responses to the two previous fiscal shocks under our baseline calibration (with CSC). First, we focus on the consumption response of the high-skilled household. After a tax shock, we see that it drops, largely influenced by the income loss. By contrast, after a government spending cut, the consumption of the high-skilled household increases due to the positive wealth effect. Second, while the spending cut seems to affect relatively more the low-skilled employment, the tax shock by adversely impacting all types of income for the high-skilled household seems to affect relatively more the high-skilled employment. Finally, the differences in the responses to a tax shock with and without CSC in the production process are more pronounced that those in the responses to a negative spending shock.

To explore how the mismatch rate reacts to other standard shocks in our DSGE model and also whether the latter generates plausible impulse responses, we also examine the responses to a negative supply shock on TFP and a positive risk premium shock. Figure 12 presents the results for a set of selected variables, while we report the full set of responses in the Online Appendix. For comparison, we also show in Figure 12 the corresponding responses to the two fiscal adjustment shocks examined earlier. Both the negative TFP shock and the risk premium shock induce an investment slump and also a reduction in hirings requiring high skills, which is translated to a decline in quits from mismatch jobs since the openings of upgraded positions decrease. Mismatch employment goes up and this finding remains robust to the alternative specification of the production function (without CSC). As in the case of the tax shock, the high-skilled household reduces the share of its members who search for a mismatch job under the baseline calibration, but increases it in the absence of CSC (alternative calibration) when the recessionary effects are reinforced. A difference from the fiscal shocks studied earlier is that firms tend more easily to reduce the share of low-skill vacancies going to mismatch applicants. However, this effect does not seem strong enough to counteract the rise in mismatch employment. In sum, the mismatch rate behaves counter-cyclically in our model for the shocks under study. This finding seems to be in line with Brunello et al. (2019) who conclude that skills mismatch is counter-cyclical in Europe as skilled individuals are more willing to take up lower skill-demanded jobs when facing the unemployment threat during a recession, and this effect tends to prevail on the cleansing effect of recessions.

Figure 12: Macro shocks and vertical skills mismatch: An overview



Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively. Shocks: Row 1: TFP (negative), Row 2: Risk premium, Row 3: Government spending (negative), Row 4: Tax rate.

7 Conclusion

Recent socio-economic changes, including increased global competition, technological development and structural change, work-force education levels, and an ageing population, have led to a labor market, where the demand for skills is changing rapidly and it is difficult to match the demanded with the supplied skills (see, e.g., Vandeplas and Thum-thysen (2019)). This has important implications for productivity, growth, unemployment, and welfare. At the same time, public debt as a ratio to GDP soared across the world during COVID-19 and is expected to remain elevated, posing a challenge for policymakers. In a recent speech (June 1, 2023), the First Deputy Managing Director of the IMF, Gita Gopinath argued that "Today fiscal policy needs to be tighter to help bring inflation down and rebuild policy buffers."

The paper investigates an overlooked question: can fiscal tightening affect labor market efficiency? We document a new link between fiscal tightening and skills mismatch. The paper provides novel empirical and theoretical evidence suggesting that fiscal tightening can lead to a rise in the mismatch rate. The main policy implication of our findings is that fiscal consolidation policy makers should take into account the potential implications for skills mismatch in the labor market.

An avenue for future research may be to combine skills mismatch and emigration in the DSGE model developed here and to also investigate the empirical link between the two. On the one hand, skills mismatch is considered to be a potential factor in emigration decisions. On the other hand, we still lack an analysis of how high-skilled and low-skilled emigration affects skills mismatch in the labor market. For instance, high-skilled emigration can free up positions requiring high skills in the sending economy. At the same time, in bad times, it could exacerbate the recession by amplifying demand losses. Finally, while this paper has focused on over-qualification, the effects of fiscal policy on under-qualification still remain an open question. We leave those issues for future work.

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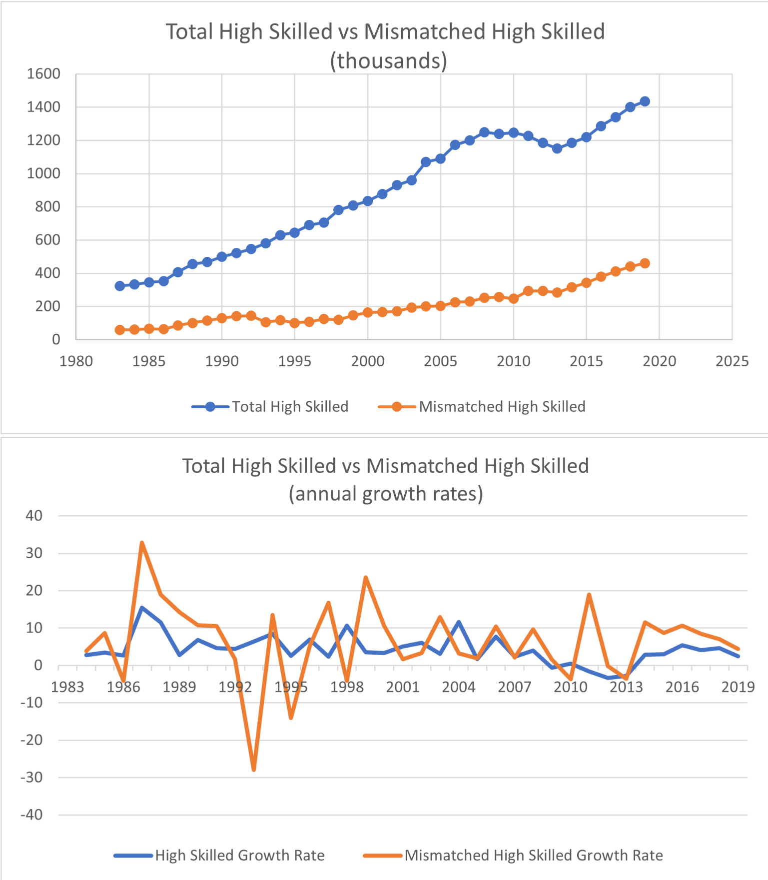
ONLINE APPENDIX

Fiscal Tightening and Skills Mismatch

Konstantinos Mavrigiannakis   Andreas Vasilatos   Eugenia Vella

Figure A.1: Nominator (mismatched employees) and denominator (total high-skilled employees) of the skills mismatch rate, 1983-2020



Notes: Authors' calculations based on micro data from the Greek Labor Force Survey of Elstat

A Decomposing the skills mismatch rate

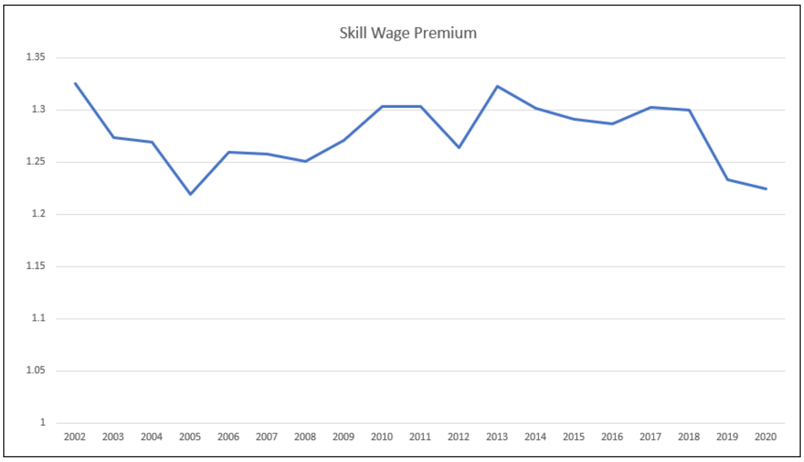
Figure A.1 shows that from 1983 to 2020 the total number of high-skilled employees (denominator of the mismatch rate) tripled, while the total number of mismatched high-skilled employees (nominator of the mismatch rate) exhibits an almost seven-fold increase.

B Skill premia for mismatched and non-mismatched workers

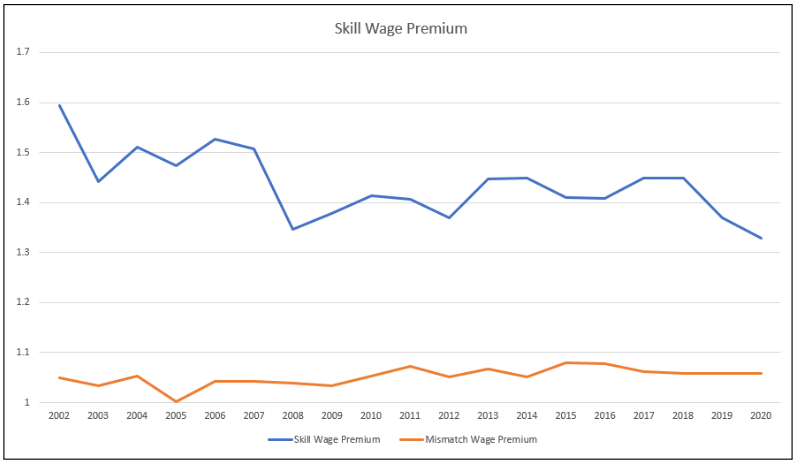
As expected, disentangling the two different skill wage premia reveals a higher premium for the non-mismatched workers relative to the case where mismatched and non-mismatched workers are aggregated (see Figure A.2).

C Bailout programs in Greece

Figure A.2: Evolution of the skill wage premium in Greece



(a) mismatched workers are included in the calculation of the mean high-skilled wage



(b) mismatched workers are excluded from the high-skilled

Notes: Authors' calculations based on micro data from the Greek Labor Force Survey

Table A.1: Economic adjustment programs during the debt crisis in Greece

1st Economic Adjustment Programme

Agreed: 2 May 2010
PM: George Papandreou, PASOK
Tenure: May 2010 – June 2013
Size: *110 bn
Contributors: GLF (Bilateral Loans pooled from the EA) (€80bn), IMF (€30bn)
Objectives:

- Implementation of sustainability-enhancing fiscal consolidation;
- implementation of financial sector policies to stabilize the system;
- Reforms in the economy's structure towards a more investment -and export-led growth model;
- Restore Greece's credibility for private investors.

Status*: €72.8bn were disbursed by March 2012, when the programme was superseded by the 2nd Economic Adjustment Programme.*

**This amount was eventually reduced by €2.7 billion, as Slovakia decided not to participate in the GLF while Ireland and Portugal stepped down from the facility as they requested financial assistance themselves.*

2nd Economic Adjustment Programme

Agreed: 1 March 2012 (PM: Lucas Papademos, ND, PASOK)
Tenure: March 2012-end 2014
Size: €130bn + €34.5bn (undisbursed amounts of the GLF)
Contributors: EFSF (€144.7bn), IMF (€19.8bn until 2016)
Objectives:

- Restructuring debt held by private creditors to bring the total debt level back to a sustainable path;
- Underpin fiscal consolidation efforts with structural reforms, to boost growth, and improve competitiveness.

Status*: The Programme was extended to 30 June 2015 and expired. The Greek government failed to repay about €1.5bn to the IMF. A three-week bank holiday took place.*

Total amount disbursed: €153.7bn *(On 27 February 2015, the €10.9bn that were earmarked but not needed for bank recapitalisation were returned by the Hellenic Financial Stability Fund (HFSF) to the EFSF)*

3rd Economic Adjustment Programme

Agreed: 19 August 2015 (PM: Alexis Tsipras, SYRIZA/ANEL)
Tenure: August 2015- August 2018
Amount: up to €86bn *
Contributors: ESM
Objectives:

- Restoring fiscal sustainability;
- Safeguarding financial stability; boosting growth, competitiveness and investment, and;
- Reforming the public administration.

Status: *Greece successfully concluded the Programme*

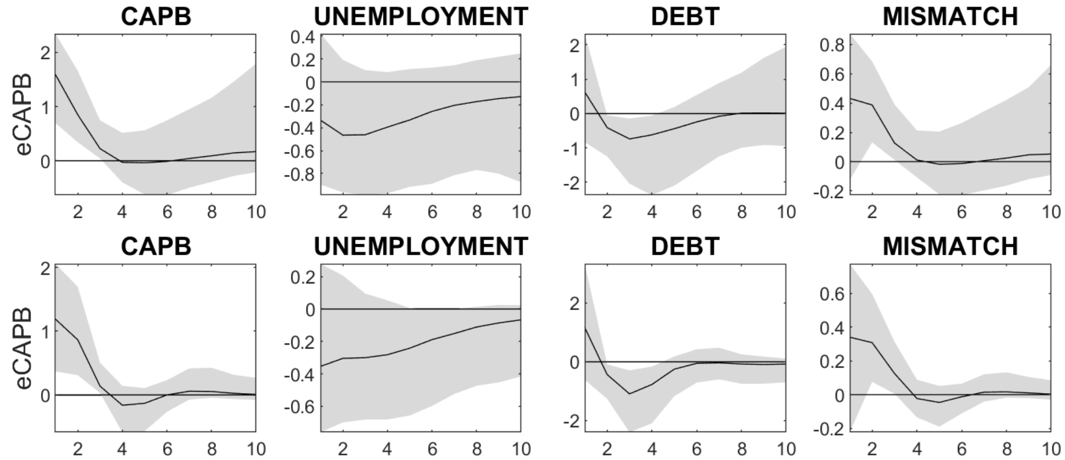
**€ 61.9 bn were disbursed*

Source: Pagoulatos (2018)

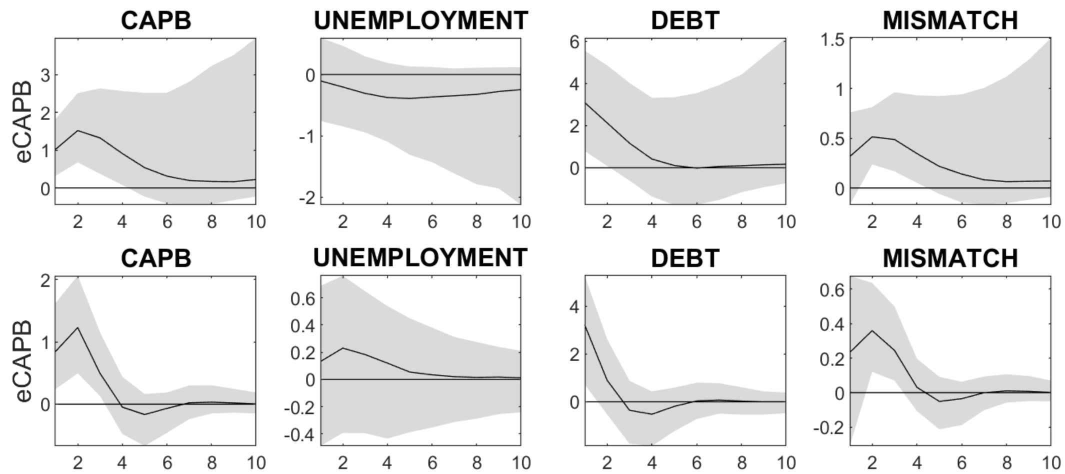
D VARs with the unemployment rate

Figure A.3: Responses to a one standard deviation shock in CAPB

(a) Cholesky (row 1: linear trend, row 2: linear and quadratic trend)



(b) Sign restrictions of Pappa et al. (2015) (row 1: linear trend, row 2: linear and quadratic trend)

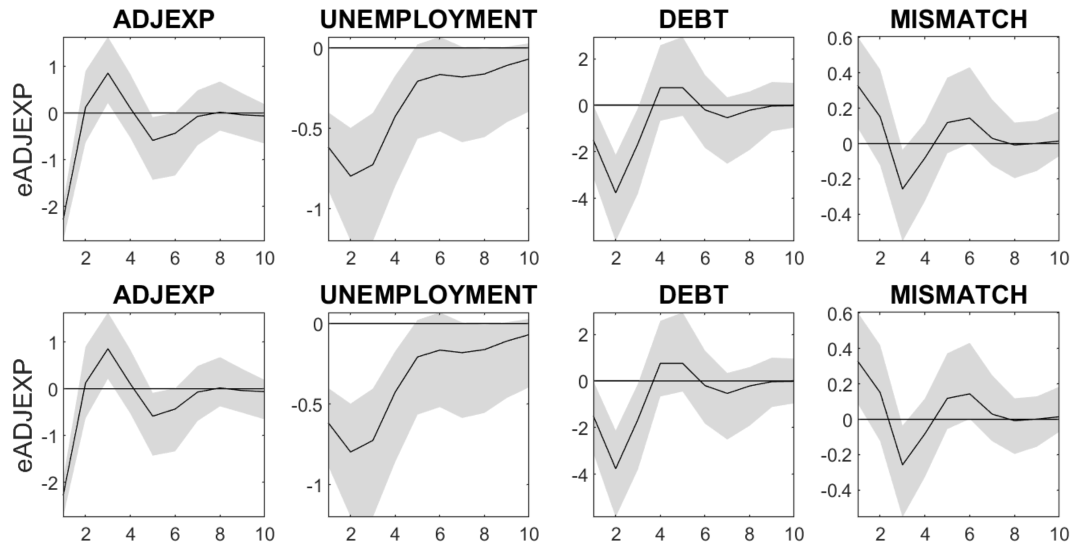


(c) Sign restrictions of Pappa et al. (2015) but with unrestricted debt ratio

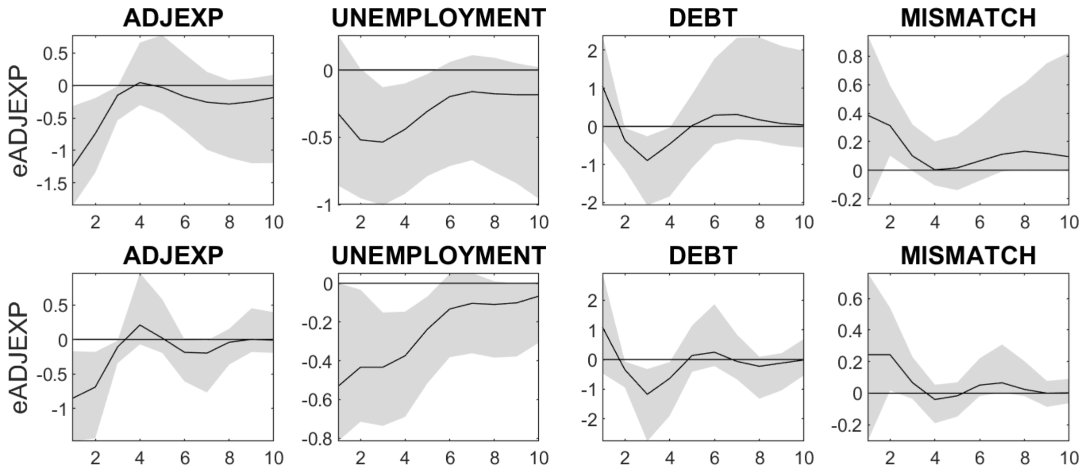
(row 1: linear trend, row 2: linear and quadratic trend)

Note: See Figure 3.

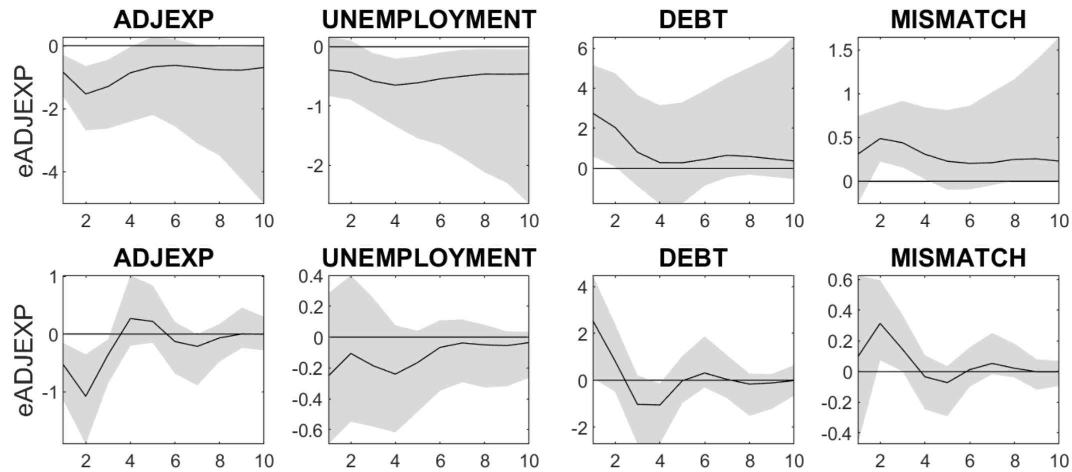
Figure A.4: Responses to a one standard deviation negative shock in cyclically adjusted primary expenses



(a) Cholesky (row 1: linear trend, row 2: linear and quadratic trend)



(b) Sign restrictions of Pappa et al. (2015) (row 1: linear trend, row 2: linear and quadratic trend)

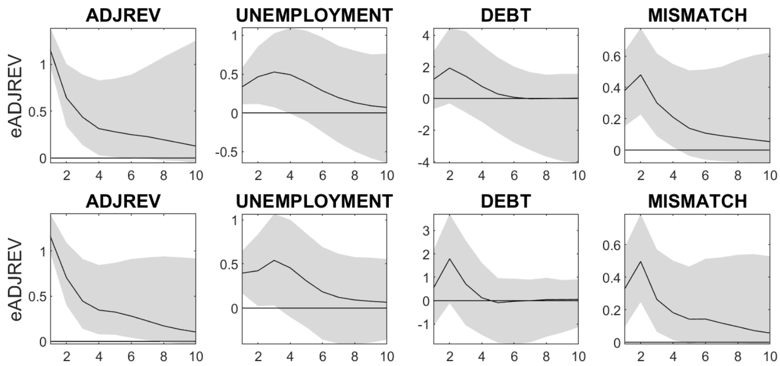


(c) Sign restrictions of Pappa et al. (2015) but with unrestricted debt ratio

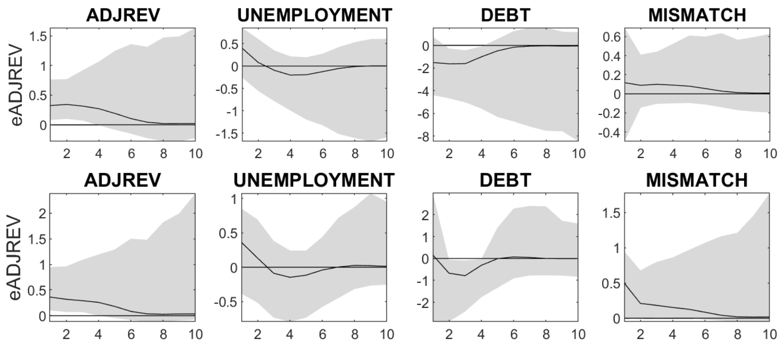
(row 1: linear trend, row 2: linear and quadratic trend)

Note: See Figure 3.

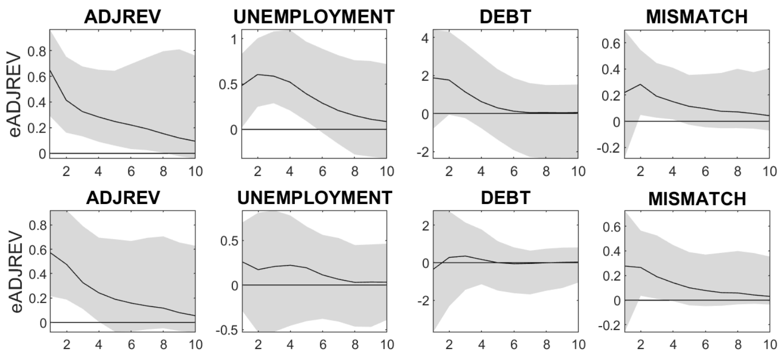
Figure A.5: Responses to a one standard deviation shock in the cyclically adjusted revenues



(a) Cholesky (row 1: linear trend, row 2: linear and quadratic trend)

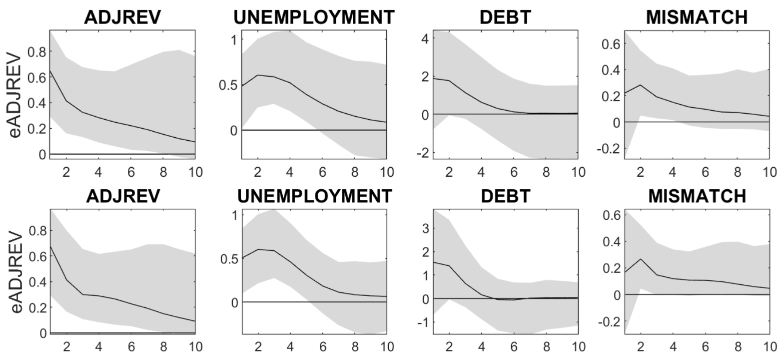


(b) Sign restrictions of Pappa et al. (2015) (row 1: linear trend, row 2: linear and quadratic trend)



(c) Sign restrictions of Pappa et al. (2015) but with unrestricted debt ratio

(row 1: linear trend, row 2: linear and quadratic trend)



(d) Sign restrictions on the unemployment rate with unrestricted debt ratio

(row 1: linear trend, row 2: linear and quadratic trend)

Note: See Figure 3.

E Household's optimization problems

E.1 The optimization problem of the representative low-skilled household

After replacing l_t^l in the utility function from equation (1), the Lagrangian is given by:

$$\mathcal{L}_t^l = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\frac{(c_t^l)^{1-\eta}}{1-\eta} + \Phi^l \frac{(1 - n_t^{l,l} - u_t^l)^{1-\phi}}{1-\phi} \\ - \lambda_{c_t^l}\Big(c_t^l - (1-\tau_t)(w_t^{l,l} n_t^{l,l}) - \bar{\omega} u_t^l - \bar{g}_t^{t,l}\Big) - \lambda_{n_t^{l,l}}\Big(n_{t+1}^{l,l} - (1-\sigma^l) n_t^{l,l} - \psi_{H,t}^{l,l} u_t^l\Big)\Bigg] \quad \text{(A.1)}$$

First order conditions

The first order conditions with respect to c_t^l, $n_{t+1}^{l,l}$, and u_t^l are:

$[c_t^l]$

$$\lambda_{c_t^l} = (c_t^l)^{-\eta} \quad \text{(A.2)}$$

$[n_{t+1}^{l,l}]$

$$\lambda_{n_t^{l,l}} = \beta E_t\Big[- \Phi^l (l_{t+1}^l)^{-\phi} + \lambda_{c_{t+1}^l}(1-\tau_{t+1}) w_{t+1}^{l,l} + \lambda_{n_{t+1}^{l,l}}(1-\sigma^l)\Big] \quad \text{(A.3)}$$

$[u_t^l]$

$$\Phi^l (l_t^l)^{-\phi} = \lambda_{n_t^{l,l}} \psi_{H,t}^{l,l} + \bar{\omega} \lambda_{c_t^l} \quad \text{(A.4)}$$

We define the marginal value for the household of having an additional member employed in a low skill position:

$$V_{n_t^{l,l}}^H = -\Phi^l (l_t^l)^{-\phi} + \lambda_{c_t^l}(1-\tau_t) w_t^{l,l} + \lambda_{n_t^{l,l}}(1-\sigma^l) \quad \text{(A.5)}$$

E.2 The optimization problem of the representative high-skilled household

The Lagrangian is given by:

$$\mathcal{L}_t = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\frac{(c_t^h + \psi g_t^c)^{1-\eta}}{1-\eta} + \Phi^h \frac{(1 - n_t^{h,l} - n_t^{h,h} - u_t^h)^{1-\phi}}{1-\phi} \\ - \lambda_{c_t^h}\Big(c_t^h + k_{t+1}^h - (1-\delta) k_t^h + \frac{\Xi}{2}\left(\frac{k_{t+1}^h}{k_t^h} - 1\right)^2 k_t^h + e_t(1 + r_t^d) d_t^h - e_t d_{t+1}^h + b(z_t) n_t^{h,l} - \bar{\omega} u_t^h - \bar{g}_t^{t,h} \\ - (1-\tau_t)(w_t^{h,h} n_t^{h,h} + w_t^{h,l} n_t^{h,l} + r_t^k k_t^h) + \pi_{d,t}\Big) \\ - \lambda_{n_t^{h,h}}\Big(n_{t+1}^{h,h} - (1-\sigma^h) n_t^{h,h} - \psi_{H,t}^{h,h}(s_t u_t^h + \phi(z_t) n_t^{h,l})\Big) \\ - \lambda_{n_t^{h,l}}\Big(n_{t+1}^{h,l} - (1 - \sigma^l - \phi(z_t)\psi_{H,t}^{h,h}) n_t^{h,l} - \psi_{H,t}^{h,l}(1-s_t) u_t^h\Big)\Bigg] \quad \text{(A.6)}$$

First order conditions:

The first order conditions with respect to c_t^h, k_{t+1}^h, d_{t+1}^h, $n_{t+1}^{h,h}$, $n_{t+1}^{h,l}$, u_t^h, s_t and z_t are:

[c_t^h]

$$\lambda_{c_t^h} = (c_t^h)^{-\eta} \tag{A.7}$$

[k_{t+1}^h]

$$\lambda_{c_t^h}\left(1+\Xi\left(\frac{k_{t+1}^h}{k_t^h}-1\right)\right) = \beta E_t \lambda_{c_{t+1}^h}\left(1-\delta+(1-\tau_{t+1})r_{t+1}^k+\frac{\Xi}{2}\left(\left(\frac{k_{t+2}^h}{k_{t+1}^h}\right)^2-1\right)\right) \tag{A.8}$$

[d_{t+1}^h]

$$\lambda_{c_t^h} e_t = \beta E_t \lambda_{c_{t+1}^h} e_{t+1}(1+r_{t+1}^d) \tag{A.9}$$

[$n_{t+1}^{h,h}$]

$$\lambda_{n_t^{h,h}} = \beta E_t\Big[-\Phi^h(l_{t+1}^h)^{-\phi}+\lambda_{c_{t+1}^h}(1-\tau_{t+1})w_{t+1}^{h,h}+\lambda_{n_{t+1}^{h,h}}(1-\sigma^h)\Big] \tag{D.14}$$

[$n_{t+1}^{h,l}$]

$$\begin{aligned}\lambda_{n_t^{h,l}} = \beta E_t\Big[-\Phi^h(l_{t+1}^h)^{-\phi}+\lambda_{c_{t+1}^h}\Big((1-\tau_{t+1})w_{t+1}^{h,l}-b(z_{t+1})\Big)+\lambda_{n_{t+1}^{h,l}}\Big(1-\sigma^l-\phi(z_{t+1})\psi_{H,t+1}^{h,h}\Big)\\ +\lambda_{n_{t+1}^{h,h}}\psi_{H,t+1}^{h,h}\phi(z_{t+1})\Big]\end{aligned} \tag{D.15}$$

[u_t^h]

$$\Phi^h(l_t^h)^{-\phi} = \lambda_{n_t^{h,h}}\psi_{H,t}^h s_t+\lambda_{n_t^{h,l}}\psi_{H,t}^{h,l}(1-s_t)+\bar{\omega}\lambda_{c_t^h} \tag{D.16}$$

[s_t]

$$\lambda_{n_t^{h,h}}\psi_{H,t}^h = \lambda_{n_t^{h,l}}\psi_{H,t}^{h,l} \tag{A.10}$$

[z_t]

$$\lambda_{c_t^h}\frac{b'(z_t)}{\phi'(z_t)} = \psi_{H,t}^{h,h}\Big(\lambda_{n_t^{h,h}}-\lambda_{n_t^{h,l}}\Big) \tag{A.11}$$

We define the marginal value for the household of having an additional member employed in a high-skill or in a mismatch position, respectively:

$$V_{n_t^{h,h}}^H = -\Phi^h(l_t^h)^{-\phi}+\lambda_{c_t^h}(1-\tau_t)w_t^{h,h}+\lambda_{n_t^{h,h}}(1-\sigma^h) \tag{A.12}$$

$$V_{n_t^{h,l}}^H = -\Phi^h(l_t^h)^{-\phi}+\lambda_{c_t^h}\Big((1-\tau_t)w_t^{h,l}-b(z_t)\Big)+\lambda_{n_t^{h,l}}\Big(1-\sigma^l-\phi(z_t)\psi_{H,t}^{h,h}\Big)+\lambda_{n_t^{h,h}}\psi_{H,t}^{h,h}\phi(z_t) \tag{A.13}$$

F Firm's maximization problem

Each firm $f = 1, ..., N_t^f$ chooses the capital, k_t, vacancies for each skill, $v_t^{l,f}$, $v_t^{h,f}$, and the fraction of low-skill positions that will be allocated to high-skilled applicants, x_t^f to maximize the discounted expected value of future profits subject to technology and the employment laws of motion. The firm solves the following problem:

$$Q(k_t^f, n_t^{l,l,f}, n_t^{h,h,f}, n_t^{h,l,f}) = \max_{k_t^f, v_t^{l,f}, v_t^{h,f}, x_t} \Big\{ p_{d,t} y_{i,t}^f - w_t^{l,l} n_t^{l,l,f} - w_t^{h,l} n_t^{h,l,f} - w_t^{h,h} n_t^{h,h,f} - r_t^k k_t^f - \kappa^l v_t^{l,f} - \kappa^h v_t^{h,f} + E_t \left[\Lambda_{t,t+1} Q(k_{t+1}^f, n_{t+1}^{l,l,f}, n_{t+1}^{h,h,f}, n_{t+1}^{h,l,f}) \right] \Big\} \tag{A.14}$$

subject to:

$$y_{i,t}^f = A_t \left[\alpha (n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{A.15}$$

$$x_{i,t}^f = \left[\zeta (k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^f)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{A.16}$$

$$n_t^{l,f} = n_t^{l,l,f} + q^h n_t^{h,l,f} \tag{A.17}$$

and the laws of motion of employment, eqs. (11)-(13) solving for vacancies:

$$v_t^{l,f} = \frac{n_{t+1}^{h,l,f} - (1 - \sigma^l - \phi(z_t)\psi_{H,t}^{h,h}) n_t^{h,l,f}}{x_t \psi_{F,t}^{h,l}} \tag{A.18}$$

$$v_t^{l,f} = \frac{n_{t+1}^{l,l,f} - (1 - \sigma^l) n_t^{l,l,f}}{(1 - x_t)\psi_{F,t}^{l,l}} \tag{A.19}$$

$$v_t^{h,f} = \frac{n_{t+1}^{h,h,f} - (1 - \sigma^h) n_t^{h,h,f}}{\psi_{F,t}^{h,h}} \tag{A.20}$$

Solving eq. (A.18) for x_t, we get:

$$x_t = \frac{n_{t+1}^{h,l,f} - (1 - \sigma^l - \phi(z_t)\psi_{H,t}^{h,h}) n_t^{h,l,f}}{v_t^{l,f} \psi_{F,t}^{h,l}} \tag{A.21}$$

Then, we substitute eq. (A.21) in eq. (A.19) to get:

$$v_t^{l,f} = \frac{n_{t+1}^{l,l,f} - (1 - \sigma^l) n_t^{l,l,f}}{\psi_{F,t}^{l,l} - \left(\frac{n_{t+1}^{h,l,f} - (1 - \sigma^l - \phi(z_t)\psi_{H,t}^{h,h}) n_t^{h,l,f}}{v_t^{l,f} \psi_{F,t}^{h,l}} \right) \psi_{F,t}^{l,l}}$$

$$\iff v_t^{l,f} = \frac{n_{t+1}^{l,l,f} - (1 - \sigma^l) n_t^{l,l,f}}{\psi_{F,t}^{l,l}} + \frac{n_{t+1}^{h,l,f} - (1 - \sigma^l - \phi(z_t)\psi_{H,t}^{h,h}) n_t^{h,l,f}}{\psi_{F,t}^{h,l}} \tag{A.22}$$

Therefore, the firm will maximize eq. (A.23) below, taking into account eqs. (A.15)-(A.17), and eqs. (A.20) and (A.22). Then, using the optimal values for $n_{t+1}^{l,l,f}$ and $n_{t+1}^{h,l,f}$, x_t will be given residually by eq. (A.21).

$$Q_t = \max_{k_t^f, v_t^{l,f}, v_t^{h,f}, x_t} \Big\{ p_{d,t} y_{i,t}^f - w_t^{l,l} n_t^{l,l,f} - w_t^{h,l} n_t^{h,l,f} - w_t^{h,h} n_t^{h,f} - r_t^k k_t^f$$

$$- \kappa^l \left(\left(n_{t+1}^{l,l,f} - (1-\sigma^l) n_t^{l,l,f} \right) + \frac{\psi_{F,t}^{l,l}}{\psi_{F,t}^{h,l}} \left(n_{t+1}^{h,l,f} - (1-\sigma^l - \phi(z_t)\psi_{H,t}^{h,h}) n_t^{h,l,f} \right) \right)$$

$$- \kappa^h \left(\frac{n_{t+1}^{h,f}}{\psi_{F,t}^h} - (1-\sigma^h) \frac{n_t^{h,f}}{\psi_{F,t}^h} \right) + E_t \left[\Lambda_{t,t+1} Q_{t+1} \right] \Big\} \quad \text{(A.23)}$$

The first-order conditions are given by:

$[n_{t+1}^{l,l,f}]$

$$-\frac{\kappa^l}{\psi_{F,t}^{l,l}} + E_t \Lambda_{t,t+1} \frac{\partial Q_{t+1}}{\partial n_{t+1}^{l,l,f}} = 0 \iff \frac{\kappa^l}{\psi_{F,t}^{l,l}} = E_t \Lambda_{t,t+1} \Big\{ p_{d,t+1} y_{i,t+1}^{l,f} - w_{t+1}^{l,l} + \frac{(1-\sigma^l)\kappa^l}{\psi_{F,t+1}^{l,l}} \Big\} \quad \text{(A.24)}$$

$[n_{t+1}^{h,f}]$

$$-\frac{\kappa^h}{\psi_{F,t}^{h,h}} + E_t \Lambda_{t,t+1} \frac{\partial Q_{t+1}}{\partial n_{t+1}^{h,f}} = 0 \iff \frac{\kappa^h}{\psi_{F,t}^{h,h}} = E_t \Lambda_{t,t+1} \Big\{ p_{d,t+1} y_{i,t+1}^{h,f} - w_{t+1}^{h,h} + \frac{(1-\sigma^h)\kappa^h}{\psi_{F,t+1}^{h,h}} \Big\} \quad \text{(A.25)}$$

$[n_{t+1}^{h,l,f}]$

$$-\frac{\kappa^l}{\psi_{F,t}^{h,l}} + E_t \Lambda_{t,t+1} \frac{\partial Q_{t+1}}{\partial n_{t+1}^{h,l,f}} = 0 \iff \frac{\kappa^l}{\psi_{F,t}^{h,l}} = E_t \Lambda_{t,t+1} \Big\{ p_{d,t+1} y_{i,t+1}^{h,l,f} - w_{t+1}^{h,l} + \frac{(1-\sigma^l - \phi(z_{t+1})\psi_{H,t+1}^{h,h})\kappa^l}{\psi_{F,t+1}^{h,l}} \Big\} \quad \text{(A.26)}$$

$[k_t^f]$:

$$r_t^k = p_{d,t} y_{i,t}^{k,f} \quad \text{(A.27)}$$

where

$$y_{i,t}^{k,f} = \frac{\partial y_{i,t}^f}{\partial k_t^f} \qquad y_{i,t}^{l,f} = \frac{\partial y_{i,t}^f}{\partial n_t^{l,l,f}} \qquad y_{i,t}^{h,f} = \frac{\partial y_{i,t}^f}{\partial n_t^{h,f}} \qquad y_{i,t}^{h,l,f} = \frac{\partial y_{i,t}^f}{\partial n_t^{h,l,f}}$$

The values of an additional unit of high-skilled, low-skilled and mismatched employment to the firm follow:

$$V_{n_t^h}^F = p_{d,t} y_{i,t}^{h,f} - w_t^{h,h} + \frac{(1-\sigma^h)\kappa^h}{\psi_{F,t}^h} \quad \text{(A.28)}$$

$$V_{n_t^{l,l}}^F = p_{d,t} y_{i,t}^{l,l,f} - w_t^{l,l} + \frac{(1-\sigma^l)\kappa^l}{\psi_{F,t}^{l,l}} \quad \text{(A.29)}$$

$$V_{n_t^{h,l}}^F = p_{d,t} y_{i,t}^{h,l,f} - w_t^{h,l} + \frac{(1-\sigma^l - \phi(z_t)\psi_{H,t}^{h,h})\kappa^l}{\psi_{F,t}^{h,l}} \quad \text{(A.30)}$$

G Wage bargaining problem

The Nash bargaining problem is to maximize the weighted sum of log surpluses for each employment status. The wages for non-mismatch and mismatch employment are thus given as the optimal solution of the following problems:

$$\max_{w_t^{j,j}} \left\{ \left(1-\theta^{j,j}\right) \ln V_t^{n_t^{j,j}} + \theta^{j,j} \ln V_t^{n_t^{j,j,f}} \right\}, \quad j = l,h \tag{A.31}$$

$$\max_{w_t^{h,l}} \left\{ \left(1-\theta^{h,l}\right) \ln V_t^{n_t^{h,l}} + \theta^{h,l} \ln V_t^{n_t^{h,l,f}} \right\} \tag{A.32}$$

where θ denotes the bargaining power of firms, $V_t^{n_t^{h,l,f}}$ and $V_t^{n_t^{j,j,f}}$ are respective value functions of an additional unit of mismatch and non-mismatch employment to each firm, and $V_t^{n_t^{j,j}}$ and $V_t^{n_t^{l,l}}$ are the marginal values of having an additional household member employed in a non-mismatch and mismatch position, respectively.

Derivation of the non-mismatch wage ($w_t^{j,j}$).

$$V^F_{n_t^{j,j}} = p_{d,t} y_{i,t}^{j,j,f} - w_t^{j,j} + \frac{(1-\sigma^j)\kappa^j}{\psi_{F,t}^{j,j}}$$

$$V^H_{n_t^{j,j}} = -\Phi^j (l_t^j)^{-\phi} + \lambda_{c_t^j}(1-\tau_t) w_t^{j,j} + \lambda_{n_t^{j,j}}(1-\sigma^j)$$

If we substitute the constraints, we get:

$$\max_{w_t^{j,j}} \Bigg\{ \left(1-\theta^j\right) \ln \Bigg(-\Phi(l_t^j)^{-\phi} + \lambda_{c_t^j}(1-\tau_t) w_t^{j,j} + \lambda_{n_t^{j,j}}(1-\sigma^j) \Bigg) + \theta^j \ln \Bigg(p_{d,t} y_{i,t}^{j,f} - w_t^{j,j} + \frac{(1-\sigma^j)\kappa^j}{\psi_{F,t}^h} \Bigg) \Bigg\} \tag{A.33}$$

Thus, the non-mismatch wage $w_t^{j,j}$ is given by:

$$w_t^{j,j} = (1-\theta^j) \Bigg(p_{d,t} y_{i,t}^{j,j,f} + \frac{(1-\sigma^j)\kappa^j}{\psi_{F,t}^{j,j}} \Bigg) - \frac{\theta^j}{\lambda_{c_t^j}(1-\tau_t)} \Bigg(-\Phi(l_t^j)^{-\phi} + \lambda_{n_t^{j,j}}(1-\sigma^j) \Bigg) \tag{A.34}$$

Derivation of the mismatch wage ($w_t^{h,l}$).

$$V^F_{n_t^{h,l}} = p_{d,t} y_{i,t}^{h,l,f} - w_t^{h,l} + \frac{(1-\sigma^l - \phi(z_t)\psi_{H,t}^{h,h})\kappa^l}{\psi_{F,t}^{h,l}}$$

$$V^H_{n_t^{h,l}} = -\Phi^h (l_t^h)^{-\phi} + \lambda_{c_t^h}\Big((1-\tau_t) w_t^{h,l} - b(z_t)\Big) + \lambda_{n_t^{h,l}}\Big(1-\sigma^l - \phi(z_t)\psi_{H,t}^{h,h}\Big) + \lambda_{n_t^{h,h}} \psi_{H,t}^{h,h} \phi(z_t)$$

If we substitute the constraints, we get:

$$\max_{w_t^{h,l}} \Bigg\{ \left(1-\theta^{h,l}\right) \ln \Bigg(-\Phi(l_t^h)^{-\phi} + \lambda_{c_t^h}\Big((1-\tau_t) w_t^{h,l} - b(z_t)\Big) + \lambda_{n_t^{h,l}}\Big(1-\sigma^l - \phi(z_t)\psi_{H,t}^{h,h}\Big) + \lambda_{n_t^{h,h}} \psi_{H,t}^{h,h} \phi(z_t) \Bigg) + \theta^{h,l} \ln \Bigg(p_{d,t} y_{i,t}^{h,l,f} - w_t^{h,l} + \frac{(1-\sigma^l - \phi(z_t)\psi_{H,t}^{h,h})\kappa^l}{\psi_{F,t}^{h,l}} \Bigg) \Bigg\}$$

Thus, the mismatch wage $w_t^{h,l}$ is given by:

$$w_t^{h,l} = (1-\theta^{h,l})\Bigg(p_{d,t}y_{i,t}^{h,l,f} + \frac{(1-\sigma^l-\phi(z_t)\psi_{H,t}^{h,h})\kappa^l}{\psi_{F,t}^{h,l}}\Bigg) - \frac{\theta^{h,l}}{\lambda_{c_t^h}(1-\tau_t)}\Bigg(- \Phi(l_t^h)^{-\phi} - \lambda_{c_t^h}b(z_t) + \lambda_{n_t^{h,l}}\Big(1-\sigma^l-\phi(z_t)\psi_{H,t}^{h,h}\Big) + \lambda_{n_t^{h,h}}\psi_{H,t}^{h,h}\phi(z_t)\Bigg) \quad \text{(A.35)}$$

The three wages are then given by:

$$w_t^{h,h} = (1-\theta^{h,h})\Bigg(p_{d,t}(1-\alpha)(1-\zeta)A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t}}{x_{i,t}}\right)^{\frac{1}{\epsilon}}\left(\frac{x_{i,t}}{t^h n_t^h}\right)^{\frac{1}{\rho}} + \frac{(1-\sigma^h)\kappa^h}{\psi_{F,t}^{h,h}}\Bigg) - \frac{\theta^{h,h}}{\lambda_{c_t^h}(1-\tau_t)}\Bigg(-\Phi^h(l_t^h)^{-\phi} + \lambda_{n_t^{h,h}}(1-\sigma^h)\Bigg) \quad \text{(D.26)}$$

$$w_t^{h,l} = (1-\theta^{h,l})\Bigg(p_{d,t}\alpha q^h A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t}}{t^l n_t^{l,l} + t^h q^h n_t^{h,l}}\right)^{\frac{1}{\epsilon}} + \frac{(1-\sigma^l-\phi(z_t)\psi_{H,t}^{h,h})\kappa^l}{\psi_{F,t}^{h,l}}\Bigg) - \frac{\theta^{h,l}}{\lambda_{c_t^h}(1-\tau_t)}\Bigg(- \Phi^h(l_t^h)^{-\phi} - \lambda_{c_t^h}b(z_t) + \lambda_{n_t^{h,l}}\Big(1-\sigma^l-\phi(z_t)\psi_{H,t}^{h,h}\Big) + \lambda_{n_t^{h,h}}\psi_{H,t}^{h,h}\phi(z_t)\Bigg) \quad \text{(D.27)}$$

$$w_t^{l,l} = (1-\theta^{l,l})\Bigg(p_{d,t}\alpha A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t}}{t^l n_t^{l,l} + t^h q^h n_t^{h,l}}\right)^{\frac{1}{\epsilon}} + \frac{(1-\sigma^l)\kappa^l}{\psi_{F,t}^{l,l}}\Bigg) - \frac{\theta^{l,l}}{\lambda_{c_t^l}(1-\tau_t)}\Bigg(-\Phi^l(l_t^l)^{-\phi} + \lambda_{n_t^{l,l}}(1-\sigma^l)\Bigg) \quad \text{(D.28)}$$

In steady-state terms, we have the following expressions:

$$w^{h,h} = (1-\theta^{h,h})\Bigg(p_d(1-\alpha)(1-\zeta)A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{x_i}\right)^{\frac{1}{\epsilon}}\left(\frac{x_i}{t^h n^h}\right)^{\frac{1}{\rho}} + \frac{(1-\sigma^h)\kappa^h}{\psi_F^{h,h}}\Bigg) - \frac{\theta^{h,h}}{\lambda_{c^h}(1-\tau_t)}\Bigg(-\Phi^h(l^h)^{-\phi} + \lambda_{n^{h,h}}(1-\sigma^h)\Bigg) \quad \text{(S.28)}$$

$$w^{h,l} = (1-\theta^{h,l})\Bigg(p_d\alpha q^h A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l} + t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} + \frac{(1-\sigma^l-\phi(z)\psi_H^{h,h})\kappa^l}{\psi_F^{h,l}}\Bigg) - \frac{\theta^{h,l}}{\lambda_{c^h}(1-\tau)}\Bigg(- \Phi^h(l^h)^{-\phi} - \lambda_{c^h}b(z) + \lambda_{n^{h,l}}\Big(1-\sigma^l-\phi(z)\psi_H^{h,h}\Big) + \lambda_{n^{h,h}}\psi_H^{h,h}\phi(z)\Bigg) \quad \text{(S.29)}$$

$$w^{l,l} = (1-\theta^{l,l})\Bigg(p_d\alpha A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l} + t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} + \frac{(1-\sigma^l)\kappa^l}{\psi_F^{l,l}}\Bigg) - \frac{\theta^{l,l}}{\lambda_{c^l}(1-\tau)}\Bigg(- \Phi^l(l_t^l)^{-\phi} + \lambda_{n^{l,l}}(1-\sigma^l)\Bigg) \quad \text{(S.30)}$$

H Marginal productivities

$$y_{i,t}^f = A_t\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}} \tag{A.36}$$

$$x_{i,t}^f = \Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}} \tag{A.37}$$

$$n_t^{l,f} = n_t^{l,l,f} + q^h n_t^{h,l,f} \tag{A.38}$$

We denote the marginal productivities of k_t^f, $n_t^{l,l,f}$, $n_t^{h,f}$ and $n_t^{h,l,f}$ to be:

$$y_{i,t}^{k,f} = \frac{\partial y_{i,t}^f}{\partial k_t^f} \qquad y_t^{l,l} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{l,l,f}}, \qquad y_t^h \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{h,f}}, \qquad y_t^{h,l} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{h,l,f}} \tag{A.39}$$

H.1 Marginal productivity of high-skilled labor

$$y_t^h \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{h,f}} = (1-\alpha)(1-\zeta)A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t}^f}{x_{i,t}^f}\right)^{\frac{1}{\epsilon}}\left(\frac{x_{i,t}^f}{n_t^{h,f}}\right)^{\frac{1}{\rho}} \tag{A.40}$$

$$\begin{aligned}
y_t^h \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{h,f}} &= A_t\frac{\epsilon}{\epsilon-1}\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}-1}(1-\alpha)\frac{\epsilon-1}{\epsilon}(x_{i,t})^{\frac{\epsilon-1}{\epsilon}-1}\frac{\partial x_{i,t}^f}{\partial n_t^{h,f}} \\
&= A_t\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}-1}(1-\alpha)(x_{i,t})^{\frac{\epsilon-1}{\epsilon}-1}\frac{\partial x_{i,t}^f}{\partial n_t^{h,f}} \\
&= A_t^{\frac{\epsilon-1}{\epsilon}}{y_{i,t}^f}^{\frac{1}{\epsilon}}(1-\alpha)(x_{i,t}^f)^{-\frac{1}{\epsilon}}(x_{i,t}^f)^{\frac{1}{\rho}}(1-\zeta)(n_t^{h,f})^{-\frac{1}{\rho}} \\
&= A_t^{\frac{\epsilon-1}{\epsilon}}\frac{y_{i,t}^f}{x_{i,t}^f}^{\frac{1}{\epsilon}}(1-\alpha)\left(\frac{x_{i,t}^f}{n_t^{h,f}}\right)^{\frac{1}{\rho}}(1-\zeta)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial x_{i,t}^f}{\partial n_t^{h,f}} &= \frac{\rho}{\rho-1}\Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}-1}(1-\zeta)\frac{\rho-1}{\rho}(n_t^{h,f})^{\frac{\rho-1}{\rho}-1} \\
&= \Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}-1}(1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}-1} \\
&= \Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}}\Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{-1}(1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}-1} \\
&= x_{i,t}^f {x_{i,t}^f}^{-\frac{\rho-1}{\rho}}(1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}-1} \\
&= {x_{i,t}^f}^{\frac{1}{\rho}}(1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}-1} \\
&= {x_{i,t}^f}^{\frac{1}{\rho}}(1-\zeta)(n_t^{h,f})^{-\frac{1}{\rho}}
\end{aligned}$$

H.2 Marginal productivity of low-skilled labor

$$y_t^{l,l} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{l,l,f}} = \alpha A_t^{\frac{\epsilon-1}{\epsilon}}{y_{i,t}^f}^{\frac{1}{\epsilon}}(n_t^{l,f})^{-\frac{1}{\epsilon}} \tag{A.41}$$

$$
\begin{aligned}
y_t^l \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{l,l,f}} &= A_t \frac{\epsilon}{\epsilon-1}\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}-1} \alpha\frac{\epsilon-1}{\epsilon}(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}-1}\frac{\partial n_t^{l,f}}{\partial n_t^{l,l,f}} \\
&= A_t\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}-1}\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}-1}\frac{\partial n_t^{l,f}}{\partial n_t^{l,l,f}} \qquad \frac{\partial n_t^{l,f}}{\partial n_t^{h,l,f}} = 1 \\
&= A_t^{\frac{\epsilon-1}{\epsilon}} {y_{i,t}^f}^{\frac{1}{\epsilon}} \alpha (n_t^{l,f})^{-\frac{1}{\epsilon}} \\
&= \alpha A_t^{\frac{\epsilon-1}{\epsilon}} {y_{i,t}^f}^{\frac{1}{\epsilon}} (n_t^{l,f})^{-\frac{1}{\epsilon}}
\end{aligned}
$$

H.3 Marginal productivity of mismatch labor

$$
y_t^{h,l} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{h,l,f}} = \alpha q^h A_t^{\frac{\epsilon-1}{\epsilon}} {y_{i,t}^f}^{\frac{1}{\epsilon}} (n_t^{l,f})^{-\frac{1}{\epsilon}} \tag{A.42}
$$

$$
\begin{aligned}
y_t^{h,l} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{h,l,f}} &= A_t \frac{\epsilon}{\epsilon-1}\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}-1} \alpha\frac{\epsilon-1}{\epsilon}(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}-1}\frac{\partial n_t^{l,f}}{\partial n_t^{h,l,f}} \\
&= A_t\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}-1}\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}-1}\frac{\partial n_t^{l,f}}{\partial n_t^{h,l,f}} \qquad \frac{\partial n_t^{l,f}}{\partial n_t^{h,l,f}} = q^h \\
&= A_t^{\frac{\epsilon-1}{\epsilon}} {y_{i,t}^f}^{\frac{1}{\epsilon}} \alpha (n_t^{l,f})^{-\frac{1}{\epsilon}} q^h \\
&= \alpha q^h A_t^{\frac{\epsilon-1}{\epsilon}} {y_{i,t}^f}^{\frac{1}{\epsilon}} (n_t^{l,f})^{-\frac{1}{\epsilon}}
\end{aligned}
$$

H.4 Marginal productivity of capital

$$
y_t^k \equiv \frac{\vartheta y_{i,t}^f}{\vartheta k_t^f} = \zeta(1-\alpha)A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t}^f}{x_{i,t}^f}\right)^{\frac{1}{\epsilon}}\left(\frac{x_{i,t}^f}{k_t^f}\right)^{\frac{1}{\rho}} \tag{A.43}
$$

$$
\begin{aligned}
y_t^k \equiv \frac{\vartheta y_{i,t}^f}{\vartheta k_t^f} &= A_t \frac{\epsilon}{\epsilon-1}\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}-1}(1-\alpha)\frac{\epsilon-1}{\epsilon}(x_{i,t})^{\frac{\epsilon-1}{\epsilon}-1}\frac{\partial x_{i,t}^f}{\partial k_t^f} \\
&= A_t\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}-1}(1-\alpha)(x_{i,t})^{\frac{\epsilon-1}{\epsilon}-1}\frac{\partial x_{i,t}^f}{\partial k_t^f} \\
&= A_t^{\frac{\epsilon-1}{\epsilon}} {y_{i,t}^f}^{\frac{1}{\epsilon}}(1-\alpha)(x_{i,t}^f)^{-\frac{1}{\epsilon}}(x_{i,t}^f)^{\frac{1}{\rho}}(k_t^f)^{\frac{1}{\rho}}\zeta \\
&= \zeta(1-\alpha)A_t^{\frac{\epsilon-1}{\epsilon}} \frac{y_{i,t}^f}{x_{i,t}^f}^{\frac{1}{\epsilon}} \frac{x_{i,t}^f}{k_t^f}^{\frac{1}{\rho}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial x_{i,t}^f}{\partial k_t^f} &= \frac{\rho}{\rho-1}\Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}-1}\zeta\frac{\rho-1}{\rho}(k_t^f)^{\frac{\rho-1}{\rho}-1} \\
&= \Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}-1}\zeta(k_t^f)^{\frac{\rho-1}{\rho}-1} \\
&= \Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}}\Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{-1}\zeta(k_t^f)^{\frac{\rho-1}{\rho}-1} \\
&= x_{i,t}^f {x_{i,t}^f}^{-\frac{\rho-1}{\rho}}(1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}-1} \\
&= {x_{i,t}^f}^{\frac{1}{\rho}}\zeta(k_t^f)^{\frac{\rho-1}{\rho}-1} \\
&= {x_{i,t}^f}^{\frac{1}{\rho}}\zeta(k_t^f)^{-\frac{1}{\rho}}
\end{aligned}
$$

H.5 Formulas used in the marginal productivities calculation

$$A_t\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}}+(1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}-1}=A_t^{\frac{\epsilon-1}{\epsilon}}{y_{i,t}^f}^{\frac{1}{\epsilon}} \tag{A.44}$$

$$\begin{aligned}A_t\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}}+(1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}-1}&=A_t\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}}+(1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}}\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}}+(1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{-1}\\&=y_{i,t}^f\left(\frac{y_{i,t}^f}{A_t}\right)^{-\frac{\epsilon-1}{\epsilon}}\\&=A_t^{\frac{\epsilon-1}{\epsilon}}{y_{i,t}^f}^{\frac{1}{\epsilon}}\end{aligned}$$

$$\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}}+(1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{-1}=\left(\frac{y_{i,t}^f}{A_t}\right)^{\frac{\epsilon-1}{\epsilon}} \tag{A.45}$$

$$\begin{aligned}y_{i,t}^f&=A_t\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}}+(1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}}\\\frac{y_{i,t}^f}{A_t}&=\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}}+(1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}}\\\left(\frac{y_{i,t}^f}{A_t}\right)^{\frac{\epsilon-1}{\epsilon}}&=\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}}+(1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]\\\left(\frac{y_{i,t}^f}{A_t}\right)^{-\frac{\epsilon-1}{\epsilon}}&=\Big[\alpha(n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}}+(1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{-1}\end{aligned}$$

$$\Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}}+(1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{-1}=\left(x_{i,t}^f\right)^{-\frac{\rho-1}{\rho}} \tag{A.46}$$

$$\begin{aligned}x_{i,t}^f&=\Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}}+(1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}}\\\left(x_{i,t}^f\right)^{\frac{\rho-1}{\rho}}&=\Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}}+(1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]\\\left(x_{i,t}^f\right)^{-\frac{\rho-1}{\rho}}&=\Big[\zeta(k_t^f)^{\frac{\rho-1}{\rho}}+(1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{-1}\end{aligned}$$

For the Decentralized Competitive Equilibrium:

$$\frac{\kappa^l}{\psi_{F,t}^{l,l}}+E_t\Lambda_{t,t+1}\Big\{w_{t+1}^{l,l}+\frac{(1-\sigma^l)\kappa^l}{\psi_{F,t+1}^{l,l}}\Big\}=E_t\Lambda_{t,t+1}\Big\{p_{d,t+1}\alpha A_{t+1}^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t+1}^f}{n_{t+1}^{l,f}}\right)^{\frac{1}{\epsilon}}\Big\} \tag{A.47}$$

$$\frac{\kappa^l}{\psi_{F,t}^{h,l}}+E_t\Lambda_{t,t+1}\Big\{w_{t+1}^{h,l}+\frac{(1-\sigma^l-\phi(z_{t+1})\psi_{H,t+1}^h)\kappa^l}{\psi_{F,t+1}^{h,l}}\Big\}=E_t\Lambda_{t,t+1}\Big\{p_{d,t+1}\alpha q^h A_{t+1}^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t+1}^f}{n_{t+1}^{l,f}}\right)^{\frac{1}{\epsilon}}\Big\} \tag{A.48}$$

Divide eq. (A.47) by eq. (A.48) and get:

$$\frac{\frac{\kappa^l}{\psi_{F,t}^{l,l}}+E_t\Lambda_{t,t+1}\Big\{w_{t+1}^{l,l}+\frac{(1-\sigma^l)\kappa^l}{\psi_{F,t+1}^{l,l}}\Big\}}{\frac{\kappa^l}{\psi_{F,t}^{h,l}}+E_t\Lambda_{t,t+1}\Big\{w_{t+1}^{h,l}+\frac{(1-\sigma^l-\phi(z_{t+1})\psi_{H,t+1}^h)\kappa^l}{\psi_{F,t+1}^{h,l}}\Big\}}=\frac{E_t\Lambda_{t,t+1}\Big\{p_{d,t+1}\alpha A_{t+1}^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t+1}^f}{n_{t+1}^{l,f}}\right)^{\frac{1}{\epsilon}}\Big\}}{E_t\Lambda_{t,t+1}\Big\{p_{d,t+1}\alpha q^h A_{t+1}^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t+1}^f}{n_{t+1}^{l,f}}\right)^{\frac{1}{\epsilon}}\Big\}} \tag{A.49}$$

$$\frac{\frac{\kappa^l}{\psi^{l,l}_{F,t}} + E_t\Lambda_{t,t+1}\left\{w^{l,l}_{t+1} + \frac{(1-\sigma^l)\kappa^l}{\psi^{l,l}_{F,t+1}}\right\}}{\frac{\kappa^l}{\psi^{h,l}_{F,t}} + E_t\Lambda_{t,t+1}\left\{w^{h,l}_{t+1} + \frac{(1-\sigma^l-\phi(z_{t+1})\psi^h_{H,t+1})\kappa^l}{\psi^{h,l}_{F,t+1}}\right\}} = \frac{1}{q^h} \tag{A.50}$$

$$\frac{\kappa^l}{\psi^{l,l}_{F,t}} + E_t\Lambda_{t,t+1}\left\{w^{l,l}_{t+1} + \frac{(1-\sigma^l)\kappa^l}{\psi^{l,l}_{F,t+1}}\right\} = \frac{1}{q^h}\frac{\kappa^l}{\psi^{h,l}_{F,t}} + E_t\Lambda_{t,t+1}\left\{w^{h,l}_{t+1} + \frac{(1-\sigma^l-\phi(z_{t+1})\psi^h_{H,t+1})\kappa^l}{\psi^{h,l}_{F,t+1}}\right\} \tag{A.51}$$

Alternatively, in the steady state we can get this expression:

$$\frac{w^{l,l}}{w^{h,l}} = \frac{\beta p_d \alpha A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y^f_i}{n^{l,f}}\right)^{\frac{1}{\epsilon}} + \frac{\kappa^l}{\psi^{l,l}_F}(\beta(1-\sigma^l)-1)}{\beta p_d \alpha q^h A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y^f_i}{n^{l,f}}\right)^{\frac{1}{\epsilon}} + \left(\frac{\kappa^l}{\psi^{h,l}_F}\beta(1-\sigma^l-\phi(z)\psi^h_H)-1\right)} \tag{A.52}$$

I Economy-wide final good

The representative final good firm aggregates the domestic intermediate good, $Y_{d,t}$, and imported aggregate goods, $Y_{f,t}$, to produce the economy-wide final good, Y_t, using a CES technology:

$$Y_t = \left[\omega^{\frac{1}{\gamma}}(Y_{d,t})^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}}(Y_{f,t})^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} \tag{A.53}$$

where ω denotes the degree of home bias and γ is the elasticity of substitution between home-produced and imported goods. The economy-wide final good firm maximizes profits, $\Pi_t = P_tY_t - p_{d,t}Y_{d,t} - p_{f,t}Y_{f,t}$, where $p_{d,t}$ and $p_{f,t}$ are the relative prices of the domestic and foreign intermediate goods respectively. This yields the following optimal demand schedules :

$$Y_{d,t} = \omega\left(\frac{p_{d,t}}{P_t}\right)^{-\gamma}Y_t \tag{A.54}$$

$$Y_{f,t} = (1-\omega)\left(\frac{p_{f,t}}{P_t}\right)^{-\gamma}Y_t \tag{A.55}$$

Combining equations (A.54) and (A.55) yields:

$$Y_{d,t} = \frac{\omega}{1-\omega}\left(\frac{p_{d,t}}{p_{f,t}}\right)^{-\gamma}Y_{f,t} \tag{A.56}$$

The associated price index is given by:

$$P_t = (\omega p^{1-\gamma}_{d,t} + (1-\omega)p^{1-\gamma}_{f,t})^{\frac{1}{1-\gamma}} \tag{A.57}$$

where we have assumed that the law of one price holds as in:

$$p_{f,t} = e_t p^*_{d,t} \tag{A.58}$$

J Derivation of the resource constraint

Let us first repeat the main equations of interest here:

Government budget constraint

$$\bar{\omega}(t^l u_t^l + t^h u_t^h) + s_t^t y_t + s_t^c y_t = \tau\Big(w_t^{l,l} t^l n_t^{l,l} + w_t^{h,l} t^h n_t^{h,l} + w_t^{h,h} t^h n_t^{h,h} + r_t^k k_t\Big)$$

High-skilled household's budget constraint

$$t^h c_t^h + i_t + e_t r_t^d d_t + b_t(z) t^h n_t^{h,l} = (1-\tau)(w_t^{h,h} t^h n_t^{h,h} + w_t^{h,l} t^h n_t^{h,l} + r_t^k k_t) + e_t d_{t+1} + \bar{\omega} t^h u_t^h + t^h s_t^t y_t$$

Low-skilled household's budget constraint

$$t^l c_t^l = (1-\tau_t) w_t^{l,l} t^l n_t^{l,l} + \bar{\omega} t^l u_t^l + t^l s_t^t y_t$$

Profits

- Intermediate-good firms' profits, $t^h \pi_t^f = 0$

$$p_{d,t} y_{i,t} = w_t^{l,l} t^l n_t^{l,l} + w_t^{h,l} t^h n_t^{h,l} + w_t^{h,h} t^h n_t^h + r_t^k k_t + \kappa^l v_t^l + \kappa^h v_t^h$$

- Economy-wide profits, $\pi_t = 0$

$$y_t = p_{d,t} y_{d,t} + p_{f,t} y_{f,t}$$

Net foreign assets law of motion

$$p_{d,t} y_{d,t}^* - p_{f,t} y_{f,t} = e_t(r_t^d d_t - d_{t+1})$$

Definition of net exports

$$nx_t = e_t(r_t^d d_t - d_{t+1})$$

Intermediate good distribution

$$p_{d,t} y_{i,t} = p_{d,t} y_{d,t} + p_{d,t} y_{d,t}^*$$

Let us now take the following steps:

1. Add the two household budget constraints:

$$t^h c_t^h + t^l c_t^l + i_t + e_t(r_t^d d_t - d_{t+1}) + b_t(z) t^h n_t^{h,l} = (1-\tau)(w_t^{l,l} t^l n_t^{l,l} + w_t^{h,h} t^h n_t^{h,h} + w_t^{h,l} t^h n_t^{h,l} + r_t^k k_t) + \bar{\omega}(t^h u_t^h + t^l u_t^l) + s^t y_t \quad \text{(A.59)}$$

2. Using the government budget constraint, substitute out $\bar{\omega}(t^h u_t^h + t^l u_t^l) + s^t y_t$ and rearrange:

$$t^h c_t^h + t^l c_t^l + i_t + e_t(r_t^d d_t - d_{t+1}) + b_t(z) t^h n_t^{h,l} + s_t^c y_t = (w_t^{l,l} t^l n_t^{l,l} + w_t^{h,h} t^h n_t^{h,h} + w_t^{h,l} t^h n_t^{h,l} + r_t^k k_t) \quad \text{(A.60)}$$

3. Using the equation for the intermediate-good firms' profits, substitute out $w_t^{l,l} t^l n_t^{l,l} + w_t^{h,l} t^h n_t^{h,l} + w_t^{h,h} t^h n_t^h + r_t^k k_t$ and rearrange:

$$p_{d,t} y_{i,t} - e_t(r_t^d d_t - d_{t+1}) = t^h c_t^h + t^l c_t^l + i_t + b_t(z) t^h n_t^{h,l} + s_t^c y_t + \kappa^l v_t^l + \kappa^h v_t^h \quad \text{(A.61)}$$

4. Substitute out $e_t(r_t^d d_t - d_{t+1})$ and rearrange:

$$p_{d,t}y_{i,t} - p_{d,t}y^*_{d,t} + p_{f,t}y_{f,t} = t^h c_t^h + t^l c_t^l + i_t + b_t(z)t^h n_t^{h,l} + s_t^c y_t + \kappa^l v_t^l + \kappa^h v_t^h \tag{A.62}$$

5. Using the intermediate-good distribution equation, substitute out $p_{d,t}y_{i,t}$ and rearrange:

$$p_{d,t}y_{d,t} + p_{f,t}y_{f,t} = t^h c_t^h + t^l c_t^l + i_t + b_t(z)t^h n_t^{h,l} + s_t^c y_t + \kappa^l v_t^l + \kappa^h v_t^h \tag{A.63}$$

6. Using the profits of the economy-wide output, substitute out $p_{d,t}y_{d,t}+p_{f,t}y_{f,t}$ to get the resource constraint:

$$y_t = t^h c_t^h + t^l c_t^l + i_t + b_t(z)t^h n_t^{h,l} + s_t^c y_t + \kappa^l v_t^l + \kappa^h v_t^h \tag{A.64}$$

K Labor transformations

High-skilled labor

$$\sum_{h=1}^{N_t^h} n_t^{h,h} = N_t^h n_t^{h,h}, \quad \sum_{f=1}^{N_t^f} n_t^{h,h,f} = N_t^f n_t^{h,h,f} \Rightarrow n_t^{h,h,f} = \frac{N_t^h n_t^{h,h}}{N_t^f} \Rightarrow n_t^{h,h,f} = n_t^{h,h} \tag{A.65}$$

Aggregate low-skilled labor

$$N_t^{l,s} = \sum_{l=1}^{N_t^l} n_t^{l,l} + \sum_{h=1}^{N_t^h} n_t^{h,l} = N_t^l n_t^{l,l} + N_t^h n_t^{h,l}, \quad N_t^{l,d} = \sum_{f=1}^{N_t^f} n_t^{l,l,f} + \sum_{f=1}^{N_t^f} n_t^{h,l,f} = N_t^f (n_t^{l,l,f} + n_t^{h,l,f}) \tag{A.66}$$

$$N_t^f n_t^{l,f} = N_t^f (n_t^{l,l,f} + n_t^{h,l,f}) \qquad N_t^f n_t^l = N_t^l n_t^{l,l} + N_t^h n_t^{h,l} \Rightarrow n_t^l = \frac{N_t^l n_t^{l,l} + N_t^h n_t^{h,l}}{N_t^f} \Rightarrow n_t^l = \frac{t^l}{t^h} n_t^{l,l} + n_t^{h,l} \tag{A.67}$$

Low-skilled labor

$$\sum_{l=1}^{N_t^l} n_t^{l,l} = N_t^l n_t^{l,l}, \quad \sum_{f=1}^{N_t^f} n_t^{l,l,f} = N_t^f n_t^{l,l,f} \Rightarrow n_t^{l,l,f} = \frac{N_t^l n_t^{l,l}}{N_t^f} \Rightarrow n_t^{l,l,f} = \frac{t^l}{t^h} n_t^{l,l} \tag{A.68}$$

Mismatch labor

$$\sum_{h=1}^{N_t^h} n_t^{h,l} = N_t^h n_t^{h,l}, \quad \sum_{f=1}^{N_t^f} n_t^{h,l,f} = N_t^f n_t^{h,l,f} \Rightarrow n_t^{h,l,f} = \frac{N_t^h n_t^{h,l}}{N_t^f} \Rightarrow n_t^{h,l,f} = n_t^{h,l} \tag{A.69}$$

L Production function

$$y_{i,t}^f = A_t \Big[\alpha (n_t^{l,f})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t}^f)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}} \tag{A.70}$$

$$x_{i,t}^f = \Big[\zeta (k_t^f)^{\frac{\rho-1}{\rho}} + (1-\zeta)(n_t^{h,f})^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}} \tag{A.71}$$

$$n_t^{l,f} = n_t^{l,l,f} + q^h n_t^{h,l,f} \tag{A.72}$$

$$y_{i,t} = A_t \Big[\alpha (t^h n_t^l)^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_{i,t})^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}} \tag{A.73}$$

$$x_{i,t} = \Big[\zeta (k_t)^{\frac{\rho-1}{\rho}} + (1-\zeta)(t^h n_t^h)^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}} \tag{A.74}$$

M Decentralized Competitive Equilibrium

Given market prices ($w_t^{l,l}$, $w_t^{h,l}$, $w_t^{h,h}$ r_t^k, r_t^d, e_t, $p_{d,t}$, $p_{f,t}$), government policy (s_t^c, s_t^t, τ_t) and economy-wide variables (A_t), each individual low skilled household, $l = 1, 2, \dots, N_t^l$, solves its problem as defined in section 3.4.1, each individual high skilled household, $h = 1, 2, \dots, N_t^h$, solves its problem as defined in section 3.4.2, each individual firm in the intermediate sector, $f = 1, 2, \dots, N_t^f$, solves its problem as defined in section 3.5, all markets clear and all constraints are satisfied. Thus the DCE, expressed in per capita terms, is given by eqs. (D.1)-(D.48):

Hiring probabilities $\{\psi_{H,t}^{l,l}, \psi_{H,t}^{h,l}, \psi_{H,t}^{h,h}\}$

$$\psi_{H,t}^{l,l} = \frac{m_t^{l,l}}{t^l u_t^l} \tag{D.1}$$

$$\psi_{H,t}^{h,l} = \frac{m_t^{h,l}}{(1-s_t)t^h u_t^h} \tag{D.2}$$

$$\psi_{H,t}^{h,h} = \frac{m_t^{h,h}}{s_t t^h u_t^h + t^h \phi(z_t) n_t^{h,l}} \tag{D.3}$$

Vacancy-filling probabilities $\{\psi_{F,t}^{l,l}, \psi_{F,t}^{h,l}, \psi_{F,t}^{h,h}\}$

$$\psi_{F,t}^{l,l} = \frac{m_t^{l,l}}{(1-x_t)v_t^l} \tag{D.4}$$

$$\psi_{F,t}^{h,l} = \frac{m_t^{h,l}}{x_t v_t^l} \tag{D.5}$$

$$\psi_{F,t}^{h,h} = \frac{m_t^{h,h}}{v_t^h} \tag{D.6}$$

Low-skilled household $\{n_t^l, l_t^l, u_t^l, \lambda_{n_t^l}\}$

$$\lambda_{n_t^{l,l}} = \beta E_t \Big[-\Phi^l (l_{t+1}^l)^{-\phi} + \lambda_{c_{t+1}^l}(1-\tau_{t+1})w_{t+1}^{l,l} + \lambda_{n_{t+1}^{l,l}}(1-\sigma^l) \Big] \tag{D.7}$$

$$\Phi^l (l_t^l)^{-\phi} = \lambda_{n_t^{l,l}} \psi_{H,t}^{l,l} + \bar{\omega}\lambda_{c_t^l} \tag{D.8}$$

$$n_{t+1}^{l,l} = (1-\sigma^l)n_t^{l,l} + \psi_{H,t}^{l,l} u_t^l \tag{D.9}$$

$$n_t^{l,l} + u_t^l + l_t^l = 1 \tag{D.10}$$

High-skilled household $\{c_t^h, i_t, k_{t+1}, d_{t+1}, n_t^{h,l}, n_t^{h,h}, l_t^h, u_t^h, s_t, z_t, \lambda_{n_t^{h,l}}, \lambda_{h_t^{h,h}}\}$

$$\left(\frac{c_{t+1}^h}{c_t^h}\right)^\eta \left(1 + \Xi\left(\frac{k_{t+1}^h}{k_t} - 1\right)\right) = \beta E_t \left(1 - \delta + (1-\tau_{t+1})r_{t+1}^k + \frac{\Xi}{2}\left(\left(\frac{k_{t+2}^h}{k_{t+1}^h}\right)^2 - 1\right)\right) \tag{D.11}$$

$$\left(\frac{c_{t+1}^h}{c_t^h}\right)^\eta e_t = \beta E_t e_{t+1}(1 + r_{t+1}^d) \tag{D.12}$$

$$k_{t+1}^h = (1-\delta)k_t^h + i_t^h - \frac{\Xi}{2}\left(\frac{k_{t+1}^h}{k_t^h} - 1\right)^2 k_t^h \tag{D.13}$$

$$\lambda_{n_t^{h,h}} = \beta E_t\Big[-\Phi^h(l_{t+1}^h)^{-\phi} + \lambda_{c_{t+1}^h}(1-\tau_{t+1})w_{t+1}^{h,h} + \lambda_{n_{t+1}^{h,h}}(1-\sigma^h)\Big] \tag{D.14}$$

$$\begin{aligned}\lambda_{n_t^{h,l}} = \beta E_t\Big[-\Phi^h(l_{t+1}^h)^{-\phi} + \lambda_{c_{t+1}^h}\Big((1-\tau_{t+1})w_{t+1}^{h,l} - b(z_{t+1})\Big) + \lambda_{n_{t+1}^{h,l}}\Big(1-\sigma^l - \phi(z_{t+1})\psi_{H,t+1}^{h,h}\Big)\\ + \lambda_{n_{t+1}^{h,h}}\psi_{H,t+1}^{h,h}\phi(z_{t+1})\Big]\end{aligned} \tag{D.15}$$

$$\Phi^h(l_t^h)^{-\phi} = \lambda_{n_t^{h,h}}\psi_{H,t}^{h,h}s_t + \lambda_{n_t^{h,l}}\psi_{H,t}^{h,l}(1-s_t) + \bar{\omega}\lambda_{c_t^h} \tag{D.16}$$

$$\lambda_{n_t^{h,h}}\psi_{H,t}^{h,h} = \lambda_{n_t^{h,l}}\psi_{H,t}^{h,l} \tag{D.17}$$

$$\lambda_{c_t^h}\frac{b'(z_t)}{\phi'(z_t)} = \psi_{H,t}^{h,h}(\lambda_{n_t^{h,h}} - \lambda_{n_t^{h,l}}) \tag{D.18}$$

$$t^h c_t^h + i_t + e_t r_t^d d_t + b(z_t)t^h n_t^{h,l} = (1-\tau_t)(w_t^{h,h}t^h n_t^{h,h} + w_t^{h,l}t^h n_t^{h,l} + r_t^k k_t) + e_t d_{t+1} + \bar{\omega}t^h u_t^h + t^h s_t^t y_t \tag{D.19}$$

$$n_{t+1}^{h,l} = \big(1-\sigma^l - \phi(z_t)\psi_{H,t}^{h,h}\big)n_t^{h,l} + \psi_{H,t}^{h,l}(1-s_t)u_t^h \tag{D.20}$$

$$n_{t+1}^{h,h} = (1-\sigma^h)n_t^{h,h} + \psi_{H,t}^{h,h}\Big(s_t u_t^h + \phi(z_t)n_t^{h,l}\Big) \tag{D.21}$$

$$n_t^{h,l} + n_t^{h,h} + u_t^h + l_t^h = 1 \tag{D.22}$$

Matches $\{m_t^{l,l}, m_t^{h,l}, m_t^{h,h}\}$

$$m_t^{l,l} = \mu_1((1-x_t)v_t^l)^{\mu_2}(t^l u_t^l)^{1-\mu_2} \tag{D.23}$$

$$m_t^{h,l} = \mu_1(x_t v_t^l)^{\mu_2}((1-s_t)t^h u_t^h)^{1-\mu_2} \tag{D.24}$$

$$m_t^{h,h} = \mu_1(v_t^h)^{\mu_2}(s_t t^h u_t^h + t^h\phi(z_t)n_t^{h,l})^{1-\mu_2} \tag{D.25}$$

Wages $\{w_t^{l,l}, w_t^{h,l}, w_t^{h,h}\}$

$$w_t^{h,h} = (1-\theta^h)\left(p_{d,t}(1-\alpha)(1-\zeta)A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t}}{x_{i,t}}\right)^{\frac{1}{\epsilon}}\left(\frac{x_{i,t}}{t^h n_t^h}\right)^{\frac{1}{\rho}} + \frac{(1-\sigma^h)\kappa^h}{\psi_{F,t}^{h,h}}\right) - \frac{\theta^h}{\lambda_{c_t^h}(1-\tau_t)}\left(-\Phi^h(l_t^h)^{-\phi} + \lambda_{n_t^{h,h}}(1-\sigma^h)\right) \tag{D.26}$$

$$w_t^{h,l} = (1-\theta^{h,l})\left(p_{d,t}\alpha q^h A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t}}{t^l n_t^{l,l}+t^h q^h n_t^{h,l}}\right)^{\frac{1}{\epsilon}} + \frac{(1-\sigma^l-\phi(z_t)\psi_{H,t}^{h,h})\kappa^l}{\psi_{F,t}^{h,l}}\right)$$
$$-\frac{\theta^{h,l}}{\lambda_{c_t^h}(1-\tau_t)}\left(-\Phi^h(l_t^h)^{-\phi}-\lambda_{c_t^h}b(z_t)+\lambda_{n_t^{h,l}}\left(1-\sigma^l-\phi(z_t)\psi_{H,t}^{h,h}\right)+\lambda_{n_t^{h,h}}\psi_{H,t}^{h,h}\phi(z_t)\right) \quad (D.27)$$

$$w_t^{l,l} = (1-\theta^l)\left(p_{d,t}\alpha A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t}}{t^l n_t^{l,l}+t^h q^h n_t^{h,l}}\right)^{\frac{1}{\epsilon}}+\frac{(1-\sigma^l)\kappa^l}{\psi_{F,t}^{l,l}}\right)-\frac{\theta^l}{\lambda_{c_t^l}(1-\tau_t)}\left(-\Phi^l(l_t^l)^{-\phi}+\lambda_{n_t^{l,l}}(1-\sigma^l)\right) \quad (D.28)$$

Intermediate-goods firm $\{y_{i,t},\, x_{i,t},\, x_t,\, n_t^l,\, y_{d,t}^*,\, r_t^k,\, p_{d,t}, v_t^l, v_t^h, \Lambda_{t,t+1}\}$

$$y_{i,t} = A_t\left[\alpha(t^h n_t^l)^{\frac{\epsilon-1}{\epsilon}}+(1-\alpha)(x_{i,t})^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \quad (D.29)$$

$$x_{i,t} = \left[\zeta(k_t)^{\frac{\rho-1}{\rho}}+(1-\zeta)(t^h n_t^h)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \quad (D.30)$$

$$n_t^l = \frac{t^l}{t^h}n_t^{l,l}+q^h n_t^{h,l} \quad (D.31)$$

$$x_t = t^h\left(\frac{n_{t+1}^{h,l}-(1-\sigma^l-\phi(z_t)\psi_{H,t}^{h,h})n_t^{h,l}}{\psi_{F,t}^{h,l}v_t^l}\right) \quad (D.32)$$

$$\frac{\kappa^h}{\psi_{F,t}^{h,h}} = E_t\Lambda_{t,t+1}\Big\{p_{d,t+1}(1-\alpha)(1-\zeta)A_{t+1}^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t+1}}{x_{i,t+1}}\right)^{\frac{1}{\epsilon}}\left(\frac{x_{i,t+1}}{t^h n_{t+1}^h}\right)^{\frac{1}{\rho}}-w_{t+1}^{h,h}+\frac{(1-\sigma^h)\kappa^h}{\psi_{F,t+1}^{h,h}}\Big\} \quad (D.33)$$

$$\frac{\kappa^l}{\psi_{F,t}^{l,l}}+E_t\Lambda_{t,t+1}\Big\{w_{t+1}^{l,l}+\frac{(1-\sigma^l)\kappa^l}{\psi_{F,t+1}^{l,l}}\Big\} = \frac{1}{q^h}\left(\frac{\kappa^l}{\psi_{F,t}^{h,l}}+E_t\Lambda_{t,t+1}\Big\{w_{t+1}^{h,l}+\frac{(1-\sigma^l-\phi(z_{t+1})\psi_{H,t+1}^{h,h})\kappa^l}{\psi_{F,t+1}^{h,l}}\Big\}\right) \quad (D.34)$$

$$\frac{\kappa^l}{\psi_{F,t}^{l,l}} = E_t\Lambda_{t,t+1}\Big\{p_{d,t+1}\alpha A_{t+1}^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t+1}}{t^l n_{t+1}^{l,l}+t^h q^h n_{t+1}^{h,l}}\right)^{\frac{1}{\epsilon}}-w_{t+1}^{l,l}+\frac{(1-\sigma^l)\kappa^l}{\psi_{F,t+1}^{l,l}}\Big\}$$

$$\frac{\kappa^l}{\psi_{F,t}^{h,l}} = E_t\Lambda_{t,t+1}\Big\{p_{d,t+1}\alpha q^h A_{t+1}^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t+1}}{t^l n_{t+1}^{l,l}+t^h q^h n_{t+1}^{h,l}}\right)^{\frac{1}{\epsilon}}-w_{t+1}^{h,l}+\frac{(1-\sigma^l-\phi(z_{t+1})\psi_{H,t+1}^{h,h})\kappa^l}{\psi_{F,t+1}^{h,l}}\Big\}$$

$$\Lambda_{t,t+1} = \beta\Big(\frac{c_{t+1}^h}{c_t^h}\Big)^{-\eta} \quad (D.35)$$

$$r_t^k = p_{d,t}y_{i,t}^k \quad (D.36)$$

$$y_{i,t} = y_{d,t}+y_{d,t}^* \quad (D.37)$$

$$y_{d,t}^* = (1-\omega^*)\Big(\frac{p_{d,t}}{e_t}\Big)^{-\gamma^*}y_t^* \quad (D.38)$$

Economy-wide final good, $\{y_t,\, y_{d,t},\, y_{f,t}\}$

$$y_t = \Big[\omega^{\frac{1}{\gamma}}(y_{d,t})^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}}(y_{f,t})^{\frac{\gamma-1}{\gamma}}\Big]^{\frac{\gamma}{\gamma-1}} \tag{D.39}$$

$$1 = (\omega p_{d,t}^{1-\gamma} + (1-\omega)p_{f,t}^{1-\gamma})^{\frac{1}{1-\gamma}} \tag{D.40}$$

$$y_{d,t} = \frac{\omega}{1-\omega}\Big(\frac{p_{d,t}}{p_{f,t}}\Big)y_{f,t} \tag{D.41}$$

Government sector $\{s_t^t\}$

$$\bar{\omega}(t^l u_t^l + t^h u_t^h) + s_t^t y_t + s_t^c y_t = \tau_t\Big(w_t^{h,l}t^h n_t^{h,l} + w_t^{h,h}t^h n_t^{h,h} + w_t^{l,l}t^l n_t^{l,l} + r_t^k k_t\Big) \tag{D.42}$$

Market clearing conditions $\{c_t^l,\, e_t\}$

$$y_t = t^h c_t^h + t^l c_t^l + i_t + s_t^c y_t + \kappa^l v_t^l + \kappa^h v_t^h + b(z_t)t^h n_t^{h,l} \tag{D.43}$$

$$p_{d,t}y_{d,t}^* - p_{f,t}y_{f,t} = e_t(r_t^d d_t - d_{t+1}) \tag{D.44}$$

Closing the SOE $\{r_t^d,\, rp_t\}$

$$r_t^d = r_t^* + rp_t \tag{D.45}$$

$$rp_t = \psi^{rp}\Bigg(exp\Big(\frac{e_t d_{t+1}}{gdp_t} - \frac{ed}{gdp}\Big) - 1\Bigg) + \epsilon_t^{rp} \tag{D.46}$$

Additional definitions $\{nx_t,\, gdp_t\}$

$$nx_t = e_t(r_t^d d_t - d_{t+1}) \tag{D.47}$$

$$gdp_t = y_t + nx_t \tag{D.48}$$

The above forms a system of 48 equations in the paths of 48 unknown endogenous variables: y_t, $y_{i,t}$, $x_{i,t}$, $y_{d,t}^*$, $y_{d,t}$, $y_{f,t}$, c_t^l, c_t^h, i_t, k_{t+1}, d_{t+1}, n_t^l, $n_t^{l,l}$, $n_t^{h,l}$, $n_t^{h,h}$, x_t, z_t, s_t l_t^l, l_t^h, u_t^h, u_t^l, $\Lambda_{t,t+1}$, $\lambda_{n_t^l}$, $\lambda_{h_t^{h,l}}$, $\lambda_{n_t^{h,h}}$, r_t^k, $w_t^{l,l}$, $w_t^{h,l}$, $w_t^{h,h}$, $m_t^{l,l}$, $m_t^{h,l}$, $m_t^{h,h}$, v_t^l, v_t^h, $\psi_{H,t}^{l,l}$, $\psi_{H,t}^{h,l}$, $\psi_{H,t}^{h,h}$, $\psi_{F,t}^{l,l}$, $\psi_{F,t}^{h,l}$, $\psi_{F,t}^{h,h}$, $p_{d,t}$, r_t^d, rp_t, e_t, s_t^t, nx_t, gdp_t.[17]

Definition of labor market participants $\{lmp_t^h, lmp_t^l\}$

$$lmp_t^h = \frac{u_t^h + n_t^{h,l} + n_t^{h,h}}{u_t^h + n_t^{h,l} + n_t^{h,h} + l_t^h} \tag{D.49}$$

$$lmp_t^{l,l} = \frac{u_t^l + n_t^{l,l}}{u_t^l + n_t^{l,l} + l_t^l} \tag{D.50}$$

Functional forms $\{b(z), \phi(z)\}$

$$b(z_t) = b_1(z_t)^{b_2} \tag{D.51}$$

$$\phi(z_t) = \phi_1(z_t)^{\phi_2} \tag{D.52}$$

[17] Note that $p_{d,t} = e_t p_{d,t}^*$, $e_t = \frac{P_t^*}{P_t}$.

N Steady-state equilibrium

In the long-run, the economy reaches an equilibrium where no shocks exist and variables remain constant. Thus, all variables satisfy that $x_{t+1} = x_t = x_{t-1} = x$. The steady-state equilibrium is given by the following equations:
Hiring probabilities $\{\psi_H^{l,l},\, \psi_H^{h,l},\, \psi_H^{h,h}\}$

$$\psi_H^{l,l} = \frac{m^{l,l}}{t^l u^l} \tag{S.1}$$

$$\psi_H^{h,l} = \frac{m^{h,l}}{t^h (1-s) u^h} \tag{S.2}$$

$$\psi_H^{h,h} = \frac{m^{h,h}}{t^h s u^h + t^h \phi(z) n^{h,l}} \tag{S.3}$$

Vacancy-filling probabilities $\{\psi_F^{l,l},\, \psi_F^{h,l},\, \psi_F^{h,h}\}$

$$\psi_F^{l,l} = \frac{m^{l,l}}{(1-x) v^l} \tag{S.4}$$

$$\psi_F^{h,l} = \frac{m^{h,l}}{x v^l} \tag{S.5}$$

$$\psi_F^{h,h} = \frac{m^{h,h}}{v^h} \tag{S.6}$$

Low-skilled household $\{n^l,\, l^l,\, u^l,\, \lambda_{n^l},\, \lambda_{c^l}\}$

$$\lambda_{c^l} = (c^l)^{-\eta} \tag{S.7}$$

$$\lambda_{n^{l,l}} = \frac{\beta}{1-\beta(1-\sigma^l)} \Big[-\Phi^l (l^l)^{-\phi} + \lambda_{c^l} (1-\tau) w^{l,l} \Big] \tag{S.8}$$

$$\Phi^l (l^l)^{-\phi} = \lambda_{n^{l,l}} \psi_H^{l,l} + \bar{\omega} \lambda_{c^l} \tag{S.9}$$

$$n^{l,l} = \frac{\psi_H^{l,l} u^l}{\sigma^l} \tag{S.10}$$

$$n^{l,l} + u^l + l^l = 1 \tag{S.11}$$

High-skilled household $\{c^h,\, i,\, k,\, d,\, n^{h,l},\, n^{h,h},\, l^h, u^h,\, s,\, z,\, \lambda_{n^{h,l}},\, \lambda_{n^{h,h}},\, \lambda_{c^h}\}$

$$\lambda_{c^h} = (c^h)^{-\eta} \tag{S.12}$$

$$1 = \beta \Big(1 - \delta + (1-\tau) r^k \Big) \tag{S.13}$$

$$1 = \beta(1 + r^d) \tag{S.14}$$

$$\lambda_{n^{h,h}} = \frac{\beta}{1 - \beta(1 - \sigma^h)}\Big[- \Phi^h (l^h)^{-\phi} + \lambda_{c^h}(1 - \tau)w^{h,h}\Big] \tag{S.15}$$

$$\lambda_{n^{h,l}} = \frac{\beta}{1 - \beta\Big(1 - \sigma^l - \phi(z)\psi_H^{h,h}\Big)}\Big[- \Phi^h (l^h)^{-\phi} + \lambda_{c^h}\Big((1 - \tau)w^{h,l} - b(z)\Big) + \lambda_{n^{h,h}}\psi_H^{h,h}\phi(z)\Big] \tag{S.16}$$

$$\Phi^h (l^h)^{-\phi} = \lambda_{n^{h,h}}\psi_H^{h,h} s + \lambda_{n^{h,l}}\psi_H^{h,l}(1 - s) + \bar{\omega}\lambda_{c^h} \tag{S.17}$$

$$\lambda_{n^{h,h}}\psi_H^{h,h} = \lambda_{n^{h,l}}\psi_H^{h,l} \tag{S.18}$$

$$\lambda_{c^h}\frac{b'(z)}{\phi'(z)} = \psi_H^{h,h}(\lambda_{n^{h,h}} - \lambda_{n^{h,l}}) \tag{S.19}$$

$$t^h c^h + i + er^d d + b(z)t^h n^{h,l} = (1 - \tau)(w^{h,h}t^h n^{h,h} + w^{h,l}t^h n^{h,l} + r^k k) + ed + \bar{\omega}t^h u^h + t^h s^t y \tag{S.20}$$

$$k = \frac{i}{\delta} \tag{S.21}$$

$$n^{h,l} = \frac{\psi_H^{h,l}(1 - s)u^h}{(\sigma^l + \phi(z)\psi_H^{h,h})} \tag{S.22}$$

$$n^{h,h} = \frac{\psi_H^{h,h}\Big(su^h + \phi(z)n^{h,l}\Big)}{\sigma^h} \tag{S.23}$$

$$n^{h,l} + n^{h,h} + u^h + l^h = 1 \tag{S.24}$$

Matches $\{m^{l,l}, m^{h,l}, m^{h,h}\}$

$$m^{l,l} = \mu_1((1 - x)v^l)^{\mu_2}(t^l u^l)^{1-\mu_2} \tag{S.25}$$

$$m^{h,l} = \mu_1(xv^l)^{\mu_2}((1 - s)t^h u^h)^{1-\mu_2} \tag{S.26}$$

$$m^{h,h} = \mu_1(v^h)^{\mu_2}(st^h u^h + t^h\phi(z)n^{h,l})^{1-\mu_2} \tag{S.27}$$

Wages $\{w^{l,l}, w^{h,l}, w^{h,h}\}$

$$w^{h,h} = (1-\theta^h)\left(p_d(1-\alpha)(1-\zeta)A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{x_i}\right)^{\frac{1}{\epsilon}}\left(\frac{x_i}{t^h n^h}\right)^{\frac{1}{\rho}}+\frac{(1-\sigma^h)\kappa^h}{\psi_F^{h,h}}\right)-\frac{\theta^h}{\lambda_{c^h}(1-\tau_t)}\left(-\Phi^h(l^h)^{-\phi}+\lambda_{n^{h,h}}(1-\sigma^h)\right) \tag{S.28}$$

$$\begin{aligned} w^{h,l} = (1-\theta^{h,l})&\left(p_d\alpha q^h A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l}+t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} + \frac{(1-\sigma^l-\phi(z)\psi_H^h)\kappa^l}{\psi_F^{h,l}}\right) \\ &- \frac{\theta^{h,l}}{\lambda_{c^h}(1-\tau)}\left(-\Phi^{h,h}(l^h)^{-\phi}-\lambda_{c^h}b(z)+\lambda_{n^{h,l}}\left(1-\sigma^l-\phi(z)\psi_H^{h,h}\right)+\lambda_{n^{h,h}}\psi_H^{h,h}\phi(z)\right) \end{aligned} \tag{S.29}$$

$$w^{l,l} = (1-\theta^l)\left(p_d\alpha A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l}+t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} + \frac{(1-\sigma^l)\kappa^l}{\psi_F^{l,l}}\right) - \frac{\theta^l}{\lambda_{c^l}(1-\tau)}\left(-\Phi^l(l_t^l)^{-\phi}+\lambda_{n^{l,l}}(1-\sigma^l)\right) \tag{S.30}$$

Intermediate good firm $\{y_i, x_i, x, n^l, y_d^*, r^k, p_d, v^l, v^h\}$

$$y_i = A\Big[\alpha(t^l n^{l,l}+t^h q^h n^{h,l})^{\frac{\epsilon-1}{\epsilon}}+(1-\alpha)(x_i)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}} \tag{S.31}$$

$$x_i = \Big[\zeta(k)^{\frac{\rho-1}{\rho}}+(1-\zeta)(t^h n^h)^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}} \tag{S.32}$$

$$x = t^h\frac{(\sigma^l+\phi(z)\psi_H^{h,h})n^{h,l}}{\psi_F^{h,l}v^l} \tag{S.33}$$

$$\frac{\kappa^h}{\psi_F^{h,h}} = \beta\Big\{p_d(1-\alpha)(1-\zeta)A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{x_i}\right)^{\frac{1}{\epsilon}}\left(\frac{x_i}{t^h n^{h,h}}\right)^{\frac{1}{\rho}} - w^{h,h} + \frac{(1-\sigma^h)\kappa^h}{\psi_F^{h,h}}\Big\} \tag{S.34}$$

$$\frac{w^{h,l}}{w^{l,l}} = \frac{\beta p_d\alpha q^h A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l}+t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} + \left(\frac{\kappa^l}{\psi_F^{h,l}}\beta(1-\sigma^l-\phi(z)\psi_H^{h,h})-1\right)}{\beta p_d\alpha A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l}+t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} + \frac{\kappa^l}{\psi_F^{l,l}}(\beta(1-\sigma^l)-1)} \tag{S.35}$$

$$\frac{\kappa^l}{\psi_F^{l,l}} = \beta\Big\{p_d\alpha A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l}+t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} - w^{l,l} + \frac{(1-\sigma^l)\kappa^l}{\psi_F^{l,l}}\Big\}$$

$$\frac{\kappa^l}{\psi_F^{h,l}} = \beta\Big\{p_d\alpha q^h A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l}+t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} - w^{h,l} + \frac{(1-\sigma^l-\phi(z)\psi_H^{h,h})\kappa^l}{\psi_F^{h,l}}\Big\}$$

$$r^k = p_d\zeta(1-\alpha)A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t}}{x_{i,t}}\right)^{\frac{1}{\epsilon}}\left(\frac{x_{i,t}}{k_t}\right)^{\frac{1}{\rho}} \tag{S.36}$$

$$y_i = y_d + y_d^* \tag{S.37}$$

$$y_d^* = (1-\omega^*)\Big(\frac{p_d}{e}\Big)^{-\gamma^*}y^* \tag{S.38}$$

Economy-wide final good $\{y,\, y_d,\, y_f\}$

$$y = \Big[\omega^{\frac{1}{\gamma}}(y_d)^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}}(y_f)^{\frac{\gamma-1}{\gamma}}\Big]^{\frac{\gamma}{\gamma-1}} \tag{S.39}$$

$$1 = (\omega p_d^{1-\gamma} + (1-\omega)p_f^{1-\gamma})^{\frac{1}{1-\gamma}} \tag{S.40}$$

$$y_d = \frac{\omega}{1-\omega}\Big(\frac{p_d}{p_f}\Big)y_f$$

$$y_{d,t} = \omega(\frac{p_{d,t}}{P_t})^{-\gamma}y_t \tag{S.41}$$

$$y_{f,t} = (1-\omega)(\frac{p_{f,t}}{P_t})^{-\gamma}y_t \tag{S.42}$$

Government sector $\{s^t\}$

$$\bar{\omega}(t^l u^l + t^h u^h) + s^t y + s^c y = \tau\Big(w^{l,l}t^l n^{l,l} + w^{h,l}t^h n^{h,l} + w^{h,h}t^h n^{h,h} + r^k k\Big) \tag{S.43}$$

$$\bar{\omega}(t^l u^l + t^h u^h) + s^t y + s^c y = \tau\Big(y - p_f y_f\Big)$$

Market clearing conditions $\{c^l,\, e\}$

$$y = t^h c^h + t^l c^l + i + s^c y + \kappa^l v^l + \kappa^h v^h + b(z)t^h n^{h,l} \tag{S.44}$$

$$p_d y_d^* - p_f y_f = ed(r^d - 1) \tag{S.45}$$

Functional forms $\{b(z), \phi(z)\}$

$$b(z) = b_1(z)^{b_2} \tag{S.46}$$

$$\phi(z) = \phi_1(z)^{\phi_2} \tag{S.47}$$

In sum, we have a system of 47 equations in 47 unknown endogenous variables:[18] y, y_i, x_i, y_d^*, y_d, y_f, c^l, c^h, i, k, d, n^l, $n^{h,l}$, $n^{l,l}$, $n^{h,h}$, l^l, l^h, u^h, u^l, x, z, s, λ_{c^l}, λ_{c^h}, λ_{n^l}, $\lambda_{n^{h,l}}$, $\lambda_{n^{h,h}}$, r^k, $w^{l,l}$, $w^{h,l}$, $w^{h,h}$, $m^{l,l}$, $m^{h,l}$, $m^{h,h}$, v^l, v^h, $\psi_H^{l,l}$, $\psi_H^{h,l}$, $\psi_H^{h,h}$, $\psi_F^{l,l}$, $\psi_F^{h,l}$, $\psi_F^{h,h}$, p_d, e, s^t, $b(z)$, $\phi(z)$.

[18] In the steady state, r^d=r^{d*} since from eq. (D.45) we get that the steady-state risk premium will be zero. Note also that the following hold: $p_f = ep_d^*$, $p_d^* = 1$, $e = \frac{P^*}{P}$.

O Calibration strategy

We set r^d=0.037 using data on long-term interest rates and find β from:

$$1 = \beta(1 + r^d)$$

Using data on k/y and i/y, we find δ from:

$$\frac{k}{y} = \frac{\frac{i}{y}}{\delta}$$

We set τ=0.3 and find r^k from:

$$1 = \beta\Big(1 - \delta + (1 - \tau)r^k\Big)$$

We set employment and unemployment rates per household type using data from the Labor Force Survey (Hellenic Statistical authority). Then, l^l and l^h are obtained residually from:

$$n^{l,l} + u^l + l^l = 1$$

$$n^{h,l} + n^{h,h} + u^h + l^h = 1$$

Setting σ^l=0.10, σ^h=0.06, $\psi_H^{h,h}$=0.14 and $\psi_H^{h,l}$=0.88, we solve the following system of equations for s and $\phi(z)$:

$$n^{h,l} = \frac{\psi_H^{h,l}(1 - s)u^h}{(\sigma^l + \phi(z)\psi_H^{h,h})}$$

$$n^{h,h} = \frac{\psi_H^{h,h}\Big(su^h + \phi(z)n^{h,l}\Big)}{\sigma^h}$$

Next, we find $\psi_H^{l,l}$ from the following steady-state law of motion:

$$n^{l,l} = \frac{\psi_H^{l,l}u^l}{\sigma^l}$$

We solve for $m^{l,l}$, $m^{h,l}$ and $m^{h,h}$ from the definitions of hiring probabilities:

$$\psi_H^{l,l} = \frac{m^{l,l}}{t^l u^l}$$

$$\psi_H^{h,l} = \frac{m^{h,l}}{t^h(1 - s)u^h}$$

$$\psi_H^{h,h} = \frac{m^{h,h}}{t^h su^h + t^h n^{h,l}}$$

We find xv^l and $\psi_F^{h,l}$ from:

$$\psi_F^{h,l} = \frac{m^{h,l}}{xv^l}$$

$$x = t^h\,\frac{(\sigma^l + \phi(z)\psi_H^{h,h})n^{h,l}}{\psi_F^{h,l}v^l}$$

Then, we find v^l (and then x) from:

$$\psi_F^{l,l} = \frac{m^{l,l}}{(1-x)v^l}$$

We solve for μ_1 and μ_2 from the matching functions:

$$m^{l,l} = \mu_1((1-x)v^l)^{\mu_2}(t^l u^l)^{1-\mu_2}$$

$$m^{h,l} = \mu_1(xv^l)^{\mu_2}((1-s)t^h u^h)^{1-\mu_2}$$

We find v^h and $\psi_F^{h,h}$ from the following system of equations:

$$\psi_F^{h,h} = \frac{m^{h,h}}{v^h}$$

$$m^{h,h} = \mu_1(v^h)^{\mu_2}(st^h u^h + t^h n^{h,l})^{1-\mu_2}$$

We control for the elasticity between physical capital and skilled labor ($\rho = 0.67$), for the elasticity between low-skilled labor and capital-skilled labor ($\epsilon = 1.12$), share of low-skilled labor ($\alpha = 0.36$) and we pin down total vacancy costs to be 1% of GDP. Furthermore, by setting the price levels P, p_d^* equal to 1, and in order to match k/y, y^f/y, y^{d^*}/y, d/y, $w^{h,h}/w^{l,l}$, $w^{h,l}/w^{l,l}$, c/y from the data, the following equations form a system of 17 equations on 17 unknowns: y, k,y_i, x_i, y_d, y_f,e, p_f, p_d, $w^{h,h}$, $w^{h,l}$, $w^{l,l}$, γ, q^h, ω,ζ and κ^l.

$$y_i = A\Big[\alpha(t^l n^{l,l} + t^h q^h n^{h,l})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(x_i)^{\frac{\epsilon-1}{\epsilon}}\Big]^{\frac{\epsilon}{\epsilon-1}}$$

$$x_i = \Big[\zeta(k)^{\frac{\rho-1}{\rho}} + (1-\zeta)(t^h n^h)^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}}$$

$$\frac{\kappa^h}{\psi_F^h} = \beta\Big\{p_d(1-\alpha)(1-\zeta)A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{x_i}\right)^{\frac{1}{\epsilon}}\left(\frac{x_i}{t^h n^{h,h}}\right)^{\frac{1}{\rho}} - w^{h,h} + \frac{(1-\sigma^h)\kappa^h}{\psi_F^h}\Big\}$$

$$\frac{w^{h,l}}{w^{l,l}} = \frac{\beta p_d \alpha q^h A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l}+t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} + \left(\frac{\kappa^l}{\psi_F^{h,l}}\beta(1-\sigma^l-\phi(z)\psi_H^h)-1\right)}{\beta p_d \alpha A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l}+t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} + \frac{\kappa^l}{\psi_F^{l,l}}(\beta(1-\sigma^l)-1)}$$

$$\kappa^l = \frac{\beta p_d \alpha q^h A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l}+t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} - w^{h,l}))}{\frac{1}{\psi_F^{h,l}}(1-\beta(1-\sigma^l-\phi(z)\psi_H^{h,h}))}$$

$$r^k = p_d\zeta(1-\alpha)A_t^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_{i,t}}{x_{i,t}}\right)^{\frac{1}{\epsilon}}\left(\frac{x_{i,t}}{k_t}\right)^{\frac{1}{\rho}}$$

$$y_i = y_d + y_d^*$$

$$y = \Big[\omega^{\frac{1}{\gamma}}(y_d)^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}}(y_f)^{\frac{\gamma-1}{\gamma}}\Big]^{\frac{\gamma}{\gamma-1}}$$

$$1 = (\omega p_d^{1-\gamma} + (1-\omega)p_f^{1-\gamma})^{\frac{1}{1-\gamma}}$$

$$y_{d,t} = \omega(\frac{p_{d,t}}{P_t})^{-\gamma} y_t$$

$$y_{f,t} = (1-\omega)(\frac{p_{f,t}}{P_t})^{-\gamma} y_t$$

$$p_d y_d^* - p_f y_f = ed(r^d - 1)$$

$$p_f = e p_d^*$$

$$k = (k/y)y$$

$$y_f = (y_f/y)y$$

$$w^{l,l} = (w^{l,l}/w^{h,h})w^{h,h}$$

$$w^{h,l} = (w^{l,l}/w^{h,l})w^{h,l}$$

Foreign output $y^\star$ and general price level P^* are obtained from the following two equations:

$$y_d^* = (1-\omega^*)\Big(\frac{p_d}{e}\Big)^{-\gamma^*} y^*$$

$$e = \frac{P^*}{P}$$

Equations (S.12), (S.15), (S.16), (S.17), (S.18), (S.20), (S.43) and (S.44) can be written as a function of only Φ^h and the government spending share in output, s^c. Hence, using these equations, the ratio c/y from the data and by setting unemployment benefits ($\bar{\omega} = 0.55w^{l,l}$) we calibrate these two parameters and solve for the variables $s^t, c^h, c^l, b(z), \lambda_\eta^{h,h}, \lambda_\eta^{h,l}, \lambda_c^h$:

$$\lambda_{c^h} = (c^h)^{-\eta}$$

$$\lambda_{n^{h,h}} = \beta\Big[- \Phi^{h,h}(n^{h,h})^{-\phi^h} + \lambda_{c^h}(1-\tau)w^{h,h} + \lambda_{n^{h,h}}(1-\sigma^h) - \lambda_l^h\Big]$$

$$\lambda_{n^{h,l}} = \beta\Big[- \Phi^h(n^{h,l})^{-\phi^h} + \lambda_{c^h}\Big((1-\tau)w^{h,l} - b(z^h)\Big) + \lambda_{n^{h,l}}\Big(1-\sigma^l - \phi(z^h)\psi_H^{h,h}\Big) + \lambda_{n^{h,h}}\psi_H^{h,h}\phi(z) - \lambda_l^h\Big]$$

$$\lambda_l = \lambda_{c^h}\bar{\omega} + \lambda_{n^{h,h}}\psi_H^{h,h}s + \lambda_{n^{h,l}}\psi_H^{h,l}(1-s)$$

$$\lambda_{n^{h,h}}\psi_H^{h,h} = \lambda_{n^{h,l}}\psi_H^{h,l}$$

$$t^h c^h + i + er^d d + b(z)t^h n^{h,l} = (1-\tau)(w^{h,h}t^h n^{h,h} + w^{h,l}t^h n^{h,l} + r^k k) + ed + \bar{\omega}t^h u^h + t^h s^t y$$

$$\bar{\omega}(t^l u^l + t^h u^h) + s^t y + s^c y = \tau\Big(w^{l,l}t^l n^{l,l} + w^{h,l}t^h n^{h,l} + w^{h,h}t^h n^{h,h} + r^k k\Big)$$

$$y = t^h c^h + t^l c^l + i + s^c y + \kappa^l v^l + \kappa^h v^h + b(z)t^h n^{h,l}$$

The previous equations yield $s^c = 0.2$ and $\Phi^h = 0.01$. We set the Frisch elasticity with respect to labor $\phi = 1.5$ and we derive firms' bargaining power $\theta^{h,h}$ and $\theta^{h,l}$ from the following two equations:

$$w^{h,h} = (1-\theta^h)\left(p_d(1-\alpha)(1-\zeta)A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{x_i}\right)^{\frac{1}{\epsilon}}\left(\frac{x_i}{t^h n^h}\right)^{\frac{1}{\rho}} + \frac{(1-\sigma^h)\kappa^h}{\psi_F^h}\right) - \frac{\theta^h}{\lambda_{c^h}(1-\tau_t)}\left(-\Phi^h(l^h)^{-\phi} + \lambda_{n^{h,h}}(1-\sigma^h)\right) \tag{S.28}$$

$$\begin{aligned} w^{h,l} = (1-\theta^{h,l})&\left(p_d\alpha q^h A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l} + t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} + \frac{(1-\sigma^l-\phi(z)\psi_H^h)\kappa^l}{\psi_F^{h,l}}\right) \\ &- \frac{\theta^{h,l}}{\lambda_{c^h}(1-\tau)}\Bigg(-\Phi^h(l^h)^{-\phi} - \lambda_{c^h}b(z) + \lambda_{n^{h,l}}\Big(1-\sigma^l-\phi(z)\psi_H^{h,h}\Big) \\ &+ \lambda_{n^{h,h}}\psi_H^{h,h}\phi(z)\Bigg) \end{aligned} \tag{S.48}$$

The remaining unknowns for the low-skilled households, $\Phi^l, \theta^l, \lambda_\eta^{l,l}, \lambda_c^l$, are derived from the following equations:

$$\lambda_{c^l} = (c^l)^{-\eta}$$

$$\lambda_{n^{l,l}} = \frac{\beta}{1-\beta(1-\sigma^l)}\Big[-\Phi^l(l^l)^{-\phi} + \lambda_{c^l}(1-\tau)w^{l,l}\Big]$$

$$\Phi^l(l^l)^{-\phi} = \lambda_{n^{l,l}}\psi_H^{l,l} + \bar{\omega}\lambda_{c^l}$$

$$w^{l,l} = (1-\theta^l)\left(p_d\alpha A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{t^l n^{l,l} + t^h q^h n^{h,l}}\right)^{\frac{1}{\epsilon}} + \frac{(1-\sigma^l)\kappa^l}{\psi_F^{l,l}}\right) - \frac{\theta^l}{\lambda_{c^l}(1-\tau)}\left(-\Phi^l(l_t^l)^{-\phi} + \lambda_{n^{l,l}}(1-\sigma^l)\right) \tag{S.49}$$

Regarding the parameters of the cost of the on-the-job search (b_1, b_2), the efficacy of this search (ϕ_1, ϕ_2) and the search effort to end mismatch z, we use the following procedure: first we normalize $\phi_1 = 1$ and we set the cost of search (b_2) to be quadratic. Finally, by using the following equations, we solve for ϕ_2, b_1, z:

$$b(z) = b_1(z)^{b_2}$$

$$\phi(z) = \phi_1 (z)^{\phi_2}$$

$$\lambda_{c^h} \frac{b'(z)}{\phi'(z)} = \psi_H^{h,h} (\lambda_{n^{h,h}} - \lambda_{n^{h,l}})$$

P More impulse responses to fiscal tightening shocks

The fiscal tightening shocks initially cause a depreciation of the exchange rate, which raises net exports on impact (see Figure A.6). In line with the decrease in imports during the first half of the time horizon, the domestically sold intermediate good decreases as well, and thus the intermediate good declines. The responses of the exchange rate and exports are reversed in the subsequent periods.

The high-skill wage rises on impact due to the participation decline but falls in later periods since labor demand (vacancies) falls (see Figure A.7). The low-skill wage slightly rises in the baseline calibration again due to the reduction of participation. The standard skill wage premium (i.e., of high-skilled versus low-skilled workers) slightly falls on impact in the baseline calibration, indicating a fall in wage inequality. Finally, there is an increase in the unemployment rate for the high-skilled household after the impact period (in which the reduction in participation dominates), essentially driven by the decline in its employment, but a decrease for the low-skilled household, coming from the decline in its labor market participation and its rather quick employment recovery.

Q Other shocks and skills mismatch

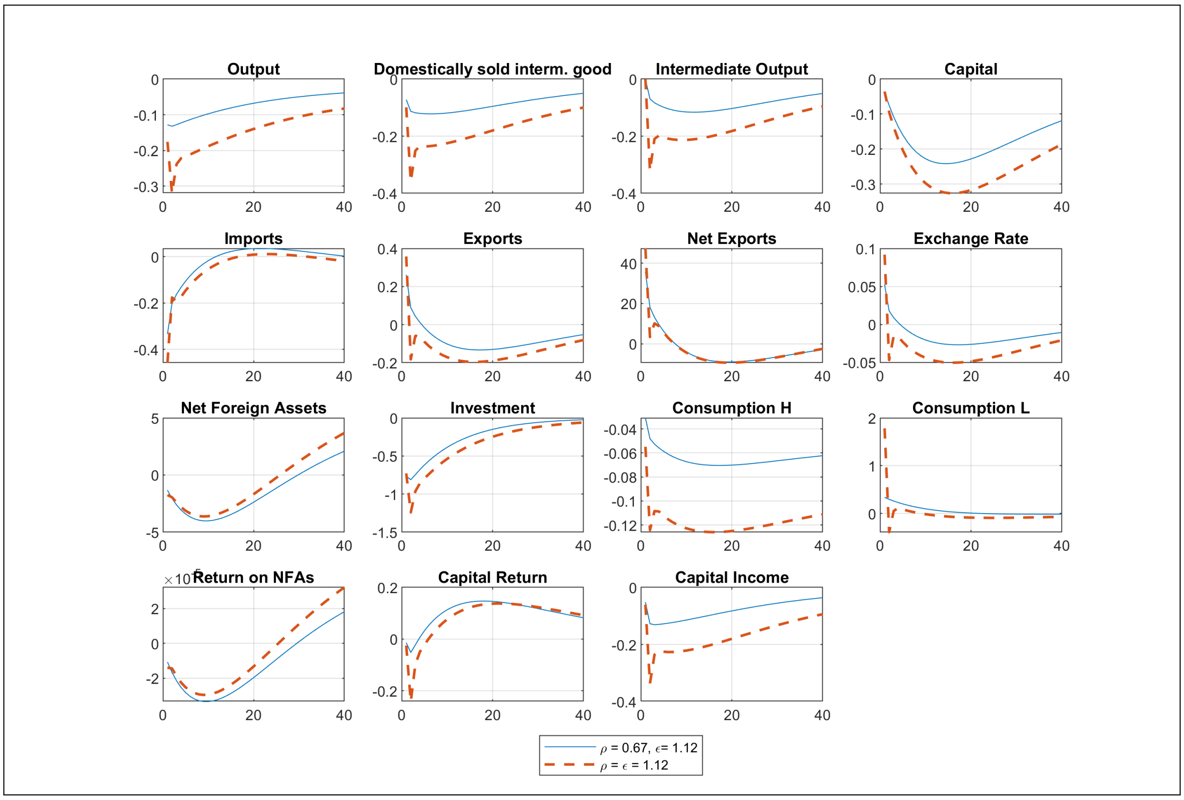
Q.1 Negative TFP shock and skills mismatch

Following a negative shock to total factor productivity, intermediate output falls as expected (see panel (a) in Figure A.8). Consumption decreases for both household types and investment drops too, resulting in a capital (income) decline. Given the fall in intermediate output and since the exchange rate appreciates, we see a fall in net exports in the baseline calibration. The high-skilled household chooses to invest more in net foreign assets in the first half of the time horizon due to the increased returns.

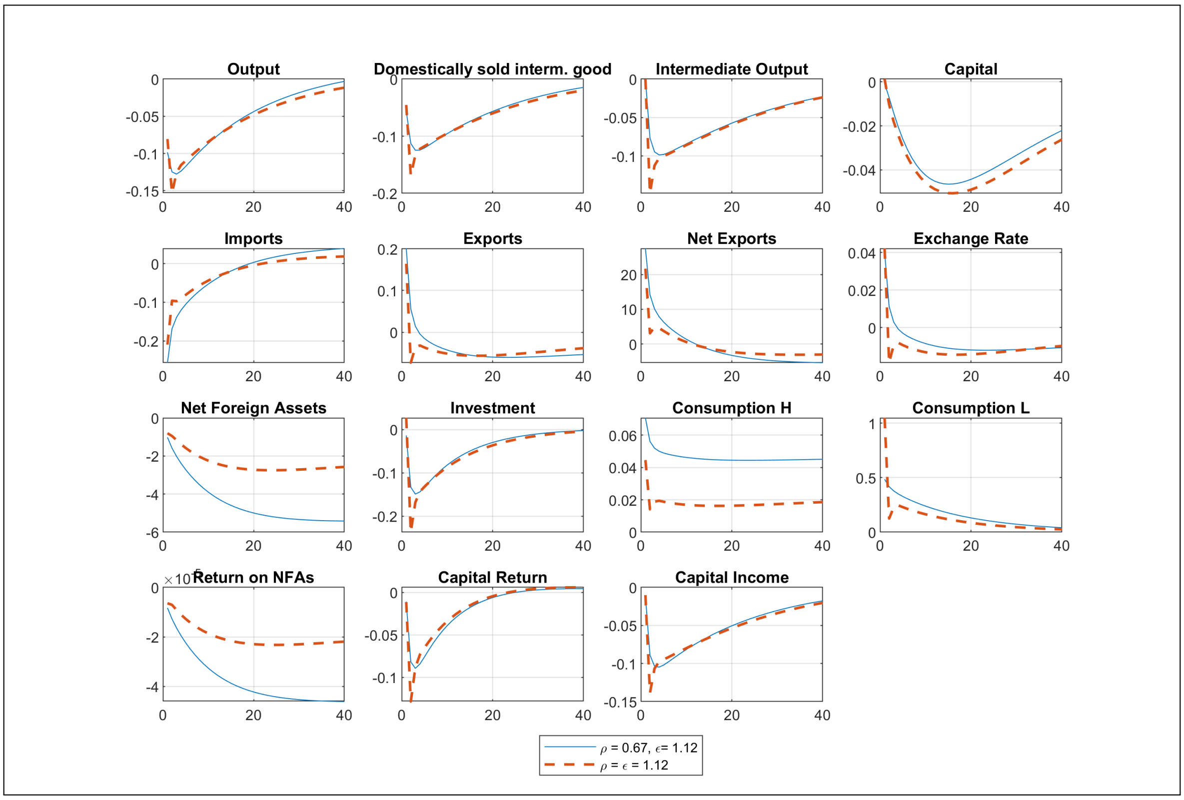
Due to the fall in the marginal product of labor (MPL), all wages decrease after the TFP shock (see panel (a) in Figure A.9). In terms of wage inequality, the gap between the high-skilled and the low-skilled widens. As mentioned previously, high-skilled households experience a drop in capital income, which is why they increase labor market participation after the impact period. Following the TFP shock and the fall in the MPL, high-skill vacancies fall on impact but low-skill vacancies rise. As a result, high-skill employment decreases but low-skilled employment increases. Labor market participation for the low-skilled households rises and so does aggregate participation. Unemployment rises temporarily for low-skilled households and more persistently, but with a delay, for high-skilled households.

Quits from mismatch jobs to take up high-skill positions decrease given the fall in the latter (see panel (b) in Figure A.9). For that reason, a negative TFP shock leads to a persistent increase in mismatch employment in our model economy. This happens despite the fact that aggregate low-skill vacancies are tilted less towards high-skilled (mismatched) applicants on impact and despite the subsequent fall in the fraction of searchers for a

Figure A.6: Responses of macroeconomic variables to fiscal tightening shocks



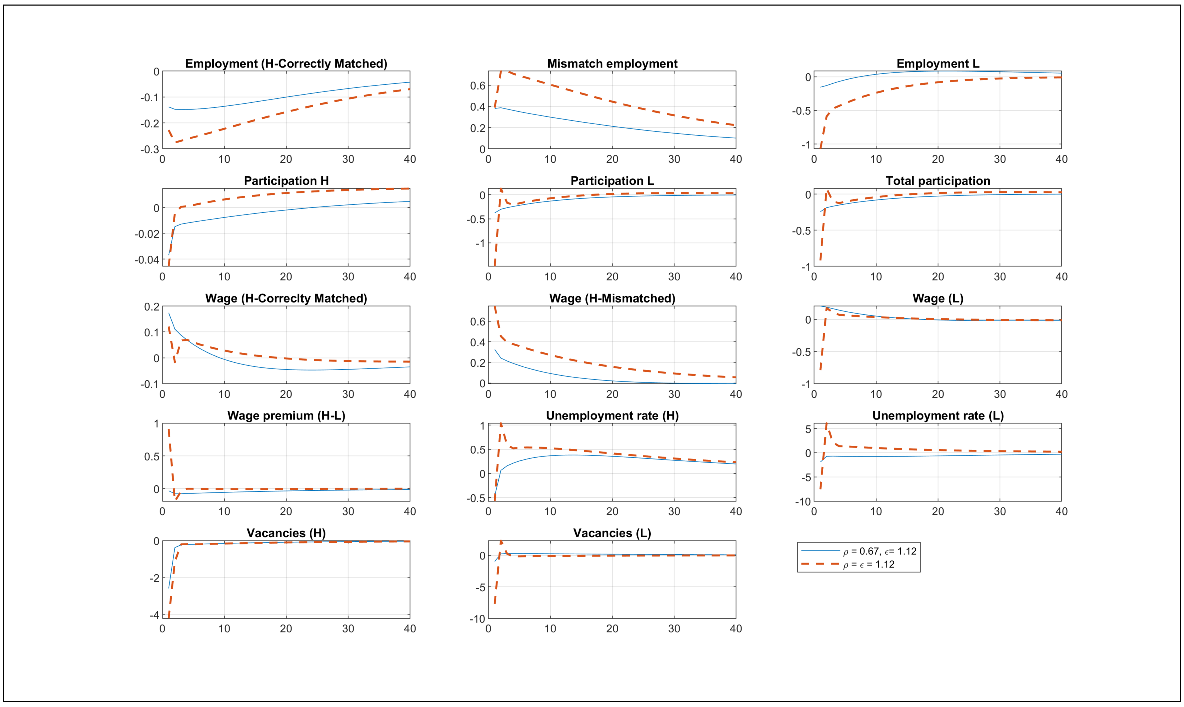
(a) 1% positive income tax rate shock



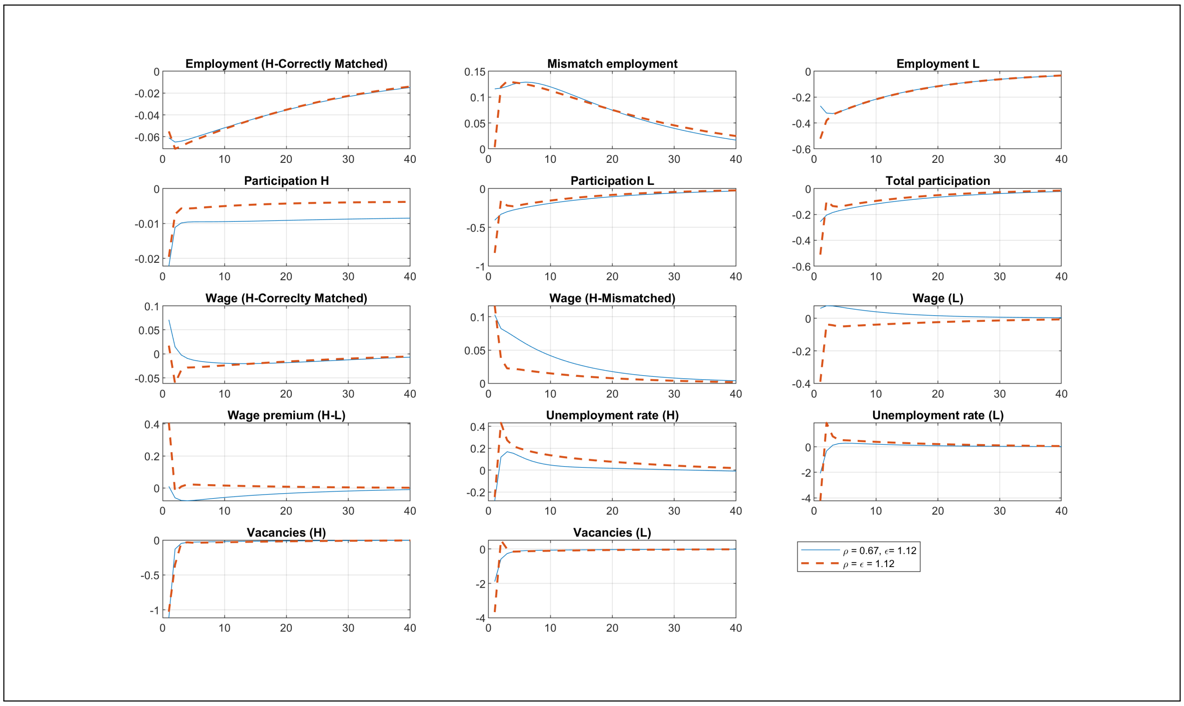
(b) 1% negative government spending shock

Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively.

Figure A.7: Responses of labor market variables



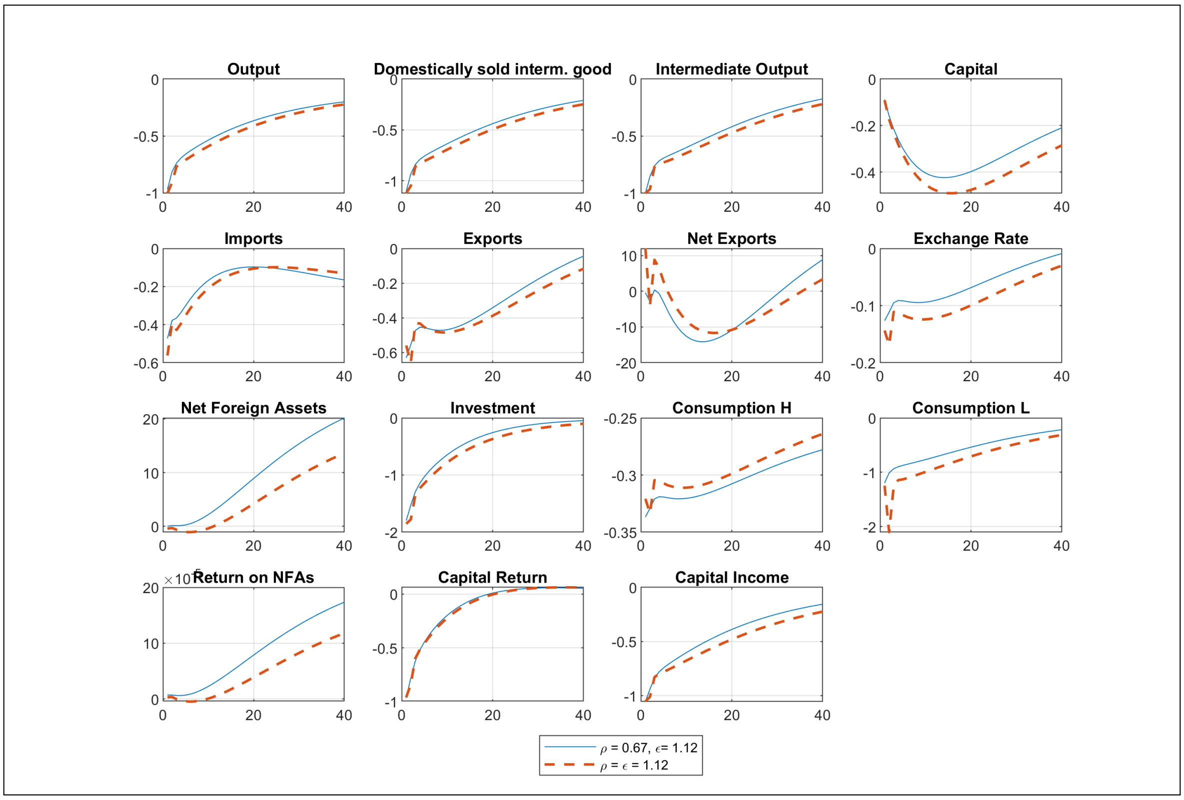
(a) 1% positive tax rate shock



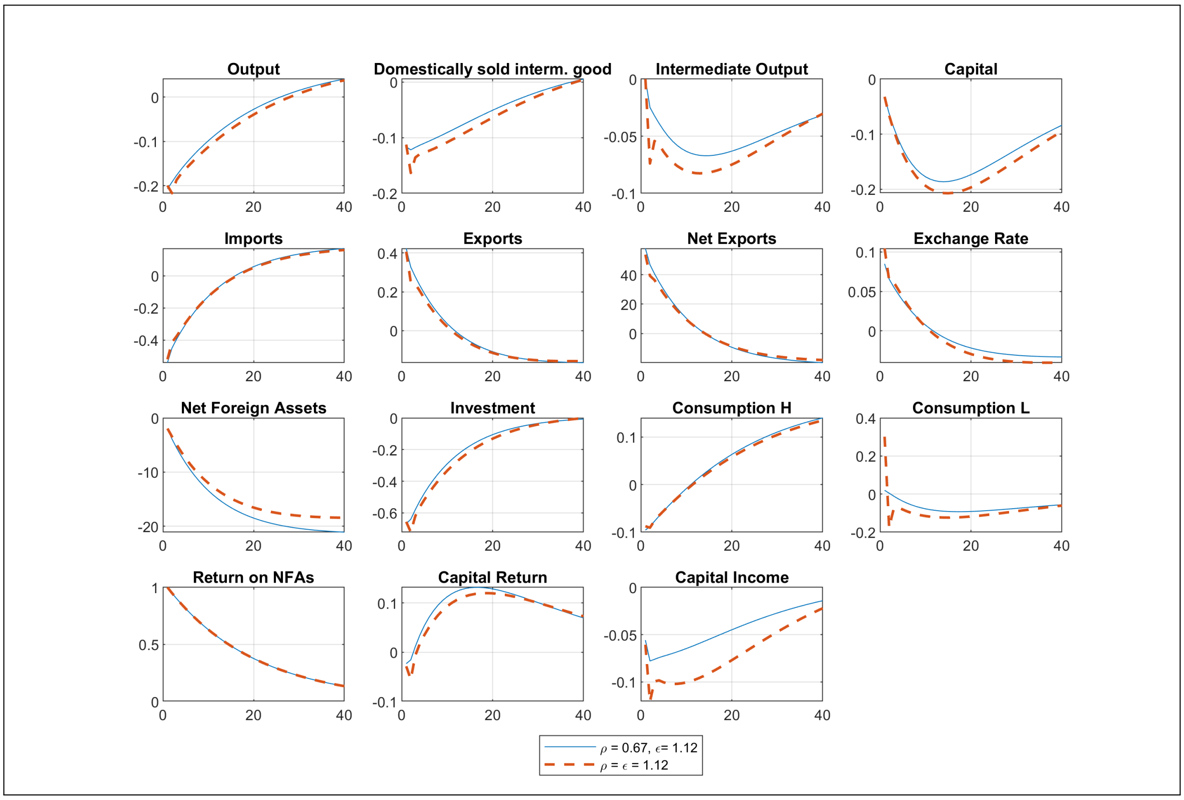
(b) 1% negative government spending shock

Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively.

Figure A.8: Responses of macroeconomic variables to other shocks



(a) 1% negative TFP shock



(b) 1% positive risk premium shock

Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively.

mismatch job. In conclusion, a negative TFP shock raises mismatch employment through a drop in mismatch quits.

Sensitivity analysis. As with the tax shocks, the rise in the mismatch rate after a TFP shock is substantially magnified in the absence of CSC in the production process (see panel (b) in Figure A.9). The high-skilled household reacts now to the shock by raising, rather than reducing, the share of searchers for mismatch jobs. The drop in output and investment are amplified. Therefore, the intuition we studied in Section 5.3 for tax shocks continues to hold here.

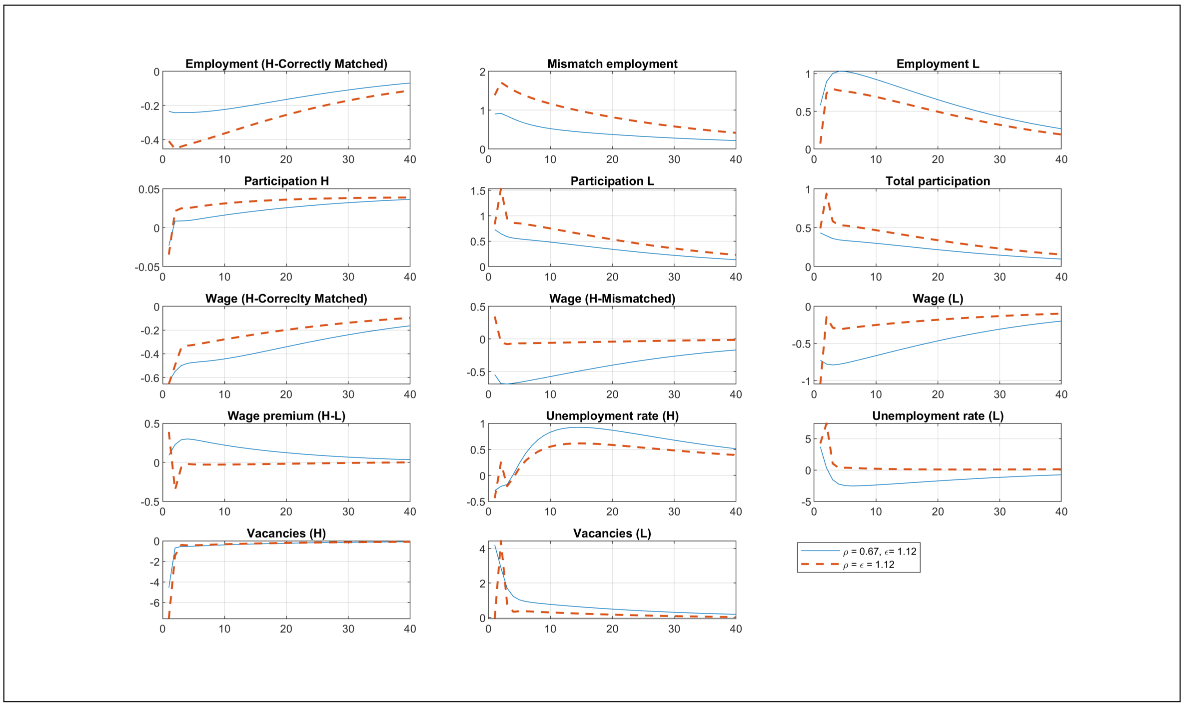
Q.2 Risk premium shock and skills mismatch

In the presence of a positive risk premium shock, the cost of participating in international markets increases, and so net foreign assets decrease (see panel (b) in Figure A.8). An exchange rate depreciation causes exports to increase and imports to decrease, resulting in an increase in net exports. Following the decrease in imports, domestically-sold output drops since both inputs are assumed complementary in the production of final output, which decreases as well. Intermediate output decreases given the decline of investment and the capital stock in the economy. The return on foreign assets increases, and so does with a lag the return on capital. Consumption of low-skilled households decreases after the impact period due to their income drop, while for high-skilled households it increases after a few periods given the increase in their income from NFAs.

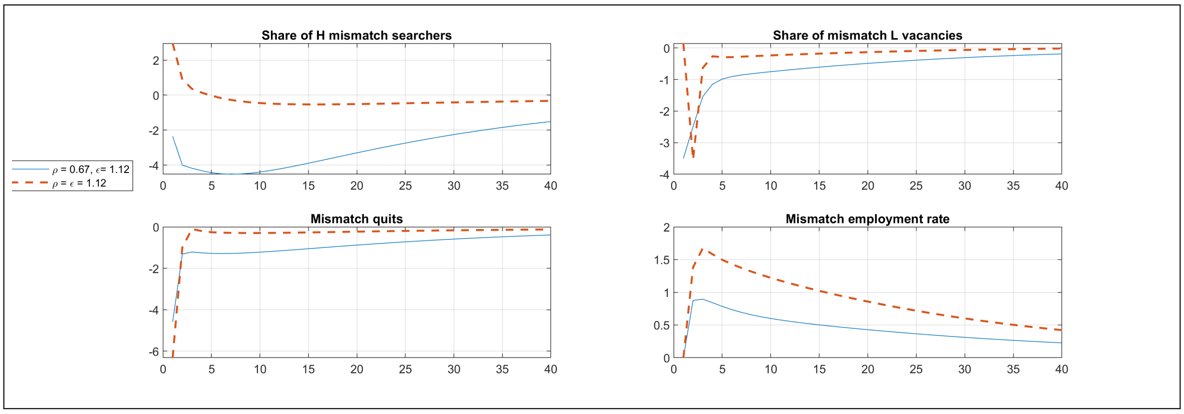
High-skilled employment decreases, but low-skilled employment rises above the steady state (see panel (a) in Figure A.10). Mismatch employment rises consistently with a drop in mismatch quits (see panel (b) in Figure A.10). Vacancies requiring high skills go down, but low-skill ones stay relatively unaffected. Labor market participation rises for both households, but the effect is short-lived for the high-skilled one. A higher fraction of low-skill vacancies is on impact allocated to mismatch searchers, thus pushing up on mismatch employment, but the effect is temporary and quickly changes sign. The share of high-skilled searching for mismatch jobs rises on impact and falls persistently later, in line with the share of mismatch vacancies.

Sensitivity analysis. As with the tax and TFP shocks, the rise in the mismatch rate after a risk premium shock is magnified in the absence of CSC in the production process (see panel (b) in Figure A.10), but the variation between the baseline and the alternative calibration seems more muted now. Again, the high-skilled household reacts to the shock by raising, rather than reducing, the share of searchers for mismatch jobs. The drop in output and investment are slightly amplified. Therefore, the intuition we studied in Section 5.3 for tax shocks continues to hold not only for TFP shocks but also for risk premium shocks in our model.

Figure A.9: Responses of labor market and mismatch variables to a 1% negative TFP shock



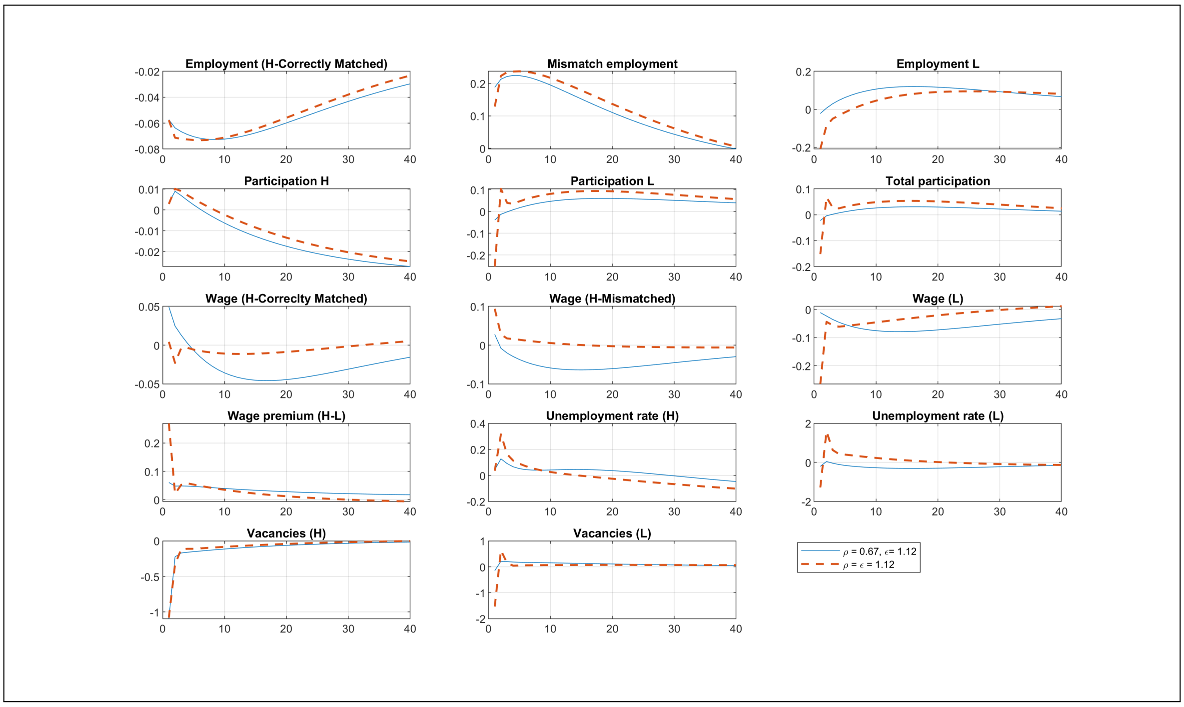
(a) labor market variables



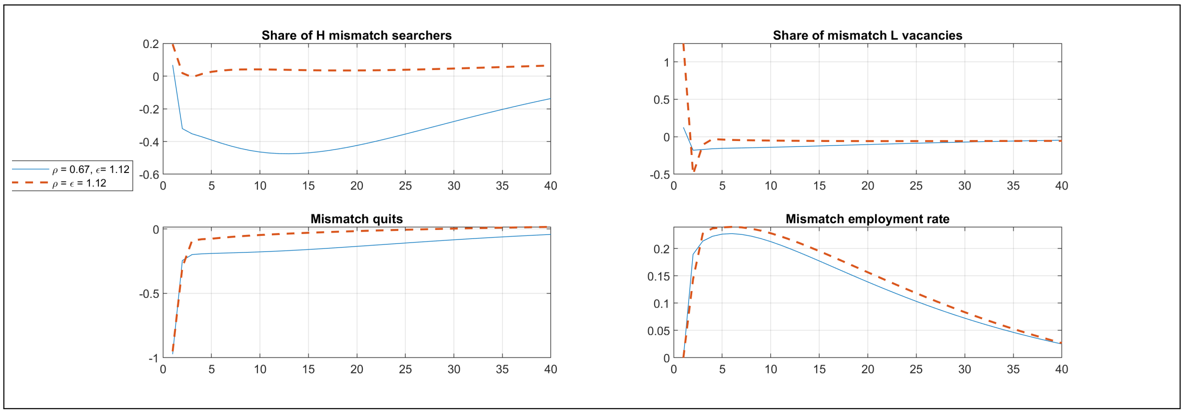
(b) mismatch variables

Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively.

Figure A.10: Responses of labor market and mismatch variables to a 1% risk premium shock



(a) labor market variables



(b) mismatch variables

Notes: The continued lines refer to the case with capital-skill complementarity (CSC), while the dashed lines represent the no CSC case. Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively.