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# Alternative Ways of Information Processing as a Source of Sustainable and Rational Peer Disagreement

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# Alternative Ways of Information Processing as a Source of Sustainable and Rational Peer Disagreement

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**Abstract**

Consider an event of interest $B$ and another event $A$ which is viewed as information for $B$. When a decision maker (DM) evaluates the effect of $A$ on $B$, she evaluates the degree to which she asserts the indicative conditional "if $A$ then $B$", written as $A \longrightarrow B$. In the context of the standard Bayesian confirmation theory, the degree of assertability, $As(A \longrightarrow B)$ is given by DM's subjective conditional probability $P(B \mid A)$. However, the Bayesian interpretation is not the only rational interpretation of $As(A \longrightarrow B)$. An alternative interpretation is that $As(A \longrightarrow B)$ goes by the probability that the proposition $A \longrightarrow B$ is true, that is by DM's unconditional probability $P(A \longrightarrow B)$. It is now widely accepted that there is no interpretation of "$\longrightarrow$" that ensures the genral validity of $P(A \longrightarrow B) = P(B \mid A)$. Hence, there are multiple truth-conditional interpretations of "$\longrightarrow$" each corresponding to a distinct way of information processing. One of these interpretations, namely the material implication of the Propositional Logic, competes favorably with the Bayesian interpretation on normative grounds. As a result, two decision makers can disagree about their posterior probabilities of $B$ even if they share the same information $A$ and have identical prior probability functions.

*Keywords: Information processing; rationality; disagreement; bayesianism; logic*

*JEL Classification*: C44, D81, D83, D89

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# 1 Introduction

In his seminal paper, Aumann (1976) proves the following proposition: If two people have identical prior probability distributions, and their posteriors are "common knowledge", then these posteriors must be equal, even if they are based on completely different information. This proposition is taken to imply that two economic agents cannot "agree to disagree". This result is based on the assumptions that (i) both agents update their priors (each based on her own private information) via Bayesian Conditionalization (BC) and (ii) each of the two agents knows that the other agent updates by BC. Hence, both agents are assumed to agree on how the "available information" (which takes the form of an event or a proposition $A$) should be processed. An interesting question that arises at this point is whether the two agents can disagree about the way in which the information should be processed. Put differently, is there more than one *rational* way of information processing? If so, then (as will be shown in this paper) two people with the same priors, the *same information* but with different ways of processing that information will not be compelled to agree.

The concept of "information processing" is a key pillar of Economics and Decision Theory.The standard view is that the processing of information $A$ for the event/proposition of interest $B$ by the decision maker (DM) is based on analyzing the probabilistic implications of $A$ for $B$, thus culminating in the formation of DM's subjective conditional probability $P(B \mid A)$. The question that we raise in this paper is whether the probabilistic implications of $A$ for $B$ are uniquely captured by $P(B \mid A)$ (i.e. the Bayesian way) or there are other "probabilistic vehicles" that may play the role of "information processors". More specifically, consider the following two propositions: $A$: "The Federal Reserve, at some specific time point $t = m$, announces an unexpected rise in the interest rate" and $B$: "The stock price index at $t = m$ declines". DM is asked whether she finds $A$ to be "useful information" or "evidence" for $B$. What are the logical steps that DM follows in order to answer this question?

As a first step, DM is likely to ask herself the question "what will happen to the stock price index at $t = m$ *if* the FED unexpectedly increases the interest rate at that time?". In other words, DM wonders whether a rise in the interest rate by the FED supports or even necessitates a decline in the stock prices. This means that the first step in DM's processing of the information $A$ for the proposition of interest $B$ is to conceive the *indicative conditional* "if $A$ then $B$", denoted by $A \longrightarrow B$, where $A$ and $B$ are referred to as "the antecedent" (the implying proposition) and "the consequent" (the implied proposition) of the conditional, respectively. The switch to this hypothetical (if-then) mode of reasoning is considered to be a necessary step that DM has to take in order to evaluate whether $A$ carries useful information for the proposition of interest $B$. This information can be "causal" or "evidential". In the aforementioned example, consider agent X who believes that a rise in interest rate by the FED ($A$) causes stock prices to fall ($B$). Another agent Y, believes that such a rise in interest rate is good evidence for (although not a cause of) a decline in stock prices. Both agents believe that $A$ is useful information for $B$, and they assert that by saying "if $A$ then $B$". Bonnefon and Villejoubert (2007) pose the issue as follows: "A natural way to express that $A$ is a reason to believe $B$ is to embed both propositions in an epistemic conditional (also called 'inferential', or 'evidential') of the form "if $A$, then $B$" (2007, p. 210, our notation). Evans et. al. (2007) emphasize the epistemic role of conditional statements as follows: "In our view, such statements are of particular importance because *if* is used to initiate the imagination and simulation of possibilities, a process that we term *hypothetical thinking*... Such imaginary thought is required whenever we entertain a hypothesis, attempt to forecast future events, or imagine the *consequences of alternative actions* to support our decision making." (2007, p. 1772, emphasis added). Ali, Chater and Oaksford (2011) argue that "hypothetical, 'what if', reasoning" is of fundamental importance "to all kinds of cognitive processes, from decision making and planning to moral reasoning" (2011, p. 403). Stalnaker (1984) explicitly ties "information processing" to the indicative conditional: "Indicative conditionals are merely expressions of conditional beliefs, that is, of the agent's dispositions to change what he accepts *in response to new information*" (1984, p. 102, emphasis added).

The second step for DM is to consider the degree to which she is justified (according to her own view) to state the conditional proposition "if $A$ then $B$". In other words, she has to consider the degree, $As(A \longrightarrow B)$, to which she *accepts* or *asserts* the conditional proposition $A \longrightarrow B$. This means that in answering the question whether $A$ conveys useful information for $B$, DM proceeds as follows: a) She envisages the conditional $A \longrightarrow B$. b) She contemplates her degree of "acceptability" or "assertability", $As(A \longrightarrow B)$, of this conditional. c) She chooses how to interpret $As(A \longrightarrow B)$ and then, based on the selected interpretation, how to ascertain it. To this end, the philosophical literature has identified two main interpretations of

$As(A \longrightarrow B)$. The first interpretation, hereafter referred to as $Int1$, identifies $As(A \longrightarrow B)$ with $DM'$s subjective conditional probability $P(B \mid A)$ :

$$As(A \longrightarrow B) = P(B \mid A).$$

As already mentioned, this interpretation is universally endorsed by the Economic literature and the Bayesian decision theory, especially regarding how DM learns from experience. In particular, a widely accepted norm of diachronic rationality is the aforementioned Bayesian Conditionalization: DM's new (posterior) probability of $B$ in the light of information $A$ should be set equal to her prior subjective conditional probability $P(B \mid A)$. An early proponent of $Int1$ is Ramsey (1926), who in a widely-cited footnote made the following claim (in our notation): "If two people are arguing 'If $A$ will $B$?' and are both in doubt as to $A$, they are adding $A$ *hypothetically* to their stock of knowledge and arguing on that basis about $B$...We can say they are fixing their degrees of belief in $B$ *given* $A$" (1926, pp. 155, emphasis added).[1] Skyrms (1980) defends $Int1$ by arguing that "The idea that the assertibility of the indicative conditional of natural language goes by the corresponding conditional probability is so attractive that it has been advanced again and again..." (1980, pp. 87-88). Edgington (1995) claims that "on the supposition of $A$" or "given $A$" seem to be "mere stylistic variations" on "if $A$". (see also Jackson 1987 and Hajek 2012 for similar views). On this view, the conditionals which are (intuitively) highly assertible such as "if the FED increases the interest rate, the stock prices will fall" will invariable be those for which the probability of the consequent ("the stock price will fall") given the antecedent (the FED increases the interest rate) is high. Similarly, conditionals with low degree of asseribility (e.g. "if the velocity of money increases, the inflation rate will decline") are those for which the corresponding conditional probability is low.

The second interpretation of $As(A \longrightarrow B)$, hereafter referred to as $Int2$, begins with the following question (see Mellor 1993): "What is it to accept the conditional 'if $A$ then $B$'?" Mellor argues that the obvious answer is "to believe it" (1993, p. 235). This means that (for all $A$, $B$ and $\gamma$), to accept (or assert) "if $A$ then $B$" to degree $\gamma$ is to have subjective probability equal to $\gamma$ *of the truth* of the indicative conditional $A \longrightarrow B$. On this view, $As(A \longrightarrow B)$ is interpreted as $P(A \longrightarrow B)$, that is, as DM's *unconditional* subjective probability that the proposition $A \longrightarrow B$ is true:

$$As(A \longrightarrow B) = P(A \longrightarrow B).$$

This interpretation assumes that $A \longrightarrow B$ is a "truth-conditional proposition", (i.e. it is a "meaningful" proposition in the sense that it is either "true" or "false") a thesis that is sometimes referred to as "propositionalism" (Rothchild 2013). Put differently, $Int2$ assumes that DM uses "if" to express a proposition, namely "if $A$ then $B$", that is evaluable in terms of truth, i.e. it can be either "true" or "false". [2]

## 1.1 Conditional Probability versus Probability of the Conditional

Are the two interpretations introduced above really different? For example, do we find it reasonable to accept $Int1$ and reject $Int2$? If we answer this question in the affirmative, then we must accept that "acceptability" or "assertability" is different than "truth", or as Stalnaker (1975) puts it "assertion aims at more than truth" (1975, p. 138). On this view, someone may assert a proposition $A$, without believing in the truth of $A$. On the other hand, if $P(A \longrightarrow B)$ were "always" equal to $P^s(B \mid A)$ (i.e. for every $B$ and $A$ in the domain of $P$, such that $P(A) > 0$, and for every possible probability function $P$) then the aforementioned question becomes irrelevant. In such a case, $Int1$ and $Int2$ are epistemically equivalent and empirically indistinguishable, which implies that the degree of assertion $As(A \longrightarrow B)$ can be identified either by means of $P(B \mid A)$ or *equivalently* by means of $P(A \longrightarrow B)$. Indeed, there are good intuitive reasons to believe that $P(A \longrightarrow B)$ is equal to $P(B \mid A)$ : "The probability that the stock prices will fall *if* the FED increases the interest rate", at first glance, sounds equivalent to "the probability that the stock prices will fall *given that* the FED increases the interest rate". The preceding discussion culminates in the following fundamental question: Does the equality

$$P(A \longrightarrow B) = P(B \mid A) \tag{1}$$

[1] The procedure for evaluating the conditional by means of $P(B \mid A)$ is usually referred to as the "Ramsey test" (see Gibbard 1981).

[2] The question of whether the indicative conditional $A \longrightarrow B$ has truth conditions has been at the center of philosophical debate since the Mid-1970s (see, for example, Adams 1975, Edgington 1995, Bennett, 2003).

hold always? An affirmative answer to this question was intitially given by Stalnaker (1970), and for this reason (1) is usually referred to as *Stalnaker's hypothesis.* This hypothesis states that the assertability of the conditional $A \longrightarrow B$ goes (as with any other truth-conditional proposition) by the unconditional subjective probability that $A \longrightarrow B$ is true, which in turn (as a free bonus) is always equal to the conditional probability of the consequent, $B$, given the antecedent, $A$.

However, this "happy coincidence" did not last long. In a famous paper, Lewis (1976) proved his "triviality results", according to which there is no conditional connective "$\longrightarrow$" that both makes $A \longrightarrow B$ truth-conditional and ensures the validity of (1) uniformly, (that is for every $A$, $B$ and $P$). If we accept this result then the ensuing question is how to interpret the failure of (1). To this end, we may distinguish two alternative interpretations. First, the conditional $A \longrightarrow B$ does not express a truth-conditional proposition. In other words, $A \longrightarrow B$ is not a proposition that admits the truth values "true" or "false". But if $A \longrightarrow B$ is not "truth-apt" then as Hajek (2012) remarks, "nor is it probability-apt" (2012, p. 147). Hence, the left-hand side of (1) does not make any sense, and this is why the equation fails to hold. On this view, the assertability of the conditional goes (solely) by $P(B \mid A)$, that is $Int1$ prevails. Second, $A \longrightarrow B$ is a truth-conditional proposition, in which case $P(A \longrightarrow B)$ does make sense, i.e. it is the probability that the proposition $A \longrightarrow B$ is true. On this interpretation, the failure of (1) means that some values of $P(A \longrightarrow B)$ must differ from $P(B \mid A)$. This in turn implies, that in general, we are left with two alternative candidates for the assertability of the conditional $A \longrightarrow B$, namely $P(A \longrightarrow B)$ and $P(B \mid A)$. Which of these two candidates represents the rational way of ascertaining $As(A \longrightarrow B)$ or, alternatively, of rationally processing the information content of $A$ for the proposition of interest $B$?

This question has been the subject of intense debate among philosophers and psychologists alike over the past 50 years.[3] For example, the philosophers Adams (1965, 1975, 1998) and Edgington (1986, 1995a, 1995b) reject the view that conditionals have truth conditions, thereby supporting $Int1$. In a similar fashion, the psychologists Evans and Over (2004) endorse $Int1$ and provide evidence that apart from its normative virtues, $Int1$ enjoys empirical support as well (see also Stevenson & Over, 1995; Liu, Lo, & Wu, 1996; Oaksford, Chater, and Larkin, 2000). More specifically, Edgington (1986) argues that the correct way for a DM to assess the conditional $A \rightarrow B$ is by ascertaining the degree to which she asserts $B$ "within the scope of the supposition, or assumption, that $A$", that is by eliciting her subjective conditional probability $P(B \mid A)$. By doing this, DM exercises "a double illocutionary force", namely "an assumption and an assertion within its scope." (1986. p. 5).

Byrne and Johnson-Laird (2009), on the other hand, argue that asserting $A \longrightarrow B$ by means of $P(B \mid A)$ is mistaken (see also Johnson-Laird and Byrne 1991). This error stems from the fact that people often misinterpret the question "In what cases is it true that if *that* happens then *this* happens" as meaning "If *that* happens then in what circumstances is it true that *this* happens?" In other words, people often "mistakenly evaluate, not the truth of the conditional, but the truth of its consequent given that its antecedent holds." (2009, p. 284). In a similar fashion, Girotto and Johnson-Laird (2004) argue that in the relevant psychological studies, respondents appear to misinterpret $P(A \longrightarrow B)$ as $P(B \mid A)$ because (a) the antecedent $A$ in the conditional $A \longrightarrow B$ is a subordinate clause and (b) whenever there is a subordinate clause, people tend to interpret propositional operators and questions that preface this clause as applying only to the main clause. Johnson-Laird and his co-authors take the view that proper conditional reasoning is dictated by the so-called "mental models theory" (Johnson-Laird, 1983; Johnson-Laird and Byrne, 1991) in the context of which the conditional $A \longrightarrow B$ is truth-bearer and therefore its assertability goes by $P(A \longrightarrow B)$. On this view, the correct interpretation of the conditional is the one implied by $Int2$.

The foregoing discussion suggests that we are left with two alternative candidates for the assertability of the conditional $I \longrightarrow A$, namely $P(A \longrightarrow B)$ and $P(B \mid A)$. Moreover, the existing empirical evidence suggests that some people assert the conditional via $P(A \longrightarrow B)$ while others via $P(B \mid A)$. As a result, we can distinguish two sets of decision makers, namely $\mathbf{S}_C$ and $\mathbf{S}_P$: The members of the first set choose to adopt $P(A \longrightarrow B)$ as the appropriate vehicle for information processing, while the members of $\mathbf{S}_P$ choose $P(B \mid A)$ for the same purpose. Given that $P(A \longrightarrow B)$ is in general different from $P(B \mid A)$ (due to the aforementioned triviality results), the agents in $\mathbf{S}_C$ seem to "disagree" with those in $\mathbf{S}_P$.

[3] It is curious that economists are completely absent from this debate. As we mentioned above, they universally adopt $Int1$ while seeming to ignore $Int2$.

Apart from the fundamental disagreement between the agents in $\mathbf{S}_C$ and those in $\mathbf{S}_P$ about the truth-conditionality of $A \longrightarrow B$, additional disagreement may arise within the members of $\mathbf{S}_C$, i.e. among the agents who assert the conditional via $P(A \longrightarrow B)$. The source of this disagreement is located in the alternative interpretations that these agents (all members of $\mathbf{S}_C$) may give to the conditional connective "$\longrightarrow$", which appears in the proposition $A \longrightarrow B$.[4] In other words, even though all the agents in $\mathbf{S}_C$ have agreed on the basic premise that $A \longrightarrow B$ is a meaningful proposition (i.e. it is a truth-bearer), they nevertheless disagree on the semantics of "$\longrightarrow$". This topic is analyzed in Section 3 of the paper. The alternative interpretations of $As(A \longrightarrow B)$ are summarized in Table 1:

| | |
|---|---|
| 1). "$\longrightarrow$" is Truth-conditional | 1). "$\longrightarrow$" is not Truth-Conditional |
| 2). $P(A \longrightarrow B)$ | 2). $P(B \mid A)$ |
| 3). Several interpretations of "$\longrightarrow$" | |

Table 1: Alternative Interpretations of $As(A \longrightarrow B)$

Is the aforementioned disagreement about the assertability of $A \longrightarrow B$ rational? Alternatively, is there more than one rational way to assert $A \longrightarrow B$? The answer to this question is the main objective of the present paper.

## 1.2 Outline and Main Results

Section 2 discusses Lewis's triviality results on the basis of which (1) is rejected.

Section 3 defines the alternative ways in which economic agents assert the conditional proposition $A \longrightarrow B$. More specifically, we consider several interpretations of "$\longrightarrow$" each of which corresponds to a different way of information processing. We show that agents' disagreement on the truth conditions of $A \longrightarrow B$ ($\mathbf{S}_C$ versus $\mathbf{S}_P$ as well as disagreement within $\mathbf{S}_C$) automatically translates into disagreement about the degree of assertability of $A \longrightarrow B$, even under the assumption that these agents share the same prior probability distribution over the "state descriptions" (or possible worlds).

Section 4 analyzes the normative status of the alternative interpretations of "$\longrightarrow$" introduced in Section 3. The main criterion on the basis of which the alternative interpretations of the conditional are assessed is that of "probabilistic validity of an inference", introduced by Adams (1975, 1998). More specifically, we consider a set $\mathcal{I}$ containing nine types of inference (argument) forms, some of which are intuitively "good" (in the sense that the conclusion "seems to follow necessarily" from the premises) others intuitively "bad" (in the sense that the premises do not seem to entail the conclusion) and the rest "controversial". Regarding the latter, some cases of a controversial inference form are intuitvely valid while some others seem suspect.[5] As a result, these inferences are in the boundary zone that seperates valid from non-valid arguments. Since these nine inference forms are the most widely analyzed ones within conditional reasoning, we can say that $\mathcal{I}$ summarizes most of our pretheoretical intuitions about "logical necessity". A rational interpretation of "$\longrightarrow$" is one that delivers all the intuitively good inferences and blocks all the faulty ones. In other words, our commonsense intuitions about good or bad inferences constitute the "data" we use to evaluate different "theories" (interpretations) of the indicative conditional. Armed with the formal notion of "probabilistic validity", we examine which interpretation of "$\longrightarrow$" renders the intuitively "good" inferences formally valid and the intuitively "bad" inferences formally invalid. Another way to put the matter is as follows: Each of the interpretations of "$\longrightarrow$" can potentially lead to two types of errors: (a) To formally reject an inference that we (informally, pre-theoretically) regard as reasonable. (b) To formally accept an inference that we intuitively regard as absurd. The results show that for none of the interpretations of "$\longrightarrow$", under examination the probabilities of the aforementioned two types of errors are both zero. Hence, there is no interpretation of the indicative conditional that translates all the intuitively good inferences of $\mathcal{I}$ into formally valid ones and all the intuitively faulty inferences of $\mathcal{I}$ into formally invalid ones. These results notwithstanding, two of the

[4] Two agents adopt a different semantic interpretation of $A \longrightarrow B$ iff they assign different truth values to the corresponding ($A \longrightarrow B$)-column of the relevant truth table.

[5] This is mainly due to the philosophers who never get tired of looking for counterexamples of inference forms that are generally considered valid.

interpretations under investigation, namely the material implication of propositional logic and the non-truth-conditional $P(B \mid A)$, seem to fare better than the rest in terms of preserving most of our commonsense intuitions about rational conditional inferences.

Building on these results, Section 5 analyzes the issue of plausible convergence of the interpretations of "$\longrightarrow$" among economic agents, referred to as "logical convergence". Consider two such agents, $S_1$ and $S_2$ who, having adopted two different interpretations of "$\longrightarrow$", end up having different degrees of assertability of $A \longrightarrow B$. It is important to note that no amount of empirical information can force these two disagreeing agents to come to an agreement, since the source of their disagreement is not, for example, that they have different prior probability distributions, but that they each process the same empirical information, $A$, in a different way. Therefore, the accumulation of information will not produce a Bayesian-style convergence of beliefs. The only way for these two agents to reach a common degree of assertability of $A \longrightarrow B$ is for each (or at least one of them) to change the way they process $A$, eventually adopting a common way of information processing. How can such a change occur? A plausible reason that make (say) $S_1$ to change her mode of information processing is to realize that her current interpretation of "$\longrightarrow$" leads her to erroneous logical inferences (accept counter-intuitive inferences as formally valid and vice versa). This type of "logical learning" may induce a change in the way in which $S_1$ interprets "$\longrightarrow$". Our analysis shows that the aforementioned logical learning is hard to achieve. The main reason for this is the one mentioned above, namely that none of the conditionals under consideration codifies all of our intuitions about "good conditional reasoning". As a result, the concept of "logical error" is not clearly defined, thus making any "learning-by-error" convergence process infeasible. In the absense of logical learning, economic agents may permanently and rationally disagree. Section 6 concludes the paper.

## 2 Triviality Results

As already mentioned, if (1) were valid, then it could serve as the starting point of a semantic analysis of the conditional $A \longrightarrow B$. In other words, this equation (if valid) could be used to shed some light on the question of what is the nature of "$\longrightarrow$". More specifically, an implicit defintion of "$\longrightarrow$ " is the following: "The connective $\longrightarrow$ is whatever makes the equation (1) valid, for every initial credence function $P$ and every pair of propositions $A$ and $B$ in the domain of $P$, with $P(A) > 0$ ".

However, Lewis (1976) demonstrated that such an analysis of "$\longrightarrow$" is futile. His main result is that there is no connective "$\longrightarrow$" for which (1) holds universally. Lewis's analysis may be summarized as follows: The first assumption that Lewis makes is that if (1) holds always then the following equation also holds always,

$$P(A \longrightarrow B \mid X) = P(B \mid A \wedge X), \tag{2}$$

provided that $P(A \wedge X) > 0$. Fitelson (2015) refers to (2) as "the Resilient equation". Lewis' s argument is based on the intuitvely appealing idea, that the class of credence functions $\mathcal{C}$ that satisfy (1) must be closed under conditionalization. Assume that upon learning $X$, DM's new credence function is $P'$, where $P'(-) = P(- \mid X)$. If (1) is meant to be a universal rationality requirement then it must hold for any credence function in $\mathcal{C}$ including $P'$. A tacit assumption in (2) is that $X$ can be any proposition such that $P(A \wedge X) > 0$. Since there are no constraints on what $X$ may be, Lewis invites us to take $X$ to be the consequent $B$ of the conditional $A \longrightarrow B$ or its negation, $\neg B$. With $B$ being taken as $X$, (2) becomes:

$$P(A \longrightarrow B \mid B) = P(B \mid A \wedge B).$$

Similarly, if $\neg B$ is taken as $X$, (2) becomes:

$$P(A \longrightarrow B \mid \neg B) = P(B \mid A \wedge \neg B).$$

It is clear that the conditional probabilities $P(B \mid A \wedge B)$ and $P(B \mid A \wedge \neg B)$ are equal to 1 and 0 respectively. Finally, by applying the theorem of total probability to the proposition $A \longrightarrow B$, (and with respect to the partition $\{B, \neg B\}$) we have

$$P(A \longrightarrow B) = P(A \longrightarrow B \mid B)P(B) + P(A \longrightarrow B \mid \neg B)P(\neg B).$$

By replacing $P(A \longrightarrow B \mid B)$ and $P(A \longrightarrow B \mid \neg B)$ with $P(B \mid A \wedge B)$ and $P(B \mid A \wedge \neg B)$, respectively and given that these conditional credences are equal to 1 and 0, respectively, we end up with

$$P(A \longrightarrow B) = P(B) \tag{3}$$

which under (1) gives rise to

$$P(B \mid A) = P(B).$$

The last equation implies that $A$ and $B$ are probabilistically independent under $P$. Lewis characterizes this result as "absurd".[6] DM will reach the conclusion that $A$ and $B$ are independent regardless of their true probabilistic relationship. Bennett (2003) remarks that this result "deprives conditionals of all their force" (Bennett 2003, p. 63), whereas Van Fraassen (1976) refers to it as a "bombshell". Indeed, no matter how strong the true connection between $A$ and $B$ is, (3) suggests that $A$ and $B$ are probabilistically independent. Alternatively, the antecedent $A$ in the conditional $A \longrightarrow B$ becomes irrelevant.

It must be emphasized that what lies at the heart of Lewis's triviality result is the Resilient equation (2). Even more important is Lewis's claim that the Resilient equation must hold for any proposition $X$ such that $P(A \wedge X) > 0$. This universal quantification that characterises (2) enables Lewis to take $X$ as the consequent $B$ of the conditional $A \longrightarrow B$. Fiteslon (2015?) argues that Lewis's triviality result is due precisely to this excessive permissiblility with respect to "admissible propositions" $X$. However, other "triviality results" that have appeared in the literature, based on different assumptions than Lewis's, have established on rather firm grounds the implausibility of (1) (see, Lewis 1986, Hajek 1989, 1994 and Milne 2003).

## 2.1 Alternative Interpretations of the Indicative Conditional $A \longrightarrow B$

Apart from the fundamental disagreement between the agents in $\mathbf{S}_C$ and those in $\mathbf{S}_P$ about the truth-conditionality of $A \longrightarrow B$, additional disagreement may arise within the members of $\mathbf{S}_C$, i.e. among the agents who assert the conditional via the unconditional probability $P(A \longrightarrow B)$. More specifically, even though all the agents in $\mathbf{S}_C$ have agreed on the basic premise that $A \longrightarrow B$ is a meaningful proposition (i.e. it is a truth-bearer), they nevertheless disagree on the semantics of "$\longrightarrow$".[7]

Before delving into the intricancies of the alternative interpretations of "$\longrightarrow$", it is necessary to introduce some elements of the formalism developed by Adams (1975, 1998). In particular, the main distinction between $\mathbf{S}_C$ and $\mathbf{S}_P$ is reflected on the difference between the "languages" of the agents of these two groups. The language of the agents in $\mathbf{S}_C$, denoted by $\mathcal{L}^C$ and referred to as "factual language ", consists of propositional variables, such as $A$ and $B$, together with the constants $T$ and $F$, denoting a logical truth and a logical falsehood, respectively. $\mathcal{L}^C$ is closed under the standard logical connectives, namely negation ($\neg$), conjuction ($\wedge$), disjunction ($\vee$), material implication ($\supset$) and material biconditional ($\equiv$). The language of the agents in $\mathbf{S}_P$, denoted by $\mathcal{L}^P$ and referred to as "conditional language" is an extention of $\mathcal{L}^C$, in the sense that it contains all propositional formulas of $\mathcal{L}^C$ along with the so-called "conditional formulas" $\varphi \Longrightarrow \psi$ (where $\varphi$ and $\psi$ are formulas of $\mathcal{L}^C$ and $\varphi$ is not logically false). As already mentioned, the conditional connective "$\Longrightarrow$" is not truth-conditional in the sense that $\varphi \Longrightarrow \psi$ does not have truth values (although it admits of the so-called "ersatz" truth values - see below). "$\Longrightarrow$" is defined via the relation

$$P(\varphi \Longrightarrow \psi) = P(\psi \mid \varphi),\ P(\varphi) > 0$$

and for this reason "$\Longrightarrow$" is referred to as "probability conditional".[8] A serious limitation of "$\Longrightarrow$", that will be discussed in the sequel, is that it renders all the formulas with probability conditionals embedded in them, meaningless (see Section 5).

[6] In the relevant literature, this result is usually referred to as "Lewis's first triviality result", proved by Lewis (1976). In this paper, Lewis provides two additional triviality results: (a) DM's assigns non-zero probabilities to at most two of any set of pairwise inconsistent propositions. (b) DM's probability function takes at most four values.

[7] Two agents adopt a different semantic interpretation of $A \longrightarrow B$ iff they assign different truth values to the corresponding ($A \longrightarrow B$)-column of the relevant truth table.

[8] As Bennett (2003) remarks: "Adams's theory tells us what indicative conditionals are by telling us what it is to accord probabilities to them." (2003, p. 136). Furthermore, $P(\varphi \Longrightarrow \psi)$ should not be interpreted as the probability that $\varphi \Longrightarrow \psi$ is true, because $\varphi \Longrightarrow \psi$ is not a truth-bearing proposition. Rather, as mentioned in Introduction, $P(\varphi \Longrightarrow \psi)$ is interpeted as the assertibility, $As(\varphi \Longrightarrow \psi$ of $\varphi \Longrightarrow \psi$.

As already mentioned the probability conditional admits of ersatz truth values. Specifically, for the cases that $A$ is true, $A \Longrightarrow B$ is (a) true (in the ersatz sense) if $B$ is true and (b) false (in the ersatz sense) if $B$ is false. When $A$ is false, the probability conditional $A \Longrightarrow B$ does not have truth values - not even ersatz. It should be emphasized that these ersatz truth values are "truth values in name only, and there is no suggestion that this kind of 'truth' is something that should be aimed at in reasoning, or that it is better than falsehood" (Adams 1998, p. 65).[9] This results in an ersatz or "defective" truth table for "$\Longrightarrow$". In other words, the agents of $\mathbf{S}_P$ treat the cases FT and FF (in which the antecedent $A$ is false) as "irrelevant" or as having no consequences for the assertability of the conditional, thus interpreting "$\Longrightarrow$" by the defective truth table. The latter was anticipated by Ramsey (1926) and de Finetti (1936), who diserned a close relation between the conditional proposition "if $A$ then $B$" and the conditional bet: "the agent bets that, if $A$ then $B$". Similarly, Egre et. al (2021) argue that there is an "isomorphism between the conditions that settle the truth of a (conditional) proposition, and the conditions that settle the winner of a (conditional) bet." (2021, p. 191). In the context of our interest rate - stock prices example, the agent wins in the case that the FED increases the interest rate and the stock prices decline ($A$ is true and $B$ is true), and looses when the FED increases the interest rate and stock prices do not decline ($A$ is true and $B$ is false). This raises the following question: What happens In the case that the FED does not increase the interest rate ($A$ is false and $B$ is true or $A$ is false and $B$ is false)? In this case, the necessary condition for the validity of the bet is not present which implies that the bet is called off. If $A$ is false, then an agent in $\mathbf{S}_P$ thinks that she neither wins nor looses which means that for her the bet is "void". Evans et.al. (2007) provide evidence, based on truth table tasks, in favor of the defective truth table (see also Politzer et. al. 2010). In these experiments, participants were asked about the truth values they assign to "if $A$ then $B$" for each of the four truth-value combinations of $A$ and $B$. For the cases in which the antecedent is false (i.e. FT and FF), the majority of the respondends could not evaluate the truth value of the conditional, thus characterizing FT and FF as "irrelevant" or "void".

All the agents under consideration are assumed to distribute subjective probabilities to the "state descriptions", $\omega_1$, $\omega_2$, $\omega_3$ and $\omega_4$, defined by the corresponding rows of Table 2:

| State Descriptions | $A$ | $B$ | Prior Probabilities |
|---|---|---|---|
| $\omega_1$ | $T$ | $T$ | $p$ |
| $\omega_2$ | $T$ | $F$ | $q$ |
| $\omega_3$ | $F$ | $T$ | $r$ |
| $\omega_4$ | $F$ | $F$ | $1-(p+q+r)$ |

Table 2: State descriptions

The set $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ is alternatively referred to as the set of "possible worlds". The way that the agents of $\mathbf{S}_C$ assign subjective probabilities to formulas formed from $A$ and $B$ differs from that of the agents in $\mathbf{S}_P$ only with respect to conditional formulas. Specifically, for factual formulas (i.e. for those which do not contain "$\Longrightarrow$") the subjective probability of any formula (for both $\mathbf{S}_C$ and $\mathbf{S}_P$) is computed as the sum of probabilities of the state descriptions in which this formula is true. For conditional propositions (i.e. for those that contain "$\Longrightarrow$" as the main connective) such as $\varphi \Longrightarrow \psi$, the agents in $\mathbf{S}_P$ may compute $P(\varphi \Longrightarrow \psi)$ in two equivalent ways: (i) $P(\varphi \Longrightarrow \psi)$ (which is equal to $P(\psi \mid \varphi)$) is the sum of the probabilities of the state descriptions in which both $\varphi$ and $\psi$ are true, divided by the sum of probabilities of the state descriptions in which $\varphi$ is true. If there are no state descriptions in which $\varphi$ is true, $P(\varphi \Longrightarrow \psi)$ is set equal to 1 (Adams, 1998, p. 64). (ii) The second way is based on the ersatz truth values of $\varphi \Longrightarrow \psi$. Specifically, $P(\varphi \Longrightarrow \psi)$ is the sum of probabilities of the state descriptions in which $\varphi \Longrightarrow \psi$ has the ersatz value $T$, divided by the sum of probabilities of the state descriptions in which $\varphi \Longrightarrow \psi$ has an ersatz value ($T$ or $F$).

The following assumption introduces a "strong source of agreement" among the agents of both groups: They all agree on the subjective probabilities they assign to the state descriptions. More specifically, all the agents (that is those in $\mathbf{S}_C$ as well as those in $\mathbf{S}_P$) share the following "common prior" subjective probability

[9] Some other authors who also support the defective truth table take the view, contrary to Adams, that the truth values of the TT and TF cases are not ersatz but genuine truth values. They also assume that the truth value of the conditional for the cases FT and FF is "I" (i.e. irrelevant), thus adopting a three-valued truth table (see, for example, de Finetti 1936).

distribution:

$$\begin{aligned} P^s(\omega_1) &= p \\ P^s(\omega_2) &= q \\ P^s(\omega_3) &= r \\ P^s(\omega_4) &= 1-(p+q+r). \end{aligned}$$

Now it is time to define in more detail the different interpretations of the indicative conditional "$\longrightarrow$" within the agent group $\mathbf{S}_C$. For each of these interpretations, we shall discuss the philosophical arguments and the psychological evidence for and against it. Next, based on these interpretations, we will analyze how each of these subsest of $\mathbf{S}_C$ computes the degree of assertability of $A \longrightarrow B$ and compare these results with those for $\mathbf{S}_P$.

(i) Some agents in $\mathbf{S}_C$ interpret "$\longrightarrow$" as the truth-functional, material implication "$\supset$" of Propositional Logic. For this subset of $\mathbf{S}_C$, hereafter referred to as $\mathbf{S}_{C1}$, $As(A \longrightarrow B)$ is given by $P(A \supset B)$. The truth-functionality of "$\supset$" means that the truth value of $A \supset B$ is a function of the truth values of $A$ and $B$ which in turn implies that the truth value of $A \supset B$ does not depend on the empirical content (or context) of $A$ and $B$. In particular, for the members of $\mathbf{S}_{C1}$, $A \supset B$ is true in the cases that (i) $A$ is T and $B$ is T, (ii) $A$ is F and $B$ is T and (iii) $A$ is F and $B$ is F. This means that the only case in which $A \supset B$ is false is when $A$ is T and $B$ is T. This assigment of truth values makes $A \supset B$ logically equivalent to the disjunction "not $A$ or $B$" ($\neg A \vee B$). It must be empasized that although "$\supset$" plays an essential role in mathematical reasoning, it suffers from the so-called "paradoxes of material implication". More specifically, the inferences (argument forms) PMI1: "not $A$" entails "if $A$ then $B$"and PMI2: "$B$" entails "if $A$ then $B$" are both (paradoxically) valid. These paradoxes have their "probabilistic versions" which will occupy us later.

Is there any psychological theory that predicts the existence of the $\mathbf{S}_{C1}$ group? In other words, what is the psychological process through which some people (in our case the members of $\mathbf{S}_{C1}$) interpret the indicative conditional as material implication? Such a theory is the "mental models theory" of Johnson-Laird and Byrne (1991) and Girotto and Johnson-Laird (2004). The key idea of the mental models theory is that conditional reasoning is based on the simulation of possibilities. More specifically, when confronted with the indicative conditional "if $A$ then $B$", people initially represent it with one "explicit" mental model representing the possibility in which $A$ and $B$ are both true (denoted by $[A]\ B$) and one "implicit" mental model representing the possibilities in which $A$ is false (denoted by $\cdots$). The square brackets surrounding $A$ indicate that people in this group hold the view that there are no other possibilities in which $A$ holds, (namely $A$ is true and $B$ is false) which in turn implies that the implicit model ($\cdots$) refers exclusively to cases in which $A$ is false. If people are asked to "flesh out" the implicit model (the three dots) in order to turn it into an explicit model, they list the following possibilities: First, "$A$ is false and $B$ is true" and second "$A$ is false and $B$ is false". For these people, therefore, the core semantics of the conditional "if $A$ then $B$" is given by the following possibilities:

$$\begin{array}{ll} A & B \\ \neg A & B \\ \neg A & \neg B. \end{array}$$

This in turn implies that the semantics of the indicative conditional is identical to that of material implication.[10]

[10] There may be agents within $\mathbf{S}_{C1}$ who, on the one hand, believe that "$\longrightarrow$" is "$\supset$" but on the other hand they hold that the assertibility of $A \longrightarrow B$ does not go by the unconditional probability $P(A \supset B)$. Why is that? Lewis explains that $A \supset B$ is true even when $A$ is false. In such a case "the speaker ought not to assert the conditional if he believes it to be true predominantly because he believes *its antecedent to be false*, so that its probability of truth consists mostly of its probability of *vacuous truth*." (1976, p. 306, emphasis added). In such a case a measure of the assertibility of $A \supset B$ is given by $P(A \supset B)$ minus a term, $a$, that Lewis calls "the diminution of assertibility":

$$a = [1 - P(A \supset B)] \times \frac{P(\neg A)}{P(A)}$$

Hence, the resultant measure of assertibility for this group of agents is given by

$$As(A \longrightarrow B) = P(A \supset B) - a.$$

(ii) There is another subset of $\mathbf{S}_C$, hereafter referred to as $\mathbf{S}_{C2}$, which consists of agents who maintain the view that "$\longrightarrow$" is truth-functional, but define it by a set of truth values that are different from those of "$\supset$". Specifically, the members of $\mathbf{S}_{C2}$ hold the view that $A \longrightarrow B$ is true only in the case that $A$ is T and $B$ is T (and false in the other three cases). Johnson-Laird and Byrne (1991) provide evidence for the presence of such a subset in their experimental studies. They explain this findings by arguing that the agents in $\mathbf{S}_{C2}$ have an initial representation of $A \longrightarrow B$, with the same explicit mental model as the conjunction $A \wedge B$. This means that the members of $\mathbf{S}_{C2}$ do not have an implicit model (the three dots mentioned above) and their representation of the conditional is limited to only the ($[A]$ $B$) possibility. Further evidence suggests that this interpretation of the conditional is mainly adopted by young children, whereas "older children list the first and the third possibilities; and still older children list all three possibilities" (Quelhas, Johnson-Laird and Juhos 2010, p. 1717). This "conjuctive conditional" will hereafter be denoted with "$\rightarrowtail$". For the members of $\mathbf{S}_{C2}$, $As(A \longrightarrow B)$ goes by $P(A \rightarrowtail B)$.

(iii) Finally, we may distinguish a third subset of $\mathbf{S}_C$, hereafter referred to as $\mathbf{S}_{C3}$, consisting of agents who take the view that "$\longrightarrow$" is truth-conditional but not truth-functional. This means that the truth values of $A$ and $B$ are not sufficient to determine the truth value of $A \longrightarrow B$. More specifically, there is universal agrrement among logicians that $A \longrightarrow B$ is false if $A$ is true and $B$ is false (TF) (see, for example, Edgington 1986). For the remaining three cases, namely $A$ is true and $B$ is true (TT), $A$ is false and $B$ is true (FT) and $A$ is false and $B$ is false (FF), the corresponding truth values of the conditional $A \longrightarrow B$ may not be unique (see Weirich 2020). This means that in the TT, FT and FF cases, the conditional $A \longrightarrow B$ can be either true or false (T/F), which is another way to say that "$\longrightarrow$" is non-truth-functional.This in turn implies that some instances of TT, or FT or FF make the corresponding conditionals $A \longrightarrow B$ true, while some others make them false. Weirich (2020) refers to such conditionals (that is conditionals with multiple truth values) as 'open conditionals'. To decide which of the open conditionals are true and which are false, each agent in $\mathbf{S}_{C3}$ has to examine the "content" of the propositions $A$ and $B$, which makes the conditional $A \longrightarrow B$ content-dependent. More precisely, an agent in this group does not merely consider the truth values of $A$ and $B$ in order to decide whether $A \longrightarrow B$ is true or false. She also looks for a "connection" (causal, evidential or causal) between $A$ and $B$, on the basis of which she will decide on the truth values of $A \longrightarrow B$. As Stalnaker (1968) puts it "If the 'connection' holds, you check the 'true' box. If not, you answer 'false'" (1968, p. 100). Markovits and Vachon (1990) provide a list of empirical studies that lend support to the hypothesis that "content significantly influences the way that people derive conclusions" (1990, p. 942).For example, an agent from $\mathbf{S}_{C3}$ may find the proposition "if the FED increases the interest rate, then the stock prices will decline" true, provided that the FED increases the interest rate and the stock prices decline. On the other hand, the same agent may find the proposition "if the FED increases the interest rate then the Earth revolves around the Sun" false even if both the antecedent and the consequent are true, simply because she thinks that these propositions are causally unrelated. This "open conditional" will hereafter be denoted with "$\triangleright$" and its assertability is given by $P(A \triangleright B)$. Of the alternative distributions of truth values in the conditional that "$\triangleright$" allows, we will only consider the case (T, F, T, F). The case (T,F,F,T) corresponds to the truth values of the "biconditional" of Propositional Logic, while the case (T,F,F,F) is the case of the "conjuctive conditional", discussed above.

The truth values of "$\supset$", "$\rightarrowtail$", "$\triangleright$" and "$\Longrightarrow$" are depicted in the columns 2-5, respectively of Table 3. At the end of these columns, the corresponding probabilities of the conditional "if $A$ then $B$" are also reported.

It is easy to show that

$$P(A \supset B) - a = P(B \mid A),$$

which implies that the assertability of the ocnditional for this group of people is identical to that of $\mathbf{S}_P$.

| | Truth-Functional | | Truth-Conditional | Non-Truth-Conditional |
|---|---|---|---|---|
| | $\mathbf{S}_{C1} : A \supset B$ | $\mathbf{S}_{C2} : A \rightarrowtail B$ | $\mathbf{S}_{C3} : A \triangleright B$ | $\mathbf{S}_{P} : A \Longrightarrow B$ |
| State Descriptions | Material Implication | Conjuctive | Open | Probabilistic Conditional |
| $\omega_1$ | $T$ | $T$ | $T/F$ | $T$ |
| $\omega_2$ | $F$ | $F$ | $F$ | $F$ |
| $\omega_3$ | $T$ | $F$ | $T/F$ | . |
| $\omega_4$ | $T$ | $F$ | $T/F$ | . |
| | $P(A \supset B) = p + q + r$ | $P(A \rightarrowtail B) = p$ | $P(A \triangleright B) = p + r$ for the (T,F,T,F) case | $P(A \Longrightarrow B) = P(B \mid A) = \frac{p}{p+q}$ |

Table 3: Alternative Sets of Truth Values for the Indicative Conditional "if $A$ **then** $B$"

Table 3 suggests that the agents' disagreement about the proper interpretation of the indicative conditional is translated into disagreemnt about the degree of assertability of this conditional. For example, assume that all the agents under consideration agree that their prior probabilities are distributed uniformly over the state descriptions, i.e. $P(\omega_1) = P(\omega_2) = P(\omega_3) = P(\omega_4) = 0.25$ for each and every agent in $S_C \cup S_p$. Then, despite their initial full agreement about their prior probabilities on $\Omega$, the agents will end up disagreeing significantly about the probabilities they attach to the indicative conditional "if $A$ then $B$", depending on how each agent interprets it, i.e.

$$\begin{aligned}&P(A \supset B) = 0.75\\&P(A \rightarrowtail B) = 0.25\\&P(A \rhd B) = 0.50\\&P(A \Longrightarrow B) = 0.50\end{aligned}$$

# 3 Normative Analysis of the Indicative Conditional

The analysis of the previous section showed that there is more than one way to interpret the conditional sentence "if $A$ then $B$", namely "$\supset$", "$\rightarrowtail$", "$\rhd$" and "$\Longrightarrow$". The question we will address in this section is whether any of these interpretations is superior to the others on normative grounds. That is, is there a uniquely rational interpretation of the indicative conditional? As already mentioned in Introduction, the answer to this question involves two steps: In the first step we shall specify the set $\mathcal{I}$ of normative criteria that any formal interpretation of conditional "$\longrightarrow$" should meet. In the second step, we will examine the relative performance of the alternative interpretations of "$\longrightarrow$" under consideration on the basis of the criteria of $\mathcal{I}$. The following analysis excludes the open conditional, since for "$\rhd$" there is no unique truth-value assignment for the (T,T), (F,T) and (F,F,) cases (see Table 3).

As mentioned above, the set $\mathcal{I}$ contains nine inference forms which summarize most aspects of conditional reasoning. Some of these forms are widely accepted as "good" others as "flawed" and the rest as "indecisive". These forms are the following[11]:

1) *Modus Ponens:*

$$\left[\begin{array}{l} A \longrightarrow B \\ A \\ \hline B \end{array}\right]$$

Modus Ponens (MP) is considered to be at the heart of conditional reasoning and is (almost) universally accepted as "good" inference. Moreover, in the relevant psychological studies, MP is the form of inference that people judge as the most obvious[12]. We take MP to be an intuitively vaid inference form.

2) *Modus Tollens:*

$$\left[\begin{array}{l} A \longrightarrow B \\ \neg B \\ \hline \neg A \end{array}\right]$$

Modus Tollens (MT) is also widely accepted as an intuitively good form of inference. Nonetheless, some counterexamples have been proposed in the literature that purport to show that MT is not universally valid (see, for example, Forrester 1984, Yalcin 2012). However, there are good reasons to believe that these counterexamples are not genuine. For example, the case suggested by Yalcin concerns a probabilistic version of MT, in which the premises are context-specific. This in turn raises doubts as to whether the proposition $B$ in the minor premise ($\neg B$) is identical to the proposition $B$ in the major premise ($A \longrightarrow B$). Besides, the validity of MT has been taken for granted since the time of Aristotle and the Stoics[13]. For example, in

[11] In all the inference forms introduced below, the horizontal line is used to seperate the "premises" from the "conclusion" of the argument. Following the notational convention adopted in the paper, the arrow "$\longrightarrow$" signifies the uninterpreted conditional of the natural language.

[12] However, even for the MP, certain objections have been raised regarding its universal validity. For example, McGee (1985) argues that when the consequent of the conditional is itself a conditional, then counter-examples to MP can be devised.

[13] The Stoics, based on the ideas of Aristotle's Prior Analytics, posited both MP and MT as "indemonstrable modes of argument", that is froms of inference whose validity was indisputable..

the first half of the 20th century, Popper developed his "falsificationist methodology" using MT as its logical basis (Popper 1959 [1935]). Given the wide acceptance of MT over time and the fact that the aforementioned counterexamples are likely to be "spurious", we classify MT as intuitively valid.

3) *Denying the Antecedent*:

$$\left[\begin{array}{l} A \longrightarrow B \\ \neg A \\ \hline \neg B \end{array}\right]$$

Denying the antecedent (DA) is universally recognized as a logical fallacy. [14]

4) *Affirming the Consequent:*

$$\left[\begin{array}{l} A \longrightarrow B \\ B \\ \hline A \end{array}\right]$$

Affirming the Consequent (AC) has also been recognized as a fallacious form of reasoning since the time of Aristotle: "Since after rain the ground is wet in consequence, we suppose that if the ground is wet, it has been raining; whereas that does not necessarily follow." (Aristotle, Sophistical Refutations, 167b1 cf. Hansen and Pinto, 1995, p. 25). Godden and Zenker (2015) claim that "...adding DA and AC to the repertoire of acceptable inference-licenses is *ruinous to a logical system.*" (2015, p. 104, emphasis added). AC and DA are classified as intuitively invalid (bad) inferences.

The next three inference forms are controversial. Some philosophers consider them "intuitively valid" while others consider them "flawed".

5) *Transitivity*:

$$\left[\begin{array}{l} A \longrightarrow B \\ B \longrightarrow C \\ \hline A \longrightarrow C \end{array}\right]$$

Transitivity (TR) of conditional reasoning (or Hypothetical Syllogism) is considered by some philosophers to be the "sine qua non" of theoretical reasoning, since the latter typically involves a long chain of arguments. For example, consider the following two indicative conditionals: $A \longrightarrow B$: "If the FED increases the interest rate, then the stock prices will decline." $B \longrightarrow C$: "If the stock prices decline, then private consumption (due to wealth effects) will fall". It seems intuitively reasonable to infer $A \longrightarrow C$ : "If the FED increases the interest rate, then private consumption will fall". A powerful argument for TR is the following (see Braine 1979, p. 36): Assume the following: (i) $A \longrightarrow B$ and (ii) $B \longrightarrow C$. Suppose $A$. Then $B$ follows by (i) and MP. Given now $B$, then $C$ follows by (ii) and MP. Hence, on the supposition of $A$, $C$ follows. As a result, $A \longrightarrow C$. This argument is based on the so-called "conditional proof", in the context of which if the assumed proposition $A$ is shown to imply the proposition $C$ then we are entitled to conclude $A \longrightarrow C$. Therefore, anyone who rejects TR (while accepting the validity of MP) must (implictly or explicitly) reject the validity of conditional proof.[15]

Several such rejections of TR have been proposed in the philosophical literature. More specifically, consider the following counterexample to hypothetical syllogism, put forward by Stalnaker (1968): $A \longrightarrow B$: "If J. Edgar Hoover had been born a Russian, then he would today be a communist". $B \longrightarrow C$: "If J. Edgar Hoover were today a communist, then he would be a traitor". $B \longrightarrow C$ (Conclusion): "If J. Edgar Hoover had been born a Russian, then he would be a traitor". Stalnaker finds it reasonable to accept the premises, $A \longrightarrow B$ and $B \longrightarrow C$ and deny the conclusion $B \longrightarrow C$. In this counterexample, it is important to notice

[14] Nonetheless, Godden and Walton (2004) argue that while $\neg A$ cannot be used to establish $\neg B$, it can be used to suggest that $B$ cannot be defended on the basis of $A$, thus implying that another reason should be put forward for asserting $B$. This interpretation may explain why people often (erroneously) endorse DA.

[15] For example, Stalnaker (1968 p.106, footnote 10) acknowledges as valid the following inference: From $A \longrightarrow B$, $B \longrightarrow C$ and $A$ we infer $C$. What he denies is the conclusion $A \longrightarrow C$, which means that he denies "conditional proof".

that the conditionals involved are not indicative but "counterfactual" or "subjunctive" ones.[16] Wright (1983) argues that this type of conditionals are context-dependent, which implies that the apparently paradoxical conclusion in the above inference should not be interpreted as a failure of transitivity, but rather be attributed to the fact that the range of worlds relevant to the assessment of the asserted counterfactuals varies from context to context. Hence, due to context dependence, the consequent of the first premiss may not be identical to the antecedent of the second premiss, implying that there is no reason to expect transitivity to hold (see Braine 1979, for more discussion on this point). Since almost all the counterexamples forwarded against HS involve counterfactual conditionals (e.g. Bennett 2003), it may be argued that indicative conditionals are immune to this kind of criticism.[17] However, although this view seems to be the prevailing one, it is not universally accepted. Plausible counterexamples to indicative conditionals have also been proposed (see, for example Bennett 2003, pp. 145-146) which leads us to classify TR as a "controversial" form of inference.

6) *Strengthening the Antecedent:*

$$\left[\frac{A \longrightarrow B}{A \wedge C \longrightarrow B}\right]$$

Strengthening the Antecedent (SA) is meant to imply that when $A$ implies $B$, then $A$ in conjuction with any other proposition $C$ continues to imply $B$. Stalnaker (1968) argues that SA implies and is implied by TR, which means that any counterexamples of SA serves as counterexamples to TR as well (see also Bennett 2003). He invites us to consider the following inference: $A \longrightarrow B$: "If this match were struck, it would light" and $A \wedge C \longrightarrow B$ : "If this match had been soaked in water overnight *and* it were struck, it would light." (1968, p. 106). Stalnaker characterizes this inference as "obviously invalid". The first thing to notice is that, as in the case of TR, the conditionals involved are all counterfactuals. However, for the case of SA, it is much easier to construct counterexamples for indicative conditionals too. Hence, the problem is not so much one of counterfactuality as one of context dependence. The important question to ask is whether "the match" figuring in $A \longrightarrow B$ is identical to that figuring in $A \wedge C \longrightarrow B$. Braine (1979) eloquently describes this problem as follows: "But obviously, if the match has been soaked in water, the rule 'if this match were struck, it would light' is not valid: if *if* p *then* q is wrong, then *if* p *and* r *then* q is not expected to hold. If the premiss were really valid, 'this match' would presumable be some new and special kind of match that continues to function despite soaking, and the conclusion would follow." (1979, p. 41). Given the aforementioned debate in the philosophical literature, as well as SA's close logical connection to TR, we classify SA as a controversial inference form.

7) *Contraposition:*

$$\left[\frac{A \longrightarrow B}{\neg B \longrightarrow \neg A}\right]$$

Contraposition (CP) is closely related to Modus Tollens. Specifically, if we accept the validity of (i) MT and (ii) conditional proof, then CP follows. Hence, if one believes that CP is a flawed form of inference while maintaining the validity of MT, one rejects the rule of conditional proof. For many conditionals of the natural language, contrapositions seems to hold. For example, consider the following indicative conditional: "If John Smith is an economist, then he knows the law of demand". From this it is reasonable to argue that "if John Smith does not know the law of demand, then he is not an economist". On the other hand, consider the following example (Bennett 2003, p. 143): $A \longrightarrow B$: "If he does not live in Paris he lives somewhere in France". From this it is absurd to argue $\neg B \longrightarrow \neg A$: "If he does not live anywhere in France, he lives in Paris". This and other similar examples make the normative status of CP debatable. For this reason, we classify CP as a controversial form of inference.

The next two inference forms, already introduced in Introduction, clearly violate our commonsense intuitions. But, "paradoxically", in Propositional Logic they are formally valid.[18]

[16] In counterfactual conditionals, as opposed to the indicative ones, the antecedent is believed to be highly improbable. As Iatridou (2000) puts it: "counterfactuality is used as a term only with respect to situations that cannot be helped anymore." (2000, p. 231).

[17] Nonetheless, Stalnaker (1968) insists that transitivity does not hold even for indicative conditionals.

[18] The logical term "paradox" is defined in Oxford English Dictionary as: "A statement or proposition which, from an acceptable premise and despite sound reasoning, leads to a conclusion that is against sense, logically unacceptable or self-

8) *Paradox of Material Implication - 1 (PMI1):*

$$\left[\begin{array}{c} \neg A \\ \hline A \longrightarrow B \end{array}\right].$$

For example, from the proposition "if the Earth is not flat" follows the proposition "if the Earth is flat then the stock prices will decline".

9) *Paradox of Material Implication - 2 (PMI2):*

$$\left[\begin{array}{c} B \\ \hline A \longrightarrow B \end{array}\right].$$

For example, from the proposition "the stock prices will decline" follows the proposition "if the Earth is flat then the stock prices will decline".

PMI-I and PMI-II are widely considered to be the main arguments against interpreting the indicative conditional of natural language by material implication.[19] In the normative analysis of the alternative interpretations of "$\longrightarrow$" that follows, the material conditional "$\supset$" will be penalized for rendering PMI-I and PMI-II as valid.

## 3.1 Probabilistic Validity

As already mentioned, the formal criterion of validity that we adopt is that of "probabilistic validity" (or p-validity) introduced by Adams (1965, 1975, 1998).

**Definition 1** *(probabilistic validity)*

*An inference is p-valid iff the uncertainty of its conclusion cannot exceed the sum of the uncertainties of its premises*" (Adams 1998, p. 131).*

The following remarks summarize Adams' theory of probabilistic validity:

(i) Adams defines the uncertainty $U(A)$ of a proposition $A$ as the probability that $A$ is false, which is equal to one minus the probability that is is true, i.e. $U(A) = 1 - P(A)$.

(ii) Probabilistic validity is an extention of the "classical validity" of Propositional Logic. Specifically, an inference is classically valid if the truth of its premises guarantees the truth of its conclusion. In other words, in the context of a classically valid inference, $\mathcal{IN}$, with premises $A_1, A_2, ...A_n$ and conclusion $C$, it is impossible for $A_1, A_2, ...A_n$ to be true and $C$ to be false. On the other hand, $\mathcal{IN}$ is p-valid, iff:

$$\sum_{i=1}^{n} U(A_i) \geq U(C)$$

for all uncertainty functions $U$.

(iii) Although probability itself admists of degrees, p-validity is an all-or-none property. An inference $\mathcal{IN}$ is either p-valid or p-invalid and "there are no gradiations in between." (Adams, 1998, p. 132).

(iv) The relationship between the probabilistic validity of an inference $\mathcal{IN}_c$ and the classical validity of its "material counterpart" $\mathcal{IN}_m$ is the following[20]: Classical validity of $\mathcal{IN}_m$ is, in general, a necessary but not sufficient condition for the probabilistic validity of $\mathcal{IN}_c$ (First Classsical Validity Theorem - FCVT, Adams 1998, p. 151) However, if the conclusion of $\mathcal{IN}_c$ is "factual" (i.e. it does not contain the probabilistic conditional "$\Longrightarrow$") then classical validity is both necessary and sufficient for probabilistic validity. (Third Classical Validity Theorem - TCVT, Adams 1998, p. 167)

contradictory".

[19] Some attempts to remove these paradoxes led to the the discovery of other notions of implications, such as "strict implication" of modal logic (see, Lewis and Langford 1932)

[20] The material counterpart $\mathcal{I}_m$ of an inference $\mathcal{I}_c$ that involves "$\Longrightarrow$" is defined as the inference derived from $\mathcal{I}_c$ when all instances of "$\Longrightarrow$" in the latter are replaced by "$\supset$".

(v) Another type of validity defined by Adams is "weak validity". This applies exclusively to inferences in which all the premises are certain. Specifically, an inference $\mathcal{IN}$ is weakly valid if whenever its premises are certain, so is its conclusion. Specifically, if

$$P(A_1) = P(A_2) = ... = P(A_n),$$

then

$$P(C) = 1.$$

This notion gives rise to Adams' Second Classical Validity Theorem (SCVT, 1998, p. 152), which proves the equivalence between classical validity and weak validity. This means that any inference $\mathcal{IN}_c$, whose material counterpart $\mathcal{IN}_m$ is classically valid, is also weakly valid.

(vi) The last remark implies that for the inferences in which the conditional connectives "$\supset$", "$\rightarrowtail$" or "$\triangleright$" occur, classical validity, probabilistic validity and weak validity are equivalent. On the other hand, for inferences involving the probabilistic conditional "$\Longrightarrow$" in their conclusion, classical validity (necessary and sufficient for weak validity) is only necessary for p-validity.

(vii) There is an interesting relationship between p-validity of an inference $\mathcal{IN}_c$ and the "probability limits" of the premises and the conclusions of $\mathcal{IN}_c$. In the First Equivalence Theorem (1998, p. 152), Adams shows that an inference $\mathcal{IN}_c$ is p-valid if and only if $P(C)$ necessarily approaches unity in the limit when all of the probabilities $P(A_i)$, $i = 1, 2, ..., n$ approach unity.

Armed with the theoretical results summarized above, we can now proceed to examine which of the nine inference forms under consideration are p-valid for each of the three alternative interpretations, "$\supset$", "$\rightarrowtail$" and "$\Longrightarrow$" of the indicative conditional "$\longrightarrow$", introduced in the previous section. As already mentioned, an interpretation of "$\longrightarrow$" is normatively appealing if it respects our intuitions about "good" and "bad" inference forms. Note that for "$\supset$" and "$\rightarrowtail$", the conclusions of all the nine inferences under consideration are factual. This means that TCVT applies which in turn implies that for all these cases p-validity is equivalent to classical validity.

(i) *The material implication "$\supset$":* It is well-known that according to "$\supset$", MP, MT, TR, SA, CP, PMI1 and PMI2 are classically valid. Moreover, DA and AC are invalid. In view of TCVT, "$\supset$" succeeds in characterizing the intuitively good inferences MP and MT as p-valid, and the intuitvely bad inferences DA and AC as p-invalid. Moreover, the three controversial inferences, TR, SA and CP are, according to "$\supset$", p-valid. The biggest drawback of "$\supset$" is that it renders the intuitively bad inferences PMI1 and PMI2 classically valid and hence p-valid.

(ii) *The conjuctive conditional* "$\rightarrowtail$": It is easy to show that for "$\rightarrowtail$", MP is classically valid and hence p-valid. For MT, however, the situation is different. The main premiss $A \rightarrowtail B$ is true only in the first row of the relevant truth table (see Table 3). The second premiss, $\neg B$ is true in the second and fourth rows of the truth tabe. Hence, the two premisses of MT are, according to "$\rightarrowtail$", *contradictory.* This means that MT is "trivially valid", since it is impossible for both its premisses to be true (contradictory propositions cannot be sumultaneously true) and its conclusion to be false. However, trivial formal validity is not the kind of validity that an interpretation of the indicative conditional should aim at. Popper (1940) comments on this issue as follows: "From two contradictory premisses, we can logically deduce anything, and its negation as well. We therefore convey with such a contradictory theory-nothing. A theory which involves a contradiction is entirely useless, because it does not convey any sort of information." (1940, p. 410). In the present case, an agent cannot be simultaneously certain of the truth of both of MT's premisses. Indeed, since $P(A \rightarrowtail B) = p$ and $P(\neg B) = 1 - (p + r)$ absolute confidence for the first premiss ($p = 1$) implies zero confidence for the second premiss. Hence, any agent who adopts "$\rightarrowtail$" will judge (by her own "probabilistic lights") any MT inference that she may encounter, *a-priori unsound*. Similar analysis shows that DA also has contradictory premisses, thus also being "trivially valid". In contrast, AC apears to be (genuinely) classically valid. Indeed, $A \rightarrowtail B$ is true in the first row of truth table whereas $B$ is true in the first and third rows of this table. These two premisses are jointly true on the first row in which the conclusion $A$ is also true. Hence, "$\rightarrowtail$" makes an intuitively bad inference to look formally valid. The analysis, so far, has shown that on normative grounds, "$\rightarrowtail$" is a very poor interpretation of "$\longrightarrow$" since with the exception of MP, it fails to preserve our intuitions about "good" and "bad" inferences. Based on these decisive results, we do not consider it necessary to continue the analysis of the conjuctive conditional for the remaining five inference forms. Its failure to preserve our intuitions about MT, DA and AC makes the conjuctive conditional an unacceptable interpretation of the indicative conditional.

(ii) *The probabilistic conditional* "$\Longrightarrow$": The first thing to note is that of the nine inference forms under consideration, the first four, namely MP, MT, DA and AC have as their conclusion a factual proposition, namely $B$, $\neg A$, $\neg B$ and $A$, respectively. Hence, TCVT applies which means that MP and MT are p-valid whereas DA and AC are p-invalid. For the remaining five inference forms, namely TR, SA, CP, PMI1 and PMI2, the conclusion is a conditional (no-factual) proposition, which in turn allows (according to FCVT) classically valid inferences to be p-invalid. Indeed, Adams (1998) shows that this is true for all five of these inference forms. For example, for the case of CP and for $p = 0.5$, $q = 0.2$ and $r = 0.2$, the uncertainty of the conclusion $U(\neg B \Longrightarrow \neg A)$ is equal to 0.666, whereas the uncertainty of the premiss, $U(A \Longrightarrow B)$ is equal to 0.285, thus establishing the p-invalidity of CP. A big advantage of "$\Longrightarrow$" over "$\supset$" is that it renders the intuitively bad inferences PMI1 and PMI2 p-invalid. For example, for PMI1 and for $p = 0.1$, $q = 0.2$ and $r = 0.2$, the uncertainty of the conclusion $U(A \Longrightarrow B)$ is equal to 0.666, whereas the uncertainty of the premiss, $U(\neg A)$ is much lower, i.e. 0.30.

Based on the above analysis, can we claim without hesitation that the probabilistic conditional is normatively preferable to the material conditional? The answer to this question is negative. This is because the intuitive validity of the three remaining inference forms, TR, AS, and CP, is controversial. This means that to one person these three inferences seem wrong, whereas to another person they are perfectly legitimate. For the first person, the probabilistic conditional (which characterizes them as p-invalid) prevails unconditionally over the material conditional. For the second person, however, "$\Longrightarrow$" is likely to be inadmissible. For this person, the avoidance of the paradoxes of material implication (ensured by "$\Longrightarrow$") comes at a very high cost, namely the characterization of three perfectly good and vital inference forms as invalid. This means that these two people can disagree about what the correct interpretation of the indicative conditional is, without having a clear normative standard to tell them who is right and who is wrong. These remarks bear on the issue of "rational logical disagreement" to which we now turn.

# 4 Rational Logical Disagreement

Let us assume that person X begins his epistemic/logical life having adopted the conjunctive conditional "$\rightarrowtail$" as her information processing vehicle. Over time and as X is faced with alternative inference tasks, she will discover that her initial commitment to "$\rightarrowtail$" causes her insurmountable problems in successfully completing these tasks. For example, she will realize that in the context of the truth-functional "$\rightarrowtail$", the premisses of any MT argument are contradictory. Hence, she is likely to switch to another conditional connective, such as "$\supset$" or "$\Longrightarrow$". In this case, the notion of "logical error" is clearly defined, and it is X's realization of this error that will cause her to change her logical mode of information processing.

The characterization of logical error is far less clear for the cases of both "$\supset$" and "$\Longrightarrow$". In particular, consider the person Y who has initially adopted the material conditional "$\supset$". This person will find that this interpretation of the indicative conditional serves her well with respect to MP, MT, DA, and AC. If Y believes that TR, AS and CP are good inference forms, then she will be satisfied with the performance of "$\supset$" in those cases as well. However, when she encounter instances of the inference forms PMI1 and PMI2, she is likely to be embarassed by the fact that her formal interpretation of the conditional makes these clearly flawed inferences appear formally valid. Assume that this predicament is sufficient to cause Y to switch to the probabilistic conditional "$\Longrightarrow$". As for MP, MT, DA, and AC, the switch from "$\supset$" to "$\Longrightarrow$" is neutral, since under both "$\supset$" and "$\Longrightarrow$", MP and MT are p-valid and DA and AC are p-invalid. Furthermore, Y will be pleased to see that now PMI1 and PMI2 are formally invalid. However, when Y encounters the intuitively good (by her own lights) inference forms TR, AS and CP she will realize that they now appear to be p-invalid, contrary to her intuitions. This may motivate her to swithich back to "$\supset$".

Next, suppose that a third person, Z, has initially adopted the probabilistic conditional "$\Longrightarrow$". Suppose also that this person recognizes TR, AS and CP as good inferences. Does this person have any reason to reconsider her commitment to "$\Longrightarrow$", despite the fact that "$\Longrightarrow$" conforms to the norms dictated by all nine inference forms under consideration? The answer is yes. The reason concerns the potential of Z's formal language $\mathcal{L}^P$ to formally express propositions that appear to be fully meaningful in natural language. This is the problem of "embedded conditionals", mentioned in the introduction. For example, consider the following proposition: "Either if the FED increases the interest rate, the stock prices will decline or if the FED increases the interest rate the stock prices won't decline". The formal counterpart of this proposition is of the form $(A \Longrightarrow B) \vee (A \Longrightarrow \neg B)$. However, this proposition is meaningless, since it involves the disjunction of two conditionals which, in the context of $\mathcal{L}^P$, is undefined. Even simpler propositions such as "It is not the case that if the FED increases the interest rate, the stock prices will decline" do not make sense in Z's language, since the negation of the conditional, $\neg(A \Longrightarrow B)$, is not defined in $\mathcal{L}^P$.[21] On the other hand, these propositions make perfect sense in the context of $\mathcal{L}^C$, since $(A \supset B) \vee (A \supset \neg B)$ and $\neg(A \supset B)$ are well defined. As a consequence, Z appears to face the following dilemma: On the one hand, if she adopts "$\Longrightarrow$", she solves the problem of the paradoxes of material implication at the cost of not being able to formally express propositions that are meaningful in natural language. On the other hand, if she adopts "$\supset$" then she can express any proposition containing embedded conditionals at the cost of having to accept PMI1 and PMI2 as formally valid.

The above discussion casts doubt on the agreement-type results of the relevant literature (Aumann 1976). Specifically, consider two individuals who have the same prior probability function and moreover their posterior probabilities are common knowledge (in the Aumann's sense). Are these two posterior probabilities equal? The answer is in general negative. Knowing each other probabilities is not sufficient to produce agreement as each person may not know the other person's way of information processing (i.e. the other person's interpretation of the conditional). Moreover, this disagreement may be permanent since there is no clear incentive for these two persons to arrive at a common interpretation of the indicative conditional.

# 5 Conclusions

The main points of the paper may be summarized as follows:

(i) When a decision maker evaluates the effect of information $A$ on the event/proposition $B$, she is essentially evaluating the degree to which she asserts the indicative conditional "if $A$ then $B$", written as $A \longrightarrow B$.

(ii) The rational interpretation of the indicative conditional "$\longrightarrow$" is not unique. As a result, how a decision maker decides to process information $B$ depends on the interpretation of "$\longrightarrow$" that decision maker adopts.

(iii) In the context of the standard Bayesian confirmation theory, the assertability of the indicative conditional $A \longrightarrow B$ goes by the conditional probability $P(B \mid A)$ (which may alternatively be written as $P(A \Longrightarrow B)$). On this view, the probabilistic conditional "$\Longrightarrow$ ” is not truth-conditional, which means that $A \Longrightarrow B$ is not a proper proposition. Instead, for the decision makers who hold the view that the indicative conditional is truth-conditional, the assertability of $A \longrightarrow B$ goes by the unconditional probability $P(A \longrightarrow B)$.

(iv) Lewis's triviality results have shown that there is no interpretation of the indicative conditional that ensures the equality $P(A \longrightarrow B) = P(B \mid A)$ for every $P$ and every pair of propositions $A$ and $B$ in the domain of $P$, with $P(A) > 0$

(v) The two dominant interpretations of "$\longrightarrow$" are: a) the Bayesian probabilistic conditional "$\Longrightarrow$" and b) the truth-functional material implication "$\supset$" of Propositional Logic.

(vi) None of the two aforementioned interpretations enjoys universal supremacy on normative grounds. The probabilistic conditional surpasses material implication in that it renders two intuitively invalid inferences, namely PMI1 and PMI2 formally invalid. On the other hand, the material implication, as opposed to the probabilistic conditional, allows propositions with embedded conditionals to be meaingful.

(vii) The normative ambiguity of "$\Longrightarrow$" and "$\supset$" is accentuated by the fact that some widespread inference forms, namely TR, SA and CP cannot be clearly classified as good or bad ones. A decision maker who finds these three inferences to be intuitively valid will prefer "$\supset$" to "$\Longrightarrow$" since the former, as opposed to the latter, makes these forms formally valid. The opposite is true for a decision maker who believes that these inferences are intuitively flawed.

(viii) As a result of the lack of unambiguous normative guidance, two decision makers can disagree about their posterior probabilities of $B$ even if they share the same information $A$ and have identical prior probability functions. Moreover, this disagreement will persist if each of the two decision makers stick to her own interpretation of the indicative conditional.

[21] Adams himself recognizes the problem of embedded cnditionals as a serious flaw in his theory (see 1998, Appendix 4)

# 6 Data availability

Data availability is not applicable to this article as no new data were created or analysed in this study.

# References


[1] Adams, E. W. (1965). 'A Logic of Conditionals'. *Inquiry* 8: 166–97.

[2] Adams, E. W. (1975). *The Logic of Conditionals*. Dordrecht: Reidel.

[3] Adams, E. W. (1998). *A Primer of Probability Logic*. Stanford, Calif.: CSLI Publications

[4] Ali, N, N. Chater and M. Oaksford (2011). 'The Mental Representation of Causal Conditional Reasoning: Mental Models or Causal Models'. *Cognition* 119: 403-418.

[5] Aumann, R. (1976). 'Agreeing to Disagree'. *The Annals of Statistics* 4: 1236-1239.

[6] Bennett, J. (2003). *A Philosophical Guide to Conditionals*. Oxford University Press.

[7] Bonnefon, J. F. and G. Villejoubert, (2007). 'Modus Tollens, Modus Shmollens: Contrapositive Reasoning and the Pragmatics of Negation'. *Thinking and Reasoning* 13: 207–22.

[8] Braine, M.D.S. (1979). 'On Some Claims about If-Then'. *Linguistics and Philosophy*, 3: 35-47.

[9] Byrne, R.M.J and P.N. Johnson-Laird (2009). '"If" and the Problems of Conditional Reasoning'. Trends in Cognitive Sciences 13: 282-287.

[10] De Finetti, B. (1936). 'La prevision: ses lois logiques, ses sources subjectives', *Annales de I'Inslitut Henri Poincare,* 7, pp. 1-68, translated by H. E. Kyburg as 'Foresight: Its Logical Laws, Its Subjective Sources', in Kyburg and Smokier (eds) (1980): 53-118.

[11] Edgington, D. (1986). 'Do Conditionals have Truth Conditions?', reprinted in Jackson (ed.) *Conditionals*. Oxford University Press: 176–201.

[12] Edgington, D. (1995a). 'Conditionals and the Ramsey Test'. *Proceedings of the Aristotelian Society* suppl. vol. 69: 67–86.

[13] Edgington, D. (1995b). 'On Conditionals'. *Mind* 104: 235–329.

[14] Egre, P., L. Rossi and J. Sprenger (2021). 'De Finettian Logics of Indicative Conditionals Part I: Trivalent Semantics and Validity'. *Journal of Philosophical Logic* 50: 187-213.

[15] Evans, J.B.T., S.J. Handley, H. Neilens and D. E. Over (2007). 'Thinking about Conditionals: A Study of Individual Differences'. *Memory and Cognition* 35: 1772-1784.

[16] Evans, J.B.T., and D.E. Over (2004). *If.* Oxford: Oxford University Press.

[17] Forrester, J. W. (1984). 'Gentle Murder, or the Adverbial Samaritan'. *The Journal of Philosophy* 81: 193–197.

[18] Gibbard, A. (1981). 'Two Recent Theories of Conditionals' in Harper et al. (eds.): 211-247.

[19] Girotto, V. and P.N. Johnson-Laird (2004). 'The Probabilities of Conditionals'. *Psychologia* 47: 207-225.

[20] Godden, D.M., and D. Walton (2004). 'Denying the Antecedent as a Legitimate Argumentative Strategy: A Dialectical Model.' *Informal Logic*, 24: 219-243.

[21] Godden, D.M., and F. Zenker (2015). 'Denying Antecedents and Affirming Consequents: The State of the Art.' *Informal Logic*, 35: 88-134.

[22] Hajek, A. (1989). 'Probabilities of Conditionals - Revisited'. J*ournal of Philosophical Logic* 18: 423–8.

[23] Hajek, A. (1994). 'Triviality on the Cheap?', in E. Eells and B. Skyrms (eds.), *Probability and Conditionals: Belief Revision and Rational Decision*. Cambridge University Press, 113–40.

[24] Hajek, A. (2012). 'The Fall of "Adams' Thesis"?'. *Journal of Logic, Language, and Information* 21: 145-161.

[25] Hansen, H. V., and R.C. Pinto, (eds.) (1995), *Fallacies : Classical and Contemporary Readings*, Pennsylvania State University Press.

[26] Harper, W.L., R. Stalnaker and G. Pearce (eds.) (1981). *Ifs*. Reidel, Dordrecht.

[27] Iatridou, S. (2000). 'The Grammatical Ingredients of Counterfactuality'. *Linguistic Inquiry*, 31: 231-270.

[28] Jackson, F. (1987). Conditionals. Oxford: Blackwell.

[29] Johnson-Laird, P. N. (1983). *Mental Models: Towards a Cognitive Science of Language, Inference and Consciousness*. Harvard University Press.

[30] Johnson-Laird, P. N. and R. Byrne. (1991). *Deduction*. Hove & London: Erlbaum

[31] Lewis, C.I. and C.H. Langford (1932). *Symbolic Logic*. New York: Denver.

[32] Lewis, D. (1976). 'Probabilities of Conditionals and Conditional Probabilities'. *The Philosophical Review* 85: 297-315.

[33] Lewis, D. (1986). 'Probabilities of Conditionals and Conditional Probabilities II'. *The Philosophical Review* XCV: 581-589.

[34] Liu, I.M., K.C. Lo and J.T. Wu (1996). 'A Probabilistic Interpretation of If-Then'. *The Quarterly Journal of Experimental Psychology* 49A: 828-844.

[35] Markovits, H. and R. Vachon (1990). 'Conditional Reasoning, Representation, and Level of Abstraction'. *Developmental Psychology*, 26: 942-951.

[36] McGee, V. (1985). 'A Counterexample to Modus Ponens'. *Journal of Philosophy* 82: 462-471.

[37] Mellor, D. H. (1993). 'How to Believe a Conditional'. *Journal of Philosophy* 90: 233–48.

[38] Milne, P. (2003) 'The simplest Lewis-style triviality proof yet?'. *Analysis* 63: 300–303.

[39] Oaksford, M., N. Chater, and J. Larkin. (2000). 'Probabilities and Polarity Biases in Conditional Inference.' *Journal of Experimental Psychology: Learning, Memory, and Cognition* 26: 883–899.

[40] Politzer, G. D.P. Over, J. Baratgin (2010). 'Betting on Conditionals'. *Thinking and Reasoning* 16: 172-197.

[41] Popper, K. (1940). 'What is Dialectic?'. *Mind* 49: 403-426.

[42] Popper, K. (1959). *The Logic of Scientific Discovery*. London: Hutchinson. This is an English translation of Logik der Forschung, Vienna: Springer (1935).

[43] Quelhas, A.C., P.N. Johnson-Laird, C. Juhos (2010). 'The Modulation of Conditional Assertions and its Effects on Reasoning.' *The Quarterly Journal of Experimental Psychology* 63: 1716-1739.

[44] Ramsey, F. P. (1926). 'Truth and Probability'. reprinted. in his 1978: 58–100.

[45] Ramsey, F. P. (1978). *Foundations* London: Routledge.

[46] Rothchild, D. (2013). 'Do Indicative Conditionals Express Propositions?', *Nous*, 47: 49-68.

[47] Skyrms, B. (1980). *Causal Necessity*. New Haven: Yale University Press.

[48] Stalnaker, R. C. (1968). 'A Theory of Conditionals', reprinted. in Harper et al. (eds.) 1981: 41–55.

[49] Stalnaker, R. C. (1970). 'Probability and Conditionals'. Philosophy of Science 37: 64-80.

[50] Stalnaker, R. C. (1975). 'Indicative Conditionals'. *Philosophia* 5: 269-286.

[51] Stalnaker, R. C. (1984). *Inquiry*. Cambridge, Mass.: MIT Press.

[52] Stevenson,R.J. and D.E. Over (1995). 'Deduction from Uncertain Premises'. *The Quarterly Journal of Experimental Psychology*, 48A: 613-643.

[53] van Fraassen, B.C. (1976). 'Probabilities of Conditionals', in W. Harper and C. Hooker (eds.), *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science*, vol. 1. Dordrecht: Reidel: 261–300.

[54] Weirich, K. (2020). 'Defending Truth Tables for Indicative Conditionals'. *Philosophical Studies*, 177: 1635-1657.

[55] Wright, C. (1983). 'Keeping Track of Nozick'. *Analysis*, 43: 134-40.

[56] Yalcin, S. (2012). 'A Counterexample to Modus Tollens'. J*ournal of Philosophical Logic*, 41: 1001-1024.