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COOPERATIVE AND NON-COOPERATIVE SOLUTIONS IN A DYNAMIC MODEL OF FOREST MANAGEMENT

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ABSTRACT - This study develops a finite-horizon optimal control model linking forest biomass, biodiversity, cumulative extraction, and stochastic disturbance shocks to assess three governance regimes: non-cooperative management with free terminal states (OLNE-Free) and two cooperative approaches - one with fixed ecological targets (Regulator-Fixed) and another with flexible endpoints (Regulator-Free). Non-cooperative harvesters prioritize short-term extraction, overlooking biodiversity's contribution to productivity and allowing extraction to accumulate. In contrast, cooperative regimes internalize ecological values and dynamically adjust harvest effort, resulting in improved ecological and economic outcomes. Cooperative management moderates harvesting intensity, enhances biodiversity, and increases overall welfare compared to non-cooperative approaches. Implementing mechanisms - such as fees, taxes, or regulations - that align private incentives with social values helps decentralize cooperation and buffers outcomes against parameter variability. Sensitivity analysis demonstrates that cooperative regimes consistently influence ecological changes and tend to promote more stable long-term dynamics. These findings highlight the critical role of biodiversity valuation and flexible cooperation in advancing sustainable forest management amid ecological and economic indeterminacies.

Keywords: Forest dynamics, Biodiversity, Finite-horizon cooperative governance.

1. Introduction

Reconciling short-run economic incentives with long-term ecological sustainability remains a central challenge in forest management. Forest ecosystems supply not only marketable goods such as timber but also critical public services, including carbon sequestration, hydrological regulation, and biodiversity maintenance (Spittlehouse and Stewart, 2004; Puettmann et al., 2012). However, conventional forest policies frequently prioritize private profitability while neglecting the underlying biophysical processes that sustain ecosystem productivity over time. Since Faustmann's (1849) foundational model of optimal rotation, forest economics has evolved to incorporate stochastic pricing, disturbance risk, and non-market values (Lembersky, 1978; Reed, 1984; Snyder and Bhattacharya, 1990; Chadès and Bouteiller, 2005). Yet two critical limitations persist. First, biodiversity is often externalized or treated as a static constraint, rather than modeled as a productive asset. Empirical studies increasingly show that species and functional diversity enhance forest growth, carbon storage, and stability through mechanisms such as niche complementarity and insurance effects (Tilman et al., 1996; Cardinale et al., 2012; Brockerhoff et al., 2017). These findings suggest that biodiversity loss not only undermines ecosystem integrity but also compromises long-term economic returns. Despite this, most bioeconomic models treat biodiversity as exogenous to the production process. Unlike most prior studies, this paper explicitly endogenizes biodiversity as a dynamic, productive input influencing forest biomass growth, rather than treating it as an exogenous or static constraint. Second, the relative performance of alternative governance regimes under ecological instability and feedback remains poorly understood. While advances in flexible management and demographic modeling have increased ecological realism (Walters and Holling, 1990; Franklin et al., 2000; Grimm et al., 2005; Caswell, 2008; Allen et al., 2011), they often fail to capture the institutional and behavioral divergence between decentralized and coordinated decision-making. This study addresses these limitations by developing a finite-horizon optimal control model in which biodiversity directly enhances biomass growth and is itself affected by cumulative degradation and stochastic ecological shocks. Within this framework, we compare three governance regimes: a non-cooperative open-loop Nash equilibrium with unconstrained terminal states (OLNE-Free), a cooperative regime with fixed ecological end targets (Regulator-Fixed), and a cooperative regime with endogenously determined terminal states (Regulator-Free). These regimes are designed not as normative prescriptions but as institutional alternatives, enabling analysis of how endogenous ecological valuation and planning flexibility shape forest dynamics under ecological variability. Simulation results reveal that non-cooperative harvesters front-load extraction, neglect biodiversity's contribution to productivity, and allow resource withdrawal to accumulate. The Regulator-Fixed scenario mitigates these tendencies through binding constraints but exhibits reduced responsiveness to ecological shifts. The Regulator-Free regime, in contrast, dynamically aligns harvesting effort with evolving ecological and economic conditions. Without overstating precision, this regime generally maintains higher biomass and biodiversity, and achieves stronger welfare outcomes. These performance differences stem from integrating biodiversity into the growth function and permitting fixable adjustments over time. While the model does not simulate specific policy instruments, the resulting shadow-price differentials offer a conceptual foundation for incentive-based strategies. A theoretical implication of the model is that the difference between cooperative and non-cooperative shadow prices can be interpreted as an implicit marginal external cost - suggesting a dynamic Pigouvian fee that adjusts over time to internalize biodiversity's role in productivity and intertemporal scarcity. However, in practice, implementing such a tax is fraught with institutional barriers, including enforcement limitations, credit constraints, and ill-defined property rights - especially in tropical forest contexts. As such, market-based compensation mechanisms such as Payments for Ecosystem Services (PES), conservation subsidies, and forest-linked financial instruments (e.g., green or forest bonds) may offer more viable and politically palatable alternatives. These approaches align with REDD+ strategies and emerging biodiversity finance initiatives, which aim to incentivize conservation by rewarding avoided degradation rather than penalizing extraction. This analysis contributes to a growing body of work that highlights the interdependence between ecological complexity and economic sustainability (Brock and Xepapadeas, 2002; 2003; Naeem et al., 2009; Vardas and Xepapadeas, 2010). It reinforces the view that biodiversity should be treated not only as a constraint or amenity but as a dynamic, productive input to forest systems. Governance structures that internalize ecological feedbacks and permit flexible responses to environmental variability are better positioned to support long-term sustainability. These insights are consistent with recent policy frameworks, such as those proposed by IPBES[1] (Bongaarts, 2019), and underscore the importance of ecological valuation in shaping future conservation and land-use strategies. The paper is organized as follows: Section 2 reviews the relevant literature; Section 3 introduces the modeling framework and governance regimes; Section 4 presents the simulation results and policy implications; Section 5 offers an in-depth discussion, followed by concluding remarks.

2. Literature

The economic foundations of forest harvest modeling trace back to Faustmann (1849), whose rotation rule conceptualized timber production as an intertemporal investment problem by discounting future revenues. Subsequent refinements incorporated replanting costs, variable interest rates, and fiscal instruments - demonstrating that higher establishment costs or lower discount rates prolong optimal harvest cycles, while higher interest rates accelerate them (Hyytiäinen and Tahvonen, 2003). Despite these advances, early models largely treated forest growth as deterministic and ecological complexity as exogenous. A second wave of research introduced stochasticity into the optimization framework, particularly through volatile timber prices and biological growth indeterminacy. Clarke and Reed (1989) co-modeled price and stock dynamics using geometric Brownian motion, deriving a stochastic stopping rule that balances volatility and biological growth drift. Willassen (1998) extended this approach in an impulse-control setting, while Buongiorno (2001) applied Markov decision processes

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[1] IPBES: Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services.

(MDPs) to simulate optimal forest policies under indeterminacy. Earlier, Lembersky (1978) demonstrated how MDPs could be used to manage forest dynamics under probabilistic transitions, laying the groundwork for discrete-state optimization. Chadès and Bouteiller (2005) further advanced this approach in multi-agent contexts, solving cooperative and competitive forest management problems using MDP frameworks. Yet, biodiversity remained absent from these models - neither as a productivity determinant nor a dynamic component of forest ecosystems. The modeling of catastrophic disturbances added further realism. Routledge (1980) and Reed (1984) showed how wildfire risk compresses optimal rotation lengths. Thorsen and Helles (1998) linked windthrow to stand structure, while Loisel (2014) incorporated storm and pest damage into stochastic frameworks. These works recognized ecological risk but continued to treat biodiversity as a passive casualty rather than a structural variable influencing stability and growth. As attention shifted to ecosystem services, forest economics began integrating non-timber values. Snyder and Bhattacharya (1990) modeled recreation benefits increasing with stand age; Swallow and Wear (1993) incorporated wildlife habitat preferences; and Koskela and Ollikainen (2000) added carbon rents in response to climate policy. Even-aged models explored optimal thinning and rotation (Haight and Monserud, 1990; Cao et al., 2006; Clark, 2010), though often assuming that amenities did not affect biological growth. More flexible modeling emerged with uneven-aged and selective logging regimes. Schreuder (1971) demonstrated how continuous forest management systems - particularly under uneven-aged regimes - could be modeled using simulation, offering an alternative to traditional rotation-based approaches. Montgomery and Adams (1995) applied Hamilton-Jacobi-Bellman equations to characterize steady-state logging rules under continuous effort, while Touza et al. (2008) used variational inequalities to derive dynamic harvesting strategies. Although these models allowed for natural regeneration and flexible harvesting schedules, biodiversity feedbacks continued to be treated as exogenous to biomass dynamics. Strategic interactions among multiple agents - particularly under open access or fragmented property rights - added further bioeconomic complexity. Differential game theory captured intertemporal externalities in decentralized settings (Başar and Olsder, 1999), but few studies addressed how biodiversity-productivity linkages affect strategic equilibria. Meanwhile, ecological studies increasingly demonstrated that biodiversity enhances productivity. Tilman et al. (1996), Cardinale et al. (2012), and Brockerhoff et al. (2017) showed that species richness and structural heterogeneity improve biomass accumulation, nutrient cycling, and ecosystem stability. Ignoring these linkages risks underestimating both ecological stability and long-term economic returns. Disturbance ecology reinforces the need for stochastic modeling. Wildfires, pest outbreaks, and extreme weather degrade biodiversity and disrupt ecosystem states (Turner, 2010; Johnstone et al., 2016). Stochastic control tools - such as jump-diffusion processes (Øksendal and Sulem, 2007; Allen, 2010) and catastrophe-risk models from fisheries (Reed, 1988) - provide a foundation for simulating abrupt ecological shocks and flexible responses. Despite these conceptual advances, most forest models still do not endogenize biodiversity within the growth process, nor do they simulate cooperative and non-cooperative governance or account for strategic behavior. Recent work by Agliardi et al. (2024), which examines biodiversity preservation under ambiguity, irreversibility, and regulatory constraints, offers a complementary perspective that moves in the direction of more integrated ecological-economic modeling. Our study addresses that gap by building a dynamic model in which biodiversity enters the biomass growth function and is itself vulnerable to cumulative harvesting and stochastic shocks, modeled as a compound Poisson process. We compare three governance regimes - non-cooperative with free terminal states (OLNE-Free), cooperative with fixed terminal constraints (Regulator-Fixed), and cooperative with endogenous terminal conditions (Regulator-Free) - applying Pontryagin's Maximum Principle to derive optimal and Nash strategies (Corriga et al., 1988; Xepapadeas, 2022). The model reveals how biodiversity influences forest dynamics and long-term outcomes through its role in the growth process and its exposure to cumulative harvesting and ecological shocks. The resulting divergence between cooperative and non-cooperative shadow values provides a theoretical foundation for policy-relevant instruments.

3. The model

This study develops a bio-economic framework to model the dynamic interactions between biomass harvesting, biodiversity conservation, and forest ecosystem sustainability within a spatially homogeneous environment. We consider a finite number of rational agents $i = 1, \ldots, n$, each maximizing their own profit by harvesting biomass $u_i(t)$ over a finite planning horizon. The spatial homogeneity assumption implies that biomass and biodiversity are uniformly distributed across the forest, enabling the use of aggregate state variables without spatial disaggregation.

The natural growth dynamics of standing biomass $x(t)$ are initially articulated using a generalized nonlinear function:

$$F(x) = \alpha x(t) + \beta x(t)^{(\theta + 1)} \quad (1)$$

In alignment with classical population ecology modeling, we simplify this representation by setting $\theta = 1$, thereby reducing it to a form resembling the logistic growth equation. Re-expressing parameters, we define $\alpha = r(B)$ and $\beta = \frac{r(B)}{K}$, with K denoting the carrying capacity of the forest ecosystem. This leads to the logistic growth model:

$$\dot{x}(t) = r(B)x(t)\left(1 - \frac{x(t)}{K}\right) - \sum_{i=1}^{n} u_i(t), \qquad (2)$$

where $r(B(t))$ is the intrinsic biomass growth rate, which depends positively on the ecological index $B(t)$ encompassing biodiversity and broader ecosystem functionality.

The formulation of biodiversity's contribution to ecosystem productivity has been a subject of considerable research. Tilman et al. (1996) introduced a saturating functional form to describe how ecosystem productivity increases asymptotically with species richness. Loreau and Hector (2001) identified two primary mechanisms - complementarity and selection effects - that reinforce biodiversity's role in promoting ecosystem functioning. Cardinale et al. (2012) synthesized empirical findings across ecosystems, supporting a nonlinear, concave relationship between biodiversity and ecosystem services. In line with these insights, we model biodiversity's effect on biomass growth using a power-law function to capture the diminishing returns of biodiversity on forest productivity. Specifically, the intrinsic growth rate is defined as:

$$r(B) = AB^{\zeta}, 0 < \zeta \leq 1, \quad (3)$$

where $A > 0$ is a scaling coefficient and $\zeta \in (0,1]$ shapes how productivity changes with biodiversity. This specification captures that increases in biodiversity tend to enhance productivity, especially at lower diversity levels, though the rate of improvement may vary and is not necessarily strongly diminishing as biodiversity increases.

The biodiversity metric $B(t)$ evolves dynamically, representing both the regenerative capacity of the forest ecosystem and the cumulative effects of harvesting, which together influence productivity in complex ways (Baumgärtner, 2006). Its dynamics are modeled as:

$$\dot{B}(t) = G\big(B(t), Z(t)\big), \quad (4)$$

where the function G captures the dynamics of biodiversity growth in response to ecological factors and cumulative harvesting. It is constructed to satisfy key ecological and economic properties. Specifically, biodiversity growth increases with species richness ($\frac{\partial G}{\partial B} > 0$), though at a diminishing rate ($\frac{\partial^2 G}{\partial B^2} \leq 0$), reflecting ecological saturation effects. Conversely, cumulative harvesting negatively affects biodiversity growth ($\frac{\partial G}{\partial Z} < 0$), with the marginal impact decreasing as harvesting intensity rises ($\frac{\partial^2 G}{\partial Z^2} \geq 0$), capturing potential robustness or threshold effects. The function is further constrained by boundary conditions: biodiversity growth ceases as cumulative harvesting reaches an upper limit $\bar{Z}$ or as species richness approaches a saturation threshold $\bar{B}$, i.e., $\lim_{Z \to \bar{Z}} G\big(B(t), Z(t)\big) = 0$ and $\lim_{B \to \bar{B}} G\big(B(t), Z(t)\big) = 0$. Collectively, these properties ensure that the function realistically represents the trade-offs between harvesting pressure and biodiversity sustainability, allowing for growth under moderate conditions but signaling collapse under excessive exploitation.

Disturbance ecology recognizes that biodiversity can experience sudden, discontinuous losses triggered by fires, wind-throws, pest outbreaks, or pathogen incursions (Turner, 2010; Johnstone et al., 2016). To model such pulse events, a compound Poisson jump term can be incorporated into the biodiversity state equation as follows:

$$\dot{B}(t) = G\big(B(t^{-}), Z(t^{-})\big)dt - k\,\dot{N}_t\,, \qquad (5)$$

where $\dot{N}_t$ is a Poisson process representing the occurrence of jump events, and k denotes the average loss in biodiversity per event (Allen, 2010). The left-limit notation t^{-} indicates that the jumps occur instantaneously, affecting the state just prior to the jump. Since the biomass growth rate - for example, $r(B) = AB^{\zeta}$ - is evaluated at the post-jump state, stochastic shocks are instantaneously incorporated into the logistic biomass growth equation (1), precipitating an immediate decline in productivity that endures until biodiversity is restored. This jump specification augments the continuous diffusive noise employed in other ecological models (Lande, 1993) and aligns with empirical evidence indicating that forest disturbances manifest as infrequent yet severe pulses, rather than as minor, recurrent fluctuations. In expectation, the Poisson jump component contributes an additive drift correction to equation (4), yielding equation (5), while the jumps concurrently modulate the Hamiltonian via their influence on the state–costate dynamics, all of which may be formulated within a deterministic or open-loop control framework that explicitly eschews reliance on martingale or arbitrage arguments.

To facilitate analytical tractability, this study adopts the simplifying assumption of a constant jump intensity, denoted by ϖ and a fixed mean loss per event, k. Under these assumptions, the expected drift in biodiversity dynamics can be expressed as

$$\dot{B}(t) = G\big(B(t), Z(t)\big) - \varpi.k, \qquad (6)$$

where the term $\varpi.k$ represents the average instantaneous reduction in biodiversity due to jump events. quantifies the average instantaneous reduction in biodiversity attributable to jump events. This formulation captures the net effect of stochastic shocks on the system while preserving model simplicity.

Here, the growth function is specified linearly as

$$G\big(B(t), Z(t)\big) = gB(t) - \delta Z(t).$$

Substituting this expression into equation (6) yields:

$$\dot{B}(t) = \; gB(t) - \delta Z(t) - \; \varpi.k, \qquad (7)$$

with $g > 0$ and δ representing the sensitivity of biodiversity to external pressures. This linear formulation preserves analytical clarity while providing sufficient flexibility to capture essential ecological interactions subject to stochastic perturbations.

The cumulative harvest $Z(t)$ records total biomass removed, increasing with harvests and decreasing as past extraction impacts fade, modeled as

$$\dot{Z}(t) = \sum_{i=1}^{n} u_i(t) - \gamma Z(t), \qquad (8)$$

where γ represents the decay rate of historical extraction effects on biodiversity. Each harvester i maximizes their individual discounted profit:

$$\pi = \max_{u_i} \int_0^{\infty} e^{-\rho t}\big[p(t)u_i(t) - C\big(u_i(t)\big)\big]dt, \qquad (9)$$

In this formulation, $p(t)$ represents the price of biomass, while $C\big(u_i(t)\big)$ denotes a convex cost function, such as a quadratic function, that captures the increasing marginal costs associated with harvesting. The parameter ρ signifies the discount rate. This mathematical representation reflects the temporal value of profits, acknowledging that future earnings are intrinsically less valuable than present earnings due to the discount factor $e^{-\rho t}$.

The corresponding Hamiltonian for each harvester in the open-loop Nash framework is:

$$H = p(t)u_i(t) - C\big(u_i(t)\big) + \lambda_i \left[r\big(B(t)\big)x(t)\left(1 - \frac{x(t)}{K}\right) - u_i(t) - \sum_{j \neq i}^{n} \bar{u}_j(t) \right]$$

$$+ \mu_i[gB(t) - \delta Z(t) - \; \varpi.k] + \nu_i \left[u_i(t) + \sum_{j \neq i}^{n} \bar{u}_j(t) - \gamma Z(t) \right], \quad (10)$$

Applying Pontryagin’s Maximum Principle, the first-order condition yields the optimal control:

$$u_i^*(t) = \frac{p(t) - \lambda_i + v_i}{\alpha_{C_i}}. \quad (11)$$

Under the assumption of symmetric harvesting, substituting Eq. (11) into the system dynamics and costate evolution equations yields the system of differential equations describing the Nash equilibrium:

$$\begin{cases} \dot{x}(t) = r(B(t))\left(1 - \frac{x(t)}{K}\right) - n\frac{p(t) - \lambda + v}{\alpha_C} \\ \dot{B}(t) = gB(t) - \delta Z(t) - \varpi.k \\ \dot{Z}(t) = n\frac{p(t) - \lambda + v}{\alpha_C} - \gamma Z(t) \\ \dot{\lambda} = \rho\lambda - \left(\lambda r(B(t))\left(1 - \frac{2x(t)}{K}\right)\right) \\ \dot{\mu} = \rho\mu - \left(\lambda r'(B(t))x(t)\left(1 - \frac{x(t)}{K}\right) + \mu g\right) \\ \dot{v} = \rho v - (-\delta\mu - \gamma v) \end{cases} \quad (12)$$

In the cooperative scenario, a regulator maximizes total discounted welfare:

$$W = \max_{u_i} \int_0^\infty e^{-\rho t}\left(\sum_{i=1}^n \left[p(t)u_i(t) - C\big(u_i(t)\big)\right] + \mathrm{V}_1\big(B(t)\big) + \mathrm{V}_2\big(x(t)\big)\right) dt, \quad (13)$$

where $\mathrm{V}_1\big(B(t)\big)$ and $\mathrm{V}_2\big(x(t)\big)$ represent the societal value attributed to biodiversity and the standing biomass, respectively. These functions are assumed concave and increasing, capturing diminishing marginal valuation and the importance of conserving higher biodiversity levels and biomass stocks. Consequently, the current-value Hamiltonian capturing the regulator’s objective is:

$$H = \sum_{i=1}^n \left[p(t)u_i(t) - C\big(u_i(t)\big)\right] + \mathrm{V}_1\big(B(t)\big) + \mathrm{V}_2\big(x(t)\big)$$

$$+ \lambda_i \left[r(B)x(t)\left(1 - \frac{x(t)}{K}\right) - u_i(t) - \sum_{j \neq i}^n \bar{u}_j(t)\right] + \mu_i[gB(t) - \delta Z(t) - \varpi.k]$$

$$+ v_i \left[u_i(t) + \sum_{j \neq i}^n \bar{u}_j(t) - \gamma Z(t)\right], \quad (14)$$

Under symmetry, the optimal control remains:

$$u_i^*(t) = \frac{p(t) - \lambda_i + v_i}{\alpha_{C_i}}. (15)$$

The costate equations in the cooperative case incorporate the marginal societal valuations:

$$\begin{cases} \dot{\lambda} = \rho\lambda - \left(V_2'\big(x(t)\big) + \lambda r\big(B(t)\big)\left(1 - \frac{2x(t)}{K}\right)\right) \\ \dot{\mu} = \rho\mu - \left(V_1'\big(B(t)\big) + \lambda r'\big(B(t)\big)x(t)\left(1 - \frac{x(t)}{K}\right) + g\mu\right) \\ \dot{v} = \rho v - (-\delta\mu - \gamma v) \end{cases} \quad (16)$$

Consequently, the subsequent ODE system is to be solved:

$$\begin{cases} \dot{x}(t) = r(B(t))\left(1 - \frac{x(t)}{K}\right) - n\frac{p(t) - \lambda + v}{\alpha_C} \\ \dot{B}(t) = gB(t) - \delta Z(t) - \varpi.k \\ \dot{Z}(t) = n\frac{p(t) - \lambda + v}{\alpha_C} - \gamma Z(t) \\ \dot{\lambda} = \rho\lambda - \left(V_2'\big(x(t)\big) + \lambda r\big(B(t)\big)\left(1 - \frac{2x(t)}{K}\right)\right) \\ \dot{\mu} = \rho\mu - \left(V_1'\big(B(t)\big) + \lambda r'\big(B(t)\big)x(t)\left(1 - \frac{x(t)}{K}\right) + g\mu\right) \\ \dot{v} = \rho v - (-\delta\mu - \gamma v) \end{cases} \quad (17)$$

The coupled systems (12) and (17) represent two control problems that are approximated by taking expectations over the jump distribution and applying a scenario-based shooting algorithm, which ensures rapid convergence for moderate jump intensities while preserving the accuracy of the deterministic framework (Kushner, 1990; Seyedi, 2010; Xepapadeas, 2022). These systems are numerically solved using suitable software (e.g., Mathematica's *NDSolve*) with a stiffness-switching solver and high working precision to accurately capture the nonlinear dynamics (Seyedi et al., 2018). Depending on the modeling context, terminal conditions are imposed either as fixed states (e.g., prescribed $x(T), B(T)$) or as free costate variables (e.g., $(T) = 0, \mu(T) = 0, v(T) = 0$), reflecting the presence or absence of boundary constraints such as regulatory policies. The shooting method is employed to iteratively adjust initial costates to satisfy these terminal conditions. To evaluate the robustness of the model's outcomes, a comprehensive sensitivity analysis is performed by systematically varying key parameters $(\delta, g, \rho, \gamma, p, \varpi k)$ to examine their influence on optimal harvesting trajectories, ecosystem stability, and long-term welfare outcomes (see **Appendix A**). Under the usual ecological-economics assumptions – strictly positive parameters, interior initial stocks, and an ecologically feasible domain – the six-dimensional forward–backward systems defined in equations (12) and (17), which govern biomass, biodiversity, cumulative harvest, and their shadow prices, form a deterministic Pontryagin system that is well posed and produces a unique solution for given starting conditions. Because the biological components – logistic growth for biomass, linear regeneration for biodiversity, and exponential decay for cumulative harvest - are continuous and locally Lipschitz within the feasible domain, the Picard–Lindelöf theorem ensures that a unique local trajectory exists from any admissible starting point. In addition, the system is globally bounded. Linear growth bounds and inward-pointing vector fields ensure that, over the finite time horizon, the state variables remain uniformly bounded within the ecological domain, consistent with the saddle-point structure of the underlying dynamics (Lee and Haddad, 2021). This is formally supported using Grönwall's lemma and Nagumo's viability criterion, which together guarantee that solutions do not blow up in finite time and remain ecologically meaningful (Nagumo, 1942; Hale and Koçak, 1991; Teschl, 2012). The inclusion of costate equations – linear in the Nash system (12), affine in the planner system (17) – preserves Lipschitz continuity, so the overall forward–backward system inherits the same global existence guarantees. Continuous dependence on initial costates also ensures that the multiple-shooting algorithms used to satisfy transversality conditions are well behaved (Bryson, 2018). A local analysis around the steady state indicates that, under typical parameter values – such as a discount rate higher than ecological regeneration and cumulative impact rates, and moderate biodiversity feedbacks –s the system behaves in a locally stable manner when guided by the optimal control path over the finite time horizon (Khalil, 2002). This structure supports numerical stability as well: A-stable backward-differentiation solvers, such as those used by Mathematica's NDSolve(), accurately capture the system's decay behavior over time (Hairer et al., 1993; Hairer and Wanner, 1996). In the context of a shooting method, the contraction property of the boundary-matching map supports robust and efficient convergence of the numerical solution (Stoer and Bulirsch, 2002). These properties ensure that, for any feasible initial condition, the system remains well behaved over the entire time horizon, providing a rigorous foundation for the policy simulations and comparative-static insights developed in the article.

4. Results

This study develops a finite-horizon optimal control analysis of a coupled biomass–biodiversity–harvest system, emphasizing the influence of key ecological and economic parameters-including the stochastic disturbance intensity ϖk - on dynamic trajectories. Each parameter $(\delta, g, \rho, \gamma, p, \varpi k)$ governs the intertemporal allocation of value across state variables. Numerical simulations were performed in Wolfram Mathematica 14.2 using NDSolve with stiffness-adaptive solvers, 14-digit working precision, and adaptive time meshes to ensure numerical convergence. All runs begin from the initial condition $(x_0, B_0, Z_0) = (500, 10, 0.1)$. Free-boundary cases adopt transversality conditions $\lambda(20) = \mu(20) = v(20) = 0$, while fixed-boundary runs enforce $x(20) = 400, B(20) = 150$, and $Z(20) = 10$. Convergence was validated through mesh refinement and residual norm diagnostics, ensuring stability under all tested scenarios.

Fig. 1 presents state and costate trajectories under stochastic biodiversity shocks ($\varpi k = 0.1$). **Panels a–c** display the evolution of ecological states. **Panel a** shows that live biomass declines most rapidly under the non-cooperative free-terminal regime (OLNE-Free). Cooperative fixed-terminal governance (Reg-Fixed) sustains higher biomass levels throughout the time horizon. The cooperative free-terminal regime (Reg-Free) exhibits the slowest decline and maintains the highest biomass levels by the end of the period, although in all cases, biomass remains below initial levels. **Panel b** shows the biodiversity index $B(t)$, which increases under all regimes. The cooperative free-terminal regime (Reg-Free) yields the highest biodiversity by the end, with a steep upward trend. OLNE–Free and Reg–Fixed also show clear improvement, with OLNE–Free slightly outperforming Reg–Fixed later in the period. While all regimes support recovery, the pace and extent vary by governance. **Panel c**

shows cumulative harvesting $Z(t)$. Reg–Fixed follows an inverted-U pattern, with rapid early harvest that declines after the midpoint. OLNE–Free rises quickly at first, then levels off. Reg–Free increases steadily throughout, ending with the highest total harvest. These patterns reflect distinct strategies: early intensive use under Reg–Fixed, front-loaded harvesting in OLNE–Free, and gradual, sustained use in Reg–Free. **Panels d–f** show the evolution of costate variables. In **Panel d**, the biomass multiplier $\lambda(t)$ is higher under cooperative regimes, especially Reg–Fixed, reflecting internalized harvest externalities. Panel E shows the biodiversity shadow price $\mu(t)$: it stays flat and negative under OLNE–Free, rises from strongly negative under Reg–Fixed, and declines from a high positive value under Reg–Free. Panel F displays the terminal multiplier $v(t)$: Reg–Fixed starts high and drops sharply into negative territory, Reg–Free rises from a large negative value toward zero, and OLNE–Free stays flat and slightly positive, consistent with a non-binding terminal condition.

Fig. 2 displays the control and performance outcomes under each regime. **Panel a** shows that the non-cooperative OLNE-Free regime applies a constant harvest rate, lacking any dynamic adjustment. Reg-Fixed begins with aggressive extraction that steadily declines, while Reg-Free follows a time-varying harvest profile - effort rises through the first half of the period and then tapers off. **Panel b** highlights the welfare implications: both cooperative regimes significantly outperform OLNE-Free in cumulative discounted benefits, with Reg-Free achieving the highest total by a small margin. These results underscore the long-term gains of coordinated strategies that adjust harvest effort over time.

Fig. A1 illustrates the local sensitivity of biomass and biodiversity to key structural parameters under three governance regimes. **Panels a and b** display the OLNE-Free case, **Panels c and d** correspond to the Regulator-Fixed regime, and **Panels e and f** present the Regulator-Free regime. Each curve represents the partial derivative of a state variable – biomass $x(t)$ in the left column and biodiversity $B(t)$ in the right column - with respect to one of six parameters: cumulative impact coefficient δ, biodiversity growth rate g, discount rate ρ, natural recovery rate γ, biomass price p, and stochastic disturbance intensity ϖk, computed using centered-difference approximations along the baseline solution paths. **Panel a** shows that under OLNE-Free, biomass is highly negatively sensitive to increases in δ, ρ, p, and particularly ϖk, all of which accelerate ecological decline. In contrast, positive sensitivity is observed with respect to g and γ, indicating that regenerative processes partially offset degradation. **Panel b** reveals similar trends for biodiversity: sharp negative responses to δ and ϖk, moderate declines with p, and strong positive influence from g, while p and γ exhibit weak or negligible effects. Together, **Panels a** and **b** highlight the ecological vulnerability of non-cooperative management to both biophysical and economic stressors. **Panels c** and **d** depict the Regulator-Fixed regime, where biomass and biodiversity sensitivities become more complex due to the influence of terminal constraints. In **Panel c**, biomass responses to δ, g, γ and are non-monotonic and oscillatory over time, reflecting the interaction between dynamic effort control and fixed end conditions. In **Panel d**, biodiversity shows a bell-shaped positive sensitivity to the cumulative impact coefficient δ, peaking mid-horizon. The growth rate g exhibits a U-shaped negative pattern, while recovery γ remains consistently negative. Stochastic disturbance ϖk shows a mild but steady positive effect. Sensitivities to the discount rate ρ and biomass price p are negligible throughout. **Panels e** and **f** present the Regulator-Free case, where sensitivity profiles appear more stable and dampened. **Panel e** shows that biomass is positively responsive to δ and γ, while g and ρ exert consistently negative influence. The effect of p and ϖk remains weak and nearly flat throughout the horizon. **In Panel f**, biodiversity shows a strong and rising positive sensitivity to growth rate g. Sensitivity to δ, ρ, and ϖk is consistently negative but less volatile than in other earlier regimes. Recovery γ has a stable positive effect, while biomass price p remains minimally influential. Taken together, **Panels a–f** illustrate that system sensitivity is strongly influenced by institutional design. OLNE-Free (**Panels a–b**) displays the greatest ecological exposure, with large, monotonic sensitivities to multiple parameters. Regulator-Fixed (**Panels c–d**) shows constraint-driven oscillations and non-monotonic behavior, especially near terminal time. Regulator-Free (**Panels e–f**) yields smoother, more symmetric, and generally attenuated responses across parameters - indicating enhanced robustness and dynamic flexibility in response to structural changes.

Figs. 1–2 and **Fig. A1** show that cooperative governance - especially with free-terminal conditions - improves biomass, biodiversity, and welfare, while reducing sensitivity to structural parameters. Regulator-Free yields stable responses across drivers, whereas OLNE-Free shows strong, destabilizing effects. Cooperation thus supports both performance and ecological stability under change.

4.1.Policy Implications

The results reveal a persistent divergence between individual and socially optimal harvesting behavior, particularly under the non-cooperative free-terminal regime (OLNE-Free). As shown in **Fig. 1** (**Panels a–c**), private harvesters deplete biomass rapidly and underinvest in biodiversity recovery, failing to internalize long-term ecological trade-offs. This behavioral misalignment is captured in the costate dynamics (**Panels d–f**), where cooperative regimes assign higher shadow values to ecological states. Specifically, the cooperative biomass multiplier $\lambda(t)$ exceeds its non-cooperative counterpart throughout the horizon, reflecting stronger incentives for conservation under centralized planning. To address this gap, we propose a time-varying Pigouvian tax defined as $\tau(t) = \lambda_{Reg}(t) - \lambda_{OLNE}(t)$, levied per unit of harvested biomass. This tax would align private harvesting with social valuations, internalizing ecological externalities in real time.

As **Fig. 2** (**Panels a and b**) demonstrates, cooperative regimes dynamically adjust harvest effort and achieve higher cumulative welfare. Under stochastic disturbance ($\varpi k = 0.1$), OLNE-Free maintains a constant harvesting rate and performs poorly, while cooperative regimes strategically delay extraction - helping stabilize biomass and biodiversity as shown in **Fig. 1**, and boosting total returns as shown in **Fig. 2**. Accordingly, $\tau(t)$ should serve not only as a shadow-price correction, but also as an ecological risk adjustment during periods of ecological volatility.

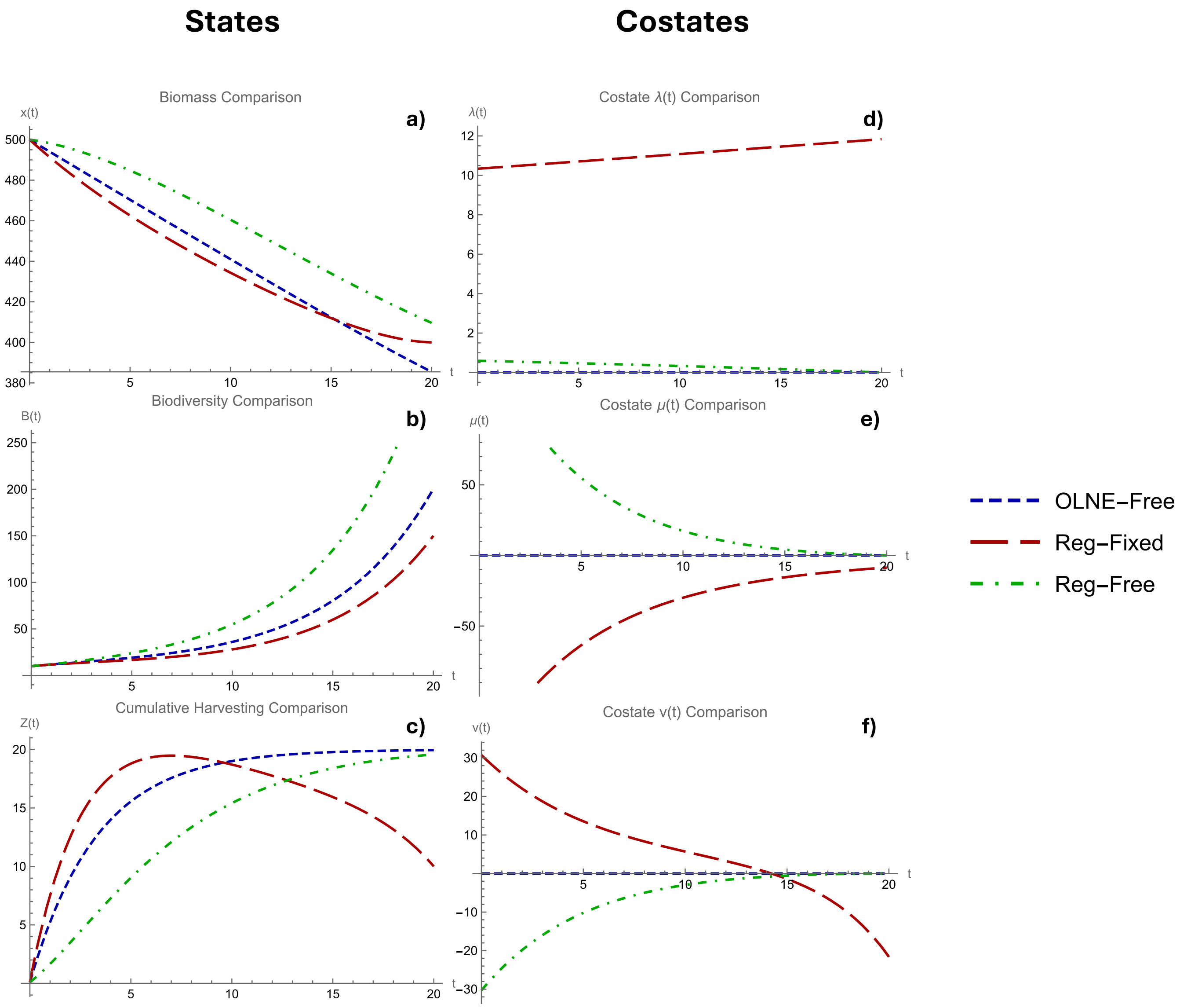

Fig. 1. Temporal Dynamics of States, and Shadow Prices in the Coupled Biomass–Biodiversity–Harvest System. Time evolution of state, and costate under three policy regimes: non-cooperative with free terminal conditions (OLNE-Free), cooperative regulation with fixed terminal targets (Reg-Fixed), and cooperative regulation with free endpoints (Reg-Free). Panels (A)–(C) show the trajectories of biomass $x(t)$, biodiversity $B(t)$, and cumulative harvest $Z(t)$, respectively. Panels (D)–(F) depict the corresponding shadow prices: biomass multiplier $\lambda(t)$, biodiversity multiplier $\mu(t)$, and terminal stock multiplier $\nu(t)$. Simulations use the following parameters: $g = 0.2, \gamma = 0.3, A = 2 \times 10^{-5}, \zeta = 1, K = 1000, \rho = 0.01, p = 40, n = 15, \delta = 0.1, \alpha_C = 100,$ with initial conditions $(x_0, B_0, Z_0) = (500,10,0.1)$. Fixed-boundary simulations impose terminal constraints $(x_f, B_f, Z_f) = (400,150,10)$ at horizon $T = 20$.

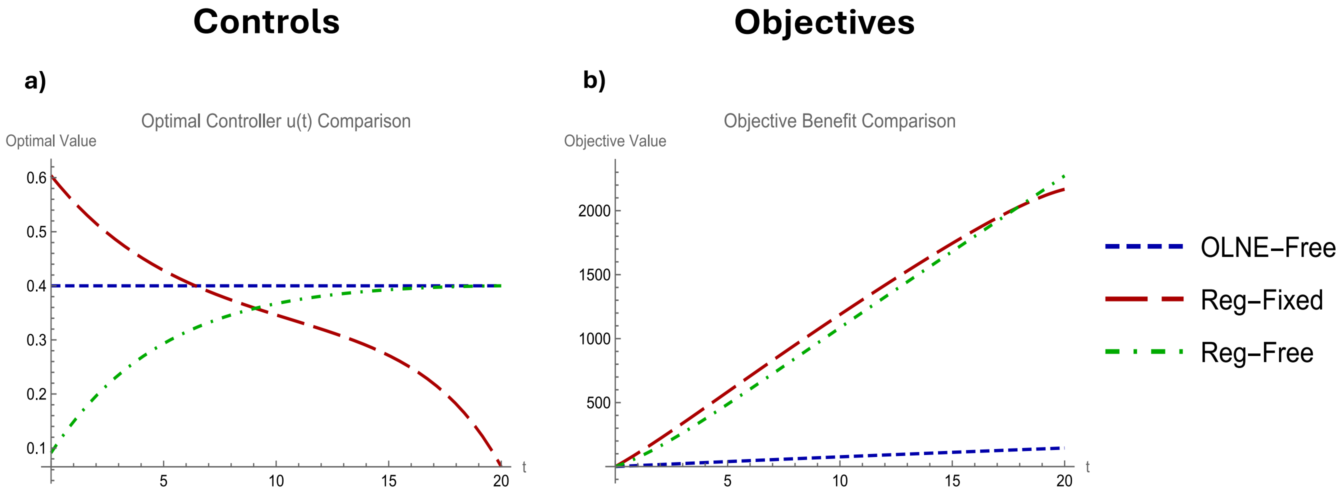

Fig. 2. Optimal harvest strategies and cumulative objective values under alternative governance regimes. Panel A shows the time path of the optimal harvest rate $u^*(t)$ for each regime. Panel B displays the corresponding cumulative present-value objective function aggregated up to time t. All simulations use the same parameter set and boundary conditions as in Fig. 1, with consistent linestyle conventions applied across panels.

Fig. A1 further reinforces this design. It shows that biomass and biodiversity exhibit regime-dependent sensitivity, with particularly strong responses to cumulative impact coefficient (δ) and biodiversity growth rate (g). OLNE-Free (**Panels a–b**) exhibits large, monotonic sensitivity profiles, indicating high ecological fragility. Regulator-Fixed (**Panels c–d**) shows non-monotonic shifts tied to terminal constraints, while Regulator-Free (**Panels e–f**) demonstrates smoother, more stable responses. These findings suggest that $\tau(t)$ should be state-contingent, rising in response to increased sensitivity, ecological fragility, or slowing biodiversity improvement. Incentive-based instruments - such as conservation subsidies, PES schemes, or ecological performance bonds - can operationalize these insights. For instance, a subsidy mechanism of the form $\sum p(t)(u^*_{Reg}(t) - u^*_{OLNE}(t))$, where $u^*_{Reg}(t)$ is the cooperative effort and $u^*_{OLNE}(t)$ is noncooperative behavior, would compensate agents who harvest less than privately optimal levels, reflecting the value of cooperation. Such instruments are particularly relevant where regulatory enforcement is limited or property rights are contested. More broadly, the findings support a shift toward flexible, context-sensitive governance tools that respond to ecological dynamics and structural indeterminacy. Cooperative regimes that adjust effort endogenously - particularly under free-terminal designs - offer a path to long-term sustainability. By stabilizing harvest trajectories and mitigating ecological shocks, such policies help reconcile individual use with collective ecological sustainability.

5. Discussion and final remarks

The present analysis explores the dynamic interplay between biomass, biodiversity, and harvesting activities within an abstract forest ecosystem, emphasizing the critical role of governance regimes in shaping ecological and economic trajectories. By employing a well-grounded and structurally rich modeling framework, this study highlights fundamental descriptive patterns that transcend specific parameter values, consistent with established theoretical bioeconomic modeling principles (Clark, 2010). This approach allows for robust insights into how biodiversity-dependent productivity influences ecosystem functionality, reflecting existing knowledge that diverse species assemblages enhance both biomass yield and system stability (Tilman et al., 1996; Cardinale et al., 2012). A central finding is that governance structures that internalize the ecological contributions of biodiversity outperform non-cooperative management in sustaining ecosystem stocks and promoting long-term welfare. Cooperative regimes, through their explicit recognition of biodiversity’s role in ecosystem growth, maintain higher biomass levels over time and foster more robust biodiversity recovery. This sustained ecological stock not only buffers the system against stochastic disturbances but also translates into significantly improved welfare outcomes, even when welfare functions are conservatively specified to avoid contentious valuations (Barbier, 2011; Costanza et al., 2014). The qualitative superiority of such cooperative management underscores the importance of integrating ecological capital into economic decision-making frameworks. The study further elucidates the mechanisms underlying these outcomes by examining the shadow prices associated with biomass and biodiversity. These costate variables reveal the implicit values assigned to ecological stocks under different governance regimes, with cooperative systems consistently attributing higher scarcity values to biomass. This valuation gap between private and social optima suggests the need for corrective economic instruments that can bridge behavioral divergences. Drawing on foundational optimal control theory and classical resource taxation literature (Reed, 1979), a dynamic, state-contingent Pigouvian tax emerges as a theoretically sound and practically relevant policy tool. Such a tax, by reflecting the evolving marginal scarcity of ecological assets, incentivizes harvesters to adjust their efforts in a manner that aligns private incentives with collective ecological sustainability. Sensitivity analyses highlight the nuanced ways in which ecological and economic parameters interact with governance structures to influence system dynamics. Cooperative regimes demonstrate flexible responses to increased cumulative impact coefficient and biodiversity growth rates, reducing harvest efforts when ecological stress intensifies and capitalizing on growth opportunities when conditions improve. In contrast, non-cooperative management tends to exacerbate ecological fragility by maintaining or increasing harvest pressure under stress, thereby weakening ecosystem stability. These findings resonate with broader theoretical insights into the vulnerability of unpriced natural capital to shocks and the challenges posed by discounting future ecological benefits (Dasgupta, 2021). Importantly, cooperative governance exhibits greater flexibility in adjusting harvest trajectories over time, mitigating the adverse effects of high discount rates and enhancing intertemporal sustainability. Institutional design emerges as a pivotal factor in balancing sustainability and flexibility. Fixed terminal constraints provide clear sustainability targets but may limit responsiveness in the face of environmental variability and indeterminacy. Conversely, governance frameworks that allow for endogenous determination of terminal states promote dynamic adjustment and responsiveness to evolving ecological conditions, reducing inefficiencies associated with rigid mandates (Seierstad and Sydsæter, 1987). This flexibility aligns with contemporary governance paradigms advocating polycentric, context-sensitive approaches that empower local actors to respond to ecological feedback through economic incentives rather than top-down controls (Ostrom, 2009; Arjomandi et al., 2022; 2024). Moreover, systems that accommodate ecological variability enhance robustness, as shown by our model's treatment of biodiversity disturbances (Tilman et al., 2006; Hong et al., 2021; Oliveira et al., 2022). The practical implementation of flexible, incentive-based instruments such as the proposed Pigouvian tax requires real-time ecological monitoring and robust data integration. Recent technological advances enable more timely and detailed ecological monitoring, supporting the development of flexible and transparent regulatory approaches (Bush et al., 2017). However, while the model provides a theoretical foundation for time-varying pricing derived from shadow-value differentials (Reed, 1979), direct implementation of such pricing mechanisms often faces institutional challenges. Difficulties in monitoring resource use, assigning responsibility, and enforcing ecological obligations can limit the practical application of dynamic fees (Tilman et al., 2006; Ostrom, 2009; Hong et al., 2021; Oliveira et al., 2022; Arjomandi et al., 2022; 2024). These findings suggest that while the tax can follow a known optimal time path, its implementation may still require state-dependent adjustments to account for ecological sensitivity, fragility, or biodiversity changes. Incentive-based tools - such as conservation subsidies, Payments for Ecosystem Services (PES), or ecological performance bonds - can translate these principles into practice by rewarding cooperative behavior that limits harvesting below private optimal levels, thus capturing the value of cooperation. These mechanisms, by focusing on compensating conservation actions rather than penalizing resource extraction, offer particularly suitable alternatives in governance settings constrained by enforcement capacity or contested property rights (Baumgärtner and Quaas, 2010; Farley and Costanza, 2010; Wunder, 2015). To further enhance the modeling framework, future research could incorporate disturbance intensity as an endogenous function of ecological state variables - for example, increasing vulnerability to shocks as forest biomass declines. This extension would capture critical feedback loops between ecosystem degradation and disturbance exposure. Moreover, integrating environmental stressors such as climate change and spatial fragmentation would enrich the realism of simulations. Incorporating ecological nonlinearities - including Allee effects, trophic cascades, and landscape connectivity - would improve the model's capacity to evaluate long-term ecosystem stability and inform the design of robust, precautionary management instruments (Scheffer et al., 2001; Rockström et al., 2009; Dasgupta, 2021). In summary, this study highlights the importance of incorporating ecological values into dynamic decision-making frameworks to promote sustainable forest use. While it does not prescribe specific policy instruments, it provides a theoretical foundation for designing interventions that align economic incentives with ecological sustainability. Flexible, incentive-compatible approaches are essential for translating ecological knowledge into actionable strategies, supporting global initiatives such as REDD+ and the biodiversity targets promoted by IPBES (Clark, 2010; Bongaarts, 2019). In practice, conservation efforts often rely on tools like Payments for Ecosystem Services (PES), which financially reward forest maintenance, and innovative instruments such as “Amazonia” bonds that link investor returns to tangible conservation outcomes (Wunder, 2005; Reuters, 2024). Though these instruments are not explicitly modeled here, the ecological shadow value insights developed in this study offer valuable guidance for designing and governing such mechanisms within complex ecological and strategic contexts.

CRediT authorship contribution statement

Seyedalireza Seyedi: Conceptualization, Methodology, Investigation, Formal analysis, Validation, Visualization, Writing - original draft, Writing - review & editing. **Elettra Agliardi:** Conceptualization, Methodology, Investigation, Formal analysis, Validation, Visualization, Supervision, Project administration, Writing - original draft, Writing – review & editing. **Anastasios Xepapadeas:** Conceptualization, Methodology, Investigation, Formal analysis, Validation, Visualization, Supervision, Project administration, Writing - original draft, Writing - review & editing.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. : Sensitivity of State Trajectories Across Scenarios

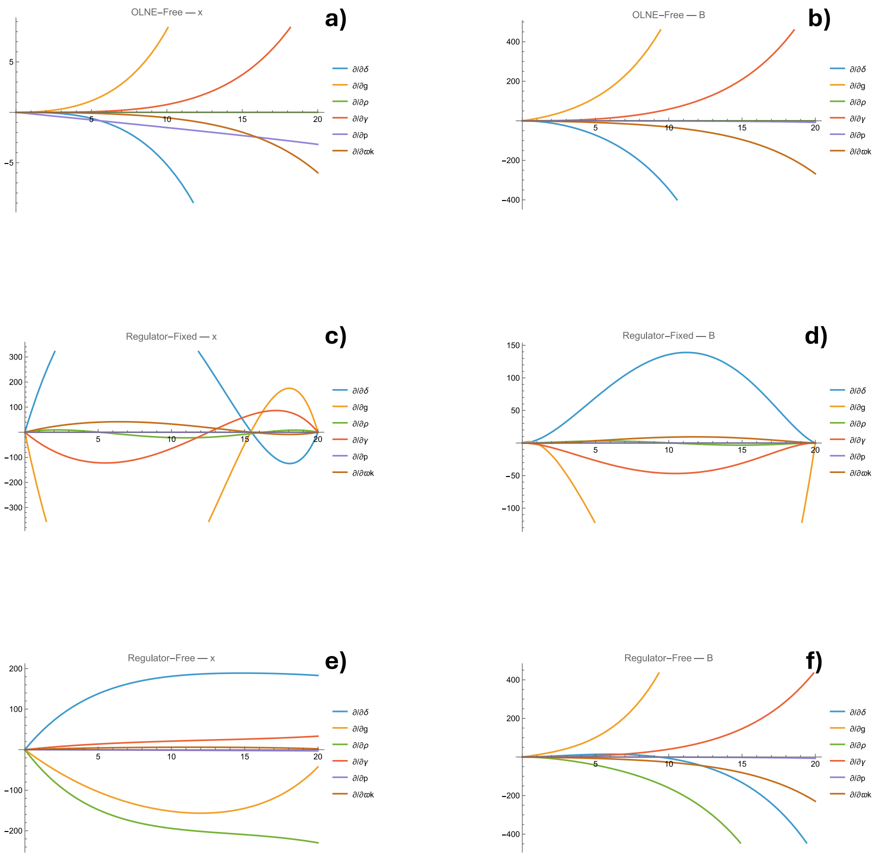

Fig. A1. Local Sensitivity of State Trajectories to Key Parameters Across Policy Scenarios. First-order sensitivity analysis of biomass $x(t)$ (left column) and biodiversity $B(t)$ (right column) with respect to key model parameters under different policy regimes. Panels (A)-(B) correspond to the OLNE-Free case, (C)-(D) to Regulator-Fixed, and (E)-(F) to Regulator-Free. Each curve shows the partial derivative of the state variable with respect to one parameter: δ (ecological depreciation), g (biodiversity regeneration rate), ρ (discount rate), γ (natural recovery rate), p (biomass price), and ϖk (stochastic disturbance intensity), evaluated along the baseline solution path. Numerical derivatives were obtained using a centered-difference scheme with step size $\varepsilon = 10^{-4}$. The results illustrate how system responsiveness varies across governance frameworks, with cooperative regimes and fixed-terminal constraints generally exhibiting dampened sensitivities, indicating enhanced robustness to parameter indeterminacy.

Appendix B. : Supplementary code

Supplementary code is available in the attached supplementary file.

References

Agliardi E, Agliardi R, Spanjers W. The Economic Value of Biodiversity Preservation. Environmental and Resource Economics. 2024 Apr;87(6):1593–610.

Allen LJS. An Introduction to Stochastic Processes with Applications to Biology. Chapman and Hall/CRC. 2010 Dec.

Allen CR, Fontaine JJ, Pope KL, Garmestani AS. Adaptive management for a turbulent future. Journal of Environmental Management. 2011 May;92(5):1339–45.

Arjomandi P, Seyedi S, Komendantova N, Hulasu EV. Vertical fit of water governing systems: A regional assessment. Current Research in Environmental Sustainability. 2024 Jan;7:100248.

Arjomandi P, Seyedi S, Nabavi E, Alikhani S. Exploring water governing system fit through a statistical mechanics approach. Water Research. 2022 May;215:118272.

Barbier EB. Capitalizing on nature: Ecosystems as natural assets. Cambridge University Press; 2011 Sep.

Başar T, Olsder GJ. Dynamic Noncooperative Game Theory. SIAM; 1999.

Baumgärtner S, Becker C, Faber M, Manstetten R. Relative and absolute scarcity of nature. Assessing the roles of economics and ecology for biodiversity conservation. Ecological Economics. 2006 Oct;59(4):487–98.

Baumgärtner S, Quaas MF. Managing increasing environmental risks through agrobiodiversity and agrienvironmental policies. Agricultural Economics. 2010 Aug;41(5):483–96.

Bongaarts J. Summary for policymakers of the global assessment report on biodiversity and ecosystem services of the Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services. Population and Development Review. 2019 Sep;45(3):680–1.

Brock WA, Xepapadeas A. Biodiversity Management under Uncertainty: Species Selection and Harvesting Rules. Edward Elgar Publishing eBooks. 2002 Oct.

Brock WA, Xepapadeas A. Valuing Biodiversity from an Economic Perspective: A Unified Economic, Ecological, and Genetic Approach. American Economic Review. 2003 Dec;93(5):1597–614.

Brockerhoff EG, Barbaro L, Castagneyrol B, Forrester DI, Gardiner B, González-Olabarria JR, et al. Forest biodiversity, ecosystem functioning and the provision of ecosystem services. Biodiversity and Conservation. 2017 Nov;26:3005–35.

Bryson AE. Applied optimal control: optimization, estimation and control. Routledge; 2018 May.

Buongiorno J. Generalization of Faustmann's formula for stochastic forest growth and prices with Markov decision process models. Forest science. 2001 Nov;47(4):466-74.

Bush A, Sollmann R, Wilting A, Bohmann K, Cole B, Balzter H, et al. Connecting Earth observation to high-throughput biodiversity data. Nature Ecology & Evolution. 2017 Jun;1(7) :0176.

Cao T, Hyytiäinen K, Olli Tahvonen, Valsta L. Effects of initial stand states on optimal thinning regime and rotation of*Picea abies*stands. Scandinavian Journal of Forest Research. 2006 Oct;21(5):388–98.

Cardinale BJ, Duffy JE, Gonzalez A, Hooper DU, Perrings C, Venail P, et al. Biodiversity loss and its impact on humanity. Nature. 2012 Jun;486(7401):59–67.

Caswell H. Matrix population models: construction, analysis, and interpretation. Sunderland, Mass.: Sinauer Associates; 2008.

Chadès I, Bouteiller B. Solving multiagent markov decision processes: A forest management example. In MODSIM05 - International Congress on Modelling and Simulation: Advances and Applications for Management and Decision Making, Proceedings. 2005. p. 1594-1600

Clark CW. Mathematical bioeconomics: the mathematics of conservation. Hoboken (New Jersey): Wiley; 2010.

Clarke HR, Reed WJ. The tree-cutting problem in a stochastic environment: The case of age-dependent growth. Journal of Economic Dynamics and Control. 1989 Oct;13(4):569–95.

Corriga G, Salimbeni D, Sanna S, Usai G. An optimal control problem in forest management. Applied Mathematical Modelling. 1988 Jun;12(3):328–32.

Costanza R, de Groot R, Sutton P, van der Ploeg S, Anderson SJ, Kubiszewski I, et al. Changes in the global value of ecosystem services. Global Environmental Change. 2014 May;26:152–8.

Dasgupta P, The economics of biodiversity: the Dasgupta review. London: HM Treasury. 2021.

Farley J, Costanza R. Payments for ecosystem services: From local to global. Ecological Economics. 2010 Sep;69(11):2060–8.

Faustmann M. Berechnung des Werthes, welchen Waldboden, sowie noch nicht haubare Holzbestande fur die Waldwirthschaft besitzen [Calculation of the value which forest land and immature stands possess for forestry]. Allgemeine Fotst-und Jagd-Zeitung. 1849;25:441-55.

Franklin JF, Lindenmayer D, MacMahon JA, McKee A, Magnuson J, Perry DA, et al. Threads of Continuity. There are immense differences between even-aged silvicultural disturbances (especially clearcutting) and natural disturbances, such as windthrow, wildfire, and even volcanic eruptions. Conservation in Practice. 2000 Jan;1(1):8–17.

Grimm V, Revilla E, Berger U, Jeltsch F, Mooij WM, Railsback SF, Thulke HH, Weiner J, Wiegand T, DeAngelis DL. Pattern-Oriented Modeling of Agent-Based Complex Systems: Lessons from Ecology. Science. 2005;310(5750):987–91.

Haight RG, Monserud RA. Optimizing any-aged management of mixed-species stands. I. Performance of a coordinate-search process. Canadian journal of forest research. 1990 Jan;20(1):15–25.

Hairer E, Wanner G, Nørsett SP. Solving ordinary differential equations I: Nonstiff problems. Berlin: Springer-Verlag; 1993.

Hairer E, Wanner G. Solving ordinary differential equations II: Stiff and differential-algebraic problems. Berlin: Springer-Verlag; 1996.

Hale JK, Koçak H. Dynamics and bifurcations. Text in Applied Mathematics 3. New York: Springer-Verlag; 1991.

Hong P, Schmid B, De Laender F, Eisenhauer N, Zhang X, Chen H, et al. Biodiversity promotes ecosystem functioning despite environmental change. Mori A, editor. Ecology Letters. 2021 Dec;25(2):555–69.

Hyytiäinen K, Tahvonen O. Maximum Sustained Yield, Forest Rent or Faustmann: Does it Really Matter? Scandinavian Journal of Forest Research. 2003 Sep;18(5):457–69.

Johnstone JF, Allen CD, Franklin JF, Frelich LE, Harvey BJ, Higuera PE, et al. Changing disturbance regimes, ecological memory, and forest resilience. Frontiers in Ecology and the Environment. 2016 Sep;14(7):369–78.

Khalil HK. Nonlinear systems. Upper Saddle River, New Jersey: Prentice Hall; 2002.

Koskela E, Ollikainen M. Forest Rotation under Spatial and Temporal Interdependence: a re-examination. Helsinki: University Of Helsinki; 2000.

Kushner HJ. Numerical Methods for Stochastic Control Problems in Continuous Time. SIAM Journal on Control and Optimization. 1990 Sep;28(5):999–1048.

Lande R. Risks of Population Extinction from Demographic and Environmental Stochasticity and Random Catastrophes. The American Naturalist. 1993 Dec;142(6):911–27.

Lee J, Haddad WM. On finite-time stability and stabilization of nonlinear hybrid dynamical systems. AIMS Mathematics. 2021s;6(6):5535–62.

Lembersky MR. The application of Markov decision processes to forest management. Dynamic Programming and its Applications. 1978;207–19.

Loisel P. Impact of storm risk on Faustmann rotation. Forest Policy and Economics. 2014 Jan;38:191–8.

Loreau M, Hector A. Partitioning selection and complementarity in biodiversity experiments. Nature. 2001 Jul;412(6842):72–6.

Montgomery CA, Adams DM. Optimal timber management policies. 1995. Pp.379–404 in Bromley DW, Eds. Handbook of Environmental Economics. Oxford: Basil Blackwell, Ltd.

Naeem S, Bunker DE, Hector A, Loreau M, Perrings C. Biodiversity, ecosystem functioning, and human wellbeing: an ecological and economic perspective. Oxford; New York: Oxford University Press; 2009.

Nagumo M. Über die Lage der Integralkurven gewöhnlicher Differentialgleichungen. Proceedings of the Physico-Mathematical Society of Japan 3rd Series. 1942 Jan;24:551–9.

Øksendal B, Sulem A. Applied Stochastic Control of Jump Diffusions. Springer Science & Business Media; 2007.

Oliveira BF, Moore FC, Dong X. Biodiversity mediates ecosystem sensitivity to climate variability. Communications Biology. 2022 Jun;5(1).

Ostrom E. A General Framework for Analyzing Sustainability of Social-Ecological Systems. Science. 2009 Jul;325(5939):419–22.

Puettmann KJ, Coates KD, Messier CC. A critique of silviculture: managing for complexity. Island press; 2012 Sep 26.

Reed WJ. Optimal escapement levels in stochastic and deterministic harvesting models. Journal of Environmental Economics and Management. 1979 Dec;6(4):350–63.

Reed WJ. The effects of the risk of fire on the optimal rotation of a forest. Journal of Environmental Economics and Management. 1984 Jun;11(2):180–90.

Reed WJ. Optimal Harvesting of a Fishery subject to Random Catastrophic Collapse. Mathematical Medicine and Biology A Journal of the IMA. 1988 Jan ;5(3):215–35.

Reuters. World Bank prices $225 million bond linked to Amazon reforestation. Reuters [Internet]. 2024 Aug 13 [cited 2025 Jul 3]. Available from: https://www.reuters.com/sustainability/sustainable-finance-reporting/world-bank-prices-225-million-bond-linked-amazon-reforestation-2024-08-13/

Rockström J, Steffen W, Noone K, Persson Å, Chapin FS, Lambin EF, Lenton TM, Scheffer M, Folke C, Schellnhuber HJ, Nykvist B. A Safe Operating Space for Humanity. Nature. 2009 Sep;461(7263):472-5.

Routledge RD. The Effect of Potential Catastrophic Mortality and Other Unpredictable Events on Optimal Forest Rotation Policy. Forest Science. 1980 Sep;26(3):389–99.

Scheffer M, Carpenter S, Foley JA, Folke C, Walker B. Catastrophic shifts in ecosystems. Nature. 2001 Oct;413(6856):591–6.

Schreuder GF. The Simultaneous Determination of Optimal Thinning Schedule and Rotation for an Even-Aged Forest. Forest Science. 1971 Sep;17(3):333–9.

Seierstad A, Sydsaeter K. Optimal Control Theory with Economic Applications. Advanced Textbooks in Economics. North Holland Inc.; 1987.

Seyedi S. Cutting angle method for optimal control problems [master's thesis]. Universiti Teknologi Malaysia, Faculty of Science; 2010 Sep.

Seyedi S, Amiri IS, Chaghervand S, Sorger VJ. The Numerical Solution of Continuous Time Optimal Control Problems with the Cutting Angle Method. Nova Science Publishers. 2018 Jun.

Spittlehouse DL, Stewart RB. Adaptation to climate change in forest management. Journal of Ecosystems and Management. 2004 Oct.

Snyder DL, Bhattacharyya RN. A more general dynamic economic model of the optimal rotation of multiple-use forests. Journal of Environmental Economics and Management. 1990 Mar;18(2):168-75.

Stoer J, Bulirsch R. Introduction to Numerical Analysis. New York, NY: Springer New York; 2002.

Swallow SK, Wear DN. Spatial interactions in multiple-use forestry and substitution and wealth effects for the single stand. Journal of Environmental Economics and Management. 1993 Sep;25(2):103-20.

Teschl G. Ordinary differential equations and dynamical systems. Providence (RI): American Mathematical Society; 2012.

Thorsen BJ, Helles F. Optimal stand management with endogenous risk of sudden destruction. Forest Ecology and Management. 1998 Aug;108(3):287–99.

Tilman D, Reich PB, Knops JMH. Biodiversity and ecosystem stability in a decade-long grassland experiment. Nature. 2006 Jun;441(7093):629–32.

Tilman D, Wedin D, Knops J. Productivity and sustainability influenced by biodiversity in grassland ecosystems. Nature. 1996 Feb;379(6567):718–20.

Touza J, Termansen M, Perrings C. A bioeconomic approach to the Faustmann–Hartman model: Ecological interactions in managed forest. Natural Resource Modeling. 2008 Oct;21(4):551-81.

Turner MG. Disturbance and landscape dynamics in a changing world. Ecology. 2010 Oct;91(10):2833–49.

Vardas G, Xepapadeas A. Model uncertainty, ambiguity and the precautionary principle: implications for biodiversity management. Environmental and Resource Economics. 2010;45:379-404.

Walters CJ, Holling CS. Large-Scale Management Experiments and Learning by Doing. Ecology. 1990 Dec;71(6):2060–8.

Willassen Y. The stochastic rotation problem: A generalization of Faustmann’s formula to stochastic forest growth. Journal of Economic Dynamics and Control. 1998 Apr;22(4):573-96.

Wunder S. Payments for environmental services: some nuts and bolts. CIFOR Occasional Paper No. 42. Bogor, Indonesia: Center for International Forestry Research; 2005.

Wunder S. Revisiting the concept of payments for environmental services. Ecological Economics. 2015 Sep;117:234–43.

Xepapadeas A. On the optimal management of environmental stock externalities. Proceedings of the National Academy of Sciences. 2022 Jun 14;119(24):e2202679119.