ESTIMATING THE EULER EQUATION FOR AGGREGATE INVESTMENT
WITH ENDOGENOUS CAPITAL DEPRECIATION

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Abstract: This paper looks at the empirical consequences of introducing endogenous capital depreciation in the standard neoclassical model with quadratic adjustment costs. To this end, we formulate an empirical specification that accommodates capital maintenance and utilization in the Euler equations for aggregate investment. The empirical estimates with data from the Canadian survey on Capital and Repair Expenditures show that, in contrast to the existing literature, the performance of the Euler equations is improved when we account for the impact of variable capital depreciation.

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1. Introduction

The consensus about the empirical performance of the standard neoclassical aggregate investment model within the context of the profit-maximizing firm facing quadratic adjustment costs is that it can be hardly considered a success story. The two major kinds of specifications for investment that have been tested in the empirical literature, namely the $q$ model and the Euler equation approach, have soundly failed with aggregate data. Specifically, the Euler equation approach that estimates the first-order condition of the firm, although originally viewed as a promising route, turned out disappointing as the empirical results have indicated that the overidentifying restrictions are strongly rejected and that high adjustment costs are implied by the estimated regressions, which in turn imply extremely slow adjustment of the capital stock (see e.g. Chirinko, 1993, and Whited, 1998). This result is corroborated by simulation evidence provided by Shapiro (1986) on the response of the demand for capital to changes in the price of capital and the required rate of return by investors. Moreover, Euler equations are found to exhibit substantial parameter instability (Oliner et al., 1996).

The purpose of this paper is to extend the Euler equation for investment by highlighting the attractive, yet unresolved, role of endogenous capital depreciation driven by maintenance spending in the determination of aggregate capital expenditures. Our starting point is that firms have two options in raising their productive capacity: by increasing their capital stock through ‘new’ investment or by raising capital services through repair or maintenance of the existing capital stock. By completely disregarding this second option, i.e. assuming exogenous capital depreciation, a large part of the literature has attempted to explain variations in aggregate investment by relying implicitly on adjustment costs with the latter found to be unreasonably high.\footnote{Several studies have investigated theoretically a setup for the relationship between endogenously determined depreciation and the optimal maintenance level, under which the central decision of the firm involves the allocation of expenditures between ‘new’ investment and maintenance, in order to maximise the discounted value of future income flows by affecting the capital accumulation process either directly or via the depreciation rate. See, among others, Schmalensee (1974), Nickell (1978), Schworm (1979) and Parks (1979) for early contributions in this literature. In turn, there are some empirical studies at the sectoral level that have confirmed that capital deterioration is endogenous and, in particular, associated with maintenance expenditure. Nelson and Caputo (1997) provide a brief survey of the empirical findings and the related literature.}

Our study then aims at assessing to what extent the failure of empirical studies of the Euler equation for aggregate investment in providing a plausible assessment of
adjustment costs can be attributed to the unexplored role of endogenous capital depreciation. Kalyvitis (2006) showed that the $q$ model with convex adjustment costs, which accounts for capital maintenance and endogenous depreciation, improved substantially the performance of the $q$ model by producing significant and plausible parameter estimates for factor demand equations. However, the parameterizations presented in Kalyvitis (2006) aim at estimating reduced-form specifications in which $q$ explains both ‘new’ investment and maintenance spending. In contrast, the approach adopted here is based on specifications of the first-order conditions through the Euler equations for the firm’s problem, which allow the identification of structural parameters, marginal adjustment costs and the variable depreciation rate.

The apparent lack of empirical studies with endogenous depreciation driven by capital maintenance is largely due to the unavailability of appropriate aggregate data on maintenance expenditures in most countries. McGrattan and Schmitz (1999) report that evidence from the only source of aggregate long-run data on capital expenditures in newly purchased assets, or ‘new’ investment, and maintenance, namely the Canadian survey on ‘Capital and Repair Expenditures’, indicates that maintenance expenditures are too big for economists to ignore: total business expenditures in ‘new’ investment and maintenance amounted to 14.1% of GDP in Canada with the average maintenance share covering 27% (3.8% of GDP).

Given these stylized facts, the present paper aims at providing a first step to understanding the implications for the empirical implementation of the Euler equation in the standard neoclassical model with quadratic adjustment costs under endogenous capital depreciation affected by the level of spending on capital maintenance. To this end, we develop a theoretical setup for the firm’s decision problem with endogenous depreciation affected by spending on capital maintenance. We then use aggregate data from the Canadian survey on ‘Capital and Repair Expenditures’ to estimate the system of structural Euler equations for ‘new’ investment and maintenance. In particular, we estimate the model using alternatively data for the business sector covering the period 1956-93 and for the manufacturing sector covering the period 1956-2005. Although our results are found to depend on the numerical assumptions for the calibrated parameters, the main finding of the paper is that the empirical performance of the Euler equations with variable capital depreciation rate is improved. Including capital maintenance in the
depreciation function produces estimates for the adjustment costs that are considerable lower than the values estimated in the aggregate investment literature, whereas we also manage to get plausible values for the average depreciation rate. A by-product of our empirical estimates is that the depreciation rate in the Canadian economy has exhibited substantial variation. In particular, our findings imply that, depending on the model used, the depreciation rate has varied over the period examined in a range between 1.7 and 3.4 percentage points in the business sector and between 0.7 and 2.6 percentage points in the manufacturing sector. The main picture persists when the depreciation rate is affected by variable capital utilization, an assumption that has been adopted in aggregate models with endogenous depreciation (see e.g. Burnside and Eichenbaum, 1996, Greenwood et al., 1988).

We stress that our findings on the low estimates for ‘new’ investment adjustment costs are not simply driven by the introduction of an additional friction in the firm’s value function, namely adjustment costs for maintenance, because these costs are found to be low even when their joint impact with investment adjustment costs is accounted for. Notably, we manage to improve the fit of aggregate investment equations by using the standard framework of convex adjustment costs, which has broadly failed in existing macroeconomic studies of aggregate investment behavior, rather than relying on alternative specifications for adjustment costs.\(^2\) Regarding the depreciation rate, our point estimates for the average capital depreciation rate across the estimated models for the business and the manufacturing sectors are found to be in the range of 3%-7%, whereas higher depreciation rates are obtained for machinery-equipment. These estimates are not far from those reported by Jorgenson (1996) and Nadiri and Prucha (1996) for the US.

The paper thus contributes in the investment literature by extending the neoclassical investment model with quadratic adjustment costs to account for the impact of endogenous capital depreciation driven by

\(^2\) For instance, Christiano et al. (2005) have shown that an adjustment cost specification that penalizes changes in the level of investment can generate plausible impulse responses to monetary policy shocks. However, Eberly et al. (2008) report that their results tend to favor models based on capital adjustment costs, which seem to outperform the Christiano et al. (2005) specification in describing investment behavior. Groth (2008) uses a translog cost function approach with convex adjustment costs to estimate the elasticity of investment with respect to \(q\) and reports plausible adjustment costs in the UK manufacturing and service industry.
spending on maintenance, a component of capital outlays that has been shown to be important in terms of size and influence, but has been largely neglected in the formulation of investment behaviour in the level of the macroeconomy. We stress however that our setup cannot necessarily characterize or test dynamics at the firm level. Given the aggregate nature of the data at hand, we aim here at assessing whether a simple model with capital spending in ‘new’ investment and maintenance by firms that face identical adjustment cost functions, can improve the performance of the investment Euler equation and add to our ability to track and understand capital depreciation at the aggregate level. We would be less optimistic about the performance of a similar approach with quadratic adjustment costs if data at the firm level were available. At the firm level additional factors, such as the presence of financial constraints for some firms or firm-years, have been suggested as possible reasons for the inadequate performance of the Euler equation. Nevertheless, financing constraints are unlikely to be responsible for such failures at the aggregate level over a long time span.³

The rest of the paper is structured as follows. Section 2 develops the theoretical model for investment with endogenous capital depreciation and derives the empirical specifications. Section 3 describes the data and the estimation method. Section 4 presents the empirical results and section 5 concludes the paper.

2. Optimal capital spending with endogenous depreciation

2.1. The firm’s problem with capital maintenance

Consider the standard partial equilibrium model for the representative firm, in which all markets are perfectly competitive and the firm takes factor prices, output prices, and interest rates as given. All input prices are normalized by the price of output. The firm maximizes its value, $V(\cdot)$, which is a function of the previous-period capital stock, and can influence the pattern of future capital accumulation by appropriately choosing ‘new’ investment and maintenance expenditures. We assume that these two components of capital expenditures have the same price implying that one unit of ‘new’ investment can be

³ See e.g. Chatelain and Teurlai (2006) for a detailed discussion.
transformed into one unit of maintenance in a costless manner.

The firm’s problem can be summarized as follows:

\[ V_i(K_{t-1}) = \max \{ R(K_i, L_i, I_i, M_i) + \beta E_i[V_{i+1}(K_i)] \} \]  

where \( K_t \) denotes the capital stock, \( L_t \) denotes labor, \( I_t \) and \( M_t \) denote ‘new’ investment and maintenance expenditures respectively, and \( \beta_t \) is the exogenous time-varying discount factor, so that financing decisions are irrelevant to the optimal path. In turn, net revenues, \( R_t \), are given by:

\[ R_t = F(K_t, L_t) - C(K_t, I_t, M_t) - w_i L_t - I_t - M_t \]  

where \( F(K_t, L_t) \) is the production function with the standard neoclassical properties, and \( C(K_t, I_t, M_t) \) denotes adjustment costs driven by spending on ‘new’ investment and maintenance, which will be determined below. In this setup, the firm chooses investment at the beginning of the period when new capital is installed, which becomes immediately operative. The firm also chooses a level of maintenance expenditures for the existing capital stock.

We assume that the law of motion for capital accumulation is given by:

\[ K_t = I_t + \left[1 - \frac{M_t}{K_{t-1}}\right] K_{t-1} \]  

where the depreciation function has the following general properties: \( \delta'(\frac{M_t}{K_{t-1}}) < 0 \), \( \delta''(\frac{M_t}{K_{t-1}}) > 0 \), \( \lim_{M \to 0} \frac{\delta(M_t)}{K_{t-1}} = \delta \), \( \lim_{M \to \infty} \frac{\delta(M_t)}{K_{t-1}} = 0 \). In this setup, \( \delta \) is the rate of depreciation when no maintenance is undertaken, whereas for simplicity we assume that the firm can decrease capital depreciation down to zero.\(^4\) Equation (3) shows that the capital depreciation rate is endogenously determined, since by using maintenance expenditures the firm can reduce the depreciation rate of its capital stock and hence carry

\(^4\) In principle we could allow the depreciation rate to approach a constant as maintenance spending tends to infinity. We abstract from this theoretical consideration as this would add an extra parameter in our estimates without adding further insights in the empirical results.
more units of useable capital to the next period.

The firm’s problem given by equations (1) to (3) reduces the infinite-horizon optimization problem to the equivalent two-period problem. The Lagrangean corresponding to the firm’s problem is given by:

\[ \Lambda_t = R(K_t, L_t, I_t, M_t) + \beta_t E_t V_{t+1}(K_t) + \lambda_t [K_t - \left(1 - \delta(\frac{M_t}{K_{t-1}})\right)K_{t-1} - I_t] \]

Under perfect competition the first-order conditions are given by:

\[ \frac{\partial R_t}{\partial K_t} + \beta_t E_t \frac{\partial V(K_t)}{\partial K_t} + \lambda_t = 0 \]  \hspace{1cm} (4)

\[ \frac{\partial R_t}{\partial I_t} - \lambda_t = 0 \]  \hspace{1cm} (5)

\[ \frac{\partial R_t}{\partial M_t} + \lambda_t \delta'(\frac{M_t}{K_{t-1}}) = 0 \]  \hspace{1cm} (6)

Equations (4) and (5) are the standard optimization conditions, which state that the shadow value of capital, i.e. the additional value for the firm from relaxing the constraint given by (3), is equal to the discounted value of current and future revenues generated by an additional unit of capital and that the shadow price of capital \( \lambda_t \) equals the marginal product of investment, which will exceed unity for positive investment in the presence of convex adjustment costs. Equation (6) then emerges as an extra efficiency condition that equates the marginal reduction in revenues due to a rise in maintenance expenditures to the marginal benefit from the reduction in the depreciation of capital, given by \(-\delta'(M/K)\), evaluated at the shadow price of capital.\(^6\)

2.2. Empirical specification

To obtain an empirical parameterization of the model, we assume an exponential form for the depreciation function given by:

\(^5\) We omit the first-order condition for labor, which does not affect the empirical specifications derived later on.
\(^6\) For similar derivations see also McGrattan and Schmitz (1999) and Boucekkine and Ruiz-Tamarit (2003).
\[ \delta \left( \frac{M_{t}}{K_{t-1}} \right) = \delta \exp[-\gamma \frac{M_{t}}{K_{t-1}}] \]  

(7)

where \( \gamma > 0 \) is a parameter that measures the sensitivity of the depreciation rate with respect to changes in maintenance expenditures.

The first-order condition for capital is now given by:

\[
\begin{align*}
&\left[ 1 - \delta \exp[-\gamma \frac{M_{t}}{K_{t-1}}(1 + \frac{M_{t}}{K_{t-1}})] \right] \left[ \frac{\partial R}{\partial K_t} + \beta E_t \frac{\partial V(K_t)}{\partial K_t} + \lambda_t \right] = 0
\end{align*}
\]

It follows that along the optimal path:

\[
\frac{\partial V(K_{t-1})}{\partial K_{t-1}} = -\lambda_t \left[ 1 - (1 + \gamma \frac{M_{t}}{K_{t-1}})\delta \exp[-\gamma \frac{M_{t}}{K_{t-1}}] \right]
\]

(4’)

To parameterize the Euler equations we assume that the firm faces convex installation costs in both types of capital expenditures, namely ‘new’ investment and maintenance, given by:

\[ C(K_t, I_t, M_t) = \Psi(K_t, I_t) + \Xi(K_t, M_t) \]  

(8)

where \( \Psi(K_t, I_t) \) and \( \Xi(K_t, M_t) \) denote adjustment costs in ‘new’ investment and maintenance respectively, related for instance to installation costs. Although some recent studies have shown that at the plant level adjustment costs are better described by lumpy capital adjustment, the convexity assumption remains reasonable at the aggregate level (Groth, 2008). Therefore, we assume that the cost-of-adjustment functions are homogeneously linear in capital and we follow the standard specification adopted by, among others, Bond and Meghir (1994) and Hubbard (1998), which is given for ‘new’ investment by:

\[ \Psi(K_t, I_t) = \frac{\psi(I_t)}{2} \left( \frac{I_t}{K_t} - \eta_t \right)^2 K_t \]  

(9)

Similarly, the adjustment cost function for maintenance is given by:
\[ \Xi(K, M) = \frac{\xi}{2} \left( \frac{M}{K} - \varepsilon \right)^2 K \tag{10} \]

The terms \( \eta \) and \( \varepsilon \) denote classical and uncorrelated technology shocks in the adjustment cost functions for investment and maintenance, while \( \psi, \xi \) are positive parameters of the adjustment cost functions.\(^7\)

We can then combine (4'), (5) and (6) to derive the Euler equations for investment and maintenance. To simplify notation we henceforth use small caps to denote variables divided by capital. Thus we have

\[ \frac{I}{K} = i, \quad \frac{M}{K} = m, \quad \text{while} \quad \frac{\Pi}{K} = \pi \] is the ratio of profits to the capital stock. The Euler equation for investment can then be written as:

\[ i_{t+1} = \frac{1}{\beta, \phi_{i,t}} \left( \frac{1 - \beta, \phi_{i,t}\psi}{\psi} i + \frac{1}{2} \frac{m^2}{\xi} + \frac{1}{2} \frac{\xi}{\psi} m^2 + \frac{1}{\xi} \pi_i \right) + v_{1,t+1} \tag{11} \]

where \( \phi_{i,t} = 1 - \delta \exp[-\gamma m_t](1 + \gamma m_{t+1}) \) and \( v_{1,t+1} = u_{1,t+1} + \eta_{i,t+1} - \frac{1}{2\beta, \phi_{i,t}} \psi, \phi_{i,t} \frac{\psi}{2\psi, \phi_{i,t}} \eta_{i,t}^2 - \frac{\xi}{2\psi, \phi_{i,t}} \psi, \phi_{i,t} \frac{\xi}{2\psi, \phi_{i,t}} \psi, \phi_{i,t} \eta_{i,t}^2 \).

The Euler equation for maintenance is in turn given by:

\[ m_{t+1} = \frac{1}{\beta, \phi_{i,t}} \left( \frac{\rho_t - \beta, \phi_{i,t}\rho_{i,t+1}}{\xi} + \rho_t m_i + \frac{1}{2} \frac{m_i^2}{\xi} \right) + v_{2,t+1} \tag{12} \]

where \( \rho_t = \frac{1}{\gamma} \exp(\gamma m_t) \). \( v_{2,t+1} = u_{2,t+1} + \epsilon_{i,t+1} - \frac{1}{2\beta, \phi_{i,t}} \psi, \phi_{i,t} \phi_{i,t} \frac{\psi}{2\psi, \phi_{i,t}} \phi_{i,t} \eta_{i,t}^2 - \frac{\rho_t}{\beta, \phi_{i,t}} \phi_{i,t} \epsilon_{i,t} \).

An attractive feature of the model is that it nests the model with exogenous depreciation, which is a special case for \( \gamma = 0 \) with a single Euler equation for investment. Hence, any test of the significance of \( \gamma \) is a test on the validity of the key assumption of endogenous depreciation. Equations (11) and (12) form a non-linear system model that is estimated below.

\(^7\) An interesting extension would be to allow the adjustment costs for ‘new’ investment and maintenance to interact. However, this extension would introduce additional restrictions that would render the empirical specification intractable.

\(^8\) See Appendix 1 for the detailed derivation of equations (11) and (12).
2.3. Empirical specification with variable capital utilization

A plausible determinant of the depreciation rate supported by some studies is the capital utilization rate.\(^9\) This mechanism is triggered by increased user costs of capital brought about by wear and tear particularly on equipment, and suggests that capital utilization should be taken into account in conjunction with maintenance expenditures within the context of endogenous capital depreciation.\(^10\)

In this vein we extend the model outlined in the previous subsections to account for the impact of variable capital utilization that affects the depreciation rate of capital and, consequently, enters in the empirical Euler equations for ‘new’ investment and maintenance expenditures. Specifically, we assume that the depreciation rate is affected by the ratio of maintenance expenditures to capital services, rather than the capital stock.\(^11\) Hence, using capital more intensively increases the rate at which capital depreciates. The modified depreciation function becomes:

\[
\delta \left( \frac{M_i}{u_i K_{r-1}} \right) = \delta \exp \left( -\gamma \frac{M_i}{u_i K_{r-1}} \right) = \delta \exp \left( -\gamma \frac{m_i}{u_i} \right) \tag{13}\]

where \(u_i\) denotes the capital utilization rate. In contrast to the case of constant depreciation, which implies a zero marginal cost of capital utilization and therefore full capital utilization, the optimality conditions cause the marginal cost of utilization to change along with the marginal product of the underlying accumulated capital stock. This implies that the marginal benefits must be weighed against the marginal costs and that in general firms will not find it optimal to fully utilize their capital stock. The first-order

\(^9\) See, for instance, Epstein and Denny (1980) and Johnson (1994), Bitros (1976) and Everson (1982) have examined empirically the joint demand of utilization, maintenance and investment. Mullen and Williams (2004) attempt to estimate the effect of capacity utilization on maintenance expenditures in Canada but do not find any substantial impact.


\(^11\) An interesting extension of the specification for the depreciation function would be to allow maintenance expenditures and utilization to enter with differential impacts; however, adding an extra parameter in the estimated specifications would be a very demanding exercise given the available number of observations. Notice that our results would not be affected if we assumed that the utilization rate enters in the production function as well, due to the linear homogeneity of the revenue function in capital. Because of this latter assumption utilization would not affect the marginal productivity of capital.
condition for investment (5) remains intact, whereas the first-order conditions for maintenance and utilization become:

$$\frac{\partial R_i}{\partial M_i} - \frac{\gamma \lambda_i \delta \exp[-\gamma \frac{M}{u_i K_{i-1}}]}{u_i} = 0$$ (14)

$$\frac{\partial F(u_i, K_i)}{\partial (u_i K_i)} \frac{\partial (u_i K_i)}{\partial (u_i)} \lambda_i \delta \frac{M_i}{u_i} \exp[-\gamma \frac{M}{u_i K_{i-1}}] = 0$$ (15)

In Appendix 2 we show that the Euler equations for ‘new’ investment and maintenance will have a similar structure with the one obtained under variable capital utilization with the terms $\phi_{t+1}$ and $\rho_t$ given now by

$$\phi_{t+1} = 1 - \delta \exp[-\gamma \frac{m_{t+1}}{u_{t+1}}](1 + \gamma \frac{m_{t+1}}{u_{t+1}}) \quad \text{and} \quad \rho_t = \frac{u_t}{\gamma \delta} \exp[\gamma \frac{m}{u_t}].$$

Denoting $\prod(u_t) \equiv p_t$ and keeping the notation introduced earlier on, the Euler equation for utilization can be written in terms of the profits to the utilization rate as:

$$p_{t+1} = \frac{1}{\beta \phi_{t+1} z_{t+1}} \left( \pi_t + \frac{\psi}{2} l_t + \frac{\xi}{2} m_t - z_t p_t - \frac{\psi}{2} n_t + \frac{\xi}{2} e_t \right)$$ (16)

where $z_t = \frac{u_t^2}{\gamma \delta M_t} \exp\left[\frac{m}{u_t}\right]$. Equations (11), (12) and (16) yield a system of three Euler equations that is estimated below.

3. Data and estimation

Equations (11) and (12) comprise the baseline model of nonlinear equations to be tested. Our main data source is the Canadian Survey on ‘Capital and Repair Expenditures’, which is the only available data set worldwide on aggregate ‘new’ investment and maintenance expenditures. We use aggregate data, available through Canada Statistics, from both the business sector and the manufacturing sector to estimate the empirical equations. The existing studies on empirical investment equations have mostly
focused on the manufacturing sector due to data availability and quality issues. The distinction made here provides a robustness test for our conceptual approach and, moreover, allows us to highlight any discrepancies, first, in the magnitude of the estimated adjustment costs for ‘new’ investment and maintenance spending and, second, in the estimated depreciation rates between the aggregate business sector and the manufacturing sector.

In particular, private firms, households and government organizations in Canada were asked in an annual survey starting in 1956 about their capital and repair expenditures on equipment and structures. The survey is a census with a cross-sectional design and a sample size of 27,000 units; the target population is all Canadian businesses and governments from all the provinces and territories in Canada and the response rate is roughly 85%. Prior to the selection of a random sample, establishments are classified into homogeneous groups (i.e. groups with the same NAICS codes, same province/territory etc). Business enterprises are defined as those firms where the government controls less than 50% of the voting rights (the remaining of the private sector consists of private institutions and households).

*Capital* expenditures are gross expenditures on fixed assets and cover spending devoted to ‘new’ investment. These include expenditures on (i) fixed assets (such as new buildings, engineering, machinery, and equipment) which normally have a life of more than 1 year, (ii) modifications, additions, major renovations, and additions to work in progress, (iii) capital costs such as feasibility studies and general (architectural, legal, installation and engineering) fees, (iv) capitalized interest charges on loans with which capital projects are financed, (v) work by own labor force. *Repair* expenditures cover spending devoted to capital maintenance and in specific: (i) maintenance and repair of nonresidential buildings, other structures, and on vehicles and other machinery, (ii) building maintenance (janitorial services, snow removal, sanding), (iii) equipment maintenance (such as oil changes and lubrication of vehicles and machinery), (iv) repair work by own and outside labor force on machinery and equipment. The survey is conducted after 1993 in an updated form that renders the data on capital and repair expenditures in the business sector non-comparable. However, we managed to obtain consistent series ending in 2005 for the
Regarding the rest of the variables that enter in equations (10) and (11), we proxied profits as a proportion of the capital stock from after-tax corporation profits divided by the end-of-previous-period capital stock. Our proxy for profits is based on estimates of factor incomes, which calculate domestic output by measuring incomes accruing to labor (wages, salaries and supplementary labour income) and capital. The average rate on prime corporate paper was used to calculate the discount factor. Table 1 gives a synoptic presentation and some descriptive statistics of the data at hand and Figures 1A and 1B plot the ‘new’ investment and maintenance series for the business and manufacturing sectors. The average ‘new’ investment to capital ratio was 6.1% and 6.8% in the business and manufacturing sectors, whereas the corresponding maintenance to capital ratio was 2.25% and 3.35%. The volatility of both the ‘new’ investment and maintenance shares has been higher in the manufacturing sector, as indicated by the Figures and the standard deviations and the relative distance between the maximum and minimum values for the periods under consideration.

Regarding the estimation of equations (11) and (12), notice that although the expectation error \( u_{t+1} \) is uncorrelated with the other two error components, \( v_{1,t+1} \), will still be serially correlated as

\[
E(v_{1,t}v_{1,t+1}) = E\left( -\frac{1}{2\beta\phi_{t+1}^3} \eta_t^3 - \frac{1}{\beta\phi_{t+1}^2} \eta_t^2 \right)
\]

that will be generally different from zero, whereas a similar structure is implied for the corresponding error term in the Euler equation for maintenance, \( v_{2,t+1} \).

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12 Expenditures in capital and repair to capital stock by the manufacturing sector compared to the business sector were roughly steady and amounted between 25% and 30% of total capital and repair expenditures for the period 1956-1993. The correlation between the ‘new’ investment and maintenance series is 0.81 for the business sector and 0.69 for the manufacturing sector. See the Data Appendix for a detailed description on the construction of the relevant series.

13 Following a referee’s advice, we also performed our regressions using the real interest rate to calculate the discount factor. Our estimates (available upon request) were similar, although slightly worse in terms of robustness, to those reported below. We chose to report the estimates with the nominal discount factor in order to follow the majority of the literature when comparing our approach to existing estimates of the investment Euler equation (see eg Chatelain and Teurlai, 2001, and the references cited therein).

14 In all ratios of the variables to the capital stock we use the previous period capital stock to account for the fact that the model requires a beginning-of-period capital stock. See the Data Appendix for a detailed description of the dataset and the relevant sources. For an extensive presentation of the data from the Canadian Survey on ‘Capital and Repair Expenditures’ see Kalyvitis (2006). Notice that all our series are found to be stationary as indicated by standard unit-root tests.
the complex structure of the system at hand, we use a non-linear system-GMM method to estimate simultaneously the two Euler equations (11) and (12). We use as instruments two to six-period lagged values of the ‘new’ investment and maintenance to capital ratios in levels and squared, the profits to capital ratio, and the discount factor. Table 2 reports the correlation coefficients between the main instruments. The correlation of the instruments with the error term is investigated with the standard $J$-test of overidentifying restrictions.

The empirical investigation of the joint determination of depreciation, maintenance expenditures, and capital utilization becomes somewhat difficult due to the lack of data on capital utilization for Canada. Ideally, we would like to have a measure of the capital workweek to approximate the capital utilization rate. In the absence of this type of data, we use here the industrial (total non-farm goods producing industries) capacity utilization rate as a proxy for utilization in the business sector and the manufacturing industries capacity utilization rate for the manufacturing sector. (See the Data Appendix for more details on the sources and the construction of these variables.)

Attempts to estimate the model with freely-varying $\delta$ (no-maintenance depreciation rate), $\gamma$ (sensitivity of depreciation to changes in maintenance to capital ratio), and $\psi$, $\xi$ (adjustment cost parameters) were unsuccessful. This is not surprising given that there is a clear identification problem between $\delta$ and $\gamma$, since both parameters are related with the curvature of the depreciation function: a higher value of $\delta$ implies that the depreciation function approaches the actual depreciation rate with a larger slope, captured by $\gamma$. As an alternative strategy we concentrated on the parameters $\gamma$, $\psi$, $\xi$, and fixed $\delta$ by using a range of plausible values, a choice that is mainly motivated by the intuitive consensus on the plausible values for $\delta$, whereas there is no corresponding evidence on $\gamma$. The starting values for parameters $\psi$ and $\xi$ were then set at 0.1, whereas the initial value for parameter $\gamma$ was chosen on the basis

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15 Shapiro (1986) emphasizes the spurious correlation between capacity utilization and capital utilization, and also notices the difficulties associated with the measurement or construction of the latter as it involves data on the work week of capital proxied by the number of workers on late shifts, which are not available for the Canadian economy at the aggregate or sectoral level. We notice that Paquet and Robidoux (2001) have introduced a measure of capital utilization in the production function and have found that the Canadian economy can be described by constant returns to scale and perfect competition. Unfortunately, their index can be constructed only from 1970 onwards.
of model convergence, which typically resulted in relatively higher initial values of $\gamma$ for higher $\bar{\delta}$.

4. Results

4.1. Aggregate 'new' investment and maintenance spending

Before turning to the estimation of the Euler equation with endogenous capital depreciation, in Table 3 we report for comparison the results from the estimation of the standard Euler equation for investment with constant capital depreciation (see e.g. Oliner et al, 1995). The first column corresponds to the business sector sample for total investment, the second and third columns to 'new' investment in construction and machinery equipment, respectively, whereas the estimated coefficient represents the inverse of $\psi$. Estimation is based on the assumption that the depreciation rate is 6.7% for the total capital stock, 5.9% in construction and 8.2% in machinery-equipment, as reported in Hwang (2002/2003). For all three specifications the reported coefficient estimates bear the wrong sign, suggesting misspecification despite the fact that the overidentifying restrictions are not rejected (similar findings were obtained by a simple OLS regression). These results further motivate our attempt to improve the fit of the standard Euler equation by endogenizing the depreciation rate following the approach of section 2.

Table 4 reports the estimates of the basic model comprising equations (11) and (12). The reported average adjustment costs represent the marginal adjustment cost (i.e. $C_I = \psi \frac{I}{K}$) evaluated at the average investment rate. The left panel reports the estimates for the business sector with the first column showing the estimates when $\bar{\delta}=0.1$. As can be seen, all three parameters $\gamma$, $\psi$, and $\xi$ have the expected sign and small standard errors, whereas the over-identifying restrictions are not rejected. The statistical significance of $\gamma$ supports the endogenous depreciation assumption. The estimated adjustment costs are found to be 44% for 'new' investment and 15% for maintenance expenditures. The estimated average depreciation rate is found to be 2.8%, a value that is somewhat low. We perform the same exercise by postulating values $\bar{\delta}=0.15$ and $\bar{\delta}=0.2$, and the results are presented in the second and third column of Table 4 respectively. We find again that the model performs well in terms of statistical tests, but the parameter $\gamma$
measuring the response of depreciation to maintenance expenditures is somewhat lower implying more reasonable average depreciation rates in the range of 6.5%-7%. The right panel of Table 4 presents similar regressions for the manufacturing sector and again we report three regressions for the same values of $\delta$. The average depreciation rate in the Canadian manufacturing sector is slightly higher compared to the business sector. The evidence corroborates those found for the business sector and are in line with the estimates provided by Jorgenson (1996) and Fraumeni (1997) for the US.\(^{16}\) whereas the broad picture indicates that adjustment costs in ‘new’ investment in the manufacturing sector are found to be roughly two times larger than those for maintenance spending, whereas those for the business sector appear relatively larger and are about three times larger than those for maintenance.

In general, the findings support the model with endogenous depreciation and the estimated adjustment costs are lower than those provided by the empirical literature on aggregate investment, that are typically found to be implausibly high.\(^{17}\) Hence, although our results for adjustment costs exhibit a variation in their magnitude depending on the calibrated value for $\delta$, they produce more plausible estimates compared to the existing literature.

To highlight the significance of our estimates for the impact of maintenance expenditures we calculate the response of depreciation when maintenance expenditures are raised by one standard deviation from their mean value, i.e. from 2.25 to 2.5 percent as a ratio of the capital stock. This rise triggers a fall in the depreciation rate that ranges roughly between 0.37 (for $\delta=0.1$) and 0.76 (for $\delta=0.2$) percentage points. Another way to assess these figures is to calculate the difference in the depreciation rate for the maximum and minimum maintenance to capital ratios for our sample, which are 1.77% and 2.84% respectively. Our estimates imply that, depending on the model used, the depreciation rate has varied in a range between 1.7

\(^{16}\) Jorgenson (1996) reports an average depreciation rate of 15% for durable equipment and 3.1% for nonresidential structures. The figures for durable equipment range between 6.6% (railroad equipment) and 27.3% (office, computing and accounting machinery). Fraumeni (1997) reports similar figures but has a more detailed categorization; for instance the depreciation rate for railroad equipment is 5.9% and for office, computing and accounting machinery it is 27.3% before 1978 and 31.2% after 1978.

\(^{17}\) An exception is the study by Barnett and Sakellaris (1999) that has estimated the costs of installing new capital to be approximately 10% to 13% of the total investment cost.
percentage points (for $\bar{\delta} = 0.1$) and 3.4 percentage points (for $\bar{\delta} = 0.2$) over the period 1956-1993. A similar picture, although somewhat smaller in magnitude, emerges when the estimates for the manufacturing sector are considered. Following a rise in the maintenance spending to capital ratio by one standard deviation from 3.35 percent (sample average) to 3.72 percent, we find that the fall in the depreciation rate is 0.17 percentage points for $\bar{\delta} = 0.1$ and 0.57 percentage points for $\bar{\delta} = 0.2$. Regarding the sample maximum and minimum maintenance to capital ratios (2.63% and 4.13% respectively) the estimates imply that the depreciation rate in the manufacturing sector has varied between 0.7 percentage points (for $\bar{\delta} = 0.1$) and 2.6 percentage points (for $\bar{\delta} = 0.2$) over the period 1956-2005.

For comparison, Table 5 reports the results for the business sector based on a second-order approximation of equations (11) and (12).\(^\text{18}\) The results point to slightly higher depreciation rates and lower adjustment costs for both maintenance and new investment. All estimates are statistically significant and the Hansen test of the overidentifying restrictions rejects misspecification of the instrument set.

Table 6 shows the estimation results when the depreciation function allows for variable capital utilization as given by equation (13). All the estimated coefficients have the correct sign and are statistically significant. In particular, for $\bar{\delta} = 0.1$ the average depreciation rate is found to be 7.4%, whereas the adjustment costs for both ‘new’ investment and maintenance are small (3.4% and 1.3% respectively). For $\bar{\delta} = 0.15$ the depreciation rate is slightly higher (8.1%) and the adjustment costs rise marginally amounting to 4.6% and 1.7%, whereas for $\bar{\delta} = 0.2$ the depreciation rate is estimated at 4.6% and the adjustment costs at 51% and 15.5%. Regarding the corresponding estimates for the manufacturing sector, the adjustment costs are found to be higher when capital utilization is taken into account and range between 24.5% and 122.4% for ‘new’ investment and between 12.5% and 50.3% for maintenance depending on the calibrated value for $\bar{\delta}$.

4.2. ‘New’ investment and maintenance spending on machinery-equipment

\(^{18}\) We thank a referee for pointing out this alternative estimation strategy.
A key assumption underlying the estimation of empirical investment models is that capital can be treated as a homogeneous good. Some studies that have relaxed this assumption (e.g. Abel and Eberly, 2002) claim that capital heterogeneity may lead to a mismeasurement of the relationship between the various forms of capital and $q$. In an empirical context, Oliner et al. (1995) have found that investment models for structures perform worse than the corresponding ones for equipment, whereas Bontempi et al. (2004) show that the standard convex costs model performs well for equipment, but not for structures where evidence of non-convex adjustment costs is found.

To account for this distinction we modify our analysis of the Euler equation for ‘new’ investment and maintenance by assuming that only capital expenditures in machinery and equipment are relevant for the firm’s decision between ‘new’ investment and maintenance. The Canadian survey on Capital and Repair expenditures distinguishes between non-residential construction and machinery-equipment expenditures. In particular, spending on machinery-equipment covers (i) automobiles, trucks, professional and scientific equipment, office and store furniture, appliances, (ii) motors, generators, transformers, (iii) capitalized tooling expenses, (iv) pre-paid progress payments. Looking at the data we find that the focus on machinery-equipment can be further motivated by the substantial disparities between the two types of assets when their decomposition in the Canadian economy is considered. The bulk of maintenance expenditures by business enterprises involves spending in machinery-equipment (78.5% of total business maintenance outlays are concentrated in machinery and equipment, whereas the corresponding share in ‘new’ investment expenditures is 58.5%). This trend is even more pronounced in the manufacturing sector where the corresponding figures are 86.4% and 80.7%. The lower panel of Table 1 gives a description of the main statistics for expenditures in machinery and equipment in the business sector and in the manufacturing sector.19

Table 7 presents the results of our estimations on firms’ expenditure for ‘new’ investment and maintenance of machinery-equipment. To identify the model parameters we need to specify values for the

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19 There is no available data for profits stemming from the two types of assets. We therefore estimate (11) and (12) after weighting profits by the corresponding share of the capital stock in machinery-equipment.
depreciation rate of capital in machinery-equipment under no maintenance, $\delta$. Hwang (2002/3) finds that the simple overall average rate of the estimated rates of depreciation for structures and machinery-equipment in the Canadian industry sectors are 5.9% and 8.2% respectively. This picture is broadly confirmed by the evidence provided in the studies by Jorgenson (1996) and Fraumeni (1997) for the US. We follow the general consensus and we assume that the depreciation rates under no maintenance are higher for machinery-equipment and we set the calibrated values for $\delta$ alternatively at 0.15, 0.2 and 0.25.

As can be readily seen, again in all cases the estimated coefficients have the predicted signs and are statistically significant. The estimates for both the business sector and the manufacturing sector produce reasonable figures for the adjustment costs with those for ‘new’ investment estimated at below 16%, being roughly two times larger than the maximum estimates for maintenance adjustment costs (8.1%). Interestingly, the results for machinery-equipment are relatively robust to the choice of $\delta$ and quite similar in magnitude across the two sectors. The average depreciation rate for machinery-equipment in the manufacturing sector is generally higher compared to the corresponding one calculated from the specifications with aggregate capital spending.

Finally, Table 8 reports results with variable capital utilization for investment and maintenance expenditures in machinery-equipment. As expected, the depreciation rates estimated here are higher for both sectors compared to the corresponding depreciation rates for total capital ranging between 6.4% and 13.5% for the business sector and between 5.3% and 11.1% for the manufacturing sector. Following the same pattern as in Table 6, adjustment costs for investment spending on machinery-equipment are higher in the manufacturing sector compared to those in the business sector ranging between 3% and 5% in the business sector and between 5% and 45% in manufacturing. This also holds for maintenance adjustment costs that are found to be between 1% and 3% in the business sector and between 3% and 21% in manufacturing.

5. Conclusions
In this paper we have specified and estimated a neoclassical investment model with convex adjustment costs, in which firms can spend on capital maintenance that in turn affects the capital depreciation rate. We have estimated jointly the Euler equations for ‘new’ investment and maintenance using data from the Canadian survey on ‘Capital and Repair Expenditures’ and we have shown that the Euler equations perform satisfactorily in terms of parameter estimates and model identification. Our model gives reasonable estimates for the adjustment costs and the average depreciation rate. These results are not affected by the inclusion of capital utilization in our empirical specifications.

We close the paper by pointing out three directions for further research. First, the approach presented here has primarily adopted the size of adjustment costs as the main criterion for the success of the model with endogenous depreciation. However, there are other criteria that have been used in the relevant literature, like temporal stability (Oliner et al., 1995) and out-of-sample forecasts. Future research in this area could focus on these aspects of the model with endogenous depreciation and address these issues using the Canadian data on capital and repair expenditures. Second, the present paper has not addressed the impact of taxation on ‘new’ investment and maintenance expenditures. Typically, maintenance expenditures are treated as current operating expenses and can therefore be fully deducted from pre-tax revenues, whereas ‘new’ investment expenditures are only deducted through depreciation allowances. Also, policymakers often pursue growth-enhancing policies, such as special tax credits to corporate investment or subsidies to investment loans, which favour spending in ‘new’ investment. Incorporating differential forms of taxation and subsidies on capital expenditures in the firm’s problem could offer new insights. Third, it would be of interest to extend the model by examining the links with employment. For instance, if adjustment costs were specified in a more general functional form with interactions between maintenance and labor, the Euler equation would include terms for employment. Likewise, one could introduce variable labor effort (labor hoarding) and assess its implications for the formulation and estimation of the Euler equations for ‘new’ investment and maintenance, due to the fact that firms may alter labor utilization by varying hours worked, perhaps jointly with capital maintenance as these two variables are likely to be linked complementarily in the production process.
APPENDIX 1. Derivation of the Euler equations for ‘new’ investment and maintenance

Since by (4') we have that
\[ E_t \frac{\partial V_{t+1}(K_t)}{\partial K_t} = E_t \left[ 1 - (1 + \gamma \frac{M_{t+1}}{K_t}) \delta \exp[-\gamma \frac{M_{t+1}}{K_t}] \lambda_{t+1} \right], \]
we can rewrite (4) as follows:

\[
\left( \frac{\partial R_t}{\partial K_t} \right) + \beta E_t \left[ (1 - \delta \exp[-\gamma \frac{M_{t+1}}{K_t}](1 + \gamma \frac{M_{t+1}}{K_t})) \lambda_t \right] + \lambda_t = 0
\]

Now using (5) and (6) to substitute out \( \lambda_t \) yields the Euler equations for investment and maintenance as:

\[
\left( \frac{\partial R_t}{\partial K_t} \right) + \left( \frac{\partial R_t}{\partial I_t} \right) = -\beta E_t \left[ (1 - \delta \exp[-\gamma \frac{M_{t+1}}{K_t}](1 + \gamma \frac{M_{t+1}}{K_t})) \left( \frac{\partial R_{t+1}}{\partial I_{t+1}} \right) \right]
\]

\[
\left( \frac{\partial R_t}{\partial K_t} \right) - \frac{1}{\gamma \delta} \exp[-\gamma \frac{M_{t+1}}{K_t}] \left( \frac{\partial R_t}{\partial M_t} \right) = -\beta E_t \left[ (1 - \delta \exp[-\gamma \frac{M_{t+1}}{K_t}](1 + \gamma \frac{M_{t+1}}{K_t})) \frac{1}{\gamma \delta} \exp[-\gamma \frac{M_{t+1}}{K_t}] \left( \frac{\partial R_{t+1}}{\partial M_{t+1}} \right) \right]
\]

Using the linear homogeneity assumption of the production function in capital and labour to write

\[
\frac{\partial F(K_t, L_t)}{\partial K_t} = F(K_t, L_t) \frac{L_t}{K_t}, \quad \frac{\partial F(K_t, L_t)}{\partial L_t} = F(K_t, L_t) \frac{K_t}{L_t} = \frac{\Pi_t}{K_t},
\]
we can derive the expressions for \( \frac{\partial R_t}{\partial K_t}, \frac{\partial R_t}{\partial I_t}, \) and \( \frac{\partial R_t}{\partial M_t} \) as:

\[
\frac{\partial R_t}{\partial K_t} = \left( \frac{\Pi_t}{K_t} \right) + \frac{a}{2} \left( \frac{I_t}{K_t} \right)^2 + \frac{\xi}{2} \left( \frac{M_t}{K_t} \right)^2 - \frac{\psi}{2} \eta_t^2 - \frac{\xi}{2} \epsilon_t^2
\]

\[
\frac{\partial R_t}{\partial I_t} = -\psi \left( \frac{I_t}{K_t} - \eta_t \right) - 1
\]

\[
\frac{\partial R_t}{\partial M_t} = -\frac{\xi}{2} \left( \frac{M_t}{K_t} - \epsilon_t \right) - 1
\]

Assuming perfect foresight and replacing these into the Euler equations we can get the parametric form of the Euler equations for \( i_{t+1} \) and \( m_{t+1} \) given by (11) and (12) in the text.
APPENDIX 2. Derivation of the Euler equations with variable capital utilization

Since utilisation becomes a choice variable that enters (through depreciation) the capital accumulation constraint the FOCs for maintenance and utilisation become:

\[
\frac{\partial \Lambda(K, L, I, u)}{\partial M_i} = \frac{\partial R_i}{\partial M_i} - \gamma \frac{\partial \bar{\lambda}_e}{\partial u_i} e^{-\frac{M_i}{u_{i,k-1}}} = 0
\]

\[
\frac{\partial \Lambda(K, L, I, u)}{\partial u_i} = \frac{\partial F(u, K, L, I)}{\partial u} \frac{\partial \Lambda}{\partial u_i} + \lambda_i \gamma \frac{M_i}{u_i^2} e^{-\frac{M_i}{u_{i,k-1}}} = 0
\]

The first-order conditions with respect to investment, maintenance, utilization and labor yield:

\[
\frac{\partial \Lambda(K, L, I, u)}{\partial I_i} = \frac{\partial R_i}{\partial I_i} - \lambda_i = 0
\]  \hspace{1cm} (A1)

\[
\frac{\partial \Lambda(K, L, I, u)}{\partial M_i} = \frac{\partial R_i}{\partial M_i} - \gamma \frac{\partial \bar{\lambda}_e}{\partial u_i} e^{-\frac{M_i}{u_{i,k-1}}} = 0
\]  \hspace{1cm} (A2)

\[
\frac{\partial \Lambda(K, L, I, u)}{\partial u_i} = \frac{\partial F(u, K, L, I)}{\partial u} \frac{\partial \Lambda}{\partial u_i} + \lambda_i \gamma \frac{M_i}{u_i^2} e^{-\frac{M_i}{u_{i,k-1}}} = 0
\]  \hspace{1cm} (A3)

\[
\frac{\partial \Lambda(K, L, I, u)}{\partial L_i} = \frac{\partial F(u, K, L, I)}{\partial L_i} - w_i = 0
\]  \hspace{1cm} (A4)

From condition (A1), \( \lambda_i \) in condition (A3) can be substituted by an expression that depends on the investment rate. Condition (A3) then gives utilization as a function of maintenance, investment and existing capital. This states that the marginal product of utilized capital must equal its rental price at any time \( t \). Condition (A4) states that the marginal product of labor equals the wage rate.

Using the fact that \( E_i \frac{\partial V_{i+1}(K_i)}{\partial K_i} = E \left( 1 - \delta e^{-\frac{M_{i+1}}{u_{i+1}K_i}} (1 + \gamma \frac{M_{i+1}}{u_{i+1}K_i}) \right) \bar{\lambda}_{i+1} \) we get:

\[
\left( \frac{\partial R_i}{\partial K_i} \right) + \beta E_i \left( 1 - \delta e^{-\frac{M_{i+1}}{u_{i+1}K_i}} (1 + \gamma \frac{M_{i+1}}{u_{i+1}K_i}) \right) \bar{\lambda}_{i+1} = 0
\]  \hspace{1cm} (A5)

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Using (A1), (A2) and (A3) to substitute out $\lambda$ we derive the Euler equations for investment, maintenance and utilization respectively:

$$\left(\frac{\partial R_i}{\partial K_i}\right) - \beta E\left[\left(1 - \delta e^{-\gamma_n K_i}\left(1 + \gamma M_{i+1}ight)\right)\left(\frac{\partial R_{i+1}}{\partial I_{i+1}}\right)\right] = 0,$$

$$\left(\frac{\partial R_i}{\partial I_i}\right) - \frac{u_i}{\gamma\delta e^{-\gamma_n K_i}}\left(\frac{\partial R_i}{\partial M_i}\right) = -\beta E\left[\left(1 - \delta e^{-\gamma_n K_i}\left(1 + \gamma M_{i+1}\right)\right)\frac{u_{i+1}}{\gamma\delta e^{-\gamma_n K_i}}\left(\frac{\partial R_{i+1}}{\partial M_{i+1}}\right)\right] = 0,$$

$$\left(\frac{\partial R_i}{\partial M_i}\right) - \frac{u_i^2}{\gamma\delta e^{-\gamma_n K_i}}\left(\frac{\partial R_i}{\partial u_i}\right) = -\beta E\left[\left(1 - \delta e^{-\gamma_n K_i}\left(1 + \gamma M_{i+1}\right)\right)\frac{u_{i+1}^2}{\gamma\delta e^{-\gamma_n K_i}}\left(\frac{\partial R_{i+1}}{\partial u_{i+1}}\right)\right] = 0.$$

The expressions for $\frac{\partial R_i}{\partial K_i}$, $\frac{\partial R_i}{\partial I_i}$ and $\frac{\partial R_i}{\partial M_i}$ can be derived as:

$$\frac{\partial R_i}{\partial K_i} = u_i \left(\frac{\Pi}{u_i K_i}\right) + \left(\frac{I_i}{2 K_i}\right)^2 + \frac{\xi \left(M_i K_i\right)^2}{2} - \frac{\psi}{2} \eta_i - \frac{\xi}{2} \zeta_i^2,$$

$$\frac{\partial R_i}{\partial I_i} = -\psi \left(\frac{I_i}{K_i} - \eta_i\right),$$

$$\frac{\partial R_i}{\partial M_i} = -\xi \left(\frac{M_i}{K_i} - \zeta_i\right).$$

To derive the expression for $\frac{\partial R_i}{\partial K_i}$ we use the linear homogeneity assumption of the production function in capital and labor to write

$$\frac{\partial F(u, K_i, L_i)}{\partial K_i} = \left(\frac{F(u, K_i, L_i)}{u_i K_i}\right) K_i = \left(\frac{F(u, K_i, L_i)}{u_i K_i}\right) L_i = \left(\frac{\Pi}{u_i K_i}\right) \frac{Y_i - w_i L_i}{u_i} = \left(\frac{\Pi}{u_i K_i}\right).$$

Replacing into (8) and assuming perfect foresight we can drop the expectations operator and solve the Euler equation for investment to capital ratio, $i_{t+1}$, as follows:
\[ i_{t+1} = \frac{1 - \beta\phi_{t+1}}{\psi \beta \phi_{t+1}} + \frac{1}{\beta \phi_{t+1}} + \frac{1}{2 \beta \phi_{t+1}} i_{t}^{2} + \frac{\xi}{2 \psi \beta \phi_{t+1}} m_{t}^{2} + \frac{1}{\psi \beta \phi_{t+1}} \pi_{t} + v_{t+1} \]

where \( m_t \) and \( \pi_{t+1} \) denote the ratios of maintenance and profits to capital, \( \phi_{t+1} = (1 - \delta e^{-m_{t+1}} (1 + \gamma \frac{m_{t+1}}{u_{t+1}})) \) and

\[ v_{t+1} = u_{t+1} + \eta_{t+1} - \frac{1}{2 \beta \phi_{t+1}} \eta_{t}^{2} - \frac{\xi}{2 \alpha \beta \phi_{t+1}} \epsilon_{t}^{2} - \frac{1}{\beta \phi_{t+1}} \eta_{t}. \]

The term \( u_{t+1} \) is an expectation error at time \( t+1 \) that comes after dropping the expectations operator in a non-linear framework. If this error is uncorrelated with the other two error components it is still possible to show that \( v_{t+1} \) will be serially correlated, as

\[ E(v_{t}, v_{t+1}) = E\left(-\frac{1}{2 \beta \phi_{t+1}} \eta_{t}^{2} - \frac{1}{\beta \phi_{t+1}} \eta_{t}^{2}\right), \]

which will be generally different from zero.

The Euler equation for maintenance is derived as follows:

\[ m_{t+1} = \frac{\rho_{t} - \beta \phi_{t+1} \rho_{t+1}}{\beta \phi_{t+1}} + \frac{\rho_{t}}{\beta \phi_{t+1}} m_{t}^{2} + \frac{\psi}{2 \beta \phi_{t+1}} i_{t}^{2} + \frac{1}{\beta \phi_{t+1}} \pi_{t} + v_{2,t+1} \]

where \( \rho_{t} = \frac{u_{t}}{\gamma \delta} e^{\frac{\delta}{\gamma}} \) and \( v_{2,t+1} = u_{2,t+1} + \epsilon_{t+1} - \frac{1}{2 \beta \phi_{t+1}} \epsilon_{t}^{2} - \frac{\psi}{2 \beta \phi_{t+1}} \eta_{t}^{2} - \frac{\rho_{t}}{\beta \phi_{t+1}} \eta_{t} \), with a similar structure as above implied for this error term.

Finally the Euler equation for utilisation in terms of profits divided by utilisation, \( p_t \), is given as:

\[ p_{t+1} = \frac{1}{\beta \phi_{t+1} z_{t+1}} \left[ \pi_{t} + \frac{w}{2} i_{t}^{2} + \frac{\xi}{2} m_{t}^{2} - z_{t} p_{t} - \frac{w}{2} \eta_{t}^{2} - \frac{\xi}{2} \epsilon_{t}^{2} \right] \]

where \( z_{t} = \frac{u_{t}}{\gamma \delta M_{t}} e^{\frac{\delta}{\gamma M_{t}}} \). The last equation corresponds to equation (16) in the text.
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DATA APPENDIX

A. ‘New’ investment and maintenance data

The following annual variables in current prices from the Canadian Survey on Capital and Repair Expenditures of Canada Statistics were used to obtain the data for capital and repair expenditures in the aggregate economy and in the manufacturing sector.

1) Capital and repair expenditures by business enterprises: variable D843800.
2) Capital expenditures by business enterprises: variable D842986.
3) Repair expenditures by business enterprises: variable D843801.
4) Capital expenditures by business enterprises in machinery and equipment: variable D842988.
5) Repair expenditures by business enterprises in machinery and equipment: variable D843803.
9) Repair expenditures in manufacturing, machinery and equipment, variable v754445 [D878256], 1994 to 2005, and variable v62550 [D843232], 1956 to 1993.

Backward values for the manufacturing sector up to 1956 were obtained by using the growth rates for capital expenditures (the growth rates for 1992 and 1993 are common for the two surveys) and then by extrapolating the series for repair expenditures through their share in total capital and repair expenditures over 1956 to 1993.

B. Other variables

1) Business capital stock: Business sector end-year gross fixed non-residential capital stock (Canada Statistics, variable v1408305, Table 031-0002, current prices).
2) Business capital stock in machinery and equipment: Business sector end-year gross fixed non-residential capital stock in machinery and equipment (Canada Statistics, variables v1408308, Table 031-0002, current prices).
3) Manufacturing capital stock: Manufacturing sector end-year capital stock, total components, variable v1071434 [D819520], 1955 to 2007 (Canada Statistics, Table 031-0002, current prices).
4) Manufacturing capital stock in machinery and equipment: Manufacturing sector end-year capital stock, variable v1071437 [D819523], 1955 to 2007 (Canada Statistics, Table 031-0002, current prices).
5) **Interest rate**: Average rate on prime corporate paper, 90 days (International Financial Statistics, variable 15660BC.ZF).

6) **After-tax corporate profits**: Nominal corporation profits after taxes, variable v647778 [D23250], 1961 to 2006 (Canada Statistics, Table 380-0029, current prices), derived by corporation profits before taxes minus (i) interest and miscellaneous investment income paid to non-residents, (ii) corporate income tax liabilities. Backward values were extrapolated by fitting a linear regression on corporation profits before taxes for all industries, variable v501082 [D11893] (Canada Statistics, Table 380-0048).

7) **Capital utilization**: Industrial (total non-farm goods producing industries) capacity utilization rate (Canada Statistics, variables v142812, Table 028-0001, percent), averaged from quarterly data available from 1962 onwards. Backward values were extrapolated by fitting a linear regression on total fixed non-residential capital stock for all industries (Canada Statistics, variable: D99027311000) divided by Canada Gross National Product (International Financial Statistics, variable 15699A.CZF).

8) **Capital utilization in manufacturing**: Manufacturing industries capacity utilization rate, variable v4331088, Table 028-0002, 1987 to 2006 (Canada Statistics, percent, averaged from quarterly data). Backward values up to 1962 were extrapolated by using the growth rate of the manufacturing industries capacity utilization rate, variable v142817, Table 028-0001 (Canada Statistics, percent, averaged from quarterly data). Backward values up to 1956 were extrapolated by fitting a linear regression on the growth rate of end-year capital stock in manufacturing total components divided by Canada Gross Domestic Product (International Financial Statistics, variable 15699B.CZF).
<table>
<thead>
<tr>
<th>Table 1. Descriptive statistics of main variables</th>
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</thead>
<tbody>
<tr>
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<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td><strong>Business sector (1956-1993)</strong></td>
</tr>
<tr>
<td>‘new’ investment to capital ratio</td>
</tr>
<tr>
<td>squared ‘new’ investment to capital ratio</td>
</tr>
<tr>
<td>maintenance to capital ratio</td>
</tr>
<tr>
<td>squared maintenance to capital ratio</td>
</tr>
<tr>
<td>discount factor</td>
</tr>
<tr>
<td>profits over capital</td>
</tr>
<tr>
<td>‘new’ investment to capital ratio in machinery-equipment</td>
</tr>
<tr>
<td>squared ‘new’ investment to capital ratio in machinery-equipment</td>
</tr>
<tr>
<td>maintenance to capital ratio in machinery-equipment</td>
</tr>
<tr>
<td>squared maintenance to capital ratio in machinery-equipment</td>
</tr>
</tbody>
</table>

Source: CANSIM database, Statistics Canada and authors’ calculations.
TABLE 2. Correlation matrix of main variables

<table>
<thead>
<tr>
<th>Business sector</th>
<th>Investment</th>
<th>Investment lag</th>
<th>Investment lag squared</th>
<th>Maintenance</th>
<th>Maintenance lag</th>
<th>Maintenance lag squared</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Investment lag</strong></td>
<td>0.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Investment lag squared</strong></td>
<td>0.70</td>
<td>0.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Maintenance</strong></td>
<td>0.79</td>
<td>0.71</td>
<td>0.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Maintenance lag</strong></td>
<td>0.54</td>
<td>0.78</td>
<td>0.76</td>
<td>0.87</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Maintenance lag squared</strong></td>
<td>0.55</td>
<td>0.79</td>
<td>0.77</td>
<td>0.87</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Profits</strong></td>
<td>0.55</td>
<td>0.80</td>
<td>0.77</td>
<td>0.60</td>
<td>0.71</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td><strong>Profits lagged</strong></td>
<td>0.20</td>
<td>0.62</td>
<td>0.59</td>
<td>0.41</td>
<td>0.68</td>
<td>0.67</td>
<td>0.76</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Manufacturing sector</th>
<th>Investment</th>
<th>Investment lag</th>
<th>Investment lag squared</th>
<th>Maintenance</th>
<th>Maintenance lag</th>
<th>Maintenance lag squared</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Investment lag</strong></td>
<td>0.66</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Investment lag squared</strong></td>
<td>0.65</td>
<td>0.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Maintenance</strong></td>
<td>0.73</td>
<td>0.69</td>
<td>0.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Maintenance lag</strong></td>
<td>0.45</td>
<td>0.71</td>
<td>0.69</td>
<td>0.80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Maintenance lag squared</strong></td>
<td>0.45</td>
<td>0.72</td>
<td>0.70</td>
<td>0.80</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Profits</strong></td>
<td>0.13</td>
<td>0.35</td>
<td>0.33</td>
<td>0.25</td>
<td>0.29</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td><strong>Profits lagged</strong></td>
<td>-0.22</td>
<td>0.06</td>
<td>0.06</td>
<td>-0.04</td>
<td>0.17</td>
<td>0.67</td>
<td>0.08</td>
</tr>
</tbody>
</table>

**Note:**
All variables are expressed as ratios to the previous-period end capital stock.
### TABLE 3. Estimated Euler equations with exogenous capital depreciation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Total capital stock</th>
<th>Construction equipment</th>
<th>Machinery equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\psi$</td>
<td>-2.86 (0.20)</td>
<td>-1.33 (0.09)</td>
<td>-4.25 (0.24)</td>
</tr>
<tr>
<td>Observations</td>
<td>37</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>$J$-statistic</td>
<td>0.22</td>
<td>0.23</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Instrument list**

\[
\frac{I}{K} \left( \frac{I}{K} \right)^\gamma C = \beta
\]

lags t-3 to t-5  

lags t-3 to t-5  

lags t-3 to t-5

**Average depreciation rate**

- 6.7%  
- 5.9%  
- 8.2%

**Notes:**

1) Standard errors are in parentheses. The values reported in the $J$ test are the probability values of the corresponding test of over-identifying restrictions with $4 \times (n-1)$ degrees of freedom, where $n$ is the number of lags in the instruments.
2) The depreciation rates are fixed at 6.7% for total capital stock, 5.9% for construction and 8.2% for machinery-equipment. Marginal adjustment costs are evaluated at the average ‘new’ investment rates.
### TABLE 4. Estimated Euler equations for aggregate ‘new’ investment and maintenance expenditures

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Business sector</th>
<th>Manufacturing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta = 0.1$</td>
<td>$\delta = 0.15$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>56.93 (1.24)</td>
<td>33.02 (0.29)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>14.59 (0.19)</td>
<td>2.03 (0.10)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>13.49 (0.09)</td>
<td>1.98 (0.09)</td>
</tr>
<tr>
<td>Observations</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>$J$-statistic</td>
<td>0.99</td>
<td>0.96</td>
</tr>
</tbody>
</table>

| Instrument list | $\frac{LM}{K} \left( \frac{I}{K} \right)^{\beta} \left( \frac{M}{K} \right)^{\gamma} C R^\beta$ | lags $t-2$ to $t-6$ | lags $t-2$ to $t-5$ | lags $t-2$ to $t-6$ | lags $t-2$ to $t-5$ | lags $t-2$ to $t-5$ |

| Estimated marginal adjustment cost for ‘new’ investment | 44.3% | 6.2% | 0.6% | 5% | 2.6% | 34.1% |
| Estimated marginal adjustment cost for maintenance | 15.2% | 2.2% | 0.2% | 2.4% | 1.3% | 14.2% |
| Average depreciation rate | 2.8% | 7.1% | 6.5% | 8.3% | 7.2% | 3.0% |

Notes:
1) Initial values for the business sector regressions are $\delta = 0.1$: $\gamma = 10$, $\delta = 0.15$: $\gamma = 30$, $\delta = 0.2$: $\gamma = 50$ and for the manufacturing sector regressions $\delta = 0.1$: $\gamma = 6$, $\delta = 0.15$: $\gamma = 20$, $\delta = 0.2$: $\gamma = 25$.
2) Standard errors are in parentheses. The values reported in the $J$ test are the probability values of the corresponding test of over-identifying restrictions with $6 \times (n-1)$ degrees of freedom, where $n$ is the number of lags in the instruments.
3) The marginal adjustment costs evaluated at the average investment and maintenance rates.
TABLE 5. Estimated Euler equations for aggregate ‘new’ investment
and maintenance expenditures: second-order approximation

<table>
<thead>
<tr>
<th>Business sector</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>δ</td>
<td>0.1</td>
<td>0.15</td>
<td>0.2</td>
</tr>
<tr>
<td>γ</td>
<td>19.23 (2.94)</td>
<td>13.14 (0.52)</td>
<td>25.03 (0.01)</td>
</tr>
<tr>
<td>ξ</td>
<td>0.20 (0.07)</td>
<td>0.46 (0.02)</td>
<td>0.20 (0.01)</td>
</tr>
<tr>
<td>ψ</td>
<td>0.09 (0.04)</td>
<td>0.03 (0.00)</td>
<td>0.02 (0.00)</td>
</tr>
<tr>
<td>Observations</td>
<td>37</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>J-statistic</td>
<td>0.99</td>
<td>0.95</td>
<td>0.93</td>
</tr>
<tr>
<td>Instrument list</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>$M$</td>
<td>$(I-K)^\gamma$</td>
<td>$(C-K)^\delta$</td>
</tr>
<tr>
<td>lags $t-2$ to $t-6$</td>
<td>lags $t-2$ to $t-5$</td>
<td>lags $t-2$ to $t-6$</td>
<td></td>
</tr>
<tr>
<td>Estimated marginal adjustment cost for ‘new’ investment</td>
<td>0.6%</td>
<td>1.4%</td>
<td>0.6%</td>
</tr>
<tr>
<td>Estimated marginal adjustment cost for maintenance</td>
<td>0.1%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Average depreciation rate</td>
<td>6.5%</td>
<td>11.2%</td>
<td>11.4%</td>
</tr>
</tbody>
</table>

Notes:
1) Initial values for the business sector regressions are $\delta = 0.1$: $\gamma = 10$, $\delta = 0.15$: $\gamma = 20$, $\delta = 0.2$: $\gamma = 25$.
2) Standard errors are in parentheses. The values reported in the J test are the probability values of the corresponding test of over-identifying restrictions with $6 \times (n - 1)$ degrees of freedom, where $n$ is the number of lags in the instruments.
3) The marginal adjustment costs evaluated at the average investment and maintenance rates.
TABLE 6. Estimated Euler equations for aggregate ‘new’ investment and maintenance expenditures with variable capital utilization

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Business sector</th>
<th>Manufacturing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \bar{\delta} = 0.1 )</td>
<td>( \bar{\delta} = 0.15 )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>10.92 (0.08)</td>
<td>22.36 (0.43)</td>
</tr>
<tr>
<td>( \xi )</td>
<td>1.13 (0.02)</td>
<td>1.51 (0.11)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>1.12 (0.02)</td>
<td>1.54 (0.11)</td>
</tr>
<tr>
<td>Observations</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>( J )-statistic</td>
<td>0.95</td>
<td>0.99</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Instrument list</th>
<th>( I, M \left( \frac{I}{K} \right)^\top \left( \frac{M}{K} \right)^\top ), C, ( \beta )</th>
<th>lags ( t-2 ) to ( t-5 )</th>
<th>lags ( t-2 ) to ( t-5 )</th>
<th>lags ( t-2 ) to ( t-5 )</th>
<th>lags ( t-2 ) to ( t-5 )</th>
<th>lags ( t-2 ) to ( t-5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated marginal adjustment cost for ‘new’ investment</td>
<td>3.4%</td>
<td>4.6%</td>
<td>51.0%</td>
<td>24.5%</td>
<td>122.4%</td>
<td>45.4%</td>
</tr>
<tr>
<td>Estimated marginal adjustment cost for maintenance</td>
<td>1.3%</td>
<td>1.7%</td>
<td>15.5%</td>
<td>12.5%</td>
<td>50.3%</td>
<td>16.3%</td>
</tr>
<tr>
<td>Average depreciation rate</td>
<td>7.4%</td>
<td>8.1%</td>
<td>4.6%</td>
<td>7.0%</td>
<td>1.8%</td>
<td>3.6%</td>
</tr>
</tbody>
</table>

Notes:
1) The estimates are based on joint estimation of the non-linear system consisting of equations (11), (12) and (16).
2) Initial values for the business sector regressions are \( \bar{\delta} = 0.1: \gamma = 9, \bar{\delta} = 0.15: \gamma = 15, \bar{\delta} = 0.2: \gamma = 25 \) and for the manufacturing sector regressions \( \bar{\delta} = 0.1: \gamma = 6, \bar{\delta} = 0.15: \gamma = 15, \bar{\delta} = 0.2: \gamma = 25 \).
3) See Table 3.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Business sector</th>
<th>Manufacturing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta = 0.15$</td>
<td>37.97 (0.21)</td>
<td>33.32 (0.28)</td>
</tr>
<tr>
<td>$\delta = 0.2$</td>
<td>29.35 (0.46)</td>
<td>19.34 (0.23)</td>
</tr>
<tr>
<td>$\delta = 0.25$</td>
<td>11.2 (0.46)</td>
<td>3.9 % (0.35)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma = 0.15$</td>
<td>12.17 (0.21)</td>
<td>9.49 (0.28)</td>
</tr>
<tr>
<td>$\gamma = 0.2$</td>
<td>25.97 (0.43)</td>
<td>29.35 (0.46)</td>
</tr>
<tr>
<td>$\gamma = 0.25$</td>
<td>29.35 (0.46)</td>
<td>29.35 (0.46)</td>
</tr>
<tr>
<td>$\xi$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi = 0.15$</td>
<td>1.59 (0.03)</td>
<td>3.21 (0.05)</td>
</tr>
<tr>
<td>$\xi = 0.2$</td>
<td>2.96 (0.11)</td>
<td>0.45 (0.02)</td>
</tr>
<tr>
<td>$\xi = 0.25$</td>
<td>3.21 (0.05)</td>
<td>2.55 (0.04)</td>
</tr>
<tr>
<td>$\psi$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.15$</td>
<td>1.51 (0.03)</td>
<td>2.97 (0.05)</td>
</tr>
<tr>
<td>$\psi = 0.2$</td>
<td>2.72 (0.09)</td>
<td>0.43 (0.02)</td>
</tr>
<tr>
<td>$\psi = 0.25$</td>
<td>2.72 (0.09)</td>
<td>2.43 (0.09)</td>
</tr>
</tbody>
</table>

Observations

J-statistic

<table>
<thead>
<tr>
<th>Instrument list</th>
<th>Business sector</th>
<th>Manufacturing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$ $\beta$ $k$ $k$ $c$ $\gamma$ $k$ $k$ $c$ $\gamma$ $k$ $k$ $c$ $\gamma$ $k$ $k$ $c$ $\gamma$ $k$ $k$ $c$</td>
<td>lags t-2 to t-4</td>
<td>lags t-2 to t-6</td>
</tr>
</tbody>
</table>

Estimated marginal adjustment cost for ‘new’ investment

Estimated marginal adjustment cost for maintenance

Average depreciation rate

Notes:
1) The estimates are based on joint estimation of equations (11) and (12) for machinery equipment only.
2) Initial values the business sector regressions are $\delta = 0.15$; $\gamma = 9$, $\delta = 0.2$; $\gamma = 20$, $\delta = 0.25$; $\gamma = 25$ and for the manufacturing sector regressions $\delta = 0.15$; $\gamma = 12$, $\delta = 0.2$; $\gamma = 20$, $\delta = 0.25$; $\gamma = 18$.
3) See Table 3.
TABLE 8. Estimated Euler equations for ‘new’ investment and maintenance expenditures in machinery-equipment with variable capital utilization

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \delta = 0.15 )</th>
<th>( \delta = 0.20 )</th>
<th>( \delta = 0.25 )</th>
<th>( \delta = 0.15 )</th>
<th>( \delta = 0.20 )</th>
<th>( \delta = 0.25 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>1.85 (0.22)</td>
<td>16.04 (0.12)</td>
<td>23.41 (0.20)</td>
<td>5.97 (0.07)</td>
<td>22.23 (0.31)</td>
<td>22.50 (0.22)</td>
</tr>
<tr>
<td>( \xi )</td>
<td>0.89 (0.04)</td>
<td>0.58 (0.003)</td>
<td>1.09 (0.04)</td>
<td>1.24 (0.01)</td>
<td>10.59 (0.26)</td>
<td>3.94 (0.04)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.82 (0.04)</td>
<td>0.57 (0.00)</td>
<td>1.06 (0.03)</td>
<td>1.16 (0.01)</td>
<td>8.60 (0.15)</td>
<td>3.49 (0.03)</td>
</tr>
<tr>
<td>Observations</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>( J )-statistic</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.97</td>
<td>0.51</td>
<td>0.20</td>
</tr>
<tr>
<td>Instrument list</td>
<td>( l^{-2}M/K \cdot (l^{-1}M/K)^2 \cdot \zeta \cdot K \cdot \beta )</td>
<td>lags ( t-2 ) to ( t-6 )</td>
<td>lags ( t-2 ) to ( t-5 )</td>
<td>lags ( t-2 ) to ( t-5 )</td>
<td>lags ( t-2 ) to ( t-6 )</td>
<td>lags ( t-2 ) to ( t-5 )</td>
</tr>
<tr>
<td>Estimated marginal adjustment cost for ‘new’ investment</td>
<td>4.3%</td>
<td>2.8%</td>
<td>5.2%</td>
<td>5.5%</td>
<td>46.6%</td>
<td>17.3%</td>
</tr>
<tr>
<td>Estimated marginal adjustment cost for maintenance</td>
<td>1.9%</td>
<td>1.3%</td>
<td>2.5%</td>
<td>2.9%</td>
<td>21.3%</td>
<td>8.7%</td>
</tr>
<tr>
<td>Average depreciation rate</td>
<td>13.5%</td>
<td>7.9%</td>
<td>6.4%</td>
<td>11.1%</td>
<td>5.3%</td>
<td>6.5%</td>
</tr>
</tbody>
</table>

Notes:
1) Initial values for the business sector regressions are \( \delta = 0.15: \gamma = 10, \delta = 0.2: \gamma = 12, \delta = 0.25: \gamma = 15 \) and for the manufacturing sector regressions \( \delta = 0.15: \gamma = 7, \delta = 0.20: \gamma = 12, \delta = 0.25: \gamma = 15 \).
2) See Table 3.
FIGURE 1A. ‘New’ investment and maintenance expenditures in the business sector

FIGURE 1B. ‘New’ investment and maintenance expenditures in the manufacturing sector