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Uncertain Evidence and the Order of Updates: Lessons for Econometrics from Philosophical Analysis

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Uncertain Evidence and the Order of Updates: Lessons for Econometrics from Philosophical Analysis

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Abstract

There are many real-world situations where evidence is uncertain, arising from factors such as noisy measurements, incomplete data, or ambiguous observations. In such cases, Bayesian Conditionalization (BC), which assumes evidence is fully certain, is not an appropriate method for belief updating. Instead, Jeffrey Conditionalization (JC) offers a flexible alternative that accommodates uncertain evidence by allowing probabilistic updates. However, a key problem with JC, not present in BC, is its non-commutative nature: the order in which evidence is received affects the resulting posterior probabilities. This feature has significant implications for the agreement of posterior probabilities between agents. Specifically, two agents with identical priors and access to the same total evidence can reach different posterior beliefs if they process the evidence in different sequences.

Keywords: Uncertain Evidence, Jeffrey Conditionalization, Order of Updating, and Disagreement

JEL Classification: C44, D81, D83, D89

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1 Introduction

Why might an agent S be uncertain about evidence E_1? One reason for such an uncertainty is the presence of poor observation conditions of E_1. Meacham (2015) gives the following example: "If a subject sees her friend through the window, and the lighting outside is poor, then it might seem like she should assign the proposition that her friend is outside a value less than 1." (2016, p. 769). Another reason refers to "measurement problems" that arise quite frequently in the scientific practice. Huttegger (2015) describes these less-than-perfect learning experiences as follows: "A scientist may not be entirely certain as to the outcome of an experiment because the underlying process is only accessible through noisy signals. Thus, the results of an observation or introspection need not lead to singling out a proposition that fully captures what has been learned." (2015, p. 621).

Uncertain evidence poses significant challenges to the standard Bayesian framework, which assumes that evidence is certain and updates are performed via Bayesian Conditionalization (BC). In many real-world scenarios, however, evidence often comes in uncertain or probabilistic forms, requiring a more general updating framework like Jeffrey Conditionalization (JC). JC extends Bayesian reasoning by allowing updates when an agent receives partial or noisy information about a partition of events, rather than definitive evidence about a single proposition.

The focus of this paper is on the non-commutative nature of JC. This property highlights that the order in which evidence becomes available to an agent can influence the resulting posterior probabilities, unlike BC, where the final result depends only on the total evidence and not its order. This has significant implications for scenarios where evidence arrives sequentially or incrementally, particularly in understanding whether agents can maintain agreement when they process the same evidence in different sequences. Such sequential updates on uncertain evidence are common in fields like finance or medicine, where agents may receive the same pieces of evidence in different orders. Another important point is that the evidence does not need to be objectively uncertain; it is sufficient for agents to treat it as such. For example, during the COVID-19 pandemic, the scientific evidence on the effectiveness of vaccines was as certain as scientific evidence can be. However, many individuals treated this evidence as uncertain, often due to misinformation, mistrust, or differing interpretations of the data.

The questions raised in this paper - concerning uncertain evidence, belief updating, and the non-commutativity of Jeffrey Conditionalization - have been extensively analyzed in the philosophical literature, particularly in the context of probability theory and formal epistemology. Philosophers have long explored the implications of updating beliefs under uncertainty, the sensitivity of posterior probabilities to the order of evidence arrival, and the broader consequences for rational decision-making. However, these results remain largely unknown within the econometrics literature, where similar challenges often arise in sequential data analysis and decision-making under uncertainty. One of the main aims of this paper is to bridge this gap by presenting key insights from the philosophical analysis of belief updating in a way that is accessible and relevant to econometricians. By doing so, we hope to encourage further dialogue between these two fields and demonstrate the practical importance of these results for econometric applications.

This paper is organized as follows: Section 2 analyzes the main differences between updating on certain versus uncertain evidence via Bayesian and Jeffrey Conditionalization respectively. Section 3 analyzes the issue of non-commutativity of JC and discusses how this feature of JC affects whether two agents who receive the same total evidence but in different order may end up disagreeing, despite having the same total evidence and despite having started with the same priors. This section also contains a discussion of real-world cases in which sequential belief updating occurs under conditions of uncertain evidence. Section 4 concludes the paper.

2 Certain versus Uncertain Evidence and Bayesian versus Jeffrey Conditionalization

Consider a Bayesian economic agent, S, who is at the beginning of his epistemic life, i.e. at time $t = 0$ when she becomes interested in the empirical phenomenon $\mathcal{E}$. At $t = 0$, S has no empirical information about phenomenon $\mathcal{E}$. In this epistemic state of zero information, S (by virtue of being Bayesian) still needs to determine her prior subjective probability function, P_0, defined on a Boolean algebra $\mathcal{F}$ of events/propositions. Next, consider, the period $t = 1$, in which the event E_a occurs. A typical Bayesian assumption is that "observing E_a" is a sufficient condition for S to increase her subjective probability from $P_0(E_a) = p < 1$ to $P'(E_a) = 1$. S's absolute confidence in the truth of proposition E_a is necessary for allowing her to adhere to the principle of Bayesian Conditionalization (BC). BC dictates the manner by which S generates her new probability function P' and may be stated as follows[1]:

$$P'(A) = P_0(A \mid E_a), \ A, E_a \in \mathcal{F} \tag{1}$$

subject to

$$P_0(E_a) > 0.$$

In other words, (1) is the rational way for S to update her prior probability of A in the light of the evidence E_a, under the assumption that at $t = 1$ S is "absolutely certain" about the truth of the evidential proposition E_a, i.e. under the assumption $P'(E_a) = 1$.

BC together with some additional assumptions produce the following well-known "agreement-type" results:

[1] To avoid uncessary notational burden, we use the same symbols to denote an event A and the corresponding proposition A : "the event A has occured". Hence, S believes that A has occured iff she believes that A is true.

(i) Two agents S_1 and S_2 sharing the same evidence E_a, (considered as *certain* by both), and having the same prior subjective probability function, P_0, defined on $\mathcal{F}$ will have the same posterior probability function P'.

(iii) Two agents S_1 and S_2 having the same same prior subjective probability function, P_0, but possessing two different pieces of evidence $E_1^{'}$ and $E_2^{'}$, respectively (considered as *certain* by both) will end up with the same posterior P', if their posteriors are "common knowledge" in Aumann's (1976) sense.

An interesting question that arises at this point is how S should revise his prior probability of A in the light of E_a when she is uncertain about the realization of E_a or, equivalently, is uncertain about the truth of the empirical proposition E_a. In short, how should S revise $P_0(A)$ when she is *uncertain* about the available evidence, that is, when $P'(E_a) < 1$?

If S is uncertain about E_a, then she cannot employ BC to update her prior. Jeffrey (1983) proposed the following rule for rational learning in the case of "uncertain evidence":

$$P'(A) = P_0(A \mid E_a)P'(E_a) + P_0(A \mid \neg E_a)P'(\neg E_a) \tag{2}$$

where $\neg E_a$ is the complement of E_a. Note that when S is certain about E_a then $P'(E_a) = 1$ and $P'(\neg E_a) = 0$ so (2) reduces to (**??**). In other words, Jeffrey Conditionalization (JC) is a generalization of Bayesian Conditionalization to include cases of uncertain evidence.

In general, consider the finite "evidence partition" $\mathcal{E}$ (of S's prior probability space Ω) that consists of the mutually exclusive and jointly exhaustive events/propositions $E_1, E_2, ..., E_n$. The general form of Jeffrey's rule is given by:

$$P'(A) = P_0(A \mid E_1)P'(E_1) + P_0(A \mid E_2)P'(E_2) + ... + P_0(A \mid E_n)P'(E_n) \tag{3}$$

Remarks:

(i) Jeffrey (1983) derives (3) by assuming that

$$P_0(A \mid E_i) = P'(A \mid E_i) \tag{4}$$

for each $E_i \in \mathcal{E}$ for which $P'(E_i) > 0$ and $\forall A \in \mathcal{F}$. Equation (4) is usually referred to as the "rigidity condition".

(ii) JC assumes two kinds of revisions to the prior probabilities of agent S: exogenous and endogenous (see Miller 2019). Exogeneous are the revisions $P_0(E_i) \to P'(E_i)$ which are triggered by S's direct "sensory experience" of E_i. On the other hand, endogenous revisions are those that are performed via the updating rule (3), after the exogenous revisions have been made.

2.1 The Issue of Commutativity

The commutativity issue, analyzed in this section, may be described as follows: Suppose that at $t = 1$, S obtains the information E_a. Then at the later time point $t = 2$, she receives E_b. Now, assume that S revises her prior probabilities by taking into account first E_a and then E_b, thus ending up at $t = 2$ with the final posterior P''. Next assume that the order in which S updates her beliefs is reversed: first she updates based on E_b, then she updates based on E_a. In this case, her final posterior at $t = 2$ is P^{**}. Question: Is P'' identical to P^{**}? If the answer is yes, then the way in which S updated her beliefs is commutative. It is well known that BC is commutative. Weisberg (2009) refers to the normative status of BC-commutativity as follows: "...the order in which information is learned should not matter to the conclusions we ultimately draw, provided the same total information is collected. It shouldn't matter whether I find the murder weapon in the maid's room first and then hear testimony about her alibi, or the other way around. Either way my ultimate attitude about her guilt should be one of guarded suspicion." (2009, p. 793). Similar claims were made by Kelly (2008): "To the extent that what it is reasonable for one to believe depends on one's total evidence, historical facts about the order in which that evidence is acquired make no difference to what it is reasonable for one to believe." (2008, p. 616). The commutativity of evidence may be expressed as follows:

$$P_0(A) \xrightarrow{E_a} P'(A) = P_0(A \mid E_a) \xrightarrow{E_b} P''(A) = P'(A \mid E_b)$$

$$P_0(A) \xrightarrow{E_b} P^*(A) = P_0(A \mid E_b) \xrightarrow{E_a} P^{**}(A) = P^*(A \mid E_a)$$

with

$$P''(A) = P^{**}(A) = P_0(A \mid E_a, E_b),\ \forall A, E_a, E_b \in \mathcal{F}. \tag{5}$$

The commutative of BC is stated by (5).

Let us know withdraw the "certain evidence" assumption and suppose instead that S is less uncertain about the truth of the evidence propositions E_a and E_b. In such a case, the rational way to update her degrees of belief is JC. Is JC commutative? The answer is generally negative. Schematically,

$$P_0(A) \xrightarrow{E_a, P'(E_a)<1} P'(A) = P_0(A \mid E_a)P'(E_a) + P_0(A \mid \neg E_a)P'(\neg E_a)$$
$$\xrightarrow{E_b, P''(E_b)<1} P''(A) = P'(A \mid E_b)P''(E_b) + P'(A \mid \neg E_b)P''(\neg E_b)$$

$$P_0(A) \xrightarrow{E_b, P^*(E_b)<1} P^*(A) = P_0(A \mid E_b)P^*(E_b) + P_0(A \mid \neg E_b)P^*(\neg E_b)$$
$$\xrightarrow{E_a, P^{**}(E_a)<1} P^{**}(A) = P^*(A \mid E_a)P^{**}(E_a) + P^*(A \mid \neg E_a)P^{**}(\neg E_a)$$

with

$$P''(A) \neq P^{**}(A) \text{ for some } A, E_a, E_b \in \mathcal{F}. \tag{6}$$

Wagner (2002) analyzes the conditions under which Jeffrey Conditionalization (JC) is commutative, building on earlier insights from Field (1978) and Diaconis and Zabell (1982). The key idea is that achieving commutativity requires a redefinition of what constitutes a "learning experience" in the context of belief updating. Rather than demanding identical posterior probabilities, Wagner argues that identical learning experiences should instead be characterized by identical Bayes factors - the ratios of updated odds to prior odds for competing hypotheses. By redefining learning experiences in terms of Bayes factors, Wagner shows that JC becomes commutative, as the changes in odds remain consistent regardless of the order in which evidence is processed. However, under the traditional view that equates learning experiences with identical posterior probabilities, JC remains non-commutative.

The non-commutativity of JC, that is, the sensitivity of JC to the order in which evidence arrives is often seen as the main objection against JC (see, for example, Field 1978, Domotor 1980, van Fraassen 1989, Doring 1999). van Fraassen, for example, argues that two persons who start with the same priors at the beginning of a given day, and have the same total evidence at the end of that day should end up with the same posteriors, regardless of the sequence in which they receive the data. If this is not the case then, van Fraassen claims, the whole idea of "learning from experience" is meaningless. However, more recent studies tend to restore the normative status of JC arguing that its non-commutativity is not a defect but rather a virtue. Lange (2000) writes: "Although Jefrey's rule is formally noncommutative, this does not represent a defect in the rule. On the contrary, this kind of non-commutativity is exactly right. The key point will be that in switching the order in which numbers are plugged into Jefrey's rule, *we are not really switching the order in which the same two sensory experiences are taken into account*. Rather, we are dealing with entirely diferent pairs of observations. That is why they should generally yield diferent final degrees of belief." (2000, p. 393, emphasis added).

2.2 An Example

Consider the random experiment of simultaneously tossing two fair coins. The sample space Ω is defined as follows:

$$\Omega = \{HH, HT, TH, TT\}$$

where H and T denote "heads" and "tails", respectively. Since Ω is finite, we can take $\mathcal{F}$ to be the power set of Ω. Consider an agent S whose prior subjective probability function is P_0, $P_0 : \mathcal{F} \to [0, 1]$. Since S knows that the coins are fair (this information is part of her overall background informtation), and since she is assumed to "defer to chance", it follows that (at $t = 0$)

$$P_0(\{HH\}) = P_0(\{HT\}) = P_0(\{TH\}) = P_0(\{TT\}) = \frac{1}{4}.$$

Assume that at $t = 1$, and $t = 2$ S receives the pieces of information $E_a = \{HH, HT, TH\}$ and $E_b = \{HH, TH, TT\}$, respectively. Suppose that S is absolutely certain of the truth of E_a and E_b. Then, S's "total evidence" at $t = 2$ is $E_a \cap E_b = \{HH.TH\}$. In such a case conditionalizing first on E_a (with posterior P') and then on E_b (with posterior P'') is equivalent to conditionalizing first on E_b (with posterior P^*) and then on E_a (with posterior P^{**}) which is equivalent to conditionalizing once on $E_a \cap E_b$ (with posterior P^+). For example, it is easy to show that

$$P''(\{HH\}) = P^{**}(\{HH\}) = P^+(\{HH\}) = \frac{1}{2}.$$

Next, let us withdraw the assumption of "certain evidence" and consider two agents, S_a and S_b, who share the same prior subjective probability function, defined on Ω. Both agents receive the same pieces of uncertain evidence, E_a and E_b but in different orders. S_a receives E_a first followed by E_b while S_b first updates on E_b and then E_a. Since their priors are identical, both agents initially assign the same probabilities to E_a and E_b, specifically 0.9 and 0.95 respectively. Given the uncertainty of the evidence, both agents employ Jeffrey Conditionalization (JC) instead of Bayesian Conditionalization (BC). For S_a, her first experience with E_a results in the following posterior probability function P':

$$\begin{aligned}
P'(\{HH\}) &= P_0(\{HH\} \mid E_a)P'(E_a) + P_0(\{HH\} \mid \neg E_a)P'(\neg E_a) \simeq 0.3 \\
P'(\{HT\}) &= P_0(\{HT\} \mid E_a)P'(E_a) + P_0(\{HT\} \mid \neg E_a)P'(\neg E_a) \simeq 0.3 \\
P'(\{TH\}) &= P_0(\{TH\} \mid E_a)P'(E_a) + P_0(\{TH\} \mid \neg E_a)P'(\neg E_a) = 0.3 \\
P'(\{TT\}) &= P_0(\{TT\} \mid E_a)P'(E_a) + P_0(\{TT\} \mid \neg E_a)P'(\neg E_a) = 0.1.
\end{aligned}$$

S_a's second experience, i.e. the one based on E_b yields:

$$\begin{aligned}
P''(\{HH\}) &= P'(\{HH\} \mid E_b)P''(E_b) + P'(\{HH\} \mid \neg E_b)P''(\neg E_b) \simeq 0.407 \\
P''(\{HT\}) &= P'(\{HT\} \mid E_b)P''(E_b) + P'(\{HT\} \mid \neg E_b)P''(\neg E_b) \simeq 0.05 \\
P''(\{TH\}) &= P'(\{TH\} \mid E_b)P''(E_b) + P'(\{TH\} \mid \neg E_b)P''(\neg E_b) \simeq 0.407 \\
P''(\{TT\}) &= P'(\{TT\} \mid E_b)P''(E_b) + P'(\{TT\} \mid \neg E_b)P''(\neg E_b) \simeq 0.136
\end{aligned}$$

Let us now shift our attention to the second agent. S_b revises P_0 to P^* in the light of E_b, with $P^*(E_b) = 0.95$:

$$\begin{aligned}
P^*(\{HH\}) &= P_0(\{HH\} \mid E_b)P^*(E_b) + P_0(\{HH\} \mid \neg E_b)P^*(\neg E_b) \simeq 0.316 \\
P^*(\{HT\}) &= P_0(\{HT\} \mid E_b)P^*(E_b) + P_0(\{HT\} \mid \neg E_b)P^*(\neg E_b) = 0.05 \\
P^*(\{TH\}) &= P_0(\{TH\} \mid E_b)P^*(E_b) + P_0(\{TH\} \mid \neg E_b)P^*(\neg E_b) \simeq 0.316 \\
P^*(\{TT\}) &= P_0(\{TT\} \mid E_b)P^*(E_b) + P_0(\{TT\} \mid \neg E_b)P^*(\neg E_b) \simeq 0.316
\end{aligned}$$

Next, S_b updates on the basis of E_a for which $P^{**}(E_a) = 0.9$:

$$\begin{aligned}
P^{**}(\{HH\}) &= P^*(\{HH\} \mid E_a)P^{**}(E_a) + P^*(\{HH\} \mid \neg E_a)P^{**}(\neg E_a) \simeq 0.417 \\
P^{**}(\{HT\}) &= P^*(\{HT\} \mid E_a)P^{**}(E_a) + P^*(\{HT\} \mid \neg E_a)P^{**}(\neg E_a) \simeq 0.066 \\
P^{**}(\{TH\}) &= P^*(\{TH\} \mid E_a)P^{**}(E_a) + P^*(\{TH\} \mid \neg E_a)P^{**}(\neg E_a) \simeq 0.417 \\
P^{**}(\{TT\}) &= P^*(\{TT\} \mid E_a)P^{**}(E_a) + P^*(\{TT\} \mid \neg E_a)P^{**}(\neg E_a) \simeq 0.1.
\end{aligned}$$

Observe that S_a's posterior P'' differs from S_b's posterior P^{**}. This divergence highlights that S_a and S_b will end up disagreeing on their posterior probabilities, even though they begin with a common prior and ultimately share the same total evidence, $E_a \cap E_b$. The source of their disagreement lies in two key factors: the uncertainty of the evidence they receive and the non-commutative nature of Jeffrey Conditionalization (JC). This order sensitivity, or non-commutativity, of JC causes the intermediate posteriors for the two agents to diverge, and these discrepancies carry forward to their final posteriors. Even though the agents share identical priors and ultimately have access to the same total evidence, the sequential nature of belief updates under uncertain evidence results in distinct posterior probabilities, leading to persistent disagreement.

2.3 Uncertain Evidence in Real-World Scenarios

Uncertain evidence frequently arises in everyday situations where observations or measurements are incomplete, noisy, or open to interpretation. This uncertainty is particularly relevant in areas like financial markets, medical diagnoses, environmental debates, and legal proceedings, where the order in which evidence is received can significantly shape conclusions.

Stock Market

In financial markets, investors and analysts often update their beliefs about future stock returns based on information regarding domestic and global economic fundamentals. These fundamentals are typically reported through economic indicators, such as inflation rates, employment data, or corporate earnings reports. However, the reliability of this information is not always straightforward. Initial reports are often provisional and subject to revision, sometimes significantly altering their implications. Additionally, sampling errors or reporting inaccuracies can introduce noise, leaving market participants less confident about the data's reliability. On top of this, economic indicators frequently conflict or point in divergent directions, creating even more ambiguity. As a result, financial evidence is often uncertain, and the order in which this evidence is received and processed can disproportionately influence the beliefs of market participants, leading to varied conclusions about market trends.

Medical Diagnosis

Uncertainty is a common feature in medical decision-making, particularly when it comes to diagnostic tests or imaging results. For example, a doctor evaluating a patient may first receive an X-ray or MRI scan, which, due to poor image quality, patient movement, or equipment limitations, offers ambiguous or inconclusive information. Such results often point to multiple possible diagnoses with varying degrees of confidence. Subsequent tests, like blood analyses or more advanced imaging, may clarify or contradict earlier findings, depending on when and how they are conducted.

The order in which this uncertain evidence is evaluated can play a critical role in the physician's reasoning. For instance, one doctor might first consider an inconclusive X-ray result, leading them to form an initial hypothesis about the patient's condition. When a blood test result arrives, this subsequent evidence might be interpreted in light of the earlier ambiguity, carrying less weight than it otherwise would. By contrast, another doctor who evaluates the blood test first and then reviews the X-ray might reach a different conclusion, even though both have the same total evidence. This sequence-dependent interpretation highlights how uncertainty, combined with the order of evidence evaluation, can lead to differing diagnoses and levels of confidence.

Environmental Issues

The role of uncertain evidence is particularly evident in debates about environmental issues, such as climate change or pollution. While much of the scientific evidence on topics like rising global temperatures and the impact of greenhouse gas emissions is robust and well-supported, individuals and groups may treat this evidence as uncertain for various reasons. These include exposure to misinformation, mistrust of scientific institutions, ideological biases, or simply the complexity of interpreting vast and nuanced datasets.

When evidence is perceived as uncertain, the order in which it is processed becomes critical in shaping beliefs. For example, one group might first encounter compelling evidence highlighting the severity of climate change, such as increased extreme weather events or rising sea levels. This initial information might nudge their beliefs toward accepting the urgency of environmental action. Any subsequent evidence, such as the economic costs of mitigation policies, is then interpreted through the lens of this prior shift. Conversely, another group might first encounter evidence emphasizing uncertainty—such as variability in climate models or disagreement over specific forecasts. This starting point may predispose them to view later evidence of environmental risks as less convincing, even if it is scientifically robust. This sequence-dependent reasoning, combined with uncertainty, can explain why individuals or groups - despite having access to the same total evidence - often form starkly different conclusions about the urgency of environmental action.

Legal Court Cases

The implications of uncertain evidence and order effects are especially striking in legal proceedings, where witness testimonies play a central role. In theory, witnesses are expected to provide truthful accounts under oath, but in practice, their testimonies are often uncertain due to the fallibility of human memory, subjective perceptions, or unintended biases. Additionally, witnesses may inadvertently introduce misinformation, further complicating the evaluation of their accounts.

In such cases, the order in which witnesses testify can significantly influence the verdict. For instance, consider two jurors with identical priors about a defendant's guilt. If the first witness delivers a compelling but uncertain testimony in favor of the prosecution, this evidence may shift the jurors' beliefs substantially. When a subsequent witness introduces evidence favoring the defense, it may not fully reverse the earlier shift because the prior testimony has already shaped intermediate beliefs. On the other hand, if the testimonies are presented in reverse order, the jurors' final beliefs may differ—even though the total evidence remains the same.

This dynamic illustrates how the non-commutative nature of belief updating, as captured by Jeffrey Conditionalization, can lead to persistent disagreement. The sequence of uncertain testimonies matters as much as their content, underscoring the need for careful structuring of legal proceedings to minimize undue influence from the order of evidence presentation.

In all these real-world cases, the uncertainty of evidence—combined with the sequence in which it is processed—plays a decisive role in shaping beliefs and conclusions. Whether in finance, medicine, environmental issues, or law, understanding this dynamic is essential for improving decision-making processes and managing disagreements that arise from the subjective treatment of evidence.

3 Conclusions

This paper has examined the implications of updating on uncertain evidence within the framework of Jeffrey Conditionalization (JC). By analyzing scenarios in which evidence is uncertain and sequentially received, we demonstrated that the non-commutative nature of JC can lead to disagreements between agents, even when they share identical priors and access the same total evidence. This result underscores the critical role of both the uncertainty of evidence and the sequence in which it is processed.

In contrast to Bayesian Conditionalization (BC), which assumes evidence is fully certain and order-independent, JC accommodates partial beliefs about evidence. However, this flexibility introduces order sensitivity, as intermediate posteriors depend on the sequence of updates. The divergence between agents who process evidence in different orders highlights the limitations of JC when consistency or agreement between agents is a concern.

The real-world implications of these findings are far-reaching. Uncertain evidence is pervasive in practical settings such as financial markets, scientific research, consumer behavior analysis, and medical diagnoses. In each of these contexts, the uncertainty stems from factors like noisy data, measurement errors, incomplete observations, or subjective interpretation. When such evidence is processed sequentially, the order of updates influences posterior beliefs, potentially leading to disagreements among agents, even if their priors and total evidence are identical.

These insights have significant implications for decision-making under uncertainty. In situations where agreement between agents is critical, as in collaborative scientific research or policymaking, the effects of evidence order should be carefully considered. One possible approach to mitigating disagreement is to standardize evidence processing sequences or employ frameworks that reduce the sensitivity to order. Alternatively, recognizing the inevitability of order-dependent beliefs may encourage greater transparency and communication about the sequence of evidence updates.

In conclusion, this study highlights the interplay between evidence uncertainty and the non-commutativity of JC, providing a deeper understanding of how rational agents can diverge in their beliefs. Future research may explore methods to minimize such disagreements or examine the implications of these findings in multi-agent systems and other applied contexts where uncertain evidence is the norm.

4 References

References

[1] Aumann, R. J. (1976). Agreeing to disagree. *The Annals of Statistics*, 4(6), 1236–1239.

[2] Diaconis, P., & Zabell, S. L. (1982). Updating subjective probability. *Journal of the American Statistical Association*, 77(380), 822–830.

[3] Domotor, Z. (1980). Probability Kinematics and Representation of Belief Change. *Philosophy of Science*, 47(3), 384-403.

[4] Doring, F. (1999). Why Bayesian Psychology is Incomplete. *Philosophy of Science*, 66, S379-S389.

[5] Field, H. (1978). A note on Jeffrey conditionalization. *Philosophy of Science*, 45(3), 361–367.

[6] Huttegger, S. M. (2015). Learning experiences and the value of knowledge. *Philosophical Studies*, 172(3), 723–739.

[7] Jeffrey, R. C. (1983). *The Logic of Decision* (2nd ed.). University of Chicago Press.

[8] Kelly, K. T. (2008). Disagreement, Dogmatism and Belief Polarization. *The Journal of Philosophy*, 105(10), 611-633.

[9] Lange, M. (2000). Is Jeffrey Conditionalization Defective by Virtue of Being Non-Commutative? Remarks on the Sameness of Sensory Experiences. *Synthese*, 123(3), 393-403.

[10] Meacham, C. J. G. (2016). Understanding Conditionalization. *Canadian Journal of Philosophy*, 45(5-6), 767-797.

[11] Miller, B.T. (2019). Holistic Conditionalization and Underminable Perceptual Learning. *Philosophy and Phenomenological Research*, 101(1), 130-149.

[12] van Fraassen, Bas. (1989). *Laws and Symmetry*. Oxford: Clarendon Press

[13] Wagner, C.G. (2002). Probability Kinematics and Commutativity. *Philosophy of Science*. 69(2), 266-278.

[14] Weisberg, J. (2009). Commutativity or Holism? A Dilemma for Conditionalizers. *The British Journal for the Philosophy of Science*, 60(4), 793-812.