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The Compatibility of Economic Rationality and General Rationality: Integrating Constraints on Beliefs and Preferences

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Abstract

This paper examines the interplay between coherence-based rationality (Rationality-1), which ensures consistency in belief systems via propositional calculus, and preference-based economic rationality (Rationality-2), governed by axioms such as completeness and transitivity. In the standard propositional framework $\mathcal{L}$, the Validity Consensus Property (VCP) - ensuring universal agreement on argument validity across different logical tests - holds universally. However, when the framework is extended to include preference propositions, forming $\mathcal{L}_p$, the imposition of rationality constraints from Rationality-2 affects the semantics of $\mathcal{L}_p$. These changes cause VCP to fail, meaning that agents using different logical tests may disagree on whether an argument is valid. Furthermore, in striving to satisfy the constraints of Rationality-2, an agent may accept conclusions that introduce contradictions into their belief system, thereby violating the consistency required by Rationality-1.

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1 Introduction

Rationality is a central concept across economics, psychology, philosophy, and decision theory, providing a framework for understanding decision-making, reasoning, and economic behavior. Crucially, rationality focuses on an agent's beliefs - the mental representations they hold about the world and its possibilities - rather than desires, emotions, or intentions. This distinction allows for the analysis of rationality as a property of an agent's system of beliefs.

This prompts a fundamental question: what defines rationality? Specifically, what property must a system of beliefs possess to be characterised rational? Two natural candidates, widely discussed in the philosophical and psychological literature, are coherence and truth. Coherence emphasizes the internal consistency of beliefs whereas truth concerns the alignment of beliefs with reality. Is coherence compatible with truth? At first glance, the answer is yes, as coherence supports truth by ensuring internal consistency. However, as Comesana (2020) observes, tensions can arise when coherence and truth conflict, revealing the nuanced relationship between these dimensions of rationality. In this paper, we focus exclusively on the coherence aspect of rationality, setting aside the interplay between coherence and truth.

We define a system of beliefs as coherent if and only if it is free of contradictions. A sufficient condition for ensuring that a system of beliefs is contradiction-free is its adherence to the rules of a formal calculus, which provides a structured logical framework to preserve consistency (see, for example, Inhelder and Piaget 1955, Audi 2004 but cf. Brown 1988 for a critique of this emphasis on formal rules). But what does it mean for a system of beliefs to "obey" the rules of a calculus? And, crucially, which calculus is appropriate in this context? .

Let us begin with the latter question: which calculus? To answer this, we must recognize that the objects of belief are propositions - statements that express claims about the world and are, in principle, either true or false. The selection of the relevant formal calculus depends on the epistemic environment within which the agent's beliefs are formed. Importantly, this distinction does not arise from differences in the nature of propositions themselves but from the agent's epistemic ability to ascertain their truth values within a given environment.

Two such epistemic environments are usually considered in the literature: certainty and uncertainty. Under certainty, the agent has the epistemic ability to determine the truth or falsity of any proposition with absolute confidence. Under uncertainty, the agent lacks the ability to definitively ascertain the truth or falsity of propositions and instead expresses degrees of confidence or probability. In this case, the relevant formal framework is the probability calculus (see, for example, Savage 1974).

In this paper, we focus exclusively on the epistemic environment of certainty and the associated propositional calculus. Hence, we are led to the following definition: A system of beliefs of an agent X, is free of contradictions (and hence rational) if it obeys the propositional calculus. But why is this the case? The answer lies in how the propositional calculus governs the relationships among the truth values assigned to propositions in X's language $\mathcal{L}$. Since X's system of beliefs can be understood as the set of truth values she assigns to the propositions of her language, "obeying the calculus" means that these truth values are distributed according to the rules of propositional logic. For example, consider the complex proposition "The earth is flat AND the moon is made of cheese." If the agent assigns the truth value "false" to both constituent propositions, the rules of propositional calculus dictate that the complex proposition must also be assigned the truth value "false." A failure to adhere to this rule would result in a contradiction within the belief system, undermining its coherence. Thus, the propositional calculus ensures that the agent's system of beliefs is logically consistent and coherent by governing how truth values are assigned and related. From now on, we shall refer to this type of rationality as Rationality-1.

In economics, a specific form of rationality emerges, distinct from the broader notion of rationality (Rationality-1) discussed so far. This is economic rationality, which we refer to as Rationality-2. Economic rationality is concerned with the agent's preferences, expressed through relational statements such as "x is weakly preferred to y", ($x \succeq y$). These preferences are subject to axioms like completeness and transitivity, which ensure that preferences are consistent and well-ordered, forming the basis for rational decision-making in economic theory.

The focus of this paper is to explore the relationship between Rationality-1 and Rationality-2. While these two forms of rationality are conceptually distinct - one governing the logical coherence of beliefs and the other the structural consistency of preferences - they may not be entirely independent. The question arises: how does Rationality-2, with its preference-based axioms, influence Rationality-1, which is concerned with the logical consistency of beliefs?

A crucial step in this direction is to observe that preference relations, $x \succeq y$ can be interpreted as declarative propositions, "x is at least as preferable as y", denoted by $P(x, y)$. This interpretation allows us to extend the original language $\mathcal{L}$ to include preference propositions, resulting in the extended language $\mathcal{L}_p$. By incorporating preference propositions into $\mathcal{L}_p$, preference relations are no longer isolated constructs; they are treated as propositions and, therefore, objects of belief. Consequently, beliefs in preference propositions are embedded within the broader system of beliefs governed by Rationality-1, subjecting them to its logical constraints and coherence requirements.

The aforementioned discussion implies that preference propositions are subject to two distinct sets of rationality constraints: the syntactic and semantic rules of propositional calculus (Rationality-1), and the axiomatic requirements of completeness and transitivity imposed by Rationality-2. This dual set of constraints raises a crucial question: How do the axioms of Rationality-2 influence the logical structure of $\mathcal{L}_p$? Specifically, in what ways do these constraints affect the semantics of preference propositions and the criteria for the validity of arguments involving them?

This paper addresses these questions by demonstrating that the imposition of Rationality-2 constraints alters the semantics of $\mathcal{L}_p$, leading to a failure of the Validity Consensus Property (VCP), a property that holds universally in the standard propositional framework $\mathcal{L}$. We show that in $\mathcal{L}_p$ there exist argument forms for which agents applying different validity tests may disagree on validity. Moreover, we demonstrate that in their effort to satisfy the axioms of Rationality-2, economic agents may endorse conclusions that introduce inconsistencies into their belief systems, thereby violating the coherence requirements of Rationality-1. This result highlights a fundamental tension between coherence-based rationality and preference-based rationality, a tension that has significant implications for decision theory and economic reasoning, where logical coherence and preference consistency are often assumed to align. Section 2 analyzes the interplay between Rationality-1 and Rationality-2. Section 3 concludes the paper.

2 The Interplay between Rationality-1 and Rationality-2: Semantic and Logical Implications of Preference Constraints

To analyze the interplay between Rationality-1 and Rationality-2, we begin by describing the object language $\mathcal{L}$. Following the standard procedure, we introduce the syntax and semantics of $\mathcal{L}$ simultaneously. The objects of $\mathcal{L}$ are standard propositions, *i.e.* statements about the world that can be assigned a truth value of either true (T) or false (F). The term "standard" is used to emphasize that $\mathcal{L}$ does not include any preference propositions. This special type of propositions will be introduced later as part of the extended language $\mathcal{L}_p$. The language $\mathcal{L}$ includes the usual truth-functional connectives $\neg$, $\wedge$, $\vee$, $\supset$ and $\longleftrightarrow$, interpreted as "NOT", "AND". "OR", "IF, THEN" and "IF AND ONLY IF", respectively. The truth-functionality of these connectives means that the truth value of any complex proposition in $\mathcal{L}$ is determined entirely by the truth values of its component propositions, according to the familiar truth tables for these connectives (see, for example, Copi & Cohen 2010, Chapters 6-7).

Central to propositional calculus is the concept of validity - the logical relationship between premises and a conclusion in an argument. An argument in $\mathcal{L}$ consists of a finite set of premises $\{P_1, P_2, ..., P_n\}$ and a conclusion Q, typically written as

$$P_1, P_2, ..., P_n \models Q$$

An argument is said to be valid if its conclusion logically follows from its premises. Validity can be understood in two standard and equivalent ways:

1) **Row-Based Definition:** An argument is valid if there is no truth assignment (row in the truth table) where all the premises $\{P_1, P_2, ..., P_n\}$ are true and the conclusion Q is false. In other words, for all truth assignments in which $P_1, P_2, ..., P_n$ are true, Q is also true.

2) **Implication-Based Definition:** An argument is valid if the proposition

$$(P_1 \wedge P_2 \wedge ... \wedge P_n) \supset Q$$

is a tautology ($\top$).

In standard propositional language (in which no preference propositions are included), the two defintions of validity are logically equivalent. This can be seen as follows:

a) From Row-Based to Implication-Based: If the row-based definition holds, then for every truth assignment where all premises $\{P_1, P_2, ..., P_n\}$ are true, the conclusion Q must also be true. This implies that $(P_1 \wedge P_2 \wedge ... \wedge P_n) \supset Q$ evaluates to true under every truth assignment, making it a tautology.

b) From Implication-Based to Row-Based: If $(P_1 \wedge P_2 \wedge ... \wedge P_n) \supset Q$ is a tautology, then under every truth assignment where $P_1, P_2, ..., p_n$ are all true, Q must also be true. Hence, there is no truth assignment where the premises are all true and the conclusion is false, satisfying the row-based definition.

This equivalence is fundamental to the logical structure of propositional calculus. It also has some important practical implications: Consider two agents, X and Y, who evaluate the validity of arguments in $\mathcal{L}$ using different criteria. Agent X employs the row-based definition, determining validity by checking if there exists a truth assignment where all premises are true, but the conclusion is false. Agent Y uses the implication-based definition, verifying whether the formula $(P_1 \wedge P_2 \wedge ... \wedge P_n) \supset Q$ is a tautology. Despite their differing methods, the equivalence of the two definitions guarantees that X and Y will always agree on the validity or invalidity of any argument in $\mathcal{L}$. This consistency eliminates any possibility of disagreement between agents who rely on these two standard definitions of validity, underscoring the practical robustness of propositional calculus. Let us refer to this equivalence result as the "Validity Consensus Property" (VCP).

With this foundational understanding of $\mathcal{L}$, we are now prepared to incorporate preference propositions and examine how the constraints of Rationality-2 affect the logical framework of Rationality-1. Special emphasis will be placed on whether the VCP still holds in $\mathcal{L}_p$, the extended language that includes preference propositions.

To extend the original language $\mathcal{L}$, and incorporate preference propositions, we begin by formalizing the relationship between preference relations and preference propositions. The result is the extended language $\mathcal{L}_p$, which includes both standard propositions and preference propositions. Consider the set $\mathcal{X}$ of alternatives available to the agent X. Weak preference is defined as a binary relation $x \succeq y$, $x, y \in \mathcal{X}$. Next, we map each weak preference relation $x \succeq y$ to a unique declarative proposition $P(x, y)$ that represents it. The language $\mathcal{L}_p$ is constructed by extending the original language $\mathcal{L}$ with the preference propositions $P(x, y)$, defined above. Formally,

$$\mathcal{L}_p = \mathcal{L} \cup \{P(x, y) \mid (x, y) \in \mathcal{X} \times \mathcal{X}\}.$$

Preference propositions are now integrated into the logical framework of $\mathcal{L}_p$. As objects of belief, they are subject to the same coherence constraints as standard propositions under Rationality-1. This means the agent's beliefs about weak preferences are embedded within a larger system of beliefs governed by both the logical structure of $\mathcal{L}_p$ and the axioms of Rationality-2 (e.g., completeness and transitivity). This formalization provides a unified framework for analyzing how Rationality-1 and Rationality-2 interact within the extended language $\mathcal{L}_p$.

The next step is to examine the propositional implications of the axioms of Rationality-2, namely completeness and transitivity, and their effects on the logical structure of $\mathcal{L}_p$. These axioms impose constraints on the semantics of preference propositions, fundamentally altering the standard truth-functional framework of Rationality-1. We begin by defining the notion of a symmetric proposition:

Definition: For a preference proposition $P(x, y)$ which represents "x is at least as preferable as y," the corresponding symmetric proposition is $P(y, x)$ which represents "y is at least as preferable as x".

Now we are ready to examine the semantic implications of completeness. As is well known, completeness eliminates incomparability among alternatives. Formally, for any $x, y \in \mathcal{X}$, either $x \succeq y$ or $y \succeq x$ or both. If both relations hold, then $x \sim y$. Translated in propositional language, completeness implies that either $P(x, y)$ is true or $P(y.x)$ is true or both of these symmetric propositions are true. This eliminates the possibility that both $P(x, y)$ and $P(y.x)$ are false simultaneously. Consequently, for any pair of symmetric proppsitions, $P(x, y)$ and $P(y.x)$, instead of four possible truth-value combinations (or models or possible worlds), as is the case of two standard propositions, namely TT, TF, FT, FF, only three combinations remain: TT, TF, FT. Hence, completeness (a constraint of Rationality-2) reflects a significant departure from the standard semantics of Rationality-1.

Similar departures are caused by transitivity. Transitivity ensures consistency among chains of preferences. Formally, for any $x, y, z \in \mathcal{X}$, if $x \succeq y$ and $y \succeq z$ then $x \succeq z$. In propositional terms, transitivity implies the following relationship:

$$P(x, y), P(y.z) \models P(x, z).$$

This means that the truth of $P(x, y)$ and $P(y.z)$ entails the truth of $P(x, z)$ in all possible worlds. This constraint affects the semantic structure of $\mathcal{L}_p$ by limiting the possible combinations of truth values for any three preference propositions $P(x, y)$, $P(y.z)$ and $P(x, z)$. Specifically, For three standard propositions in $\mathcal{L}$, there are $2^3 = 8$ possible truth-value combinations (or rows in the truth table). Under transitivity, one of these combinations becomes invalid: the case where the truth values of $P(x, y)$, $P(y.z)$ and $P(x, z)$ are T,T, and F, respectively. Hence, transitivity reduces the rows for triples $P(x, y)$, $P(y.z)$ and $P(x, z)$ from eight to seven.

The reduction in the number of possible worlds, incurred by completeness and transitivity, may be interpreted as a departure from general-purpose logical frameworks toward a specialized system tailored to the needs of preference reasoning. This suggests that $\mathcal{L}_p$ operates under a refined logic where domain-specific axioms influence the permissible semantic structures, effectively creating a hybrid between propositional and modal-like logics.

2.1 Effects of Completeness and Transitivity on the Inferences About Validity

The constraints introduced by completeness and transitivity in $\mathcal{L}_p$ have significant implications for the inferences about the validity of arguments. These constraints alter the semantic structure of $\mathcal{L}_p$ by eliminating specific rows from truth tables, thereby narrowing the range of possible models in which arguments are evaluated.

1. Valid Arguments in the Standard Framework

For arguments that are valid in the standard framework, these remain valid in $\mathcal{L}_p$. The reason is straightforward: the elimination of rows in the truth table does not introduce new counterexamples where the premises are true, but the conclusion is false. Since validity is defined as the truth of the conclusion in all models where the premises hold, reducing the number of models cannot invalidate an argument that was previously valid.

2. Invalid Arguments in the Standard Framework

The situation becomes more nuanced for arguments that are invalid in the standard framework. Row elimination may remove models that serve as counterexamples to the argument's validity. Specifically, if all the rows where the premises are true and the conclusion is false are eliminated, the argument may appear valid in $\mathcal{L}_p$, even though it was invalid in the standard framework. This shift occurs because the semantic constraints of Rationality-2 (e.g., completeness and transitivity) restrict the logical possibilities available for evaluating arguments.

It is now time to examine whether the Validity Consensus Property (VCP) continues to hold in the modified framework $\mathcal{L}_p$. To make the analysis clear, consider two agents: Agent X, who adheres to the row-based definition of validity, and Agent Y, who adheres to the implication-based definition of validity. Under the standard framework of $\mathcal{L}$ the two agents would universally agree on the validity or invalidity of any argument because the two definitions of validity are logically equivalent. The question is whether this agreement, guaranteed in $\mathcal{L}$ still holds universally over all arguments in the extended framework $\mathcal{L}_p$ where the semantic constraints of Rationality-2 (e.g., completeness and transitivity) modify the truth tables by eliminating certain rows. This analysis is crucial for understanding whether the integration of Rationality-1 and Rationality-2 affects the coherence of inferential reasoning within $\mathcal{L}_p$.

Universal agreement means that VCP holds for every argument in $\mathcal{L}_p$. This means that if we can identify even a single argument where X and Y disagree about its validity, then we will have demonstrated the non-universality of VCP in $\mathcal{L}_p$. Such a counterexample to the universality of VCP is the *modus tollens* argument form, based on symmetric propositions. Consider any two standard propositions, $P, Q \in \mathcal{L}$. The modus tollens argument form is as follows:

$$\begin{bmatrix} P \supset Q \\ \neg Q \\ \therefore \\ \neg P \end{bmatrix}.$$

Under the standard framework, $\mathcal{L}$ both agent X (row-based validity) and agent Y (implication-based validity) agree that modus tollens is valid for any standard propositions P, Q. This agreement reflects the universality of VCP in $\mathcal{L}$. Now consider the modified framework $\mathcal{L}_p$ and examine a restricted version of modus tollens where the propositions involved are the symmetric preference propositions $P(x, y)$ and $P(y, x)$. In this case, the restricted modus tollens argument form becomes:

$$\begin{bmatrix} P(x,y) \supset P(y,x) \\ \neg P(y,x) \\ \therefore \\ \neg P(x,y) \end{bmatrix}.$$

To examine VCP, we must construct the following truth table, which because of completeness, is a three-row one:

$P(x,y)$	$P(y,x)$	$\neg P(x,y)$	$\neg P(y,x)$	$P(x,y) \supset P(y,x)$	$(P(x,y) \supset P(y,x)) \wedge \neg P(y,x)$	$((P(x,y) \supset P(y,x)) \wedge \neg P(y,x)) \supset \neg P(x,y)$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T

For row-based validity, adopted by agent X, we compare the truth values in the sixth column (Premises 1 and 2 combined) with those in the third column (the negation of $P(x, y)$, i.e. the conclusion). The row-based definition states that the argument is valid if, in every row where the conjunction of the premises is true, the conclusion is also true. In this case, there is no row where the conjunction of the premises (sixth column) is true. Hence, the argument is invalid according to the row-based test.

For implication-based validity, chosen by agent Y, we examine the truth values in the seventh column, which represent the material implication whose antecedent and conseqient are the conjuction of the premises and the conclusion, respectively. Here, the implication is true in all three rows. Hence, according to the implication-based definition, the argument is valid.

From the above discussion, it is clear that X and Y disagree about the validity of the restricted modus tollens argument form. This disagreement affects the Rationality-1 of Y only. In particular, since Y recognizes the aforementioned argument as valid, she accepts his conclusion, namely she believes that $\neg P(x, y)$ is true. However, she has also accepted the second premiss, namely $\neg P(y, x)$ as true. Hence, she ends up believing that both $\neg P(x, y)$ and $\neg P(y, x)$ are true, or equivalently that $P(x, y)$ and $P(y, x)$ are false. But, by her adherence to completeness, she also believes that $P(x, y)$ and $P(y, x)$ cannot both be false simultaneously. Hence, she ends up with a contradictory set of beliefs, which implies that she violates Rationality-1.

3 Conclusions

The analysis in this paper underscores the nuanced interplay between Rationality-1 and Rationality-2, illustrating how the introduction of preference propositions into $\mathcal{L}_p$ impacts the logical framework of rational belief systems. The key findings include: (i) Semantic Effects of Completeness and Transitivity: These axioms fundamentally alter the standard truth-functional semantics by eliminating certain rows from truth tables, thereby constraining the range of possible models. (ii) Failure of the Validity Consensus Property (VCP): In the extended framework $\mathcal{L}_p$ disagreement between agents relying on row-based and implication-based validity tests demonstrates that VCP no longer holds universally. This failure is exemplified by the restricted modus tollens argument form. (iii) Implications for Rationality-1: The logical consequences of Rationality-2's constraints can lead agents to form contradictory beliefs, violating Rationality-1. Specifically, adherence to completeness and acceptance of certain argument conclusions may result in incoherent belief systems. (iv) Unified Framework Challenges: The findings highlight the tension between the axiomatic requirements of Rationality-2 and the coherence demands of Rationality-1. This suggests the need for further refinement in how preferences and beliefs are integrated into a cohesive rational framework.

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