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Conceptions of Rationality and their Justifications

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Abstract

Economic rationality puts constraints on the preferences and/or degrees of belief (subjective probabilities or credences) of a decision maker (DM). This paper focuses on a belief-based definition of rationality (referred to as BEL): BEL requires DM's credences to be precise, unique, ascertainable, coherent and asymptotically accurate. We distinguish two types of DMs, the "expert" (DM_s) and the "naive" (DM_o) ones, and ask whether and how BEL may be achieved by either of them. To answer this we define two sets of cognitive/epistemic properties, $\mathcal{Y}_s$ and $\mathcal{Y}_o$ for DM_s and DM_o, respectively and show that $\mathcal{Y}_s$ and $\mathcal{Y}_o$ form the basis of the corresponding processes (referred to as Bayesian Confirmation (BC) and "trial and error, frequency-based (TEFB), respectively) by which DM_s and DM_o reach BEL. This means that on the assumption that $\mathcal{Y}_s$ and $\mathcal{Y}_o$ are empirically valid, the naive decision maker thinks probabilistically "as-if" she were the expert. An important difference between this paper and the related literature concerns the "obscurity" of the "as-if" process. In our approach, this is a concrete process, namely TEFB, instead of an unspecified, "black box" one. We also argue that some of the assumptions in $\mathcal{Y}_o$ (on which standard arguments of rationality – such as the Dutch Book and Arbitrage arguments - are based) are empirically questionable. Finally, we suggest that although BEL is the normative standard against which beliefs must be measured and judged, the actual rationality of decision makers comes in degrees (graded rationality). The smooth functioning of the economic system requires decision makers who are "sufficiently" rather than "perfectly" rational.

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1 1. Introduction

The notion of rationality is fundamental to several disciplines, including economics, philosophy of science and psychology. The first question an analysis of rationality must address is "what does it mean for someone to be rational?" In other words, how is rationality defined? Is there more than one concept/definition of rationality? To answer these questions, we need to decide on the characteristics of the agent's personality to which the intended definition of rationality refers. Economists, mainly the ones with behaviouristic inclinations, believe that rationality concerns agent's *preferences* over *acts* (Savage 1954) or *propositions* (Jeffrey 1965). To the extent that a person's preferences are related to this person's choices, this *behaviouristic view* implies that rationality should be defined in terms of *how people act.*

The majority of philosophers and psychologists (with some notable exceptions), on the other hand, interpret agent's beliefs and desires as real mental states, residing in agent's mind. Lewis (1974), for example, argues that knowing what X *believes* and what X *desires* is necessary for coming to know X as a person. In fact, he puts forward the stronger claim that all other attitudes of X "are analyzable as patterns of belief and desire, actual or potential." (1974, p.332). On this *mentalistic view*, rationality is about beliefs, thus characterising *how people think or reason.*[1] On the other hand, although the mentalisitc view recognizes desires as real mental entities, it places no rationality constraints on them. This thesis stems from the so-called Humean theory of human motivation, according to which desires cannot be "reasonable or unreasonable" (see Hazlett 2021). In short, the mentalistic view identifies the concept of rationality with that of "rational beliefs", while not recognizing as legitimate the concept of "rational desires".

Beliefs refer to propositions or events and come in degrees. These degrees are usually referred to as "credences" (or subjective probabilities).[2] A DM's credence, $Cr(A)$, in a proposition A is usually interpreted as the degree to which DM is confident in the truth of A. It is usually assumed that A belongs to a Boolean algebra, $\mathcal{F}$, of propositions, describing events that are relevant for the decision problem at hand. On the mentalistic view, DM's beliefs/credences and desires/utilities are treated as *primitives*, that is unanalyzable concepts, which cause/explain DM's preferences and choices. Jeffrey (2004) says: "(Subjective) probability is a mode of judgment. From this point of view probabilities are "in the mind" the subject's, say YOURS...". Eriksson and Hajek (2007) concur: "We should not be afraid of taking 'degree of belief' as a primitive in our con-

[1] Some prominent philosophers who deviate from the mainstream mentalistic view are Ramsey (1926), Eells (1982) and Maher (1999) For example, Maher (1999) argues: "I suggest that we understand attributions of probability and utility as essentially a device for interpreting a person's preferences. On this view, an attribution of probabilities and utilities is correct just in case it is part of an overall interpretation of the person's preferences that makes sufficiently good sense of them and better sense than any competing interpretation does." (1999, p. 9).

[2] The terms "subjective probabilities" (used mainly in the economic literature) and "credences" (used mainly in the philosophical literature) are identical. In this article, for reasons of economy, we use the one-word term "credences".

ceptual apparatus." (p. 185). The primacy of beliefs over preferences was also emphasized by Tversky and Kahneman (1974): "It should perhaps be noted that, while subjective probabilities can sometimes be inferred from preferences among bets, they are normally not formed in this fashion. A person bets on team A rather than on team B because he believes that team A is more likely to win; he does not infer this belief from his betting preferences. Thus, in reality, *subjective probabilities determine preferences among bets and are not derived from them*, as in the axiomatic theory of rational decision." (1974, p. 1130, emphasis added).

The opposite is true according to the behaviouristic view. In the most radical version of behaviourism, DM's choices is the only entity that is directly observable by an independent observer. Credences, utilities, even preferences are not observable, which in turn implies that they must be treated as fictional, albeit useful, entities, which may serve as representations of DM's choices strictly in an "as-if" manner. On this view, credences and utilities are merely *instruments* that are not represented in DM's cognitive system. Ahmed (2014) expresses this view as follows: "But the behaviouristic comments that I made about personal probability ... do also apply to utility. We have no preference-independent way to identify an agent's utilities and probabilities. So we shouldn't, for instance, think of utility as directly measuring intensity of pleasure, or amounts of money, or anything else at which a rational agent might aim. Nor should we think of utilities and probabilities as jointly causing or otherwise explaining what the agent does. Nor should we think of SEU-maximization as an internalizable principle that might guide deliberation. It is not as though, prior to making any decisions about what to do, the agent can first identify his utilities ..., and his probabilities ... and then distribute preferences across acts in accordance with SEU-maximization. Rather, his own probabilities and utilities are not identifiable as such, even to him, prior to the formation of any practical dispositions. Crudely: if he takes the umbrella then he thinks rain likely; but if he doesn't then that is not a mistake, because if he doesn't then he already thinks that rain is unlikely." (2014, p.28). [3]

Is rationality (however defined) something that economists take for granted? In other words, is rationality a primary and independent theoretical concept ? Or, alternatively, is it a secondary, derivative concept arising logically from other more primitive cognitive and epistemic properties that characterize DM? The majority of economists treat rationality as an axiom in their theories. Consequently, as is usually the case with axioms, rationality does not need special justification. Besides, as Oaksford and Chater () remind us, "...humans are, almost by definition, *rational* animals." (????, p. 2). Mc Fadden summarizes this view as follows: "...it is hard for many economists to imagine that *failures*

[3] Although preferences are not directly observable, they are, nonetheless, connected to observed choices in a fairly tight manner, and in any case they are closer to choices than utilities and credences are. For example, Maher (1999) gives the following behavioristic definition of preferences: "You prefer g to f just in case you are disposed to choose g when presented with a choice between f and g." (1999, pp. 12-13). On this view, when we talk about "preferences", we effectively talk about "dispositions to choose".

of rationality could infect major economic decisions or *survive market forces*" (1999, p. 74, emphasis added). In this paper, we take the alternative route of treating rationality as a "secondary" theoretical concept derivable from a set, $\mathcal{Y}$, of DM's primitive cognitive and epistemic features. The main questions of the paper are the following: Are there more than one set $\mathcal{Y}$ on the basis of which rationality (as defined in the paper) is reached? If the answer to this question is affirmative (as we shall argue in the sequel) then what are the processes (with each process corresponding to a particular set $\mathcal{Y}$) by which a DM endowed with a particular set $\mathcal{Y}$ arrives at rationality? Finally, are the assumptions comprising each set $\mathcal{Y}$ realistic?

The rest of the paper is organized as follows: Section 2 analyzes and compares two concepts of rationality; one based on the preferences (PRE) and one based on the beliefs (BEL) of a decision maker (DM). Both of these concepts are based on the notion of "internal or structural consistency" of preferences/beliefs (internal criteria of rationality). Section 3 raises the question of whether rationality concerns only internal consistency of preferences/beliefs or it further requires a specific relation between the DM's preferences/beliefs and some aspects of the empirical world (external criteria of rationality). We argue that such an additional relation is mandatory to obtain a concept of rationality that respects our pre-theoretical intuitions about what "it is to be rational". Specifically, we take the view that this relation should be based on the notion of "accuracy" and as a result we focus on BEL, since it is meaningful to talk about "accurate beliefs", but it does not make much sense to talk about "accurate preferences"[4]. Section 4 introduces the theoretical framework within which the main questions of the paper are analyzed. Sections 5 and 6 define two types of decision makers: the "expert" DM_s (a.k.a. the "scientist") and the "ordinary" ("naive") decision maker DM_o with no formal probabilistic training and no special epistemic skills, respectively. They also define the sets $\mathcal{Y}_s$ and $\mathcal{Y}_o$ of cognitive/epistemic properties of DM_s and DM_o, respectively that are required for BEL and analyze the processes by which DM_s and DM_o, (endowed by the epistemic properties $\mathcal{Y}_s$ and $\mathcal{Y}_o$, respectively), reach BEL. Specifically, in Section 5 we argue that DM_s employs a standard "formal" procedure known as Bayesian Confirmation (BC) to attain BEL, driven by the scientific motivation of "seeking the truth". In Section 6 we argue that DM_o utilizes a pragmatic "trial and error, frequency-based" procedure (hereafter referred to as TEFB) to attain BEL, motivated by the pragmatic pursuit of "hunting for profits", yet behaving *as if* she were probabilistically sophisticated despite a lack of literacy. Finally, in Sections 5 and 6 we analyze which of the assumptions in $\mathcal{Y}_s$ and $\mathcal{Y}_o$ on which BC and TEFB are based respectively, are realistic. To that end, in Section 5 we identify two critical assumptions in $\mathcal{Y}_s$ whose realism seems at least questionable, namely "the problem of the priors" and "The problem of specifying the wrong chance theory". In Section 6, we argue that there are also two assumptions in $\mathcal{Y}_o$ that

[4] In order to define the concept of "accuracy of preferences", we must first define the concept of "objective preferences", against which to evaluate the accuracy of subjective preferences. However, objective preferences are very difficult to determine. After all, objective preferences pave the way for "objective ethics," a notoriously controversial concept.

seem to be doubtful, namely "The Dutch Book Problem" and "The Reference Class Problem". Section 7 raises the issue of the normative status of BEL in the face of overwhelming evidence against its descriptive adequacy. We also advocate the idea that "rationality comes in degrees" (distances from BEL), which gives rise to the concept of "graded rationality". The smooth functioning of the economic system requires decision makers who are "sufficiently" rather than "perfectly" rational. Section 8 concludes the paper.

2 2. Preference-based versus Belief-based Rationality

From the introductory discussion it is apparent that there are two candidate notions of rationality, one that is based on preferences and another one that is based on beliefs. A common element in both these notions is that rationality requires either preferences or beliefs to obey a formal calculus: A DM is rational iff her preferences comply with, for example, Savage's (1954) axioms. Or, a DM is rational iff her probabilistic beliefs satisfy, for example, the axioms of Kolmogorov's (1930) mathematical theory of probability. In either case, what is normatively appealing (the normative standard) is dictated by what is formally admissible. Moreover, the preference-based definition of rationality (PRE) is related to the beliefs-based one (BEL) through the so-called *representation theorems*. A typical representation theorem has the following structure: Iff DM's preferences satisfy certain axioms (e.g. Savage's) then there exists a unique probability function, P, defined on the field, $\mathcal{F}$, of events relevant for the decision problem and a unique (up to a positive linear transformation) utility function, U, defined on the set of outcomes, such that DM prefers action f to action g iff the expected utility of f is greater than the expected utility of g, where the expected utilities of f and g are computed by means of P and U. It must be emphasized that this mathematical theorem does not assert that the functions P and U correspond to DM's subjective credence function Cr and subjective utility function U_a (see, Christensen 2001, Okasha 2014). This creates the following asymmetry in the relation between PRE and BEL: BEL implies (via the representation theorem) PRE. This is beause on the mentalistic view, Cr and U_a not only exist, but they explain/cause DM's preferences as well. The mentalistic view has no problem to map the theoretical entitites P and U to the empirical ones Cr and U_a, respectively. On the other hand, the only entities the behaviouristic view acknowledge are P and U, which are treated as "mere instruments", not to be found (or at least measured) anywhere in the empirical world. On this interpretation, PRE implies that P is coherent, but it does not entail anything about Cr (since the latter does not exist). This means that under the behaviouristic view, PRE does not explain BEL.[5]

[5] This particular interpretation, namely the distinction between Cr and P and between U_a and U is necessary if one wishes to maintain (as the typical behaviourist does) the "ordinal" interpretation of utility that underlies the traditional economic theory: Specifically, the fact that Savage's RT delivers a cardinal U does not entail the cardinality of U_a, since these two

3 3. Internal and External Criteria of Rationality

Should rationality be defined solely in terms of the consistency properties of preferences or beliefs (*internal* criteria of rationality)? Or should rationality be enhanced by empirical considerations as well (*external* criteria of rationality)? More specifically, our intuitions suggest that "being rational" must somehow be related with "being right" (to borrow the title of Juan Comesana's recent book, 2020)[6]. We do not merely wish to have some beliefs, we desire true beliefs. In more pragmatic terms, Mellor (1995) argues: "We want our beliefs to be true in order to make the actions they combine with our desires to cause succeed, i.e. achieve the object of those desires." (1995, p. 45).The "accuracy of beliefs" as an explicit rationality constraint is particularly prevalent in macreoeconomics in the context of the so-called Rational Expectations Hypothesis (REH). Specifically, REH (in its strictest form) requires the economic agents to set (at each point in time) their subjective credences equal to the corresponding objective (model-implied) probabilities (see, for example Sheffrin 1996, Kirman 2014).

However, if we wish to define rationality in a way that includes the notion of accuracy, then defining rationality in terms of beliefs seems to be the only available option. This is due to the fact that the notion of "objective probabilities" or "chances" (in terms of which the external criteria on beliefs are codified) has no counterpart in terms of preferences: whereas objective probabilities (for example, relative frequencies) may be used as a guide to credences, the obscured notion of "objective preferences" cannot play a similar role in regulating subjective preferences.

Based on the above considerations, we are now ready to give a comprehensive, belief-based definition of rationality:

Definition 1 ***(Belief-based Definition of Rationality (BEL))*** *A decision maker (DM) is rational iff:*

a) For each point in time t, $t = 0, 1, 2, \ldots$, DM is capable of eliciting a sharp and unique credence function Cr_t defined on $\mathcal{F}$.

b) For each point in time t, $t = 0, 1, 2, \ldots$, DM's credence function Cr_t is coherent, that is it obeys the rules of probability calculus.

c) For $t = m$, with m being sufficiently largre, DM's credence function Cr_m is "sufficiently close" to the corresponding chance function Ch (also defined on $\mathcal{F}$).

Remarks

(i) BEL combines "coherence requirements" often encountered in the microeconomics literature (for, example in the context of the representation theorems)

entities are different (in spite of the fact that they are both referred to as "utility"). On the traditional interpretation, U is just a cardinal index and U_a does not exist.

[6] For example, we would be unwilling to call a person rational if she assigns a probability of 0.65 to the event "the sun will rise tomorrow" even if she attaches a probability of 0.35 to its complement.

with "accuracy requirements" often met in the macroeconomics literature (for example in the context of the rational expectations hypothesis).

(ii) The first rationality constraint requires DM to be capable of eliciting or ascertaining her own sharp and unique credences. Levi (2000) distinguishes between "imprecise probabilities", arising from the decision maker's *inability* to access her credal state and find out what is actually there and "indeterminate probabilities", corresponding to the case in which DM's credal state *is* actually vague. Hence, imprecise probabilities arise from difficulties in elicitation: "an agent has a unique subjective probability function, but she (or another ascriber) cannot figure out exactly what it is. Her credal state is in fact perfectly sharp, but there is some epistemic obstacle to accessing it; she (or the ascriber) simply doesn't know her mind." (Hajek and Smithson 2012, p. 4-5, emphasis added). In Salmon's terminology, credences that can be elicited with full sharpness are called *ascertainable*: "This criterion requires that there be some method by which, in principle at least, we can ascertain values of probabilities. It merely expresses the fact that a concept of probability will be useless if it is impossible in principle to find out what the probabilities are." (1966, p. 64). The issue of "ascertainability" of DM's credences is analyzed in detail in the next sub-section.

(iii) The second condition of coherence of DM's credence function for every t is not as strict as it appears at first reading. As will be shown later, DM can meet this condition if she ensures the coherence of her prior credence function, Cr_0, only once at her epistemic life, that is, at the initial time point $t = 0$, and then forms her subsequent posterior credence functions, Cr_1, $Cr_2, ...$ by conditionalizing on the incoming information I_1, $I_2, ...$, respectively (for more discussion on this point, see section 6).

(iv) The third condition may be thought of as requiring the "asymptotic accuracy" of DM's credences. DM's prior credence function Cr_0 can be arbitrarily inaccurate (due to no information available) and still DM may be rational. This means that accuracy is a property of DM's credences that will emerge in the future, when relevant probabilistic information becomes available. This view is correct up to a certain point. Although it is pointless to raise the issue of the accuracy of Cr_0 *per se*, it is still important to distinguish the properties that Cr_0 must have, in order for some future credences Cr_m (that emanate from Cr_0) to be sufficiently accurate (see section 6).

What are the cognitive abilities/epistemic properties that a DM must possess in order for the rationality constraints of BEL to be satisfied? Sections 4-7 will try to answer this question. To do so, we will distinguish between two types of decision makers. These two types differ with respect to (a) their deductive reasoning abilities, (b) their knowledge of the formal probability calculus, and (c) whether they theorize about chance. The first type, denoted by DM_o, is the "ordinary" (or "naive") decision maker who has no formal probabilistic training. The second type, denoted by DM_s, is the "expert" decision maker (a.k.a. the "scientist"). We will present the epistemic properties each of these types of decision makers must possess, in order for the rationality constraints of BEL to be satisfied.

4 4. Theoretical Setup

Section 4 aims at introducing the theoretical material necessary for the analysis that follows in sections 5-7. We assume throughout the paper that the objects of belief are *propositions* of a $\sigma-$algebra $\mathcal{F}^*$, made out of a countable set of atomic propositions and the truth functional connectives "$\neg$" (negation) and "$\vee$" (disjunction).[7] The $\sigma-$algebra $\mathcal{F}_s^*$ (or the language) of DM_s is broader than the $\sigma-$algebra $\mathcal{F}_o^*$ of DM_o. More specifically, $\mathcal{F}_s^*$ contains three types of propositions: a) Simple propositions: Propositions that describe *random or chancy events*. For example, A_1: "the coin lands Heads in the next toss" and A_2 : "the coin lands Heads in the first toss and Tails in the second" are simple propositions. Simple propositions belong to $\mathcal{F}$. b) Chance propositions: Propositions that state the chances of simple propositions. For example. A_{ch} : "the chance of A is equal to 0.5". c) Theoretical propositions, H_i, $i = 1, 2, ..., n$. Each H_i defines a family of chance propositions, with this family being represented by chance function, Ch_i, defined on $\mathcal{F}$.

As already mentioned, DM_o does not consider theoretical propositions, which means that $\mathcal{F}_o^*$ contains simple propositions as well as chance propositions, provided that the latter propositions are interpreted by DM_o as stating relative frequencies. Both DM_s and DM_o are interested in the chance propositions A_{ch}, $A_{ch} \in \mathcal{F}_s^*$, where for DM_o, the proposition A_{ch} states the relative frequency of A within a selected "reference class" (see Section 7). The rest of the discussion will focus exclusively on DM_s, although the main points apply also to the case of DM_o.

Since DM_s is interested in A_{ch}, she forms credences in these propositions, namely $Cr(A_{ch})$, $A_{ch} \in \mathcal{F}_s^*$, which implies that DM_s's credence function Cr (being either the prior credence function Cr_0, or any posterior credence function Cr_m) is defined over $\mathcal{F}_s^*$ (and not just $\mathcal{F}$). As already mentioned, a chance proposition is a statement about the chance of A, e.g. $\langle Ch(A) = x_A \rangle$, $x_A \in [0, 1]$. This raises the question of what kind of entity the chance $Ch(A)$ of A is. It is generally accepted that $Ch(A)$ is an objective property of A, which is independent of what anyone believes about A. Joyce (2009) argues that $Ch(A) = x_A$ is true if propositions with A's "overall epistemic profile" are true (roughly) x_A proportion of the time.[8] For the rest of the paper, a chance function Ch will be assumed to be a proper probability measure defined on $\mathcal{F}$, (but not on $\mathcal{F}_s^*$).[9] This means that the chance, $Ch(A)$, of A makes formal sense, whereas

[7] Given that $\mathcal{F}^*$ is σ-algebra, it is closed under countable conjunctions "$\wedge$" as well.

[8] One leading interpretation of chance, defines $Ch(A)$ as the *relative* or *limiting relative (hypothetical) frequency* at which the proposition A is true (or the event A occurs) within a given reference class (see, for example, Reichenbach 1949, von Mises 1957). Another interpretation, defines $Ch(A)$ as the *propensity* of the underlying chance mechanism to produce outcomes that comprise the event A (see, Popper 1957).A third interpretation, defines chance indirectly, namely via the role it plays in guiding our credences (see Lewis 1980). Specifically, $Ch(A)$ is whatever in the objective world we turn to in order to get advise for forming $Cr(A)$.

[9] The question of whether relative frequencies or limiting relative frequencies satisfy the axioms of Kolmogorov does not have a simple answer. An early result by Reichenbach (1956) shows that (finite) relative frequencies satisfy finite additivity and (trivially) countable addi-

$Ch(\langle Ch(A) = x_A \rangle)$ does not.

On the other hand, Cr is defined on $\mathcal{F}_s^*$. Note that the $\sigma-$algebra $\mathcal{F}$ is embedded in the larger $\sigma-$algebra $\mathcal{F}^*$.[10] The reason is that a large part of the ensuing discussion will make extensive use of the concept of "DM_s's credences about chance propositions". This is because chance propositions carry direct probabilistic information (to which DM_s might find it useful to defer).

5 5. Rationality of the Expert

5.1 5.1 Cognitive and Epistemic Properties of the Expert

(Set of Assumptions $\mathcal{Y}_s$):

(S1) DM_s knows elements of formal probability calculus, for example the theorem of total probability and the Bayes' theorem.

(S2) DM_s acknowledges the epistemic role of "chance" in the formation of her credences. This means that DM_s always "defers to chance", whenever the latter is known. Moreover, DM_s's knowledge of the chance of A "screens off" any other probabilistic information from her credence in A.[11]

(S3) (a) DM_s approaches the problem of the formation of a rational credence function, by conceiving a chance theory $\mathcal{H} = \{H_1, H_2, ..., H_n\}$. (b) The true hypothesis, H^*, is included in $\mathcal{H}$.

(S4) For all $i = 1, ..., n$, $0 < Cr_0^s(H_i) < 1$, where $Cr_0^s(H_i)$ is her credences in H_i formed at the beginning of her epistemic life at $t = 0$. Moreover, $\sum_{i=1}^{n} Cr_0^s(H_i) = 1$.

(S5) DM_s is able to derive the relation between each hypothesis H_i and the corresponding chance function Ch_i that it entails.

(S6) DM_s is a Bayesian conditionalizer.

Remarks:

(i) Bayesian Conditionalization ensures the *dynamic consistency* of DM_s's beliefs.

(ii) The second epistemic attitude of DM_s (S2) is that she acknowledges a specific role to objective probabilities (chances), namely that they should serve (if known) as a guide to the corresponding credences. Deference to Chance, takes the form of a "probabilities coordination principle", the most prominent of which is Lewis's (1976) Principal Principle (PP). More precisely, assume that a) the chance $Ch(A)$ of A is equal to x_A and b) the proposition $\langle Ch(A) = x_A \rangle$ is known to DM. PP states that for every $A \in \mathcal{F}$ and for every empirical

tivity. On the other hand, de Finetti (1970) proves that limiting relative frequencies violate countable additivity (see Hajek 2???)

[10] Again for notational simplicity, we use F^* to represent both F_s^* and F_o^*. Similarly, whenever there is no need to distinguish between DM_s and DM_o, we shall use the common notation "DM".

[11] For example if DM_s knows that the coin is fair, she disregards any other information (e.g. a series of outcomes of past tosses) bearing on the "chance of heads".

proposition E that is *admissible* for A :[12]

$$Cr_0^s\left[A \mid \langle Ch(A) = x_A \rangle \wedge E\right] = x_A \tag{1}$$

provided that $Cr_0^s\left[\langle Ch(A) = x_A \rangle \wedge E\right] > 0$. van Fraasen (1983) put forward the concept of "credal frequentism", that is, the view that "credences should constitutively aim at being close to relative frequencies" (Caie, ????, p. 14).

(iii) It should be noted that "deference to chance" is not a universally accepted epistemic attitude. In fact, some decision makers do not acknowledge the epistemic role of chance, described above. Specifically, they do not feel the need to hunt $Ch(A)$ in order to form an accurate credence $Cr(A)$. For such DMs, the issue of accuracy is not an issue at all. Some go so far as to deny the existence of chances altogether. They fully embrace de Finetti's famous aphorism that "(objective) probability does not exist". Specifically, de Finetti argues that the very notion of objective probability is nothing but a projection of some specific properties of our subjective beliefs (e.g. symmetries) onto the real world: "The abandonment of superstitious beliefs about the existence of the Phlogiston, the Cosmic Ether,...or Fairies and Witches was an essential step along the road to scientific thinking. Probability, too, if regarded as something endowed with some kind of objective existence, is no less a misleading misconception, an illusory attempt to exteriorize or materialize our true probabilistic beliefs." (1974, p. x). Long before de Finetti, Jevons (1877) made a similar claim, namely that "probability belongs wholly to the mind" (1877/1913, p. 197-198).

(iv) S4 aims to eliminate the "problem of the priors" (see section 6). As will be shown in section 6, $Cr_0^s(H_i)$, $i = 1, 2, ..., n$ are the only input that DM_s has to supply directly. The rest of her credences, $Cr_0^s(A)$, $A \in \mathcal{F}$ will be calculated mechanically in the context of Bayesian Conditionalization.

5.2 5.2. The Road to BEL for the Expert (Bayesian Confirmation Process)

Given the cognitive and epistemic abilities of DM_s, she ends up having unique, sharp, coherent and accurate credences (hence, satisfying the BEL criteria) by pursuing the following Bayesian Confirmation process (BC).

[12] When is E admissible for A? Lewis does not give a precise definition of admissibility of E. He explicitly states that he has "no definition of admissibility to offer". Instead, he gives a working characterisation of admissibility in the form of "sufficient (or almost sufficient) conditions for admissibility" (1980, p. 92). According to this characterisation, E is admissible for A if it furnishes no information about the truth of A over and above the information that is already contained in the chance of A. This means that E is inadmissible for A if it carries some specific information for A that $Ch(A)$ lacks. An example of inadmissible evidence is the proposition E : "The outcome of the next toss of the coin is heads", which may be thought of as information "coming from the future". Lewis suggests that all empirical propositions that refer to the past are admissible for propositions that refer to the present or future (1980, p. 275).

5.2.1 5.2.1. Uniqueness, Sharpness and Coherence of DM_s's prior credence function Cr_0^s for t=0

The following four steps of BC establish the uniquencess, sharpness and coherence of DM_s's prior credence function,Cr_0^s, formed at the beginning of her epistemic life, namely at $t = 0$.

Step 1:

In the first step, DM_s develops a "chance theory" (S3a). As already mentioned, a chance theory consists of an exhaustive list $\mathcal{H} = \{H_1, H_2, ..., H_n\}$ of alternative hypotheses about the objective probabilities of the propositions in $\mathcal{F}$[13]. Which hypotheses are included in $\mathcal{H}$, is a matter of DM_s to decide. For the moment, we assume that the *true* chance hypothesis H^* is included in $\mathcal{H}$ (S3b). Once DM_s has conceived $\mathcal{H}$ and given (S5) and (S1), she can proceed in the second step of BC:

Step 2:

This step is based on the supposition that each hypothesis H_i, $i = 1, 2, ..., n$ assigns a physical probability (i.e. chance), $x_{A,i}$, to each and every A, $A \in \mathcal{F}$, that is[14]

$$A \xrightarrow{H_i} x_{A,i},\ x_{A,i} \in [0, 1],\ A \in \mathcal{F},\ i = 1, 2, ..., n. \tag{2}$$

Put differently, each H_i makes, as Hawthorne (1984) puts it, "a statistical claim" about (the event that is described by) the proposition A, for every $A \in \mathcal{F}$. We also assume that for each pair of i and j, $i, j = 1, 2, ..., n$, H_i and H_j are empirically distinct, in the sense that there are some A for which $x_{A,i} \neq x_{A,j}$. The relations (2) imply that each chance hypothesis H_i entails an objective probability $x_{A,i}$ for every $A \in \mathcal{F}$, thus defining a unique chance function Ch_{H_i} (or, in a simpler notation, Ch_i) on $\mathcal{F}$. Moreover, these chance functions, Ch_i, $i = 1, 2, ..., n$ are distinct. Specifically,

$$H_i \models < Ch_i(A) = x_{A,i} >,\ A \in \mathcal{F},\ i = 1, 2, ..., n. \tag{3}$$

The "objective" relations (3) are the main reason why DM_s finds it indispensable to develop a chance theory. As Hawthorne (1994) observes: "The principal epistemic role of theoretical hypotheses is to underwrite *relatively objective* probabilities for individual events" (1994, p. 243). The relations (3) imply that each theory H_i is sufficiently precise to determine the chances of every $A \in \mathcal{F}$. This means that H_i specifies (among other things) the process by which the data are generated. For example, consider the chance hypothesis, H_1, part of

[13] The set of alternative hypotheses may be infinite. Here, we make the simplified assumption that the aforementioned set is finite.

[14] Some authors argue that the relationship between H_i and A is a "partial entailment" relationship, which is treated as an extention of the entailment relationship of the standard logic. On this interpretation, H_i entails A to the degree $x_{A,i}$, that is $H_i \overset{x_{A,i}}{\models} A$. For example, assume that H is the conjuction of the premises $\Pi 1$: "All A are B" and $\Pi 2$: "X is an A". Then H entails the conclusion Π_c :"X is a B" to the degree 1. Now instead of the universal generalization $\Pi 1$, assume that we have the statistical law, $\Pi 3$: "70% of A are B" Then the conjuction $H_S : \Pi 3 \wedge \Pi 2$ entails Π_c to the degree 0.7. In other words, $\Pi 3$ and $\Pi 2$ assign an inductive, objective probability of 0.7 to Π_c.

which is "the coin is fair and the repeated tosses of the coin are independent and identically distributed". Hence, H_1 entails the chances of propositions that describe finite sequences of coin tosses, as well as those of propositions that describe limiting properties of such sequences. In short, H_1 assigns objective probabilities to all possible pieces of evidence. DM_s is assumed to be able to uncover the mapping from $\mathcal{F}$ to $[0,1]$ that each H_i implies (S5).[15] Once these derivations are completed, DM_s can invoke PP and set her conditional credences $Cr_0^s(A \mid H_i)$, usually refered to as the *likelihoods*, equal to the corresponding objective probabilities $x_{A,i}$ (S2).

Step 3:

In the third step of the credence-building process, DM_s must decide about her subjective prior credences, $Cr_0^s(H_i)$, that assigns to H_i $i = 1, 2, ..., n$ (S4).

Step 4:

Finally, in the last step of BC, DM_s calculates her credence in A, $A \in \mathcal{F}$ as a weighted average of the likelihoods $Cr_0^s(A \mid H_i)$ of A using her prior credences, $Cr_0^s(H_i)$, of H_i as the relevant weights.[16]

$$Cr_0^s(A) = \sum_{i=1}^{n} Cr_0^s(A \mid H_i) Cr_0^s(H_i) \overset{(??)}{=} \sum_{i=1}^{n} Ch_i(A) Cr_0^s(H_i) = \sum_{i=1}^{n} x_{A,i} Cr_0^s(H_i) \tag{4}$$

Implicit in (4) is the assumption that DM_s knows the theorem of total probability and applies it accordingly (S1).

The four steps described above ensure that DM_s ends up with a sharp, unique and coherent *prior* credence function Cr_0^s (defined on $\mathcal{F}^*$). Note that the prior credence function Cr_0^s may or may not be accurate. In the next subsection we analyze whether (S1-S6) ensure sharpness, uniqueness, coherence and accuracy for the subsequent time periods, $t = 1, 2, ...$

5.2.2 5.2.2. Uniqueness, Sharpness, Coherence and Accuracy of DM_s's Posterior Credence Function Cr_t^s for t=1,2,...

Once DM_s has accomplished the task of producing a coherent prior credence function, she is well equipped to start her "learning experience". Specifically, as evidence becomes available, DM_s reallocates her credences accordingly, via Bayesian Conditionalization (S6). If the conditions (S1-S6) hold, this learning process leads to sufficient accuracy of DM_s's credences, via Bayesian convergence, sooner or later. In fact, once the major task of manufacturing a coherent prior credence function, Cr_0^s, is achieved, the only activity in which DM_s must

[15] This is not always a trivial task. If H_i, $i = 1, 2, ..., n$ are all simple hypotheses with a narrow scope (e.g. the coin is fair and the tosses are i.i.d.), then obtaining (3) might be relatively straightforward. If, however, each H_i represents a broad and complex theory, then obtaining (3) might require special skills on the part of DM^s. Hence, it is tacitly assumed that DM^s knows (apart from the probability calculus) all the calculi that are necessary for the derivation of (3).

[16] It must be emphasized that by following this method of constructing her prior credence function, DM has already imposed the constraints on this function that are implied by PP.

engage is that of "observation": each time that she receives a piece of information I_t, all that she has to do is to visit her prior credal state, check what her probabilistic commitements were at that moment (conditional on I_t) and adopt them as her current credences.

Assume that at time t, DM_s learns that the empirical proposition I_t is true (equivalently, she observes the data I_t). Given that DM_s is a Bayesian conditionalizer (S6), her posterior credence function, Cr_t becomes:

$$Cr_t^s(A) = Cr_0^s(A \mid I_t) = \sum_{i=1}^{n} Cr_0^s(A \mid H_i, I_t) Cr_0^s(H_i \mid I_t). \tag{5}$$

Assume that I_t is "admissible" in the sense that it provides no information for A over and above the information contained in H_i. This is admissibility in Lewis's (1980) sense: "Once the chances are given outright, ...evidence bearing on them no longer matters." (1980, p. 276). In other words, DM_s's knowledge of the objective probability of A screens off any other probabilistic information I_t from her credence in A (S2). This means that:

$$Cr_0^s(A \mid H_i, I_t) = x_{A,i}. \tag{6}$$

Substituting $Cr_0^s(A \mid H_i, I_t)$ from (6) in (5) yields,

$$Cr_t^s(A) = Cr_0^s(A \mid I_t) = \sum_{i=1}^{n} x_{A,i} Cr_0^s(H_i \mid I_t). \tag{7}$$

Next, focus on the conditional credences $Cr_0^s(H_i \mid I_t)$. To that end, we first note that

$$Cr_0^s(H_i \mid I_t) = \frac{Cr_0^s(I_t \mid H_i) Cr_0^s(H_i)}{\sum_{j=1}^{n} Cr_0^s(I_t \mid H_j) Cr_0^s(H_j)}.$$

Substituting the last expression for $Cr_0^s(H_i \mid I_t)$ in (7), we finally arrive at

$$Cr_t^s(A) = \sum_{i=1}^{n} x_{A,i} \frac{Cr_0^s(I_t \mid H_i) Cr_0^s(H_i)}{\sum_{j=1}^{n} Cr_0^s(I_t \mid H_j) Cr_0^s(H_j)} = \sum_{i=1}^{n} x_{A,i} \frac{1}{1 + \sum_{j \neq i} \frac{Cr_0^s(I_t \mid H_j) Cr_0^s(H_j)}{Cr_0^s(I_t \mid H_i) Cr_0^s(H_i)}} \tag{8}$$

The term $Cr_0^s(I_t \mid H_i)$ in (8) is the likelihood of I_t under H_i, $i = 1, 2, ..., n$. Given that DM_s respects PP, and given that each H_i entails an objective probability $z_{I_t,i}$, respectively for I_t, she will set her likelihoods of I_t equal to the corresponding chances,

$$Cr_0^s(I_t \mid H_i) = Ch_i(I_t) = z_{I_t,i}. \tag{9}$$

Hence,

$$Cr_t^s(A) = \sum_{i=1}^{n} x_{A,i} \frac{1}{1 + \sum_{j \neq i} \frac{z_{I_t,j} Cr_0^s(H_j)}{z_{I_t,i} Cr_0^s(H_i)}}. \tag{10}$$

The last equation shows that since DM_s's prior credences $Cr_0^s(H_i)$ are unique and coherent (S4), her posterior credence function Cr_t^s defined on $\mathcal{F}$ will be a proper probability function as well.

Next let us turn our attention to the issue of asymptotic accuracy of Cr_t, that is for $t \to \infty$. To that end, let us first assume that the experiment is repeated under independent and identically distributed (*i.i.d.*) conditions.[17] In this case, the law of large numbers applies, which means that relative frequencies converge to the corresponding objective probabilities. As a result, we obtain the so-called "likelihood ratio convergence". Specifically, assume that H_1 is the true hypothesis and H_j, $j = 2, 3, ..., n$ are all false hypotheses and consider the following ratio,

$$\frac{z_{I_t,1}}{z_{I_t,j}} \times \frac{Cr_0^s(H_1)}{Cr_0^s(H_j)}. \tag{11}$$

Since H_1 is the true hypothesis, the ratio $\frac{z_{I_t,1}}{z_{I_t,j}} \to \infty$ (equivalently, $\frac{z_{I_t,j}}{z_{I_t,1}} \to 0$), as more information accumulates. This means that

$$\begin{aligned}
\frac{1}{1 + \sum_{j \neq i} \frac{z_{I_t,j} Cr_0^s(H_j)}{z_{I_t,i} Cr_0^s(H_i)}} &\longrightarrow 1,\ i = 1 \\
\frac{1}{1 + \sum_{j \neq i} \frac{z_{I_t,j} Cr_0^s(H_j)}{z_{I_t,i} Cr_0^s(H_i)}} &\longrightarrow 0,\ i = 2, 3, ..., n,
\end{aligned}$$

and finally from (10) and the above asymptotic relationships,

$$Cr_t^s(A) \to x_{A,1},\ A \in \mathcal{F}.$$

The last equation shows that, given that I_t describes a long series of outcomes, DM_s's credence in A is approximately equal to the objective probability, $x_{A,1}$, that the true hypothesis H_1 assigns to A. Moreover, DM_s's credence in A does not depend on DM_s's prior credences $Cr_0^s(H_i)$ in H_i. This result is usually referred to as the "washing-out of priors".

5.3 5.3. Realism of the Assumptions $\mathcal{Y}_s$

What can stand in the way of DM_s's path to rationality? An obvious answer is "the empirical failure of some of the assumptions in $\mathcal{Y}_s$". This raises the question of how realistic these assumptions are. Regarding this question, we distinguish two assumptions, that even a sophisticated decision maker may not be able to satisfy, namely S4 and S3b. In the context of Bayesian Confirmation Theory, failure of S4, that is, DM_s's inability to elicit unique and coherent prior credences $Cr_0^s(H_i)$, is typically discussed under the rubric "the problem of the priors". Failure of S3b means that DM_s has specified the wrong chance theory. These two potential problems for the successful implementation of BC, are analyzed in following two sub-sections.

[17] The "*i.i.d*" assumption is overly strict and is made only for simplicity. The results that follow can be extended to cover non *i.i.d.* cases (see, for example, Hawthorne ????)

5.3.1 5.3.1. The Problem of the Priors (S4)

There may be cases in which DM_s is unable to assign unique and sharp prior credences, $Cr_0^s(H_i)$ to H_i $i = 1, 2, ..., n$. This difficulty is compounded (or caused) by the fact that DM_s, being at the information-free period $t = 0$, posseses no empirical data. Specifically, in the absense of any empirical information pertaining to the plausibility of H_i, $i = 1, 2, ..., n$ at $t = 0$, DM_s might not be able to give any credence in H_i (thus causing a "probability gap") or may be inclined to give multiple credences in each H_i.

One way for DM_s to get out of this predicament is to subscribe to the so-called Objective Bayesianism (OB). At first sight, OB appears to be an effective as well as an intuitively appealing solution to the problem of the priors. On this view, DM_s's prior credences $Cr_0^s(H_i)$ must be objectively determined by her "state of knowledge", which in the present case of no available data, is the state of "complete ignorance". The only credence function, in the set $\mathcal{C}(\mathcal{H})$ of all possible credence functions defined on H_i, that is consistent with this state of knowledge (so the argument goes) is the *uniform* credence function, $Cr_0^{s,u}$. This function assigns the same credence to every $H_i \in \mathcal{H}$, that is

$$Cr_0^{s,u}(H_i) = \frac{1}{n},\ i = 1, 2, ..., n. \tag{12}$$

The justification for adopting $Cr_0^{s,u}$ is based on the Principle of Indifference (POI) and runs as follows: Since DM_s does not possess any empirical information, she is not in an epistemic position to favor one hypothesis over any other. Put differently, in order for DM_s to assign a different credence in H_i than in H_j, $i \neq j$, she must have some evidence, upon which her credence differential is grounded. As Norton (2006) remarks: "...beliefs must be grounded in reasons, so that when there are no differences in reasons there should be no differences in beliefs" (2006, pp. 3-4). However, DM_s possesses no such evidence because she is at the begining of her epistemic life, that is $t = 0$. Hence, (12) seems to be the only sensible way to form her prior credences in H_i, $i = 1, 2, ..., n$.[18]

Even though it seems natural, POI is far from being an uncontroversial principle. Indeed, DM_s might be uncertain of how to use POI, because she realizes that there are more than one partitions to which POI may be applied. For example, instead of $\mathcal{H}$, DM_s may consider applying POI to the partition of "elementary outcomes" of Ω,

$$\Omega = \{\omega_1, \omega_2, ..., \omega_l\},$$

[18] More formally, $Cr_0^{s,u}$ is the (unique) credence function in $\mathcal{C}(\mathcal{H})$ that maximizes *Shannon's Entropy SE* (see Shannon 1948, Jaynes 1957). Maximization of SE (MaxEnt) is an epistemic principle which states that the credence function that best represents DM_s's state of knowledge is the one that maximizes SE. Hence, if DM_s views *MaxEnt* as a normatively appealing principle, then her adoption of $Cr_0^{s,u}$ follows logically. Any prior credence function other than $Cr_0^{s,u}$ implies information that DM does not have. The MaxEnt principle may be thought of as a descendant of Laplace's Principle of Insufficient Reason or Keynes's (1921) Principle of Indifference (POI).

in which case,

$$Cr_0^{s,\omega}(\omega_j) = \frac{1}{l},\ j = 1, 2, ..., l. \tag{13}$$

Depending on the selected partition, DM_s will end up with either $Cr_0^{s,u}$ or $Cr_0^{s,\omega}$.[19] Is there any argument supporting the application of POI to some particular partition $\mathcal{H}$, hereafter referred to as $POI_{\mathcal{H}}$? To answer this question we must first analyze the relationship between Principal Principle (PP) and $POI_{\mathcal{H}}$. In particular, if PP entails $POI_{\mathcal{H}}$, that is

$$PP \models POI_{\mathcal{H}} \tag{15}$$

then a decision maker that finds PP apealing, she has to embrace $POI_{\mathcal{H}}$ too. Given that PP is admitedly a widely endorsed principle of rationality, the question of whether DM_s adopts $POI_{\mathcal{H}}$ boils down to whether (15) holds. To that end, Hawthorne et. al (2015) argue in favor of (15), Pettigrew (2018) argues that the two principles are independent, while Gallow (2020) claims that PP and POI are incompatible. In Appendix we offer our own view on the validity of (15).

The above discussion has shown that both the "objective Bayesian" and the "non-Bayesian" decision makers, end up having the uniform credence function $Cr_0^{s,u}$ as their prior. This has left out the case in which DM_s is a "subjective Bayesian", in the sense that she feels unconstrained to adopt any prior credence function that she likes, as long as this function is coherent. Such a DM_s is likely

[19] To clarify this issue consider the following example. There is an urn containing black (B) and red (R) balls. DM_s is interested in assigning a credence in the proposition that "the next draw from the urn is R". DM_s does not posses any probabilistic information concerning the history of past draws. However, based on theoretical considerations (background information) she has conceived the chance theory $\mathcal{H}_A = \{H_1, H_2, H_3\}$, where H_1 H_2 and H_3 are the hypotheses that the urn contains (50B,50R), (60B,40R) and (10B,90R), respectively. Furtermore, assume that the true hypothesis is indeed included in $\mathcal{H}_A$.The question is the following: Given DM_s's ignorance about the objective probability, $Ch(R)$, of R, what should her rational credence in R be? First, by applying POI on $\mathcal{H}_A$, DM_s assigns credence equal to 1/3 to each of H_1 H_2 and H_3. Furtermore, POI on $\mathcal{H}_A$ in conjuction with PP and the theorem of total probability (TOTP), entail that

$$Cr_0^{s,u}(R) = \frac{1}{3} \times 0.5 + \frac{1}{3} \times 0.4 + \frac{1}{3} \times 0.9 = 0.6. \tag{14}$$

On the other hand, if DM_s were asked to decide which of the two colours is the most probable outcome of the next draw, she will realize that she possesses no evidence pointing to any of the two directions: Specifically, she has no information concerning past draws and of course, she does not know the relative frequence of red balls in the urn. Hence, she has no reason to believe more strongly that "the next draw is red" than "the next draw is black" and vice versa. This means that DM_s's epistemic attitude towards the two colours must be identical to that towards the three hypotheses. This in turn implies that DM_s is licenced to apply POI on the "elementary outcomes" partition, $\Omega = \{B, R\}$. In such a case, POI on Ω (alone) implies that

$$Cr_0^{s,\omega}(R) = 0.5.$$

Hence, DM_s ends up with two different credences for the same proposition, R, namely $Cr_0^{s,u}(R) = 0.6$ and $Cr_0^{s,\omega}(R) = 0.5$. This, of course, violates the rationality critierion of "uniqueness" (BEL-a). POI's sensitivity on the selected partition was first identified by Van Fraassen (1989).

to defend her choice on the basis of the aforementioned "washing-out of priors" results, according to which the choice of the prior is asymptotically inconsequential (provided that this DM_s continues to subscribe to PP). In conclusion, DM_s has at her disposal an acceptable solution, namely POI, to ensure the validity of S4. In any case, however, the choice of the prior is asymptotically inconsequential (provided that this DM_s continues to subscribe to PP).

5.3.2 5.3.2. The Problem of Specifying the Wrong Chance Theory (S3b)

Let us now examine the realism of (S3b). Specifically, we analyze the effects on DM_s's failure to include the true hypothesis H^*in the set of candidate hypotheses $\mathcal{H}$. Assume that $H^* = H_{n+1}$. More specifically, instead of $\mathcal{H} = \{H_1, H_2, ..., H_n\}$, the true chance theory $\mathcal{H}^*$ is,

$$\mathcal{H}^* = \{H_1, H_2, ..., H_n, H_{n+1}\}.$$

What are the effects of DM's erroneous specification of the relevant chance theory ($\mathcal{H}$ instead of $\mathcal{H}^*$) on the coherence and accurancy properties of her credences formed at t? With respect to coherence, the fact that DM_s believes that $\mathcal{H}$ is indeed a partition (although it is not) makes her to assign prior credences $Cr_0^s(H_i)$, $i = 1, 2, ..., n$ that sum up to one. As a result, her prior credence function Cr_0^s, produced by means of (**??**), remains coherent even under DM_s's adoption of the false chance theory $\mathcal{H}$. On the other hand, the incorrect specification of the relevant chance theory has detrimental effects on the accuracy of DM's credences. If the correct chance hypothesis is excluded from the set of candidate hypotheses, then the aforementioned likelihood ratio convergence result does not obtain, which in turn implies that DM_s's credences do not converge to the corresponding objective probabilities. This means that regardless of how much information, I_t, DM_s possesses, her credences will never be sufficiently accurate (although they will always be coherent). The root of the problem is the following: By excluding H_{n+1} from the set of possible alternatives, DM_s has effectively assigned zero prior credence in H_{n+1}. However, as Gelman and Shalizi (2013) remark: "For the Bayesian agent the truth must, so to speak, *be always already partially believed* before it can become known" (2013, p. 16, emphasis added).

One might argue that given her level of sophistication, DM_s is likely to foresee the possibility of "leaving out" the correct chance hypothesis from her chance theory. As a result, DM_s is prudent enough to include the so-called "catch-all" hypothesis, H_{ca},

$$H_{ca} : \neg(H_1 \vee H_2 \vee ... \vee H_n) \equiv \neg H_1 \wedge \neg H_2 \wedge ... \wedge \neg H_n,$$

in the set of candidate hypotheses under consideration. Hence, DM's chance theory, $\mathcal{H}^{ca}$, becomes,

$$\mathcal{H}^{ca} = \{H_1, H_2, ..., H_n, H_{ca}\}.$$

$\mathcal{H}^{ca}$as opposed to $\mathcal{H}$ is a proper partition. However, DM_s's move to include $\mathcal{H}^{ca}$ in her set of candidate chance hypotheses does not offer her much help in solving the problem of credence inaccuracy at t. This is due to the fact that H_{ca} does not entail a chance function, Ch_{ca} on $\mathcal{F}$, which in turn implies that the likelihoods $Cr_0^s(A \mid H_{ca}, I_t)$ and $Cr_0^s(I_t \mid H_{ca})$ are not objectively determined. This is because if H_{ca} is unknown, it cannot entail any chance function whatsoever. As a result, the likelihood ratio convergence result does not obtain, which means that a result analogous to (**??**) is not feasible.

So where does the process converge in the case that the correct hypothesis is not included in the hypothesis space? Autzen (2018) provides a simple coin-toss example (a modification of the fair-balance paradox) in which there are two false candidate models/hypotheses that are equidistant from the truth (and not arbitrarily close to the true model/hypothesis) and proceeds to show that the Bayesian approach tends to confirm a false candidate model/hypothesis when the data size grows infinitely. As Autzen (2018) concludes, "the fair-balance paradox and its modification reveal an unattractive feature of the Bayesian approach to scientific inference".

6 6. Rationality of the Naive Decision Maker

6.1 6.1. Cognitive and Epistemic Properties of DM_o

As already mentioned, DM_o is not endowed with the epistemic/cognitive properties of DM_s. Specifically, DM_o does not know formal probability theory and does not start the process of building her credence function by conceiving a chance theory $\mathcal{H}$. Nonetheless, she is assumed to exhibit the following epistemic properties (Set of Assumptions $\mathcal{Y}_o$):

(O1) DM_o uses frequency information to form her credences. More specifically, DM_o is capable to perform "direct inference" (see Thorn ????). O1 may be stated as follows:

$$Cr_0^o\left[A \mid \langle f(A \mid K) = x_A \rangle\right] = x_A, \tag{16}$$

where Cr_0^o denotes DM_o's prior credence function and $f(A \mid K)$ stands for the "relative frequency" of A within the reference class K.[20]

O1 is quite plausible even for the non-sophisticated decision maker. Williamson (2021) argues that normal people perform direct inference all that time without thinking about it, whereas Bastos and Taylor (2020) provide evidence that this ability starts from infancy.

[20] For example, if in a long series of coin tosses, (that is, within the reference class K) the relative frequncy of "Heads" is 50%, DM_o sets her credence in "Heads" to 0.5. Compare DM_o's direct way of credence formation with DM_s's indirect, two-step way: In the first step, DM_s uses the existing data for "Heads" and "Tails" to identify the "true" hypothesis H^* among the set of $\mathcal{H}$ competing hypotheses. Once this step is completed, DM_s adopts as her credence of "Heads", the corresponding chance implied by H^*. For DM_o the aforementioned first step is absent.

(O2) DM_o is able to identify the "correct" reference class K^*, arising from partitioning the initial reference class K with respect to the factors $X_1, X_2, ..., X_n$ that are objectively relevant for A ("epistemic competence" of DM_o). O2 implies that K in (16) is always identical to K^*.

(O3) DM_o is dynamically consistent with respect to her probabilistic commitements.

Beyond O1-O3, DM_o is also assumed to exhibit the following "betting dispositions":

(O4) DM_o is willing to bet on every $A \in \mathcal{F}$. Moreover, her credence in A is given by her betting price on the truth of A.

O4 alludes to the ascertainability of DM_o's credences. Specifically, as will be analyzed below, one (but not the only) way for DM_o to elicit her credence in A, is to discover her "betting disposition" (actual or hypothetical) towards A.

(O5) DM_o has both the disposition and the ability to avoid any financial loss that is a-priori certain.

As will be shown below, O5 lies at the heart of the so-called "Dutch Book Argument" for the coherence of DM_o's prior credence function.

(O6) DM_o has the incentive to detect and exploit existing profit opportunities.

O6 will play an instrumental role in arguing for the asymptotic accuracy of DM_o's credences. Put differently, O6 is of paramount importance in establishing the rational expectations hypothesis (see Muth 1961). As will be shown below, O6 gives DM_o the *incentive* to perform direct inference as well as to identify K^*. The *ability* of DM_o to do so is implied by O1 and O2.

In the remainder of the paper, we shall address the following questions: Under the cognitive and epistemic assumptions (properties, characteristics, and features) of DM_s and DM_o, postulated above, is it plausible for DM_s and/or DM_o to reach rationality in the sense of BEL? The arguments we will propose for the rationality of DM_s are completely different in nature and structure from those we will propose for the rationality of DM_o. In particular, the main argument for DM_s's rationality is that her cognitive and epistemic sophistication (assumptions $\mathcal{Y}_s$) enables her to design and follow a *formal process* of credence formation (BC), which will lead her to BEL. In other words, DM_s is assumed to be actively building her own credal system by following BC.

On the other hand, given the poor epistemic properties with which DM_o is endowed (mainly her complete lack of theorizing about chance), we do not expect her to build her own credence function is a way similar to that of DM_s. On the contrary, DM_o's main motivation for reaching rationality is to avoid certain financial losses and/or exploit any profit opportunities that she may detect. This comparison unearths a fundamental difference between DM_s's incentives for pursuing rationality and those of DM_o : DM_s recognizes the epistemic virtues of rationality, that is she believes that having coherent and accurate credences is either desirable in itself or lead to better decisions. For DM_s, it is clear that Kolmogorov's axioms (at least up to finite additivity) form a set of unquestionably desirable rules to which her probabilistic reasoning must conform. On the other hand, DM_o sees no intrinsically desirable features in rationality. However, she is assumed to believe that it is irrational for her to accept a bet that leads to sure financial losses (or to leave existing profit opportunities unexploited). As a result, she employs all her cognitive and epistemic capabilities to avoid such undesirable monetary outcomes. It is this pragmatic process, already referred to above as TEFB, that will lead DM_o to rationality (provided that conditions $\mathcal{Y}_o$ are true). The result is that DM_o eventually behaves *as if* she had the cognitive and epistemic capabilities of DM_s.

6.2 6.2. The Road to BEL for the Naive Decision Maker (Trial and Error, Frequency-Based Process)

The process (TEFB) by which DM_o arrives at BEL - provided that the assumptions $\mathcal{Y}_o$ are true - is quite different than that of DM_s. Specifically, TEFB consists of the following three steps:

Step 1:

The first step concerns solely the initial period $t = 0$ at which DM_opossesses no empirical information. Given the absence of any data, DM_o cannot apply any frequency principle in order to form her credences. Hence, in the first step of TEFB, DM_ohas to build her prior credence function Cr_0^o using a procedure that is totally different than that used for the formation of her posterior credence functions Cr_t^o, $t = 1, 2, ...$In order to achieve ascertainability and coherence of Cr_0^o, DM_o has to view each A, $A \in \mathcal{F}$ as a potential bet and identify her credence in A with her betting price on A. This betting attitude of DM_o is ensured by O4. In addition, DM_omust ensure that the aforementioned system of betting prices is coherent. In order to achieve this, she employs a "trial and error" process that constitutes the first component of TEFB. This process is analyzed in sub-section 6.3.2.

Step 2:

This step is implemented from $t = 1$ onwards. At each point in time, $t \geq 1$, DM_ocalculates the relative frequency of A within the "correct" reference class K^*,

$$f_t(A \mid K^*) = \frac{|A \cap K^*|}{|K^*|}.\text{[21]} \tag{17}$$

It is obvious that $f_t(A \mid K^*)$ denotes the frequency with which members of the reference class K^* are members of the "target class" A.[22] Why does DM_o feel the need to calculate $f_t(A \mid K^*)$? In other words, what is the incentive of DM_o to calculate the relative frequency of A? The answer to this question is that DM_o has the incentive to detect and exploit existing profit opportunities as postulated by O6. An example of unexploited profit opportunities is the following: Consider the random experiment of tossing a coin. DM_o's credence

[21] In general, for any set S, $|S|$ denotes the number of the elements of S, that is the "cardinality" of S.

[22] For notational simplicity, we use the same symbol A to denote the proposition "The next quarter's output growth in the US is positive" and the target class "the quarters with positive output growth".

in "heads" is 0.5. At the same time, there exists a history of past tosses (available to DM_o) suggesting that the relative frequency of heads within the initial reference class K is 0.6. Assuming that K coincides with K^*, we conclude that the coin is objectively biased. If DM_o does not align her credence in heads to the aforementioned relative frequency of heads within K^*, i.e. if DM_o "does not respect the frequencies", then she ignores relevant statistical information, thus leaving profit opportunities unexploited. It must be noted that DM_o's aversion towards leaving profit opportunities unexploited lies at the heart of the so-called Arbitrage Arguments, often employed in the literature as justifications for the Rational Expectations Hypothesis (see, Muth 1961, Hausman 1989). O6 provides DM_o with the incentive but not the ability to exploit existing profit opportunities. The ability of DM_o to do so is ensured by O2, which asserts that $DM_o$is always able to identify the objectively relevant reference class K^*. Note that O2 is similar to S3b, since both of them imply epistemically omnipotent decision makers.

Step 3:

The third step of TEFB is also implemented from $t = 1$ onwards. For example, take the time t, at which DM_o possesses empirical data I_t informing her that the relative frequency of A within K^* is x_A. Then, she "defers" to this relative frequency, that is, she sets her posterior credence $Cr_t^o(A)$ equal to her prior conditional credence in A,

$$Cr_t^o(A) = Cr_0^o\left[A \mid \langle f_t(A \mid K) = x_A \rangle\right] = x_A. \tag{18}$$

From (18), it is obvious that the properties of DM_o's credence functions Cr_t^o, $t = 1, 2, ...$ are those of the corresponding relative frequency functions f_t. This means that if f_t obeys the rules of probability calculus then the corresponding Cr_t^o will be coherent. It is well known that for any finite t, the finite relative frequency function f_t satisfies finite additivity (and trivially countable additivity - see Hajek (????) interpretations of probability - stanford encyclopedia).[23]

The next question, pertaining to the asymptotic accuracy of DM_o's posterior credences, concerns the asymptotic behaviour of $f_t(A \mid K^*)$. In particular, the question is whether the relative frequency $f_t(A \mid K^*)$ converges to the corresponding objective probability (chance) $Ch(A)$ as $t \to \infty$, that is

$$\lim_{t \to \infty} f_t(A \mid K^*) = Ch(A). \tag{19}$$

Whether (19) holds (and given that K^* is the correct reference class) depends on the process by which the data I_t is generated. If the data generating process is such that it ensures the application of the law of large numbers (for example i.i.d,) , then (19) holds and DM_o's posterior credences are accurate in the BEL sense.

[23] However, for $m \longrightarrow \infty$, the corresponding limiting relative frequencies violate countable additivity (see de Finetti 1972).

6.3 6.3. Realism of the Assumptions $\mathcal{Y}_o$

The preceding discussion has revealed three potential impediments to DM_o's path towards BEL: First, DM_o may fail to ascertain her prior credence $Cr_0^o(A)$, $A \in \mathcal{F}$ by eliciting her betting price on A (assumption O4 fails). Second, DM_o may find it hard to implement the "trial and error" component of TEFB, thus failing to end up with a coherent prior credence function Cr_0^o (assumption O5 fails). Third, DM_o may fail to identify the correct reference class K^*, which is likely to have detrimental effects on the accuracy of her posterior credences (assumption O2 fails). The realism of O4, O5 and O2 are analyzed in the following three sub-sections.

6.3.1 6.3.1. Ascertainability of DM_o's Prior Credence Function Cr_0^o (O4)

DM_o may ascertain her prior credences by one of the following two ways:

(i) DM_o can access her credal state via introspection and elicit her (determinate) credences located there. This procedure places very strong demands on DM_o's "self-awareness" capabilities. This raises the question of whether there exists an alternative procedure - other than pure introspection - by which DM_o can ascertain her credences. The answer to this question brings us to the second "more operational" way of ascertaining DM_o's credences implied by O4:

(ii) Some authors suggest that the best environment for DM_o to elicit her credences is the one in which her money is at stake (see, for example, de Finetti 1970, 1990, Jeffrey 2004). Specifically, in measuring her prior credence in A, DM_o is invited to discover her "betting disposition" (actual or hypothetical) towards A. This of course pre-supposes that DM_o is willing to bet on A for every $A \in \mathcal{F}$ (assumption O3). In answering the question "what is your prior credence in A?", DM_o is instructed to assess (in the absense of any information) what is the maximum price, p_A, that she is willing to pay to buy a lottery that offers \$1 if A proves to be true (equivalently, if the event A occurs) and \$0 otherwise (if $\neg A$ is true). Once, p_A is identified, DM_o can simply equate $Cr_0^o(A)$ with p_A. This strategy is supposed to offer an operational definition of $Cr_0^o(A)$ by providing a specific procedure of measuring $Cr_0^o(A)$. The "betting price" interpretation of credences is based on the following two implicit assumptions: First, DM_o ranks the available alternative courses of action (e.g. "take the bet" versus "deny the bet") by subscribing to the Subjective Expected Utility Maximization principle (SEUM). Second, DM_o's utility function is of the very special form, $U(x) = x$. Under these assumptions, DM_o's utility of p_A is equal to her SEU of the lottery, that is

$$\begin{aligned} U(p_A) &= Cr_0^o(A) \times U(1) + Cr_0^o(\neg A) \times U(0) \Longrightarrow \\ p_A &= Cr_0^o(A) \times 1 + Cr_0^o(\neg A) \times 0 = Cr_0^o(A). \end{aligned} \tag{20}$$

Remarks:

(i) SEUM is treated as an autonomous, primitive rationality principle, which is not derived from DM_o's preferences via a representation theorem. Good

(1952), for example, views SEUM as an obvious or natural principle of rationality, which as such, requires no further justification. This point is not hard to swallow, especially in view of the fact that SEUM was in place long before any representation theorem was proved. As is well known, SEUM was introduced by Daniel Bernoulli in 1736 as a means to solve the St. Petersburg paradox. This interpretation of SEUM is necessary for any argument supporting the view that beliefs (taken as primitive) cause preferences. Specifically, DM_o *prefers* to pay up to \$$p_A$ to buy the lottery (instead of the status quo) *because* she believes that A is true to the degree $Cr_0^o(A)$. The independent existence of SEUM generates belief-based preferences, as opposed to the case in which SEUM is derived within a representation theorem, which gives rise to preference-based beliefs. Eriksson and Hajek (2007) are quite explicit on this point. They favor the explanation "these are your credences and desirabilities; thus, those are your betting dispositions, or preferences" over the reverse explanation, "these are your betting dispositions or preferences; thus, those are your credences and desirabilities." (p. 207-208).

(ii) The assumption, $U(x) = x$ is, admittedly, a very restrictive one, since it allows the identification of $Cr_0^o(A)$ with p_A only for risk neutral DM_os (for whom the buying price of the lottery is also equal to the selling price of the same lottery). The question which naturally arises at this point is the following: Can $Cr(A)$ be identified with p_A for any utility function? Ramsey (1926) answers this question in the affirmative: DM_o can price the lottery in terms of her utilities of monetary outcomes, instead of the monetary outcomes themselves. This move, however, depends on another tacit assumption: DM is capable of eliciting her own utilities, $U(1)$ and $U(0)$ of the monetary outcomes 1 and 0, respectively. Under this assumption, we have[24]

$$\begin{aligned} p_A &= Cr_0^o(A) \times U(1) + Cr_0^o(\neg A) \times U(0) \Longrightarrow \\ p_A &= Cr_0^o(A) \times U(1) + (1 - Cr_0^o(A)) \times U(0), \end{aligned}$$

from which, solving for $Cr_0^o(A)$, yields

$$Cr_0^o(A) = \frac{p_A - U(0)}{[U(1) - U(0)]}. \tag{21}$$

The foregoing discussion shows that for the general case in which DM_o has an arbitrary utility function, the strategy of identifying credences with betting prices trades one problem for another. However, there is no reason why utilities may be "more measurable" than credences. Hence, the problem of the realism of O4 has been replaced by the problem of the realism of the assumption that DM_o is able to ascertain $U(1)$ and $U(0)$. In conclusion, we find that DM_o's strategy of eliciting her credences by equating them with the corresponding betting prices is not always operational, which in turn makes O4 empirically questionable.

[24] Note that the following derivation is based on another assumption, namely $Cr_0^o(\neg A) = 1 - Cr_0^o(A)$. This implies that in order to identify credences with betting prices, we must place some a priori "probabilistic structure" on them.

6.3.2 6.3.2. Coherence of Cr_0^o: Dutch Book Argument (O5)

Dutch Book Argument (DBA), introduced by Ramsey (1926) and de Finetti (1937), is based on the so-called Dutch Book theorem, which shows that a DM_o who does not have coherent credences (i.e. betting prices) is susceptible to a Dutch Book.[25] The latter is a set of bets, each of which appears to be fair to DM_o (by her own beliefs) but all together assure that DM_o will incur a net financial loss, whatever the outcome turns out to be. Since a rational DM_o does not exhibit this type of susceptibility (so the argument goes) it follows logically (by a modus tollens argument) that she entertains coherent beliefs.[26]

It is worth emphasizing that in the context of DBA, DM_o's aversion to certain financial losses forces her to obey a calculus (e.g. Kolmogorov's) that she may not know or might have never heard of. In other words, DM_o's attempts to avoid being Dutch-booked make her develop a system of "thought police" that "clubs her into line when she violates certain principles of right reasoning" (Garber 1983, p. 101). This means that DBA is based on the fundamental (albeit implicit) assumption that $DM_o$is capable to implement the aforementioned trial and error process, (the TE part of TEFB) that yields coherent beliefs. TE may be outlined as follows:

DM_o begins with a prior probability function, Cr_0^o, and examines whether, under Cr_0^o, she is susceptible to a Dutch Book. If she is, then she denies that Book and adjusts her initial Cr_0^o (corrects her initial error) until her susceptibility disappears. This in turn implies that TE may be thought of as a rational adaptation processes for achieving some specific goals which, by its very nature, applies to any decision maker, regardless of her level of expertise in the relevant subject matter. In other words, TE is a purely a-priori process, and as such it does not depend on the presence of any empirical information. In addition, the actual presence of the "cunning bookie" who aims at pumping up DM_o's money is not necessary. As Hayek (????) remarks: "The irrationality that is brought out by the Dutch Book argument is meant to be one internal to your degrees of belief, and in principle *detectable by you by a priori.*" (???? p.7, emphasis added). This means that DBA retains its force even in cases where no actual bets or bookies are around. In effect, what DBA does is to invite DM_o to convert whatever actual epistemic situation she faces into an equivalent betting situation and check whether her prior credences render her susceptible to a sure financial loss. If this self-testing procedure reveals such a susceptibility, then $DM_o$is supposed to invoke TE until that susceptibility disappears.

To sum up, the only properties that DM_o is required to have in order for her prior credence function to be coherent are, first, an aversion to suffering a sure monetary loss and, second, the analytical/computational skills to implement TE. Both of these properties are ensured by O5, which may be equivalently

[25] The Converse Dutch Book Theorem (CDBT) proved by Lehman 1955 and Kemeny 1955, shows the converse is also true. DBT and CDBT may be joined together as follows: "DM_o's system of betting prices on the propositions of $\mathcal{F}$ does not violate the rules of probability calculus, *if and only if* there is no Dutch Book consisting of bets at those prices".

[26] A similar type of arguments, usually referred to as "Money Pump Arguments" are usually employed to justify the "internal consistency" (e.g. transitivity) of preferences.

re-stated as follows: "DM_o has both the *disposition* and the *ability* to detect *every* Dutch Book made against her". The obvious question is whether O5 is realistic. It is important to keep in mind that O5 has two components: The first refers to DM_o's disposition while the second refers to her ability to detect each and every Dutch Book (actual or hypothetical) made against her. As will be shown below, although the first component of O5 is quite plausible, the second is not. The plausibility of the second component of O5 is analyzed below.

The Dutch Book Problem refers to the realism of O5 and may be stated as follows: Does DM_o have the skills to carry out the necessary calculations that are involved in TE? The answer to this question depends on the degree of complexity of the Dutch Book that DM_o is faced with. For simple Dutch Books, DM_o is likely to have the computational capacity to detect the aforementioned incoherence. However, in order for DM_o to be deemed as rational, she must be able to repel *any conceivable* Dutch Book made against her, simple or complex.

Let us analyze TE in more detail, starting with a simple Dutch Book: Assume that a clever betting opponent (the Dutch bookie) has detected a certain type of initial incoherence in DM_o's credences with respect to the truth/falsity of the proposition H and designs a simple Dutch Book, $db1$, accordingly. For example, DM_o has initially assigned probabilities equal to p_1 and p_2 to the truth and falsity of H, respectively with $p_1 + p_2 < 1$. Before accepting $db1$, DM_o performs "the necessary calculations" and realizes that she is about to be Dutch booked. As a result, she corrects her initial error, in the sense that she now forms new subjective probabilities p_1' and p_2' such that $p_1' + p_2' = 1$, thus restoring coherence. In this case, DM_o has implemented successfully TE.

The specific Dutch Books that are usually employed in the literature (in order to motivate their supportive role for rationality) are extremely simple. Specifically, they are based on the truth or falsity of a very small number of (mutually exclusive and/or exhaustive) propositions, which render the number, n, of the rows of the corresponding pay-off matrix relatively small. Such simple books imply a quite feasible TE. However, as the complexity of the DB increases, the DM_o's ability to implement TE becomes questionable.

The following Proposition, shows how the number of rows of the corresponding pay-off matrix can grow exponentially if the propositions are not mutually exclusive:

Proposition 2 *Consider a Dutch Book that is based on the truth or falsity of k propositions, H_1, $H_2, ..., H_k$. If these propositions are mutually exclusive then, the number of rows of the corresponding pay-off matrix is $n = k + 1$. On the other hand, if no pair of H_1, $H_2, ..., H_k$ is mutually exclusive, then $n = 2^k$.*

Proof. Each proposition H_i, can be either True or False. Assume that propositions, H_1, $H_2, ..., H_k$ are not mutually exclusive. Then, the number of possible k-tuples is $n = 2^k$. If on the other hand, the propositions are mutually exclusive, there can be at most one proposition that is True for each k-tuple. This means that there are k possible k-tuples, where only one proposition is True. Also, there is one more, where all propositions are False. Therefore, the total number of rows of the corresponding pay-off matrix is $n = k + 1$. ■

This Proposition clarifies the nature of the Dutch Book problem: It is not enough for DM_o to be able to detect some simple Dutch Books to be considered rational - she must be able to detect *all* Dutch Books no matter how complex may be (i.e. regardless of how large is n). As we move up in the scale of complexity, DBA runs out of steam. It is worth emphasizing that the existence of even a single undetected Book sufficies for characterizing DM_o as irrational. In such cases, TE breaks down, which in turn implies that DM_o's aversion to monetary losses by itself (the first component of O5) is not enough to ensure the coherence of her beliefs. This means that the second component of O5 presupposes incredible logical skills on the part of DM_o, an assumption commonly referred to as "logical omniscience" (see. for example, Hacking 1967, Garber 1983). Thus, logical omniscience (tacitly assumed in O5) is the main reason for suspecting that O5 is empirically implausible.

6.3.3 6.3.3. The Reference Class Problem (O2)

Consider a DM_o, who is about to form a credence for the proposition A: "The next quarter's output growth for the US is positive". Assume that DM_o posseses a sample of size n of historical data on quarterly output growth for US. DM_o is assumed to satisfy O3 which means that she will set her credence $Cr_t(A)$ equal to the relative frequency $f(A)$. But how should DM_o calculate $f(A)$? One option is to calculate $f(A)$ as the ratio between the number of quarters with a positive growth and the total number, n, of quarters in her sample (positive or negative). If DM_o chooses this option, then she has decided to relativise A to the "initial reference class" K_1 consisting of all the quarters in her sample. Specifically,

$$f(A \mid K_1) = \frac{|A \cap K_1|}{|K_1|} = \frac{|A \cap K_1|}{n}. \tag{22}$$

Is K_1 the only reference class that DM_o may consider? The answer is negative. For example, assume that DM_o has observed that at the last point in her sample, namely at $t = T$, the central bank unexpectedly increased the interest rate by 25 basis points. DM_o thinks that unanticipated changes in the monetary policy are likely to have implications for next period's output growth. Hence, she thinks, that the relevant reference class is not K_1 but narrower reference class K_2, $K_2 \subset K_1$ consisting of all the quarters at which the interest rate increased by 25 basis points. In such a case, the relative frequency of A with respect to K_2 is given by

$$f(A \mid K_2) = \frac{|A \cap K_2|}{|K_2|}. \tag{23}$$

If an unanticipated increase in the interest rate is indeed a relevant factor for the next quarter's output growth, we have that

$$f(A \mid K_1) \neq f(A \mid K_2). \tag{24}$$

Remark

It is worth noting that the inequality (24) is necessary and sufficient for K_2 to be a "relevant partition" of the initial reference class K_1 (see Salmon ????). If $f(A \mid K_1) = f(A \mid K_2)$ then K_2 is an irrelevant partition of K_1.

The foregoing discussion must have clarified the nature of the "reference class problem". This problem was first identified by Reichenbach (1949) who describes it as follows: "If we are asked to find the probability holding for an individual future event, we must first incorporate the case in a suitable reference class. An individual thing or event may be incorporated in many reference classes, from which different probabilities will result. This ambiguity has been called the problem of the reference class." (1949, pp. 374). In a similar fashion, Hajek (2007) says: "What we want is the unconditional probability of (the event) E; but what we have are a host of unequal conditional probabilities of the form $P(E$, given $A)$, $P(E$, given $B)$, $P(E$, given $C)$, etc. Relativized to the condition A, E has one probability; relativized to the condition B it has another; and so on. What then is the probability of E?"

It is now clear that DM_o is faced with the following epistemological problem: Find the objective reference class K^* with respect to which the proposition of interest A should be relativised. This "priviledged" reference class K^* is generated by all the factors, X_1, $X_2, ..., X_k$ that are "objectively relevant" for for the individual event/proposition under study. Hence, the epistemological problem of DM_o is to identify these (and only these) factors. To that end, Reichenbach's proposal to utilize all the factors for which reliable statistics are available does not offer much help. For example, it might be the case that of all the k factors that are objectively relevant for A, only the first two have been reliably measured. This means that although the objective relative frequency of A is given by $f(A \mid K^*)$, DM_o will end up with $f(A \mid K_2)$, where K_2 is the reference class generated by X_1 and X_2.

It is worth noting that DM_o's problem to identify the objectively relevant reference class K^* is analogous to the problem of DM_s to include the correct chance hypothesis H^* in $\mathcal{H}$. If DM_s, with all her superior knowledge of the subject runs the risk of leaving H^* out of $\mathcal{H}$, then it is much more difficult for DM_o to identify the objectively relevant reference class K^*. DM_o's incentive to exploit any available profit opportunities (i.e. assumption O6) does not ensure her ability to identify those opportunities (i.e., to identify K^* - assumption O2). As a result, the asymptotic accuracy of DM_o's credences Cr_m appears to be an overly strict rationality constraint, mainly due to the implausibility of O2.

7 7. On the Normative and Descriptive Status of BEL: Graded Rationality

Let us consider the conditional proposition P1: "If the decision makers (of DM_s and DM_o types) are not rational then the economic system will be dysfunctional most of the time" and the proposition P2: "the economic system is not dysfunctional most of the time". Let us assume that both propositions are true. Then based on a *modus tollens* argument, it follows that the decision makers DM_s and DM_o are rational. Does the general concept of rationality that appears in P1 coincides with BEL? This in turn implies (via the previous modus tollens argument) that DM_s and DM_o successfully implement the rationality-oriented procedures BC and TEFB, respectively. However, as we have discussed above, these procedures are based on some cognitive and epistemic assumptions, which, especially for DM_o, (i.e. logical omniscience and epistemic omnipotence) were deemed to be a-priori implausible. Nonetheless, a-priori implausibility does not necessarily mean a-posteriori impossibility. DM_o can after all repel every complicated Dutch book suggested to him as well and determine the objectively relevant reference class in every direct inference problem she may encounter. This means that the factual status of BEL is an empirical matter and for this reason we should turn our attention to the relevant psychological studies.

In this respect, there is overwhelming evidence that when DM_o is in a "laboratory environment", she exhibits systematic deviations from the BEL rules of probabilistic reasoning (see, for example, Kahneman 1981; Kahneman & Tversky 1973, 1983; Baron 1994, 1996; Koehler 1996; Stein 1996; Evans & Over 1996; Stanovich and West 2000). Kahneman's and Tversky's as early as 1973 reach the following conclusion: "In making predictions and judgments under uncertainty, people do not appear to follow the calculus of chance or the statistical theory of prediction" (1973, p. 237).[27] These findings seem to confirm our suspicions about the a priori implausibility of DM_o's assumptions of logical omniscience and epistemic omnipotence. Hence, the concept of rationality appearing in P1, that is, the type of rationality that is responsible for the proper functioning of the economic system, does not coincide with BEL. But then, if BEL is not the type of actual rationality that is responsible for the smooth functioning of the economic system, then what is this type of rationality? Is it completely different from BEL? In other words, can the decisions DM_o make under uncertainty not be based on her credences, i.e. can we have "non-belief-based decisions"?

If the above questions are answered in the affirmative then the descriptive status of BEL is poor because its normative status is irrelevant. If actual decisions are not based on beliefs then the rationality of these decisions does not depend on the properties of these beliefs. In this respect, G. Gigerenzer and his co-authors have proposed the idea that decision making is not based on DM_o's beliefs but rather on DM_o's "fast-and-frugal heuristics" for solving practical problems within her real-life environment (see, for example Gigerenzer et. al. 1999). Stevens (2010) describes this idea as follows: "Organisms did not evolve to follow a mathematically tractable set of principles - rather, natural selection favored decision strategies that resulted in greater survival and reproduction" (2010, p. 110). Hence, the rationality of a decision maker should not be judged by the coherence and accurancy of her credences but rather by her ability to achieve certain goals. In this regard, BEL should be replaced by the so-called "Ecological Rationality" (or "Naturalized Rationality") as the norm

[27] Tversky and Kahneman (1974) argue that not only DM_o but also DM_s exhibits biases in her probabilistic thinking when operating outside the standard BC process: "Experienced researchers are also prone to the same biases-when they think intuitively." (1974, p. 1130)

against which human rationality should be judged, or to put it more succinctly, "homo economicus" should be replaced by "homo heuristicus" (see, for example, Gigerenzer and Brighton 2008).

Alternatively, we can insist that rational decisions should be based on rational beliefs, but the following question arises: Is BEL the one and only rational ideal to which every decision maker should aspire? Or is BEL so demanding that ordinary people are a priori unlikely to ever fully satisfy it? Do we need a rationality canon with, as Jeffrey (1983) puts it, "a human face"? There are two answers to this question, the Bayesian and the Non-Bayesian (or Quasi-Bayesian) ones, which we analyze below starting with the latter:

(i) The Non-Bayesian Answer: Walley (1991) complains that BEL places too strong constraints on DM_o's credences. The requirement that DM_o has unique and precise credences for all the propositions of $\mathcal{F}$ even at $t = 0$ in which she does not have any information is absurd. Walley calls this requirement "the Bayesian dogma of precision" and considers it mistaken. On the contrary, he argues that there are epistemic situations in which the lack of information forces DM_o to adopt not one but multiple credences for the same event: "Indeed, you ought to do so (i.e. to adopt multiple credences) when you have little information on which to base your assessments." (1991, p. 3). On this view, BEL is the wrong normative canon and therefore DM_o's actual beliefs should not be evaluated against it.

Another view that also supports the normative appeal of multiple credences is the one that appears in the large literature of the so-called "ambiguity aversion". This view emanated from the "Ellsberg paradox" according to which in certain epistemic situations DM_o (and in some cases DM_s too) exhibits behavior that violates Savage's postulates, thus giving rise to multiple or incoherent credences. Despite this, DM_o is not characterized as non-rational. On the contrary, the aforementioned behavior is the appropriate (rational) one in the given epistemic context of ambiguity. This rational behavior of DM_o is encoded in the form of a set of axioms different from those of Savage (see Al-Najjar and Weinstein 2009 for a critical survey of this literature). Hence DM_o is rational even though she has multiple credence functions (Gilboa and Schmeidler 1989) or a non-additive credence function (Schmeidler 1989).

(ii) The Bayesian Answer: More than two centuries ago, Laplace had clearly stated that "The theory of probability is at bottom nothing more than good sense reduced to a calculus which evaluates that which good minds know by a sort of instinct, without being able to explain how with precision " (1814/1951 p. 196). According to this view, the probability calculus is nothing more than the codification of human intuitions about the "algebra" of credences. This algebra represents what Boole describes as "the laws of thought". In a similar tone, Ramsey (1927) argues that "the laws of probability are laws of consistency, an extension to partial beliefs of formal logic, the logic of consistency." (1927, p. ??). If we accept this view, then the normative status of BEL (which requires uniqueness and coherence of beliefs even in the absence of any empirical information) seems reasonabe. But before we fully embrace this view, let us examine it a little more carefully in the light of modern probability theory. The first step is to focus our attention on the axioms of this theory and examine their intuitive relevance. As is well known, these axioms are: (i) $\forall A \in \mathcal{F}$, $P(A) \geq 0$. (ii) $P(\Omega) = 1$, where Ω is the universal set or the "sample space". (iii) For every mutually exclusive sets A and B (subsets of Ω), $P(A \cup B) = P(A) + P(B)$. It is clear that all three axioms express propositions consistent with our intuition (the Laplacian "good sense") about how DM_o's credences should behave.

The above discussion leads us to the following conclusions: (a) The mathematical function P defined on $\mathcal{F}$ is an admissible way of representing DM_o's credences. (b) The properties with which P is endowed (the three axioms) are consistent with our untrained intuitions about the basic properties that seem to characterize credences. In fact, we could claim that the "degree of intuitiveness" of these three axioms is comparable to that of the axioms of Euclidean Geometry, or to that of Peano's axioms for arithmetic, and probably greater than the corresponding degree of intuitiveness of Zermelo's axioms for set theory. Consequently, based on the above arguments, Bayesians insist on the normative attractiveness of BEL.

But now Bayesians must answer the following question: If the axioms of the probability calculus are so intuitively obvious, then why do decision makers so often violate the rules of this calculus? In our opinion, the answer to this question should be sought in the *theorems* of probability calculus. Specifically, individuals tend to violate not the aforementioned three axioms but some of the theorems entailed by these axioms. For example, the theoretical result that the probability of the intersection of two events (or the conjunction of two propositions) A and B is always less than or equal to the individual probabilities $P(A)$ and $P(B)$ is not so intuitively obvious. This results in individuals committing the so-called "conjuction fallacy" (see, for example, Kahneman and Tversky ????). But this cannot be considered as something that threatens the normative status of coherence, just as any arithmetical or geometrical mistakes made by untutored agents do not affect the normative appeal of the rules of "arithmetic" or "Euclidean geometry", respectively. In the opposite case, i.e. when one defines or modifies the normative standard according to what is observed in practice she commits the following fallacy identified by Damer (2005): “This fallacy consists in assuming that because something is now the practice, it ought to be the practice. Conversely, it consists in assuming that because something is not now the practice, it ought not to be the practice” (Damer 2005, p. 127). This implies that the normative domain should be independent of the empirical domain, a claim that is often referred to as "the autonomy thesis" (see, for example, Behrens 2021). Hence, it is our view that the normative status of BEL (at least assumptions a and b in BEL) is autonomous and independent of how often BEL is violated in real life. BEL remains the normative standard against which the credences of both DM_o and DM_s should be measured and judged.

However, if we accept the normative role of BEL then, given the aforementioned empirical evidence, we are led to the conclusion that the decision makers are quite irrational. Nevertheless, the evidence of psychological studies is not the only evidence we have regarding the properties of actual beliefs. There is also evidence to suggest that with the exception of some episodes of significant market failure (such as the great financial crisis of 2007-08), the economic system works in a fairly satisfactory manner. Also as Cohen (1981) reminds us: "Yet, from this apparently unpromising material - indeed, from the very same students who are the typical subjects of cognitive psychologists' experiments - sufficient cadres are recruited to maintain the sophisticated institutions of modern civilization." (1981, p. 317). This means that some *degree* of BEL-type rationality of the participants in the economic system must be responsible for the aforementioned smooth functioning of the Economy as well as that of the "sophisticated institutions". In other words, although the actual rationality of the decision makers is not identical to BEL, it cannot be arbitrarily distant from it. So what is the rationality concept we are looking for (i.e., the one featuring in P1)?

In our view this concept is that of "graded rationality". In particular, rationality is not an all-or-nothing concept.[28] Slote (1989) argues that all that is required of a decision maker is to be rational enough, say to a degree p. Anything beyond p is "rational supererogation". On this view, rationality comes in degrees. The degree of rationality p may differ among reasoning tasks of different degrees of complexity (for the same decision maker) or among different decision makers (for the same reasoning task). For example, consider two decision makers who know that the probabilities of the events A and B are 0.4 and 0.5 respectively. They are asked to calculate the probability of the conjunction of A and B. The first answers 0.21 and the second 0.35. As far as BEL is concerned, both are irrational. But are they on the same level of irrationality? Probably not. The notion of "less than absolute rationality" can be recast in terms of the set of assumptions $\mathcal{Y}_o$: DM_o's degree of rationality, p, is determined by the degree of her ability to detect various Dutch Books designed against her, as well as the degree of her competence to identify the correct reference classes in the epistemic situations she encounters. Accordingly, a concept of rationality that seems to accomodate both the normative requirements of BEL and the findings of the aforementioned psychological studies is that of "graded rationality".[29] Indeed, such a notion of rationality is implicitly suggested by the findings of the aforementioned psychological studies, in which a decision maker does not appear to perform equally well in all reasoning challenges she may face. As Kahneman and Tversky (1973) observe, our reasoning "sometimes yield reasonable judgments and sometimes lead to severe and systematic errors" (1973 p. 48). Of course, defining and measuring p is far from easy. What is required is to define a "distance" between actual rationality displayed by DM_o in each of her reasoning tasks and absolute rationality, BEL. Based on the concept of

[28] Sorensen (2006) disagrees with this view. He believes that a decision maker cannot be rational without being fully rational: "'Rational' is an absolute term. 'Rational' means the absence of irrationalities, just as 'flat' means the absence of bumps, curves or irregularities. If two surfaces differ in flatness, then at least one of them is not flat. If two individuals differ in rationality, then at least one of them is not rational. To be rational is to be perfectly rational." (2006, p. 216).

[29] An alternative term is that of "satisficing rationality" (see Slote 1989). "Satisfiers" are not rational in the BEL sense, and hence they do not make the optimal decisions. However, their sub-optimal decisions are "good enough".

graded rationality, and its implication that choices can be sufficiently rational without being perfectly rational, P1 can be expressed as follows: "If the economic agents are not on average sufficiently rational in the BEL sense then the economic system will be dysfunctional most of the time".

8 8. Conclusions

The main points of the paper are summarized as follows:

(i) If rationality is based only on the concept of "internal consistency" then it can refer to both preferences and beliefs. However, if the definition of rationality is extended to include the notion of "external accuracy", (a relation between the decision maker (DM) and the empirical world), then one can only speak of rational beliefs. This is because while it is meaningful to talk about 'accurate beliefs' it does not make much sense to deliberate on the concept of 'accurate preferences'.

(ii) The definition of rationality adopted in this paper (refered to as BEL) refers exclusively to the subjective degrees of belief (credences) of DM. Specifically, a DM is rational if (and only if) her credences are sharp, unique, coherent and asymptotically accurate.

(iii) BEL is a normative rule that DM's credences must adhere to. Beyond its normative role, the next question is whether BEL can actually be achieved and if yes in which way. To answer this question, we need to define the type of DM that BEL refers to and especially her cognitive and epistemic endowment. In this paper, we defined two types of decision makers, namely the expert (DM_s) and the naive decision maker (DM_o), who differ in their cognitive and epistemic backgrounds. These backgrounds are defined in terms of the sets of assumptions $\mathcal{Y}_s$ for DM_s and $\mathcal{Y}_o$ for DM_o.

(iv) The next question is to design a potential process (consistent with $\mathcal{Y}_s$) through which DM_s will reach BEL. This procedure (referred to as BC) is based on the theory of Bayesian Confirmation. Next, we ask the same question for DM_o, namely whether there is a process (consistent with $\mathcal{Y}_o$) by which DM_o will achieve BEL. The answer to this question is in the affirmative and takes the form of a trial and error, frequency-based process, referred to as TEFB.

(v) The main difference between BC and TEFB is found in the way in which DM_s and DM_o "theorize about chance", in order to arrive at their credences: DM_s, in the context of BC, follows a two-stage "indirect inference" procedure. In the first stage, DM_s uses the available data to identify the true hypothesis H^* (among a set $\mathcal{H}$ of alternative hypotheses) that describes the statistical behavior of the phenomenon of interest. In the second stage, DM_s derives the objective probability $Ch(A)$ implied by H^* and sets her credence $Cr(A)$ equal to it. On the other hand, DM_o skips the first step of specifying a chance theory. Instead, she uses the empirical data available at t to directly calculate the relative frequency, $f_t(A \mid K^*)$ of A within the "objectively relevant reference class" K^*, and then adopt $f_t(A \mid K^*)$ as her own credence $Cr_t^o(A)$ in A.

(vi) Next, we address the question of the realism of the sets of assumptions

$\mathcal{Y}_s$ and $\mathcal{Y}_o$ for DM_s and DM_o, respectively. If both of these sets of assumptions were empirically plausible then we could argue that BEL is feasible for both types of decision makers and also that DM_o reasons probabilistically "as-if" she were DM_s. It is worth noting that the process by which the "as-if" claim is justified is not an unspecified black-box process (as is usually the case in the literature). Rather, it is the well-defined processs TEFB.

(vii) Two of the assumptions in $\mathcal{Y}_s$ are empirically questionable. The first refers to "the problem of the priors", that is, the ability of DM_s to assign precise, unique, ascertainable and coherent prior credences in the chance hypotheses H_i, $i = 1, 2, ..., n$ in $\mathcal{H}$ (assumption S4). The problem of the priors can be effectively solved if DM_s joins the camp of Objective Bayesians, thus adopting the Principle of Indifference. The second empirically questionable assumption concerns the epistemic competence of DM_s to include the true hypothesis H^* in $\mathcal{H}$ (assumption S3b). However, the ability of DM_s to always specify the correct chance theory $\mathcal{H}$ is questionable, as it tacitly assumes DM_s's "epistemic omniscience".

(viii) Two of the assumptions in $\mathcal{Y}_o$ are dubious (or outright implausible), namely O5 and O2. The first aims to establish the coherence of DM_o's prior credence function solely on the basis of pragmatic considerations, such as DM_o's aversion to suffer a certain financial loss. However, O5 does not simply assume that DM_o has the incentive to avoid such a loss; it makes the stronger claim that she also has the ability to do so. As we argued in the paper, this ability is far from guaranteed especially in the cases that DM_o is faced with a complex Dutch Book. Similar scepticism surrounds O2, that is DM_o's epistemic ability to identify the correct reference class K^*, on the basis on which she claculates the relative frequencies of the events of interest.

(ix) The aforementioned implausibity of hypotheses O5 and O2 seems to be supported by the findings of psychological studies in which decision makers appear to violate BEL. This micro-level evidence raises the question of whether BEL is the wrong normative benchmark against which rationality is assessed. This would be the case if rationality had nothing to do with beliefs. However, such a claim is hard to swallow: Decisions are based on beliefs which means that rational decisions are based on rational beliefs. Hence, we insist that BEL should retain its normative status despite its violations observed at the micro level.

(x) However, beyond the empirical evidence at the micro level, we also have evidence at the macro level which points to the opposite direction. Indeed the economy (macro-level) seems to work satisfactorily most of the time. This means that at least some degree of belief-based rationality is responsible for this result. We argue that the rationality required for the smooth functioning of the Economy is not the BEL, but some other "factual rationality" sufficiently close to BEL. Against that type of rationality, assumptions O5 and O2 do not look particularly implausible. Perhaps, all that is required is that DM_o detects the relatively simple Dutch Books and identify the relatively obvious reference classes.

9 Appendix

We provide two alternative ways to justify (15), using the following example. There is an urn containing black (B) and red (R) balls. DM_s is interested in assigning a credence in the proposition that "the next draw from the urn is R". DM_s does not posses any probabilistic information concerning the history of past draws. However, based on theoretical considerations (background information) she has conceived some chance theory $\mathcal{H}_A = \{H_1, H_2, H_3\}$.

(i) The first argument for (15) is based on White (2010) and relies on the interpretation of objective chances as relative frequencies: DM_s knows that $H_i \in \mathcal{H}_A$, $i = 1, 2, 3$, that is she knows that each H_i "has the property of belonging in $\mathcal{H}_A$" or, more simply, "has the property $\mathcal{H}_A$". Moroever, suppose that G is the attribute that "a proposition is true". DM_s is asked to decide the number of actual occurrences of G within $\mathcal{H}_A$ (or the proportion of H_i's that have the "property G"). In other words, she is asked to determine the relative frequency of G within the reference class $\mathcal{H}_A$, that is $f(G \mid \mathcal{H}_A)$. Since DM_s believes that exactly one hypothesis in $\mathcal{H}_A$ is true, her answer to the last question is $f(G \mid \mathcal{H}_A) = 1/3$. To justify this, think of an urn, $\mathcal{H}_A$, that contains n "balls" $H_1, H_2, ..., H_n$ with only one of them carrying the indicator "true" and consider the random experiment of drawing a ball at random with replacement from $\mathcal{H}_A$. In this setting, $f(G \mid \mathcal{H}_A)$ is interpreted as the number of draws with the sign "true" over the total number of draws. Now, if DME is willing to interpet "the relative frequency of true hypotheses in $\mathcal{H}_A$" as "the chance of any of these hypothesis in $\mathcal{H}_A$ being true", then she concludes that $Ch(H_1) = Ch(H_2) = Ch(H_3) = 1/3$. Hence, by appealing to PP, $Cr_0^{s,u}(H_1) = Cr_0^{s,u}(H_2) = Cr_0^{s,u}(H_3) = 1/3$ which means that (15) holds.

(ii) The second argument begins by admiting that in the absence of any empirical information, it is difficult for DM_s to elicit any of the $Cr_0^s(H_i)$'s. Such a DM_s is not Bayesian. Is she irrational? Some authors argue that DM_s's inability to elicit $Cr_0^s(H_i)$, $i = 1, 2, ..., n$ does not reflect any kind of irrationality on her part, but rather it is precisely the credal state at which DM_s is (or has to be) if she possesses no probabilistic information. For example, Walley (1991) puts forward the following thesis: "A state of complete ignorance, meaning a total absence of relevant information, can be properly modelled by *vacuous probabilities*, which are maximally imprecise, but not by any precise probabilities." (1991, p. 4, emphasis added). If we subscribe to Walley's thesis - and since we maintain the assumption that DM_s is in a state of complete ignorance - then we must accept that DM_s is neither capable nor willing to express any *precise* probabilistic assesments about H_i, $i = 1, 2, ..., n$. If DM_s were asked "what is your credence in H_i?" her only reasonable response would be "I do not know". However, even under Walley's thesis, DM_s knows that for every $A \in \mathcal{F}$, she knows the chances $Ch_i(A) = x_{A,i}$, $i = 1, 2, ..., n$ entailed by the corresponding H_i, $i = 1, 2, ..., n$. This means that under PP, DM_s credal state for A is represented by the following n credences $Cr_{0,i}^s(A) = x_{A,i}$, $i = 1, 2, ..., n$. Now, assume that DM_s is willing to summarize the information of the aforementioned n credences via their arithmetic mean. That is,

$$\overline{Cr}_0^s(A) = \frac{1}{n}\sum_{i=1}^{n} x_{A,i}. \tag{25}$$

But now observe that (25) is identical to (5) plus $POI_{\mathcal{H}}$. This means that the prior credence function of a decision maker who subscribes to TOTP and $POI_{\mathcal{H}}$ is identical to that of another decision maker who subscribes to Walley's thesis, but she is also willing to represent her "mean" credence in A as the arithmetic mean of her credences in A. In other words, DME elicit her credences in the propositions of $\mathcal{F}$ "as if" she had subscribed to $POI_{\mathcal{H}}$. This means that even for a non-objective Bayesian, (15) "effectively" holds.