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Managing the Global Commons: Taxes, Bonds, and the Equivalence Between Fiscal and Financial Instruments

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Abstract

Global commons, such as the climate system and large-scale biodiverse ecosystems, generate benefits with public good characteristics that are not reflected in market prices, leading to inefficient resource use. While Pigouvian taxes provide a benchmark solution by aligning private and social incentives, their implementation is often constrained by political, institutional, and informational limitations. This paper examines whether financial instruments can serve as effective alternatives to conventional fiscal tools in managing global commons. Specifically, it shows that appropriately designed instruments—such as green bonds, sustainability-linked bonds, and resilience bonds—can attain the environmental outcomes achieved by optimal taxation. The proposed mechanism mobilizes external funding to compensate local agents for conservation, linking financial returns to ecosystem service flows and thus internalizing global externalities. Under certain conditions, these instruments decentralize the socially optimal environmental outcome without relying on taxation. The analysis provides a conceptual foundation for using financial mechanisms as policy tools in environmental management and climate policy.

Key words: Ecosystem management, Climate change, Pigouvian taxes, Green-Sustainability-Resilience bonds, Asset pricing.

JEL Classification: G12, Q20, Q54, Q58

1 Introduction

Global commons refer to natural systems whose benefits are non-excludable and extend beyond national borders, such as the climate system and large-scale ecosystems with high biodiversity, including tropical forests like the Amazon. These systems generate ecosystem services—such as carbon sequestration and climate regulation—that accrue to the global community rather than solely to local users. Biodiversity, by sustaining the underlying ecological stock, supports the generation of services such as productivity, resilience, and insurance against adverse shocks (e.g. Dasgupta 2021). As a result, global commons exhibit characteristics of global public goods.

A central challenge in managing global commons is that a significant share of the value of the services they generate is not captured by market prices. Local agents base decisions—such as harvesting or converting ecosystems—on private returns, while the global benefits derived from the underlying ecological stock are not reflected in their objectives. This leads to systematic under-provision of conservation and over-exploitation of natural resources, resulting in socially inefficient unregulated market outcomes. Achieving the socially optimal use of these systems therefore requires regulatory intervention, which in standard economic approaches to environmental externalities relies on fiscal instruments in the form of taxes, or equivalent cap-and-trade policies.

Harvesting taxes have long been associated with resource management in forests and fisheries. In forest management, stumpage fees, timber concession taxes, and log export levies serve as regulatory instruments across a range of institutional contexts. In climate policy, carbon taxes constitute the core policy instrument, while in fisheries, common instruments include landing fees, quota royalties, and resource rent taxes.

Despite their theoretical appeal, however, harvesting or global environmental taxes face significant practical limitations. A growing literature emphasizes that, despite their theoretical efficiency, Pigouvian taxes are often constrained by political and institutional factors. Carbon pricing policies, for example, are frequently set below optimal levels due to distributional concerns, lobbying, and limited public acceptance (Aldy, 2017; Jenkins, 2014). More broadly, the design of climate policy reflects binding political constraints, which necessitate the use of complementary or alternative instruments (Jenkins and Karplus, 2016; Baranzini et al., 2017). These challenges are particularly pronounced in the context of global commons and ecosystem management, where weak institutions and the absence of observable tax bases further limit the feasibility of conventional fiscal instruments (Garcia et al. 2018). Moreover in climate policy, although carbon taxes and related instruments such as cap-and-trade systems are central to mitigation, they may be less well suited to addressing adaptation challenges that exhibit strong public good characteristics.

These limitations raise the question of whether alternative instruments can replicate the role of taxes in aligning private incentives with the social value of global commons. In particular, when fiscal instruments are infeasible or ineffective, can other mechanisms attain the socially optimal environmental allocations ? This paper addresses this question by examining whether financial instruments—such as aid-funded green bonds, bond-based claims on ecosystem services, sustainability-linked bonds, and resilience bonds—can be designed to achieve outcomes equivalent to those implemented by Pigouvian taxation.

The central idea in this paper is to explore whether it is feasible and desirable to shift the focus of environmental regulation from taxation to financing. Rather than directly penalizing excess resource use through taxes, the proposed mechanism mobilizes external funding—potentially from international investors and donor governments—to compensate local agents for conservation efforts. By linking financial flows to the value of ecosystem services generated by the underlying ecological stock, these bond-type instruments can support a regulatory system that effectively aligns the social marginal cost of resource extraction with incentives faced by local agents. When appropriately designed, this mechanism induces behavior consistent with the social optimum.

While the equivalence between alternative environmental policy instruments—such as taxes and quantity-based mechanisms—is well established, much less attention has been given to the role of financial instruments in decentralizing optimal environmental policy. Existing work on green bonds which are debt instruments used to finance green projects that deliver environmental benefits (OECD, 2016), focuses primarily on pricing and portfolio considerations (e.g. Flammer 2020; Flammer et al. 2025; Harstad 2025), rather than their potential to substitute for Pigouvian taxation when designed with a specific structure. Yet in many conservation contexts, direct taxation of resource users is politically or institutionally infeasible. The bond-based mechanisms developed in this paper provide an alternative approach that can achieve the same environmental objectives while relying on financial rather than fiscal instruments.

The main contribution of the paper is to show that, under specific conditions and structure such financial instruments can replicate environmental outcomes, achieved by optimal taxation. In this sense, green bonds and related instruments are not merely sources of financing but can be understood as policy tools that implement optimal environmental regulation. This equivalence result provides a conceptual foundation for the design and use of financial instruments in managing global commons, particularly in settings where traditional fiscal approaches are constrained by institutional, political, or informational limitations.

The remainder of the paper is organized as follows. Sections 2 and 3 characterize the socially optimal and privately optimal management of the ecosystem, respectively. Section 4 derives the optimal harvesting tax and shows that it can replicate the socially optimal allocation. Section 5 introduces three types of financial instruments, in the form of bonds, and establishes their equivalence to optimal taxation. Section 6 discusses resilience bonds and their role in financing socially optimal abatement activities with public good characteristics. Section 7 concludes.

The analysis is conducted in a deterministic framework in order to clearly illustrate the proposed financial mechanisms and their equivalence to fiscal instruments. Extending the analysis to a stochastic environment is left for future research.

2 Socially optimal ecosystem management

We consider an ecosystem that generates services which are: (i) directly productive services and represent an input in the production function and (ii) services which do not enter the production function but are generated by the ecosystem and have a positive affect on social welfare (e.g. biodiversity). Some of these services could be potentially monetized in a market economy. We consider first the problem of a social planner that seeks to maximize discounted welfare by choosing consumption and ecosystem use as an input in production, subject to the standard economic budget constraint and the constraint stemming from the evolution of the ecosystem. The planner solves

$$\max_{C_t,h_t} \int_0^\infty e^{-\rho t} \left[\log C_t + \alpha v(S_t)\right] dt \tag{1}$$

subject to

$$\dot{K}_t = AK_t^{\beta_1} h_t^{\beta_2} - C_t - eh_t - \delta K_t,\ K_0 \text{ fixed} \tag{2}$$

$$\dot{S}_t = G(S_t) - h_t - qS_t, S_0 \text{ fixed} \tag{3}$$

where $\log C_t$ is utility from consumption and $\alpha v(S_t)$ is utility derived from the current state S_t of the ecosystem with natural growth function $G(S_t)$, and decay rate q. It is assumed that natural growth is logistic with $G'' < 0$ and that $v' > 0, v'' \leq 0$. The directly productive input from the ecosystem is denoted by h_t , i.e. harvesting which is extracted at unit cost e . The production function is a standard Cobb-Douglas with $0 < \beta_1 + \beta_2 < 1$, $\beta_j > 0$, $j = 1, 2$. Furthermore, $0 \leq \alpha \leq 1$, The value $\alpha = 1$ indicates full valuation of ecosystem services by a social planner or regulator, while $\alpha < 1$ can be associated with a market economy which partially values these services or does not value them at all for $\alpha = 0$. The current value Hamiltonian for the planner's problem is:

$$\mathcal{H} = \log C_t + \alpha v(S_t) + \lambda_{st} \left(AK_t^{\beta_1} h_t^{\beta_2} - C_t - eh_t - \delta K_t \right) \tag{4}$$

$$\mu_t \left[G(S_t) - h_t - qS_t\right]$$

where the costate variables λ_s, μ have the usual interpretation as the shadow values of capital stock and ecosystem respectively.

Following the maximum principle, and omitting subscript t to ease notation, the optimality conditions for the the controls, C, h, and the modified dynamic Hamiltonian system (MHDS) are:

$$C_s : \frac{1}{C} = \lambda_s \Rightarrow C_s = \frac{1}{\lambda_s} \tag{5}$$

$$h_s : \lambda_s \left(\beta A K^{\beta_1} h^{\beta_2 - 1} - e\right) = \mu \Rightarrow h_s = \left(\frac{e + \frac{\mu}{\lambda_s}}{\beta_2 A K^{\beta_1}}\right)^{\frac{1}{\beta_2 - 1}} \tag{6}$$

$$\dot{\lambda}_s = \lambda_s \left[\rho + \delta - \beta_1 A K^{\beta_1 - 1} h_s^{\beta_2}\right] \tag{7}$$

$$\dot{\mu} = \mu \left[\rho + q - G'(S)\right] - \alpha v'(S) \tag{8}$$

$$\dot{K} = A K^{\beta_1} h_s^{\beta_2} - \frac{1}{\lambda_s} - e h_s - \delta K \tag{9}$$

$$\dot{S} = G(S) - h_s - qS, \tag{10}$$

along with the transversality conditions at infinity

$$\lim_{t \to \infty} e^{-\rho t} \lambda_{st} K_{st} = 0, \ \lim_{t \to \infty} e^{-\rho t} \mu_t S_{st} = 0.$$

Using (5) and (7) the Keynes-Ramsey rule is

$$\frac{\dot{C}_{st}}{C_{st}} = r_t^s - \rho - \delta = \beta_1 A K_{st}^{\beta_1 - 1} h_{st}^{\beta_2} - \rho - \delta \tag{11}$$

and the rate of return on capital, or the social discount rate, along the socially optimal (SO) path is

$$r_t^s = \rho + \delta + \frac{\dot{C}_{st}}{C_{st}} \tag{12}$$

The steady state $\left(K_s^*, S_s^*, \lambda_s^* = \frac{1}{C_s^*}, \mu_s^*\right)$ can be obtained by solving directly the modified Hamiltonian system for $\dot{K} = \dot{S} = \dot{\lambda}_s = \dot{\mu} = 0$. To characterize the stability of the steady state we linearize the MHDS around the steady state. The linearized system can be written as

$$\begin{pmatrix} \dot{\tilde{K}} \\ \dot{\tilde{S}} \\ \dot{\tilde{\lambda}}_s \\ \dot{\tilde{\mu}} \end{pmatrix} = \mathbf{J} \begin{pmatrix} \tilde{K} \\ \tilde{S} \\ \tilde{\lambda}_s \\ \tilde{\mu} \end{pmatrix}$$

where $\tilde{()}$ denotes deviations from the corresponding steady state, and

$$\mathbf{J} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}$$

where $\mathbf{A} = \partial(\dot{K}, \dot{S})/\partial(K, S)$, $\mathbf{B} = \partial(\dot{K}, \dot{S})/\partial(\lambda, \mu)$, $\mathbf{C} = \partial(\dot{\lambda}, \dot{\mu})/\partial(K, S)$, $\mathbf{D} = \partial(\dot{\lambda}, \dot{\mu})/\partial(\lambda, \mu)$.

Proposition 1. *Assume that $v(S)$ is concave and increasing in S; $G(S)$ is concave, initially increasing then decreasing; $G(0) = 0$, and there is S^+ such that $S^+ > 0$ and $G(S^+) = 0$; all functions involved in the Hamiltonian*

(4) are sufficiently smooth Then an optimal steady state (OSS) S^* at the declining part of the inverted U shaped $G(S^*)$ has the local saddle point property for small positive utility discount rate $\rho > 0$.

For the proof see Appendix.

The saddle point property implies that then given initial values for the state variables (K_0, S_0) and steady state values for state and costates, solution of the two-point boundary value problem will determine the SO paths $(K^*_{st}, S^*_{st}, C^*_{st}, h^*_{st}, \lambda^*_{st}, \mu_t)$ along the two dimensional stable manifold.

The socially optimal paths can be determined in the neighborhood of the steady state by the solution of the two-point boundary value problem for the linearized MHDS with initial values for the state variables and terminal values for states and costates. For the for full nonlinear MHDS numerical shooting methods can be used.

Notice that the costate variable for the biosphere at the socially optimal steady state

$$\mu^*_s = \frac{\alpha v'(S^*_s)}{[\rho + q - G'(S^*_s)]},$$

is the present value of the stream of the steady state biodiversity services discounted at the rate $\rho + q - G'(S^*)$, while the ratio

$$P^*_s = \frac{\mu^*_s}{\lambda^*_s} = \frac{\alpha v'(S^*_s) C^*_s}{[\rho + q - G'(S^*_s)]}$$

expresses the same value in consumption terms, since λ is the marginal utility of consumption from (5).

3 Privately optimal ecosystem management

We consider now the case of an unregulated market in which a representative individual with appropriate property rights does not take into account biosphere services, that is $\alpha = 0$ and considers only harvesting costs e. The individual, using as utility function $U(C_t) = \log C_t$, solves the problem

$$\max_{C_t, h_t} \int_0^\infty e^{-\rho t} [\log C_t] \, dt$$

$$\dot{K}_t = A K_t^{\beta_1} h_t^{\beta_2} - C_t - e h_t - \delta K_t, K_0 \text{ fixed}$$

with optimality conditions

$$C_m : \frac{1}{C} = \lambda \Rightarrow C_m = \frac{1}{\lambda_m} \tag{13}$$

$$h_m : \lambda_m \left(\beta_2 A K^{\beta_1} h^{\beta_2 - 1} - e\right) = 0 \Rightarrow h_m = K \left(\frac{e}{A\beta_2 K^{\beta_1}}\right)^{\frac{1}{\beta_2 - 1}} \tag{14}$$

$$\dot{\lambda}_m = \lambda_m \left[\rho + \delta - \beta_1 A K^{\beta_1 - 1} h_m^{\beta_2}\right] \tag{15}$$

$$\dot{K} = A K^{\beta_1} h_m^{\beta_2} - \frac{1}{\lambda_m} - e h_m - \delta K \tag{16}$$

The privately optimal (PO) steady state (K_m^*, λ_m^*) can be obtained by solving (15),(16) for K, λ_m with $\left(\dot{K} = 0, \dot{\lambda}_m = 0\right)$.

From (6), (14) it follows that a comparison between socially optimal and the privately optimal harvesting paths implies that $h_{mt}^* > h_{st}^*$ as long as $K_{mt}^* \geq K_{st}^*$. Both solutions have the same initial stock of capital, therefore initially market harvesting exceeds social harvesting. Since the benefits from biosphere services are not taken into account it follows that $h_{mt}^* \neq h_{st}^*$, and the privately optimal paths $(K_{mt}^*, C_{mt}^*, h_{mt}^*, \lambda_{mt}^*)$ will be different than the socially optimal paths.

The deviations in harvesting between the social optimum and the market solution are defined as:

$$\Delta h_t = h_{mt}^* - h_{st}^*. \tag{17}$$

4 The optimal harvesting tax

The attainment of the social optimum requires regulation. To implement the socially optimal solution the social planner can introduce a harvesting tax, τ combined with lump sum transfer, Tr. In this case the individual solves

$$\max_{C_t, h_t} \int_0^\infty e^{-\rho t} \left[\log C_t\right] dt$$

$$\dot{K}_t = A K_t^{\beta_1} h_t^{\beta_2} - C_t - e h_t - \tau_t h_t + Tr_t - \delta K_t, K_0 \text{ fixed}$$

The optimality condition for regulating harvesting becomes

$$\beta_2 A K_t^{\beta_1} h_t^{\beta_2 - 1} - e = \tau_t.$$

while the optimality condition for the SO from (6) is

$$\beta_2 A K_t^{\beta_1} h_t^{\beta_2 - 1} - e = \frac{\mu_t}{\lambda_{st}}.$$

Combining the two optimality conditions imply that optimal harvesting tax is:

$$\tau_t^* = \frac{\lambda_{st}^*}{\mu_t^*} \tag{18}$$

Proposition 2. *The optimal harvesting tax will implement the socially optimal paths* $(K_{st}^*, S_{st}^*, C_{st}^*, h_{st}^*, \lambda_{st}^*)$.

Proof. Regulated harvesting under the optimal tax becomes

$$h_t^R = \left(\frac{e + \frac{\mu_t^*}{\lambda_{st}^*}}{\beta_2 A K^{\beta_1}} \right)^{\frac{1}{\beta_2 - 1}}. \tag{19}$$

Then (15) at the regulated steady state becomes

$$\rho + \delta = (\beta_1 A K^{\beta_1 - 1} \left(\frac{e + \frac{\mu_t^*}{\lambda_{st}^*}}{\beta_2 A K^{\beta_1}} \right)^{\frac{\beta_2}{\beta_2 - 1}},$$

which implies, by comparing with (7) that the steady state capital stock at the regulated private optimum will be the same as the social optimum, or $K_s^* = K_R^*$. The government's balanced budged implies $\tau_t^* h_t^R = Tr_t$, therefore the capital accumulation equation under regulation becomes

$$\dot{K}_{Rt} = A K_{Rt}^{\beta_1} \left(h_t^R \right)^{\beta_2} - \frac{1}{\lambda_{Rt}} - e \left(h_t^R \right) - \delta,$$

which implies that since $K_s^* = K_R^*$, at the steady state $\lambda_R^* = \lambda_s^*$. Therefore the optimal tax reproduces the socially optimal steady state for the capital stock and the costate variable which is the marginal utility of consumption. The Hamiltonian system for the regulated privately optimal problem becomes

$$\dot{\lambda}_{Rt} = \lambda_{Rt} \left[\rho + \delta - \beta_1 A K_{Rt}^{\beta_1 - 1} \left(h_t^R \right)^{\beta_2} \right] \tag{20}$$

$$\dot{K}_{Rt} = A K_{Rt}^{1-\beta} \left(h_t^R \right)^{\beta} - \frac{1}{\lambda_{Rt}} - e - \delta K_{Rt}, \tag{21}$$

and the terminal conditions for this system are the socially optimal steady state values. (6),(7) and (9) imply that the socially optimal solution for (K, λ) is also a solution for the regulated private ecosystem management. Since the social and the private optimization problems are strictly concave they have unique solutions. Hence if the socially optimal solution is a solution for the regulated problem it is the only solution. Therefore

$$K_{st}^* = K_{Rt}^*, h_{st}^* = h_t^{R*}, C_{st}^* = C_{Rt}^*.$$

Substitution of the path h_t^{R*} into (10) will reproduce the socially optimal path S_{st}^*, since the ecosystem's initial condition S_0 for the socially optimal problem and the regulated problem are the same. ■

4.1 Numerical illustration of optimal regulation

To provide a clear picture of optimal regulation we consider a numerical example based on the following parametrization:

$$v(S_t) = S_t^g, g = 0.8$$

$$G(S_t) = r_0 S_t \left(1 - \frac{S_t}{Z}\right) - qS_t, r_0 = 0.05, Z = 100, q = 0.01$$

$$\beta_1 = 0.7\beta_2 = 0.15, e = 0.1, \delta = 0.03, \alpha = 1, \rho = 0.02, A = 0.5$$

Socially optimal steady state The steady state of the social planner's problem is given by:

$$K_s^* = 123.7, S^* = 58.8, \lambda_s^* = 0.195, \mu^* = 7.259, h_s^* = 0.03555.$$

Note that the steady state $S^* = 58.8$ is on the declining part of the $G(S)$ curve that has a maximum at $S = 50$.This implies consistency with Proposition 1. This implies a steady-state optimal tax $\tau^* = \frac{\mu^*}{\lambda_s^*} = 37.1875$.The steady state exhibits the saddle point property since the eigenvalues of the Jacobian matrix are:

$$\{0.0485, 0.0330, -0.0295, -0.0187\},$$

indicating two positive and two negative eigenvalues. Using the linearized MHDS the socially optimal paths for K_s, S, and the associated paths for λ_s, μ converging to the steady state along the stable linear manifold with is tangent to the nonlinear stable manifold of the nonlinear problem at the steady state are shown in Figures 1a and 1b. The paths for initial values $K_0 = 100, S_0 = 40$ are:

$$K_{st}^* = 123.7 + 41.51e^{-0.0295t} - 65.2e^{-0.0187t}$$

$$S_t^* = 58.8 - 18.42^{e-0.0295t} - 0.369e^{-0.0187t}$$

$$\lambda_{st}^* = 0.195 - 0.051e^{-0.0295t} + 0.091e^{-0.0187t}$$

$$\mu_t^* = 7.259 + 1.992e^{-0.0295t} + 0.046e^{-0.0187t}$$

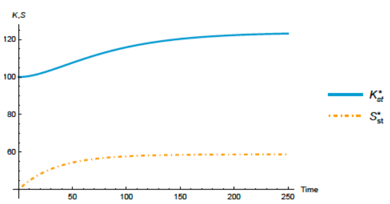

Figure 1a: Converging paths for K_s, S

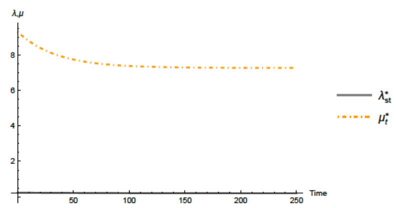

Figure 1b: Converging paths for λ_s, μ

The socially optimal harvesting path is shown in figure 2.

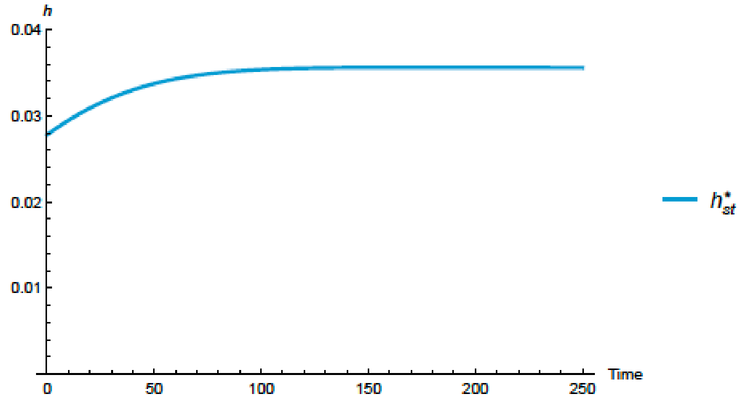

Figure 2: The socially optimal harvesting path

Unregulated private equilibrium The unregulated privately optimal solution yields

$$K^*_m = 46126.5, \lambda^*_m = 21,0841, h^*_m = 4912.12$$

This outcome reflects overexploitation of the resource. Given the specified resource dynamics, such a trajectory is not sustainable. In fact, both the resource stock and output would collapse before the system could reach this steady state. Hence, under this parametrization, the privately optimal solution is not feasible.

Regulated equilibrium The time dependent optimal tax is

$$\tau^*_t = \frac{\mu^*_t}{\lambda^*_{st}}$$

with its path shown in Figure 3.

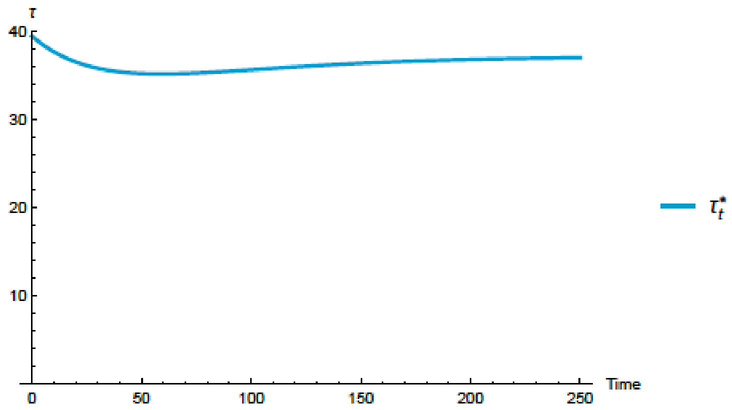

Figure 3: The socially optimal tax

To provide a clear interpretation we consider imposing the optimal steady-state tax

$$\tau^* = \frac{\mu^*}{\lambda_s^*} = 37.1875,$$

the regulated steady state becomes:

$$K_R^* = 123, 7, \lambda_R^* = 0.213, h_R^* = 0.03555,$$

This shows that regulation successfully replicates the socially optimal steady state in terms of capital accumulation and harvesting. Therefore, the regulated equilibrium also supports the socially optimal ecosystem state.The regulated system also exhibits saddle-point stability, with eigenvalues of the corresponding Jacobian.

$$\{0.0293, -0.0093\}.$$

The optimal steady-state tax provides a simple, time-invariant policy instrument that can implement the socially optimal steady state. It should be noted that the steady-state tax alone does not replicate the full optimal transition path. To fully decentralize the optimal allocation over time, the associated two-point boundary value problem for the regulated system should be solved under the time dependent optimal tax with path shown in figure 3. Nevertheless, the steady-state tax remains a practical and easily implementable policy tool when regulation through fiscal instruments is feasible.

5 The use of financial instruments

We consider the case in which the planner managing the ecosystem, instead of using taxes seeks to implement the social optimum that is harvesting h^*_{st} and flow of ecosystem services $v(S^*_{st})$ using financial instruments in the form of bonds.

We consider three cases distinguished by the holders of the bond, the source of coupon payments, and by the degree to which the bond's financial performance is linked to conservation outcomes.

5.1 Aid-funded Green bonds

The domestic social planner, by using an environmental valuation process associates a monetary value $v(S^*_{st})$ to ecosystem services which are not valued by private markets ($\alpha = 0$). Rather than using a harvesting tax to align market harvesting to the socially optimal level, the planner issues a green bond to international investors and pays them a fixed coupon $\bar{c}$. The coupon reflects a combination of investor preferences for environmental impact and donor-funded transfers that align private returns with the global social value of ecosystem services. The flow of ecosystem services at the social optimum is $v(S^*_{st})$. Thus optimal coupon payment should be the flow of ecosystem services.

$$c_t = v(S^*_{st}).$$

A private investor with $\alpha = 0$, will price the bond as

$$P_0 = \int_0^\infty e^{-\rho t} \left(\frac{\lambda_{mt}}{\lambda_{m0}}\right) c_t dt = \int_0^\infty e^{-\rho t} \left(\frac{C_0}{C_{mt}}\right) c_t dt \tag{22}$$

where $e^{-\rho t} \left(\frac{\lambda_{mt}}{\lambda_{m0}}\right)$ is a deterministic discount factor or a deterministic market pricing kernel[1] (e.g. Cochrane, 2009). The different representations of the kernel follow from the optimality conditions (13).

The present value of the bond with dividend flow $v(S^*_{st})$ is defined as

$$P_0 = \int_0^\infty e^{-\int_0^t r_u du} v(S^*_{st}) dt \tag{23}$$

[1] The analysis is conducted in a deterministic setting to isolate the core mechanism linking ecosystem service valuation, asset pricing, and conservation incentives. Introducing uncertainty would affect risk premia and bond pricing but would not alter the central equivalence result.

where r is the market rate of interest. Note that the social value of the bond at the social discount rate of the domestic country will be

$$P_0^s = \int_0^\infty \exp\left[-\int_0^t \left(\rho + \delta + \frac{\dot{C_{su}}}{C_{su}}\right) du\right] v(S_{st}^*)dt$$

The no-arbitrage condition requires the total instantaneous return on any asset to equal the market return r_t,or

$$\frac{\dot{P}_t}{P_t} + \frac{c_t}{P_t} = r_t, \tag{24}$$

which implies that the price of the green bond is determined by

$$\dot{P}_t = r_t P - v(S_{st}^*)\text{ , } P_0 \text{ given by (23).}$$

As mentioned earlier the sovereign bond pays a fixed coupon set at issuance. In the case of the green bond discussed here the annuitized fixed coupon $\bar{c}$ that has the same present value as the variable stream $v(S_t^{s*})$ is

$$\bar{c} = \frac{P_0}{A}$$

where A is the annuity factor $A = \int_0^\infty M_t dt$, $M_t = e^{-\int_{u=0}^t r_u du}$.

To provide a simple quantitative illustration, consider estimates of ecosystem service values reported in the literature (see, IPBES 2018, p. 180). For North America, Mesoamerica, the Caribbean, and South America, these values are approximately 4,056; 4,754; 7,081; and 7,872 US$/ha/year, respectively. Interpreting these as constant flows, the implied present value of one hectare of ecosystem services in perpetuity, discounted at a rate of 3%, is approximately 135,200; 158,467; 236,033; and 262,400 US$, respectively.

Bond proceeds P_0 can be used to set up a conservation fund which could finance general conservation investment and payment for ecosystem services (PES) payments to domestic agents to reduce their harvesting from the market level h_{mt} to the socially optimal level h_{st}^*. The individual harvesters of the ecosystem are asked to adjust their harvesting by Δh_t and receive, after verification, a compensation, i.e. PES. the compensation should the social value og the reduction in harvesting in consumption terms, or :

$$PES_t = \frac{\mu_t^*}{\lambda_{st}^*}\Delta h_t = \tau_t^* \Delta h_t.$$

This compensation will secure the socially optimal harvesting since the individual harvester optimal choice under the bond scheme will be

$$h^B = \arg\max_h \left[AK_t^{\beta_1} h_t^{\beta_2} - e h_t + \tau_t^*(h_{mt} - h_t)\right]$$

which results in harvesting h_{st}^*,since the optimality conditions implies

$$\beta_2 A K_t^{\beta_1} h_t^{\beta_2 - 1} = e + \tau_t^*$$

Under the bond-financed PES scheme, the individual harvester faces an effective marginal cost of not extracting at the socially optimal level equal to $e + \tau_t^*$, which coincides with the social marginal cost. Therefore, the decentralized choice of harvesting replicates the socially optimal allocation. This establishes the equivalence between a harvesting tax and the aid-funded green bond mechanism in terms of attaining the social optimal solution as shown in proposition 2.

The scheme is not self-financing. The regulator's obligations—including coupon payments, principal repayment, and PES transfers—may exceed the returns generated by the conservation fund. This gap is assumed to be covered by international aid, reflecting the global public good nature of ecosystem services. Thus, donor governments are assumed to provide transfers that ensure the financial viability of the scheme, consistent with the interpretation of ecosystem services as a global public good.

5.2 Bond-based claims on ecosystem services

In alignment with Lucas (1978) "tree model" consider the situation in which the ecosystem is explicitly regarded as an asset that generates services with a positive affect on social welfare. For example if we consider S as a biodiversity index measured in terms of some composite of species richness, Shannon or Simson index, or diversity function, then the value of biodiversity services along the socially optimal path of ecosystem management in consumption terms can be defined as $b_t = \frac{v(S_{st}^*)}{\lambda_{st}^*}$. Consider the case of the representative harvester that holds a claim on the flow b_t which is generated by the asset S. This claim can be represented by a bond with a price P and coupon:

$$D_t = \epsilon b_t + \tau_t^* \left(h_{mt}^* - h_{st}^*\right), \ 0 \leq \epsilon \leq 1.$$

The bond holders receive a compensation for reducing their harvesting to the socially optimal level, $\tau_t^* \left(h_{mt}^* - h_{st}^*\right) m$ and a dividend, ϵb_t, for maintaining the flow of non-market biodiversity services at the socially optimal level. Denoting by Z_t the asset holding of the individual and dropping subscript t to simplify notation, the budget constraint for the individual holding the asset is:

$$C + \frac{dK}{dt} + P\frac{dZ}{dt} + \delta K = AK^{\beta_1}h^{\beta_2} - eh + DZ$$

The Hamiltonian for the optimal control problem with controls, C, $\frac{dK}{dt}$, $\frac{dZ}{dt}$

, and utility function $U(C)$ is

$$\mathcal{H} = U(C) + \gamma \left(AK^{\beta_1} h^{\beta_2} - eh + DZ - C - \frac{dK}{dt} - P\frac{dZ}{dt} - \delta K \right) + \mu_K \frac{dK}{dt} + \mu_Z \frac{dZ}{dt}$$

Optimality conditions for the controls imply:

$$C : U'(C) = \gamma \tag{25}$$

$$\frac{dK}{dt} : \gamma = \mu_K \tag{26}$$

$$\frac{dZ}{dt} : P\gamma = \mu_Z = PU'(C) \tag{27}$$

The equations for the costate variables are

$$\dot{\mu}_K = \rho\mu_K - \gamma \left[\beta_1 AK^{\beta_1 - 1} h^{\beta_2} - \delta \right] \tag{28}$$

$$\dot{\mu}_Z = \rho\mu_Z - DU'(C) \tag{29}$$

Taking the time derivative of (28) and substituting in (29) it can be seen that the solution of the problem satisfies the no-arbitrage condition

$$\frac{dP/dt}{P} + \frac{D}{P} = \rho - \frac{dU'(C)/dt}{U'(C)} = r$$

Using $U(C) = \log C$ the no-arbitrage condition becomes

$$\frac{\dot{P}}{P} + \frac{D}{P} = \rho + \frac{\dot{C}}{C} = r. \tag{30}$$

The harvesters hold the bond as an asset. They pay P at $t = 0$, they receive the flow of dividend D_t and they are liable to pay $\tau_t^*(h_{st}^* - h_t)$ is a coupon penalty in case the harvester receives the coupon but does not follow the socially optimal harvesting rule. Under the coupon payment harvesting is chosen such that

$$h = \arg\max_h \left\{ AK_t^{\beta_1} h_t^{\beta_2} - eh_t + \left[D_t + \tau_t^*(h_{st}^* - h_t) \right] Z \right\}.$$

Since $Z = 1$ in equilibrium the optimality condition for the choice of h is

$$\beta_2 AK_t^{\beta_1} h_t^{\beta_2 - 1} - e = \tau_t^*$$

which will provide the socially optimal harvesting.

The financial resources for buying the bond could come from international aid in case of small budget constrained users of the ecosystem or from the users themselves in the case of big industrial users. Since the bond satisfies the no-arbitrage condition (30) there are no disincentives for buying the bond. As in the case of aid-funded bonds, the proceeds of this bond can be used to set up a conservation fund which could finance general conservation investment and coupon payments to domestic agents. If there is a financing gap foreign aid could potentially cover it.

5.3 Sustainability linked bonds

Sustainability-linked bonds (SLBs) are financial instruments whose returns are explicitly tied to the issuer's environmental performance. In particular, SLBs condition the cost of financing on the achievement of pre-defined sustainability targets. In the context of this model, SLBs are linked to the state of the ecosystem and the bond coupon depends on the evolution of the ecosystem which in our model is represented by S_t. The desired state could be either the socially optimal path or, in applied settings (e.g. the Uruguay SLB), a target environmental indicator.

We consider a decentralized economy in which the government acting as a domestic social planner issues a sovereign SLB that is purchased by international investors. Coupon payments to investors are financed through domestic lump-sum taxation. As a result, the bond affects households indirectly via taxes that depend on environmental performance. The proceeds from bond issuance can be allocated between the general budget, conservation expenditures or payments for ecosystem services (PES) with the objective of inducing socially optimal harvesting.

The mechanism operates as follows. Harvesting above the target level reduces the ecosystem stock, increases coupon payments to international investors, and therefore raises future taxes. This corresponds to the coupon step-up. Conversely, harvesting below the target—i.e., overachievement—leads to lower coupon payments and thus lower taxes, corresponding to the coupon step-down. A key assumption underlying the step-down feature is that international investors are willing to accept lower returns in exchange for improved environmental outcomes, particularly when these affect global commons. A real-world example is the Uruguay sovereign SLB, which will be further discussed below.

To formalize this mechanism, we assume that the government issues sovereign SLB with face value, B, at time $t = 0$ and pays a state-contingent coupon of the form, $r(S_t)B$, to international investors. Interest payments are financed through lump-sum taxes, $\tau_t = r(S_t)B$. The step-up/step-down structure of the coupon can be defined as:

$$r(S_t) = r^m + \xi_U \phi(S^*_{st} - S_t) - \xi_D \phi(S_t - S^*_{st}) \tag{31}$$

where, S, is the observed state of the ecosystem, S^*_{st}, denotes the socially optimal (or target) level of the ecosystem, and, $\phi(\cdot)$, is an increasing and differentiable function.

The parameters, $\xi_U > 0$, and, $\xi_D > 0$, govern the strength of the step-up and step-down adjustments, respectively. In particular, ξ_U, captures the penalty applied when environmental performance falls short of the target, $S_t < S^*_{st}$, while, ξ_D, captures the reward associated with overachievement, $S_t > S^*_{st}$.

Because the coupon of the sustainability-linked bond depends on future environmental outcomes, its price at issuance is determined by the expected present value of state-contingent payments. As a result, the bond might not be issued at par, i.e. $P_0 \neq B$, with price defined as

$$P_0 = \mathbb{E}_0 \int_0^\infty M_t^F r(S_t) B dt, \tag{32}$$

where M_t^F is the stochastic discount factor of the foreign investors. Thus the government receives P_0 and and pays coupons , $r(S_t)B$ where $r(S_t)$ reflects the coupon structure. Coupon payments are financed by taxation and the government's balanced budget implies

$$\tau_t^B = r(S_t)B. \tag{33}$$

The household pays the tax τ_t^B which depends on the state of ecosystem which in turn depends on private harvesting. Thus the household should take into account ecosystem dynamics, and after using (33) the corresponding current value Hamiltonian for the household is written as

$$\mathcal{H} = \log C_t + \lambda_t^B \left[AK_t^{\beta_1} h_t^{\beta_2} - C_t - eh_t - \delta K_t - r(S_t)B \right]$$
$$+ \mu_t^B \left[G(S_t) - h_t - qS_t \right]$$

The optimality conditions for the controls and the costate variables imply

$$C : \frac{1}{C_t} = \lambda_t^B$$
$$h : \beta_2 AK_t^{\beta_1} h_t^{\beta_2 - 1} - e = \frac{\mu_t^B}{\lambda_t^B}$$
$$\dot{\lambda}_t^B = \lambda_t^B \left[\rho + \delta - \beta_1 AK_t^{\beta_1 - 1} h_t^{\beta_2} \right]$$
$$\dot{\mu}_t^B = \mu_t^B \left[\rho + q - G'(S) \right] + \lambda_t^B r'(S_t) B \tag{34}$$

The condition for the costate variable μ, which is the shadow value for the ecosystem, at the social optimum is

$$\dot{\mu}_{st} = \mu_t \left[\rho + q - G'(S) \right] - \alpha v'(S_t) \tag{35}$$

Comparing (34) with (35) it can be seen that if the structure of the coupon and the face value of the bond are chosen such that

$$-\lambda_t^B r'(S_t)B = \alpha v'(S_{st}^*), or \; - r'(S_t)B = C_{st}^* \alpha v'(S_{st}^*) \tag{36}$$

then the socially optimal management is achieved. The condition means that the marginal borrowing costs equals the marginal social value of ecosystem services in consumption terms, However the implementation of this rule might need further fine tuning. For the coupon structure (31) the left hand side of (36) with $\phi'(\cdot) = 1$, is $(\xi_U + \xi_D) B$ which is constant . This means that the full implementation might not be possible. Partial implementation can be obtained however in alternative ways:

1. Implementation with respect to a target such as the social optimal steady state: $B = \frac{C_s^* \alpha v'(S_s^*)}{(\xi_U + \xi_D)}$. This will generate a deviation between the socially optimal harvesting path and the regulated path before the system reaches the steady state. This deviation is given by $\theta_t = h_{st}^* \left(C_{st}^* \alpha v'(S_{st}^*)\right) - h_t^R \left(\frac{C_s^* \alpha v'(S_s^*)}{(\xi_U + \xi_D)}\right)$.
2. Implement with respect to a target and use the proceeds of the bond in the form of PES to correct for any harvesting deviations. This can be in the form of a PES compensation $\frac{\mu_t^*}{\lambda_{st}^*} \theta_t$.

The structure of the financial instrument analyzed in this section is closely related to recently issued sovereign sustainability-linked bonds, such as Uruguay's 2022 bond, which ties coupon payments to environmental performance indicators including greenhouse gas emissions and native forest preservation. The bond was issued at a size of approximately USD 1.5 billion, with a maturity of 12 years (due 2034), and includes a step-up/step-down mechanism of about 15 basis points depending on the achievement of pre-specified environmental targets. As a standard sovereign instrument, bond proceeds are allocated to the general government budget, and coupon payments are serviced through general fiscal revenues, i.e. current or future taxation. While such instruments are designed as outcome-contingent contracts with discrete coupon adjustments, the framework developed here provides a continuous-time representation in which payments are directly linked to the flow value of ecosystem services and shows that under a specific design it can produce equivalent result with direct Pigouvian taxation.

6 Regulating Climate change

6.1 Mitigation-Adaptation

The Paris Agreement on climate change underscores the importance of addressing both the causes and the consequences of climate change. The former are tackled through substantial reductions in greenhouse gas emissions—commonly referred to as mitigation—while the latter are managed through efforts to strengthen climate resilience, known as adaptation. Accordingly, climate policy has traditionally been structured around these two central pillars. paper.[2] However, even under an optimally designed mitigation policy, with carbon pricing based on the social cost of carbon, climate change damages cannot be fully avoided. According to the Intergovernmental Panel on Climate Change, limiting global warming to 1.5°C significantly reduces risks compared to higher temperature scenarios, yet does not eliminate them. The IPCC's Special Report on 1.5°C makes clear that "residual damages" will persist across ecosystems, human health, and economic systems even under stringent mitigation pathways (IPCC, 2018).

The economic rationale for optimal carbon taxation is well established in the literature (e.g. Golosov et al. 2014). Mitigation reduces future damages by lowering temperature increases, but does not eliminate the effects of past emissions or the inertia embedded in the climate system due to irreversibility of climate change (Solomom 2009). As a result, a certain level of climate change and associated damages is effectively locked in . Hsiang et al. (2017) provide estimates of climate damages across sectors in the United States, highlighting substantial projected losses even under moderate warming scenarios. These impacts are already observable at current warming levels, reinforcing the conclusion that adaptation is required not only in the future but also in the present.

Contributions by Fankhauser (1995, 2017) and Tol (1995) emphasized that adaptation can substantially reduce net damages. This result has been derived by integrated assessment models (e.g. de Bruin et al. 2009). Similarly, Bosello, et al. 2010, demonstrate that adaptation reduces the marginal damage of climate change, thereby altering optimal carbon pricing trajectories. Agrawala, et al. (2011), , show that adaptation can offset a substantial

[2] The consideration of emerging climate policy approaches, such as solar radiation management, lies beyond the scope of the present

share of damages in the near to medium term, when mitigation effects are still unfolding.

Adaptation needs are highly heterogeneous across regions and sectors. Developing countries, which are often more exposed to climate risks and have lower adaptive capacity, face disproportionately higher damages (Stern, 2007; IPCC, 2022). This spatial heterogeneity implies that even globally optimal carbon pricing does not address local vulnerabilities. Adaptation is therefore essential to reduce inequities in climate impacts.

Adaptation activities can be separated into private and public. Private adaptation refers to actions taken by individuals and firms to manage the impacts of climate change in their own interest. Although private adaptation can yield significant benefits, there are valid concerns that individuals and firms may not respond effectively to climate risks. Several factors can limit the effectiveness of private adaptation. These include high transaction costs, limited access to reliable information about climate risks, and the public good nature of some adaptation measures, which may reduce individual incentives to act.

The limitations of private adaptation create a rationale for public intervention to address market and institutional failures. Many adaptation benefits have the characteristics of public goods, meaning they are non-excludable and shared by multiple individuals or firms. Examples include coastal protection, water resource management, conservation efforts, and climate information services (Collier et al., 2008). Physical infrastructure such as sea walls, levees and water supply systems, and climate-resilient transport networks also provide widespread benefits (Ranger et al., 2013; Dietz et al., 2016). Public adaptation can also support health systems through disease control and emergency response

Public adaptation to climate change can be financed through sovereign and sub-sovereign debt instruments, particularly within the expanding market for green and climate-related bonds. Beyond conventional green bonds, instruments such as climate resilience bonds and adaptation bonds have been proposed to target investments that reduce climate vulnerability. Resilience bonds are specifically designed to finance investments that reduce exposure to climate hazards such as flood protection. These bonds can generate financial benefits, including avoided costs from climate damages and reduced insurance costs, enabling partial monetization of avoided damages.

A notable example is Fiji's 2017 sovereign green bond, issued with support from the International Finance Corporation (World Bank Group 2017). As the first sovereign green bond by a small island developing state, it financed both mitigation and adaptation projects, including climate-resilient infrastructure and water management. It is widely regarded as a landmark case for accessing capital markets for adaptation finance.

6.2 Modeling mitigation-adaptation to climate change

The model developed in section 2 can be, with appropriate modifications, can be used to study mitigation-adaptation to climate change. The planner's utility is defined as

$$U(C_t, T_t) = \log C_t - D(T_t,\alpha_t),$$

where $D(T_t, \alpha_t)$ represents damages from climate change. These damages depend on T_t which is the global mean temperature anomaly relative to the preindustrial period, with $T_0 = 0$, that is $T_t = T_{current} - T_{preindustrial}$, and adaptation activities α_t that reduce damages form climate change at a given anomaly level. It is assumed, with subscripts denoting partial derivatives, that $D_T > 0, D_{TT} \geq 0, D_\alpha < 0, D_{T\alpha} < 0$. The sign of the cross partial derivative indicates the adaptation lowers the sensitivity of damages to the temperature anomaly. [3] $D(T_t, 0)$ are damages without adaptation and $D(T_t, \alpha_t)$ are damages after adaptation or residual damages. In our model climate change generates damages that enter directly into the utility function (e.g. Weitzman ; Barnett et al. 2020). In this context adaptation is public.

Output is produced from the Cobb-Douglas production function $AK_t^{\beta_1} h_t^{\beta_2}$,with h representing now energy input or emissions after an appropriate choice of units. For the evolution of the temperature anomaly instead of the multi-component carbon cycle of DICE (e.g. Barrage and Nordhaus 2024) we adopt the relative simpler approach of Mathews at al. (2009) in which the temperature anomaly at given point in time is approximately proportional to cumulative GHG emissions up this point. The constant of proportionality between change in temperature and cumulative emissions is called the tran-

[3] Adaptation can also be introduced as a stock. We choose the flow representation in order to not to increase the number of state variable. This simplification does not change the main results.

sient carbon response to cumulative CO_2 emissions (TCRE)[4] The evolution of the anomaly can be written as

$$\dot{T}_t = \Lambda h - mT_t \, , T_0 = 0 \tag{37}$$

where Λ is the TCRE parameter and $m > 0$ is heat dissipation parameter.

Assuming that adaptation costs are given by an increasing convex function $\phi(\alpha_t)$,$\phi' > 0, \phi'' \geq 0$,the planner's budget constraint is given by:

$$\dot{K}_t = AK_t^{\beta_1} h_t^{\beta_2} - eh_t - C_t - \phi(\alpha_t) - \delta K_t. \tag{38}$$

where e represent energy/emission costs. In this set up the planner's problem is:

$$\max_{C_t, h_t} \int_0^\infty e^{-\rho t} \left[\log C_t + D(T_t, \alpha_t)\right] dt$$

subject to (38) and (37).

In our model climate change generates damages that enter directly into the utility function. This specification is motivated by the observation that many climate impacts which are more relevant in the discussion of adaptation activities—such as excess mortality, increased morbidity, heat stress, and other non-market damages—affect welfare directly rather than solely through reductions in aggregate output or total factor productivity (see, for example, Barrage and Nordhaus, 2024, and earlier work by Nordhaus and co-authors). In such models, higher temperature anomalies reduce effective output, and welfare losses arise indirectly through lower consumption. In this case the production function is specified as $(1 - \Psi(T_t))AK_t^{\beta_1} h_t^{\beta_2}$, where $\Psi(T_t)$ is a damage function that affects aggregate output. The Utility-based representations of climate damages in this paper this provide a more direct accounting of welfare losses. Allowing for both damage channels would not alter our main results regarding adaptation policies. We focus on the direct impact on the utility function to simplify the analysis and to present the implications for resilience bonds more transparently.

The planner's current value Hamiltonian is written as

$$\mathcal{H} = \log C_t - D(T_t, \alpha_t) + \lambda \left[AK_t^{\beta_1} h_t^{\beta_2} - eh_t - C_t - \phi(\alpha_t) - \delta K_t\right] + \mu \left(\Lambda h_t - mT_t\right) \tag{39}$$

Optimal conditions for controls (C, h, α) imply:

[4]See Brock and Xepapadeas (2017) for an early application of this approach to a coupled model of climate and economy).

$$h : \lambda \left[A\beta_2 K^{\beta_1} h^{\beta_2 - 1} - e \right] + \mu\Lambda = 0 \quad (40)$$

$$\alpha : -D_\alpha(T, \alpha) - \lambda\phi'(\alpha) = 0 \quad (41)$$

$$C : \frac{1}{C} = \lambda \quad (42)$$

with costate equations

$$\dot{\lambda}_t = \lambda_t \left(\rho + \delta - \beta_1 A K_t^{\beta_1 - 1} h_t^{\beta_2} \right) \quad (43)$$

$$\dot{\mu}_t = (\rho + m)\mu_t + D_T(T_t, \alpha_t) \quad (44)$$

The problem of the "climate planner" has the same structure at the 'ecosystem planner" problem of section 2, with the additional simplification of linear climate dynamics. The climate problem can be therefore solved in the same way as problem (1)-(3).

In a regulated market solution emissions are taxed by a carbon tax τ, but adaption is not taken account since it is public adaptation. The relevant Hamiltonian is

$$\mathcal{H} = \log C_t + \lambda_{mt} \left[A K_t^{\beta_1} h_t^{\beta_2} - (e + \tau) h_t - C_t - \delta K_t \right] \quad (45)$$

Optimal controls

$$h : \lambda \left[\beta_2 A K^{\beta_1} h^{\beta_2 - 1} - e - \tau \right] = 0 \quad (46)$$

$$C : \frac{1}{C} = \lambda^m \quad (47)$$

$$\alpha : \alpha = 0 \quad (48)$$

Combining (40) and (46) it can be easily seen that the optimal carbon tax is

$$\tau_t^* = \frac{-\mu_t \Lambda}{\lambda_t}$$

that has a similar structure with the optimal harvesting tax. Let $(K_{st}^*, T_{st}^*, C_{st}^*, h_{st}^*, \alpha_{st}^*, \lambda_{st}^*, \mu_{st}^*)$ be the socially optimal paths obtained in the same way as in the optimal harvesting problem, and let $(K_{mt}^*, T_{mt}^*, C_{mt}^*, h_{mt}^*, 0, \lambda_{mt}^*, \mu_{mt}^*)$ the corresponding paths under the optimal carbon tax.

Solving the costate equation for μ_t forward with transversality condition at infinity $\lim_{t \to \infty} e^{-\rho t} \mu_t T_t = 0$ we obtain

$$\mu_{st}^* = -\int_t^\infty e^{-(\rho + m)(u - t)} D_T(T_u, \alpha_u) du$$

Using (42) the optimal carbon tax in consumption units is defined along the socially optimal paths as.

$$\tau_t^* = \frac{-\mu_{st}^* \Lambda}{\lambda_{st}^*} = \Lambda C_t \int_t^\infty e^{-(\rho + m)(u - t)} D_T(T_{su}^*, \alpha_{su}^*) du$$

When this tax is applied in market equilibrium the socially optimal temperature anomaly is attained, but no adaptation is taking place.

The planner's problem is to finance the adaptation costs $\phi(\alpha^*_{st})$ in order to attain residual damages $D\left(T^*_{st}, \alpha^*_{st}\right)$ instead of damages $D\left(T^*_{mt}, 0\right)$ under the optimal carbon tax.

Since adaptation is a public good one way to solve the problem is to provide the public good using the carbon tax revenues as the source of finance. In this case mitigation and adaptation policies are complementary. The regulator issues a resilience bond to finance adaptation expenditures. The bond's face value equals the market value of future adaptation costs:

$$P_0 = \int_0^\infty M_t \phi(\alpha^*_{st}) dt,$$

where $M_t = e^{-\int_{u=0}^t r_u du}$ is the discount factor and the market interest rate satisfies the Keynes-Ramsey condition:

$$r_t = \rho + \delta + \frac{\dot{C}_t}{C_t}$$

which can be obtained using (42) and (43).

A constant coupon for the resilience bond can be defined as

$$c = \frac{P_0}{A},$$

where A is the annuity factor $A = \int_0^\infty M_t dt$.

Revenue from carbon tax is given by $\tau^*_{st} h^*_{st}$, and adaptation generates monetized avoided damages

$$d_t = D\left(T^*_{mt}, 0\right) - D\left(T^*_{st}, \alpha^*_{st}\right)$$

The self-financing condition requires

$$\int_0^\infty M_t \left(\tau^*_t h^*_{st} + d_t\right) dt \geq \int_0^\infty M_t \phi(\alpha^*_{st}) dt$$

If self financing is not possible international aid may be required to cover the deficit.

7 Concluding remarks

The regulation of global commons has traditionally been analyzed within the framework of Pigouvian taxation, the standard fiscal instrument for addressing environmental externalities. By contrast, financial instruments such as green bonds have primarily been viewed as sources of financing for projects that generate environmental benefits. However, the implementation of fiscal environmental instruments is often constrained by political and institutional limitations. This paper shows that financial debt instruments can be designed in a way that allows them to attain the same environmental targets as Pigouvian taxes, while simultaneously providing financial support for green activities.

Using a growth model in which ecosystem services enter directly as inputs in production and the ecological stock provides direct welfare benefits, we demonstrate how the coupon structure of different types of bonds can be designed to replicate the environmental outcomes achieved under optimal Pigouvian taxation.

Beyond its theoretical contribution—establishing a novel equivalence between fiscal and financial environmental policy instruments—this framework has practical implications for the design of financial mechanisms that can achieve specific conservation and climate adaptation targets in settings where fiscal instruments are infeasible.

An important extension of the analysis would be to introduce uncertainty in ecosystem dynamics, allowing for the characterization of ecosystem-related risk premia. Another extension would be the introduction of the state of the ecosystem in the production function $AK^{\beta_1}h^{\beta_2}S^{\beta_3}, 0 < \beta_1 + \beta_2 + \beta_3 < 1, \beta_j > 0, j = 1, 2, 3$ to reflect the positive externalities from the ecosystem or the biosphere to output production.

Appendix

Proof of Proposition 1.

We use a heuristic argument. Consider the optimal steady state Ramsey problem with $\rho = 0$.

$$\max_{C,h} H = \max_{C,h} \left\{ \log C + \alpha v(S) + \lambda_s \left(AK^{\beta_1} h^{\beta_2} - C - eh - \delta K \right) + \mu \left[G(S) - h - qS \right] \right\}$$

optimality conditions for the costate λ imply:

$$\lambda^* = \frac{1}{C^*} > 0$$

Assume that e is very high and $\beta_2 > 0$ but small, then optimal harvest h^* will be small, so S^* will be close to $\hat{S}$ that solves

$$G\left(\hat{S}\right) - q\hat{S} = 0,$$

S^* will be at the declining part of $G(S)$ and $q - G'(S^*) > 0$. Then the optimality condition for μ implies

$$\mu^* = \frac{\alpha v'(S^*)}{q - G'(S^*)} > 0$$

With positive costates at the steady state, the maximized Hamiltonian is concave in the state variables (K, S) and convex in the costates (λ, μ). Therefore for the linearization around the OSS of the Hamiltonian system for the Ramsey problem with $\rho = 0$, the associated Jacobian matrix evaluated at the OSS there will have two eigenvalues with negative real part and two with positive real parts. [5]Assume now that $\rho > 0$, then $\rho + q - G'(S^*) > 0$ since S^* will be at the declining part of $G(S)$, and

$$\mu^* = \frac{\alpha v'(S^*)}{\rho + q - G'(S^*)} > 0$$

The smoothness assumption imply that the OSS(ρ) will be near the Ramsey problem, OSS($\rho = 0$). Therefore the eigenvalues of the Jacobian matrix $\mathbf{J}$ will come in pairs, two with negative real parts and two with positive real parts for ρ close to zero but positive and the OSS($\rho > 0$) will have the saddle point property.

[5]The case of $\det \mathbf{J} \neq 0$ is excluded.

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