Spatial Climate-Economic Models in the Design of Optimal Climate Policies across Locations[1]

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August 26, 2012

Abstract

We couple a one-dimensional energy balance climate model with heat transportation across latitudes, with an economic growth model. We derive temperature and damage distributions across locations and optimal taxes on fossil fuels which, in contrast to zero-dimensional Integrated Assessment Models, account for cross latitude externalities. We analyze the impact of welfare weights on the spatial structure of optimal carbon taxes and identify conditions under which these taxes are spatially nonhomogeneous and are lower in latitudes with relatively lower per capita income populations. We show the way that heat transportation affects local economic variables and taxes, and locate sufficient conditions for optimal mitigation policies to have rapid ramp-up initially and then decrease over time.

Keywords: One-dimensional energy balance model, heat transport, latitudes, temperature distribution, damage distribution, social planner, competitive equilibrium, local welfare weights, optimal taxes.

JEL Classification: Q54, Q58, R11

[1]This research has been co-financed by the European Union (European Social Fund – ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) - Research Funding Program: "Thalis – Athens University of Economics and Business - Optimal Management of Dynamical Systems of the Economy and the Environment."

W. Brock is grateful for financial and scientific support received from The Center for Robust Decision Making on Climate and Energy Policy (RDCEP) which is funded by a grant from the National Science Foundation (NSF) through the Decision Making Under Uncertainty (DMUU) program. He is also grateful to the Vilas Trust for financial support. None of the above are responsible for any errors, opinions, or shortcomings in this article.

The authors thank K. Judd and E. Moyer for valuable discussions and J. Stefan for editing assistance.

1 Introduction

The impact of climate change is expected to vary profoundly among geographical locations in terms of temperature and damage differentials.[2] The spatial dimension of damages can be associated with two main factors: (i) Natural mechanisms which produce a spatially *non-uniform* distribution of the surface temperature across the globe; and (ii) economic-related forces which determine the damages that a regional (or local) economy is expected to suffer from a given increase in the local temperature. These damages depend primarily on the production characteristics (e.g. agriculture vs services) or local natural characteristics (e.g. proximity to the sea and elevation). The interactions between the spatially non-uniform temperature distribution and the spatially non-uniform economic characteristics will ultimately shape the spatial distribution of damages.

Existing literature and in particular the RICE model (e.g. Nordhaus, 2007a,b, 2010, 2011) provides a spatial distribution of damages in which the relatively higher damages from climate change are concentrated in the zones around the equator.[3] However, this model as well as other Integrated Assessment Models (IAMs) does not account for the first factor, the natural mechanism generating temperature distribution across the globe.

In climate science terminology, IAMs with a carbon cycle and no spatial dimension are zero-dimensional models which do not include spatial effects due to heat transportation across space. In contrast, the one- or two-dimensional energy balance climate models (EBCMs) model heat transport across latitudes or across latitudes and longitudes (e.g. Budyko, 1969; Sellers, 1969,1976; North, 1975 a,b; North et al., 1981; Kim and North, 1992; Wu and North, 2007). Since prediction of climate at various spatial scales plays an important role in policy analysis, approaches other than EBCMs have been developed for approximating temperature fields. These are based on more complex and computationally costly models, such as pattern scaling (Lopez et al., 2012) or emulation theory (e.g. Challenor et al., 2006). However, because the purpose of this paper is to construct the simplest coupled climate economy model with a climate feedback response mechanism in space that responds to changes in the spatiotemporal structure of taxes on fossil fuels, and which is still analytically tractable, we considered the EBCMs framework as most appropriate.[4] It should be noted, however, that for a comprehensive analysis of regional climate change and prediction of future

[2] See "Climate observations, projections and impacts" at http://www.metoffice.gov.uk/climate-change/policy-relevant/obs-projections-impacts.

[3] Recent papers by Hassler and Krusell (2012) and Desmet and Rossi-Hansberg (2012) also introduce space and regional issues into models of climate change.

[4] Furthermore, models such as pattern scaling may not be suitable when there are strong nonlinear feedbacks present, such as "snow-albedo feedback at high latitudes" (Challenor et al., 2006). Since we want to allow these nonlinear type of feedbacks, which can be modelled using EBCMs, we did not use pattern scaling.

regional climates, one must turn to the large literature that deals with just that. An approach such as the MAGICC/SCENGEN model, for example, could be considered a very sophisticated combination of an energy-balance model plus pattern scaling, although this is far too simple a description of this kind of work (see Meinshausen et al., 2011). We stress that the purpose of our own work is more modest. We seek a framework simple enough for a pencil and paper analysis to expose, for example, potentially important forces that shape Pareto optimal carbon tax schedules in the face of different possibilities for international transfers. Our framework has not yet been developed to the point where it can deal with important dynamics of the actual climate system, e.g. the time-response of ocean heat uptake, which is needed for a more detailed analysis of economic impacts on the climate system. We hope this kind of exercise will prove useful for economists who are used to working with simple analytical models, but also wish to include more features of the dynamic spatial climate system than is usual in these kinds of models.

One-dimensional EBCMs predict a concave temperature distribution across latitudes with the maximum temperature at the equator. In this paper we study the economics of climate change by coupling a one-dimensional EBCM with heat transport and albedo differentiation across latitudes, with an economic growth model. This approach integrates solution methods for one-dimensional spatial climate models, which may be new to economics, with methods of solving economic models. It may therefore provide new insights regarding issues such as the spatiotemporal structure of optimal policies and the spatial distribution of damages, relative to the zero-dimensional IAMs with carbon cycle, which ignore cross latitude externalities due to heat transport.

The literature on climate and economy is so large that a complete literature review is beyond the scope of this paper. Many scholars besides Nordhaus have written extensively on coupled economy and climate models.[5] However, to our knowledge, there has been no analysis of the shape of socially optimal tax structures in models that have a spatial heat transport mechanism that shapes the dynamics of the temperature field, as we attempt to do. Thus the main contribution of our paper is to couple spatial climate models with economic models, and then use these spatial climate models to achieve three objectives.

The first objective is to show the role of heat transport across latitudes in the prediction of the spatial distribution and the corresponding temporal evolution of temperature and damages. Our results show that heat transport explicitly affects the spatial distribution of temperature and damages, thus its omission from zero-dimensional models which rely on mean global temperature may introduce a bias. As far as we know, this is the first time

[5] See Nordhaus' (2011) review for coverage of some of this work.

that the spatial distribution of surface temperature and damages, and their temporal evolutions, have been determined endogenously by accounting for the interactions between local temperature and regional damages. We therefore believe this to be a contribution of our paper relative to the traditional IAMs with regional disaggregation but without the natural mechanism of heat transport across locations.

The second objective is to provide insights into the optimal spatial and temporal profile for current and future mitigation, when thermal transport across latitudes is taken into account. Regarding the spatial profile of fossil fuel taxes, our results suggest higher tax rates for wealthier geographical zones due to the practical inability of implementing without cost the international transfers needed to implement a competitive equilibrium associated with the Pareto optimum, or when Negishi welfare weights are not used. Our one-dimensional model allows us to study how heat transport across geographical zones impacts the degree of spatial differentiation of fossil fuel taxes between poor and wealthy regions. The result that, in the absence of international transfers, a spatially uniform optimal mitigation is not possible was first noted by Chichilnisky and Heal (1994). Our results provide new insights into this issue by characterizing the spatial distribution of fossil fuel taxes and linking the degree of spatial differentiation of optimal fossil fuel taxes to heat transport.

With regard to the temporal profile of optimal mitigation, the debate among economists dealing with climate change on the mitigation side appears to have basically settled on whether to increase mitigation efforts (that is, carbon taxes) gradually (e.g. Nordhaus, 2007a, 2010, 2011) or rapidly (e.g. Stern, 2006; Weitzman, 2009 a,b). In this paper we locate sufficient conditions for profit taxes on fossil fuel firms to be decreasing over time and for unit taxes on fossil fuels to grow over time more slowly than the rate of return on capital. We also locate sufficient conditions for the tax schedule to be increasing according to the gradualist approach.

The third objective is to introduce spatial EBCMs with heat transport and endogenous albedo into economics as a potentially useful alternative to simple carbon cycle models in studying the economics of climate change.[6]

[6] Another issue that can be addressed by latitude dependent climate models is damage reservoirs. Damage reservoirs in the context of climate change can be regarded as sources of climate damages which will eventually cease to exist when the source of the damages is depleted. Ice lines and permafrost can be regarded as such reservoirs. As the ice lines move closer to the Poles due to climate change, we might expect that marginal damages from this moving will be large at first and then diminish as the ice line approaches the Poles. When there is no ice left at the Poles, this damage reservoir will have been exhausted. The presence of an endogenous ice line in the EBCM allows us to model these types of damages explicitly, given the relevant information. Permafrost is soil at or below the freezing point of water for two or more years. The permafrost feedback suggests that permafrost carbon emissions could affect long-term projections of future temperature change. Studies indicate that up to 22% of permafrost could be thawed already by 2100. Once unlocked under

Since these models are new in economics, we proceed in steps that we believe make this methodology accessible to economists. In section 2 we present a basic one-dimensional EBCM[7] which incorporates human impacts on climate resulting from carbon dioxide (CO_2) accumulation due to use of fossil fuels, which blocks outgoing radiation. In developing the model we follow North (1975 a,b) and use his notation. We use the model to expose solution methods and especially the two-mode approximating approach used in section 3 to numerically approximate latitude dependent temperature and damage functions. Section 4 couples the spatial EBCM with an economic growth model. We solve the model for the social planner and for the competitive equilibrium with taxes. In section 5 we derive the optimal taxes, their spatial structure and their temporal profiles, while in section 6 we show how heat transport affects local economic variables. The last section concludes.

2 An Energy Balance Climate Model with Human Inputs

In this section we develop a one-dimensional EBCM with human inputs. The term "one-dimensional" means that there is an explicit one-dimensional spatial dimension in the model so that our unified model of the climate and the economy evolves both in time and space.[8] We follow North (1975a,b) and North et al. (1981) in this development.

Let x denote the *sine* of the latitude. For simplicity we will just refer to x as "latitude". Following North (1975a,b), let $I(x,t)$ denote infrared radiation to space measured in W/m^2 at latitude x at time t and $T(x,t)$ denote surface (sea level) temperature measured in °C at latitude x at time t. The outgoing radiation and surface temperature can be related through

strong warming, thawing and decomposition of permafrost can release amounts of carbon until 2300 comparable to the historical anthropogenic emissions up to 2000 (approximately 440 GTC) (Schneider et al., 2011). EBCMs, by explicitly introducing the spatial dimension in the climate module of the problem, can help in the understanding of these types of latitude dependent damages which may have an important effect on the temporal and spatial structure of policy instruments, because of the 'front loading' character of damages and the possible relations with tipping points and thresholds. The modeling of damage reservoirs is beyond the scope of this paper, but we believe it represents an important area for further research. For some first results pointing out the role of damage reservoirs in generating nonlinearities and multiple steady states, see Brock et al. (2012).

[7] For more on EBCMs see, for example Pierrehumbert (2008, chapters 3 and 9, especially sections 9.2.5 and 9.2.6 and surrounding material). North et al. (1981) provide a very informative review of EBCMs while Wu and North (2007) is a recent paper on EBCMs.

[8] In contrast, a "zero-dimensional" model does not explicitly account for the spatial dimension. On the other hand more complicated spatial structures could include two-dimensional spherical models. Our methods can easily be applied to a two-dimensional spherical world as in Wu and North (2007).

the empirical formula[9]

$$I(x,t) = A + BT(x,t),\ A = 201.4W/m^2,\ B = 1.45W/(m^2)(^\circ\text{C}). \quad (1)$$

The basic energy balance equation developed in North (1975a, equation (29)) can be written, with human input added, as:

$$\frac{\partial I(x,t)}{\partial t} = QS(x)\alpha(x,x_s(t)) - [I(x,t) - h(x,t)] + D\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial I(x,t)}{\partial x}\right] \quad (2)$$

where $x = 0$ denotes the Equator, $x = 1$ denotes the North Pole, and $x = -1$ denotes the South Pole; Q is the solar constant[10] divided by 2; $S(x)$ is the mean annual meridional distribution of solar radiation which is normalized so that its integral from -1 to 1 is unity; $\alpha(x,x_s(t))$ is the absorption coefficient or co-albedo function which is one minus the albedo of the earth-atmosphere system, with $x_s(t)$ being the latitude of the ice line at time t; and D is a heat transport coefficient. This coefficient is an adjustable parameter which has been calibrated to match observed temperatures across latitudes. It is measured in $W/(m^2)(^\circ\text{C})$ or can be expressed in dimensionless form as D/B (North et al., 1981).

Equation (2) states that the rate of change of outgoing radiation is determined by the difference between the incoming absorbed radiant heat $QS(x)\alpha(x,x_s(t))$ and the outgoing radiation $[I(x,t) - h(x,t)]$. Note that the outgoing radiation is reduced by human input $h(x,t)$. Thus human input at time t and latitude x can be interpreted as the impact of the accumulated CO_2 that reduces outgoing radiation.

We define $h(x,t) = \xi \ln\left(1 + \frac{M(t)}{M_0}\right)$ where M_0 denotes the preindustrial concentration of atmospheric carbon dioxide (CO_2) in the atmosphere, $M(t)$ the concentration at time t , and $\xi = 5.35$ is a temperature-forcing parameter ($^\circ$C per W per m^2). The stock of CO_2 evolves according to:

$$\dot{M}(t) = \int_{x=-1}^{x=1} \beta q(x,t)\,dx - mM(t),\ M(0) = M_0 \quad (3)$$

where $\beta q(x,t)$ are emissions generated at latitude x, with emissions being proportional to the amount of fossil fuels used by latitude x at time t.[11] $M(t)$

[9] It is important to note that the original Budyko (1969) formulation cited by North parameterizes A, B as functions of fraction cloud cover and other parameters of the climate system. North (1975b) points out that due to non-homogeneous cloudiness, A and B should be functions of x. There is apparently a lot of uncertainty involving the impact of cloud dynamics (e.g. Trenberth et al., 2010 versus Lindzen and Choi, 2009). Hence robust control in which A, B are treated as uncertain may be called for, but this is left for further research.

[10] The solar constant includes all types of solar radiation, not just the visible light. It is measured by satellite to be roughly 1.376 W/m^2.

[11] β could be changing over time due to technical progress.

should be interpreted as the stock of man-made CO_2 in the atmosphere. With preindustrial as the baseline this means that the interpretation becomes CO_2 in excess of preindustrial. Hence, if q is set to zero, then when $M(t)$ goes to zero it signifies a return to preindustrial levels of atmospheric CO_2.

We assume that the total stock of fossil fuel available is fixed or,

$$\int_{x=-1}^{x=1} q\left(x,t\right) dx = q\left(t\right) \ , \ \int_{0}^{\infty} q\left(t\right) = R_0 \tag{4}$$

where $q\left(t\right)$ is total fossil fuels used across all latitudes at time t, and R_0 is the total available amount of fossil fuels on the planet. Thus in this model, use of fossil fuels generates emissions and emissions increase the stock of atmospheric CO_2, which in turn increases the temperature by blocking the outgoing radiation.

Returning to equation (2), in equilibrium the incoming absorbed radiant heat at a given latitude is not matched by the net outgoing radiation and the difference is made by the meridional divergence of heat flux which is modelled by the term $D\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial I(x,t)}{\partial x}\right]$ (North,1975b). This term explicitly introduces the spatial dimension stemming from the heat transport into the climate model. The ice line is determined dynamically by the condition (Budyko, 1969; North, 1975 a,b):

$$\begin{array}{ll} T > -\widetilde{T}°\mathrm{C} & \text{no ice line present at latitude } x \\ T < -\widetilde{T}°\mathrm{C} & \text{ice present at latitude } x \end{array} \tag{5}$$

where $-\widetilde{T}$ is empirically determined (e.g. -10°C). Below the ice line absorption drops discontinuously because the albedo jumps discontinuously. For example North (1975a) specifies a discontinuous co-albedo function:

$$\alpha(x, x_s) = \begin{cases} \alpha_0 = 0.38 & |x| > x_s \\ \alpha_1 = 0.68 & |x| < x_s \end{cases} \ . \tag{6}$$

2.1 Approximating Solutions for the Basic Energy Balance Equation

We turn now to a more detailed analysis of the solution process in which, although equation (2) is a PDE, the climate problem can be reduced to the optimal control of a small number of "modes" where each mode follows a simple ODE. We believe this decomposition is another contribution of our paper to the study of coupled economic and climate models.

North (1975 a,b) approached the solution of (2) by using approximation methods.[12] The solution is approximated as $\hat{I}(x,t) = \sum_{n \text{ even}} I_n(t) P_n(x)$,

[12] For a general approach to approximation methods see, for example, Judd (1998).

where $I_n(t)$ are solutions to appropriately defined ODEs and $P_n(x)$ are even-numbered Legendre polynomials. A satisfactory approximation of the solution for (2) can be obtained by the so-called two-mode solution where $n = \{0, 2\}$. We develop here a two-mode solution given the human forcing function $h(x,t)$. Since the temperature will be the basic state variable, we redefine (2) using (1), in terms of temperature $T(x,t)$ as:

$$\begin{aligned} B\frac{\partial T(x,t)}{\partial t} &= QS(x)\alpha(x,x_s) - [(A + BT(x,t)) - h(x,t)] + \\ & \quad DB\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T(x,t)}{\partial x}\right]. \end{aligned} \tag{7}$$

Using the approximation $\hat{T}(x,t) = \sum_{n \text{ even}} T_n(t)P_n(x)$, where now $T_n(t)$ are solutions to appropriately defined ODEs, the two-mode solution is defined as:

$$\hat{T}(x,t;D) = T_0(t) + T_2(t;D)P_2(x) \tag{8}$$

$$\begin{aligned} B\frac{dT_0(t)}{dt} &= -A - BT_0(t) + \\ & \quad \int_{-1}^{1}\left[QS(x)\alpha(x,x_s) + \xi \ln\left(1 + \frac{M(t)}{M_0}\right)\right]dx \end{aligned} \tag{9}$$

$$\begin{aligned} B\frac{dT_2(t)}{dt} &= -B(1+6D)T_2(t) + \\ & \quad \frac{5}{2}\int_{-1}^{1}\left[QS(x)\alpha(x,x_s) + \xi \ln\left(1 + \frac{M(t)}{M_0}\right)\right]P_2(x)dx \end{aligned} \tag{10}$$

$$T_0(0) = T_{00}, T_2(0) = T_{20}, P_2(x) = \frac{(3x^2-1)}{2} \tag{11}$$

$$S(x) = 0.5\left[1 + S_2P_2(x)\right]\ ,\ S_2 = -0.482. \tag{12}$$

The derivation of the solution is presented in Appendix 1.[13] Given the definitions of the functional forms, the two-mode solution is tractable and can be calculated given initial conditions T_{00}, T_{02} which are determined by the initial climate state. In the two-mode solution, the ice line function $x_s(t)$ which determines the co-albedo solves the equation $I_s = I(x_s(t),t)$. In terms of temperature and using the two-mode solution, the ice line function solves

$$\hat{T}(x,t;D) = T_0(t) + T_2(t;D)P_2(x_s(t)) = T_s,\ T_s = -\widetilde{T}°\text{C} \tag{13}$$

[13] The two-mode solution is an approximating solution. We can develop a series of approximations of increasing accuracy by solving this problem for expansions using a "two-mode" solution, a "three mode" solution and so on. North's results suggest that the two-mode solution is an adequate approximation for nonoptimizing models. We use the two-mode approximation in our optimal control setting. Further research could investigate how many modes are needed for a good quality approximation in an optimal control setting.

and the ice line function is given by a solution of (13), i.e.

$$x_s(t) = P_+^{-1}\left(\frac{T_s - T_0(t)}{T_2(t;D)}\right), \tag{14}$$

where the subscript "+" denotes the largest inverse function of the quadratic function $P_2(x) := (1/2)(3x^2 - 1)$. Notice that the inverse function is unique and is the largest one on the set of latitudes $[-1, 1]$. Thus there exists a nonlinear feedback from changes in temperature to the co-albedo through the endogeneity of the ice line. This feedback can be simplified by making the co-albedo function $\alpha(x, x_s)$ a smooth function of the temperature, $\alpha\left(x, \hat{T}(x,t;D)\right)$, which can be highly nonlinear around $-\widetilde{T}$°C. For example the co-albedo function

$$\alpha(x, T(x,t)) = c_0 + c_1 \tanh(T(x,t) + 10) \text{ for } (c_0, c_1) = (.525, .195) \tag{15}$$

provides a good approximation of the discontinuous function (5) at $-\widetilde{T}$°C=−10°C. A more simplified and tractable specification of the co-albedo, but without the nonlinear feedback, is the one introduced by North et al. (1981, p. 95 equation (18)), where

$$a(x) = 0.681 - 0.202 P_2(x). \tag{16}$$

2.1.1 Use of global mean temperature and potential bias

In the two-mode solution that defines the climate module by (8)-(12), and (3)-(4), spatial interactions are incorporated through the mode-2 part of the solution, i.e. the ODE (10). Thus the contribution of the second mode to the full solution can be regarded as the "importance of space" through heat transport, in the analysis of climate change. This can be seen by the following argument.

The size of the coefficient D determines the speed of transport of heat from warm areas to cool areas in (7). Assume $D = 0$, then if extra heat energy is emitted by latitude x, damages stay in latitude x, that is there are no cross latitude externalities. If D is positive, then an increase in heat energy in a given latitude will cause damages in other latitudes due to heat transfer. If D is infinite, externalities are "uniform" in the sense that any extra heat energy emitted by latitude x is uniformly distributed over the whole planet and the heat transport across latitudes is not relevant for our problem. In this case, the mode-2 solution vanishes. To show this, note that since the total amount of fossil fuel is finite and the contributions to the stock of atmospheric CO_2 are due to the use of fossil fuels, the stock of CO_2 $M(t)$ must be bounded above. Thus the second term of the right hand side of (10) is bounded above. Then the following proposition can be stated.

Proposition 1 *Assume that* $\int_{-1}^{1}\left[QS(x)\alpha(x,x_s)+\xi\ln\left(1+\frac{M(t)}{M_0}\right)\right]P_2(x)dx=\Phi(t)\leq UB<\infty$, *and that* $D\to\infty$. *Then the solution* $T_2(t)$ *of (10) vanishes.*

For the proof see Appendix 2. Thus for a given transport $D<\infty$, the relative contribution of $T_2(t)$ to the solution $\hat{T}(t)$ can be regarded as a measure of whether the heat transport is important in the solution of the problem. This result suggests that the use of the global mean temperature alone in IAMs may introduce a bias. The global mean temperature is obtained for the two-mode approximation as $m_T=\int_{-1}^{1}\hat{T}(x,t)dx=\int_{-1}^{1}\left[T_0(t)+T_2(t)P_2(x)\right]dx=T_0(t)$, since $\int_{-1}^{1}P_2(x)dx=0$. This result, along with Proposition 1, indicates that the zero-dimensional IAMs can be regarded as a special case of a one-dimensional model when $D\to\infty$. In these models the second mode that provides the spatial distribution of temperature is omitted. Since evidence indicates that D is less than infinity (e.g. North et al., 1981), our result suggests that omitting the second mode introduces a bias because cross latitude externalities are ignored. In our paper we attempt to correct for this underlying bias by keeping that second mode, and to provide a basis for a quantitative representation of this bias.

3 Temperature and Damage Functions: Approximations and Calibrations

In this section we use the two-mode approximating solution, along with some additional simplifications of the climate model, to provide analytically tractable results regarding latitude dependent local temperature and damage functions.

3.1 Simplifications of the Climate Model

We use the two-mode approximating solution with two simplifications: (i) the co-albedo function does not explicitly depend on $T(t)$ and can be written as $a(x)=a_0-a_1P_2(x)$; and (ii) $S(x)=0.5\left[1-s_0P_2(x)\right]$ (North et al., 1981). Using the the inner product notation $\langle f(x)g(x)\rangle=\int_X f(x)g(x)dx$, the two-mode approximating ODEs become:

$$\dot{T}_0 = -\frac{A}{B}-T_0(t)+\frac{1}{B}\left[\langle QS(x)\alpha(x),1\rangle+\xi\ln\left(1+\frac{M(t)}{M_0}\right)\langle 1,1\rangle\right] \quad (17)$$

$$\dot{T}_2 = -(1+6D)T_2(t)+\frac{5}{2B}\langle QS(x)\alpha(x),P_2(x)\rangle. \quad (18)$$

A possible parameterization is shown in table 1.

Table 1: Parametrization*

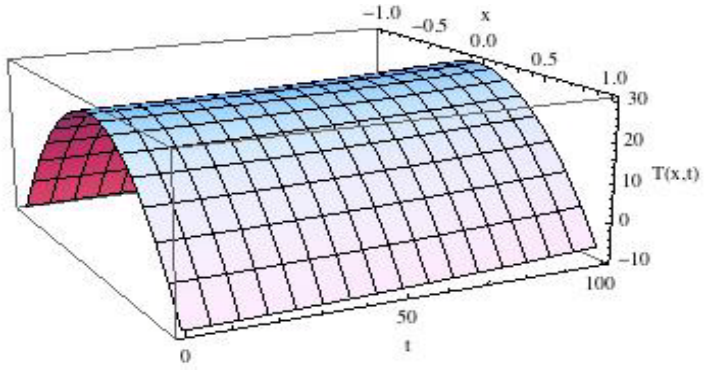

Figure 1: The temperature function

Parameter	Value	Parameter	Value
a_0	0.681	Q	340 W/m^2
a_1	0.202	M_0	596 GtC
s_0	0.477	M (2011)	831 GtC
A	221.6 W/m^2	ξ	5.35 °C (W/m^2)
B	1.24 W/(m^2)(°C)	g	1.178% IPCC A1F1 scenario
D	0.3 W/(m^2)(°C	m	0.83%

(*)Values for the dimensionless α_0, a_1, s_0 have been obtained by North et al. (1981). Values for A, B, D have been obtained by calibration so as to reproduce current global temperature. Their values are very close to those appearing in the work of Budyko and North. $g = 1.178\%$ is the average annual growth of total CO_2 emissions corresponding to the IPCC scenario A1F1 (http://www.ipcc-data.org/sres/ddc_sres_emissions.html)

Assume that T_0 and T_2 are evolving in a faster time scale than M and that they relax fast to their respective steady states, so we assume $\frac{dT_0}{dt} = \frac{dT_2}{dt} = 0$. Then temperature can be expressed as a function of M as:

$$\hat{T}(x,t;D) = C_0 + C_1 \ln\left(1 + \frac{M(t)}{M_0}\right) - \frac{C_2}{(1+6D)} P_2(x) \ \ , C_0, C_1, C_2 > 0. \tag{19}$$

The corresponding temperature function is shown in figure 1 with $t = 0$ corresponding to year 2011.

This temperature function implies a current average temperature of approximately 27°C for the equator and -9.5°C for the Poles. The predicted temperature increase for a horizon of 100 years is 3.2°C for the IPCC A1F1 emissions scenario. It is worth noting that similar temperature functions have been derived by climate scientists (e.g. Sellers, 1969, 1976), but without the impact of human activities on climate. In our case this impact is realized by the increase in the concentration of atmospheric carbon dioxide. When $D \to \infty$, the temperature function is spatially homogeneous or "flat" across latitudes around 14.8°C for 2011 and 14.4°C for the period 1951-1980.

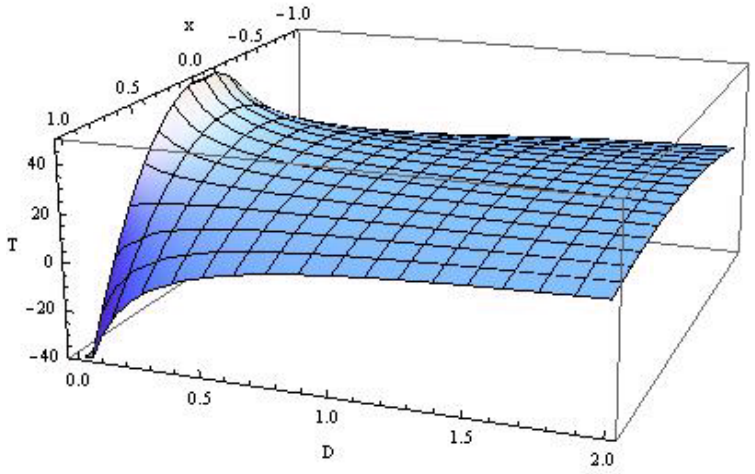

Figure 2: Temperature and thermal transport

Note that NASA's estimate of the absolute global mean temperature for the base period 1951-1980 is 14°C (data.giss.nasa.gov/gistemp/). The impact of thermal transport is made clear in figure 2, which depicts temperature as a function of latitude and the thermal transport coefficient D. As D increases, the temperature function becomes spatially flat as the increase in transport speed results in homogeneity. Thus an increase in D will warm the Poles in the two-mode solution more (and cool x near zero more) for a given increase in $M(t)$ as in figure 2 (or figure 4 of North et al., 1981, page 95).[14] The distinction between a latitude dependent and a flat temperature field provides a first sign of the impact that thermal transport may have on the temperature and damage functions.

Current empirical work based on zero-dimensional IAMs defines damages in terms of changes in the mean global temperature with respect to a base period (e.g.1890-1900). Since we are interested in the implications of thermal transport across latitudes we define damages in terms of the levels of the temperature. In this way we can trace the impact of the thermal transport on damages and perform meaningful comparative statics with respect to D.

Let $\Omega(x,t;D) = \exp\left(\gamma(\mathbf{A}(t),t)\hat{T}(x,t;D)\right), \gamma < 0$ denote the proportion of GDP available at a latitude x and time t after damages due to climate change have been accounted for. The elasticity of marginal damages with respect to the temperature is $\gamma(\mathbf{A}(t),t)\hat{T}$, where $\mathbf{A}(t)$ denotes adaptation expenses with $(\partial\gamma/\partial\mathbf{A}) < 0$. Thus an increase in temperature will increase damages when adaptation is fixed.[15] We calibrate the parameter γ so that:

[14] This is in accordance to NASA's concept of temperature anomaly (data.giss.nasa.gov/gistemp/).

[15] This formulation could be useful in an extension of the present economic model that

(i) currently (i.e. for $t = 0$) $\Omega(x, 0; D) = 1$ for all x, which means there are no damages at the current temperature level, and (ii) at $t = 90$, when the temperature function of figure 1 predicts an increase in temperature of approximately 2.5°C, $\Omega(0, 90; 100) = 0.9609$. This means that when D is very large, as in a zero-dimensional IAM, damages (i.e. $1 - \Omega$) at the equator are 3.91% of GDP. This value corresponds to a regional damage of 3.91% in output for Africa after a 2.5°C increase in global mean temperature obtained by Nordhaus and Boyer (1999, pp. 4-44). In the absence of information about adaptation expenses, the requirements for this calibration imply that $\gamma(\mathbf{A}, t) = \gamma t$, and result in $\gamma = -0.0000252$. This damage function is shown in figure 3.

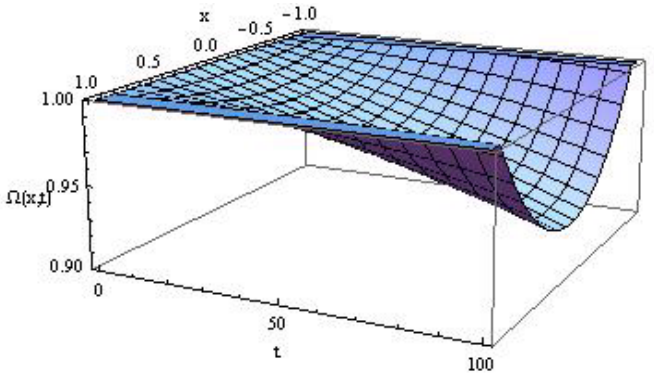

Figure 3: The damage function

It should be noted that the temperature effect around the equator combined with the regional damage effect predicts damages around 6.5% of GDP after 90 years.[16] Figure 4 depicts the damage function at $t = 90$ as a function of the thermal transport coefficient and shows that an increase in thermal transport tends to equalize damages across latitudes.[17]

included adaptation expenses as a choice variable.

[16] Using historic data on temperature fluctuations, Dell et al. (2012) find that higher temperatures substantially reduce economic growth in poor countries.

[17] The impact of thermal transport in the climate model suggests other potential sources of damages. An increase in D will warm the Poles and cool x near the equator for a given increase in $M(t)$ as in figure 2. This may trigger an accelerated melting of the ice sheets and release a "flood" that engulfs the inhabitants of x near zero who are assumed to live in low lying coastal areas, e.g. Bangladesh. Damages to x near zero increase more the bigger the flood. Notice that this mechanism models an "event" near the Poles that happens to increase damages near $x = 0$ and an increase in D makes that event happen with more intensity. A similar event could be a larger increase in permafrost melt with a larger D for a given increase in $M(t)$ as heat is transferred from the equator to the Poles. The endogenous ice line, defined in (15), is a nonlinear feedback that affects the co-albedo function. As D increases and warming near the Poles increases as well, the co-albedo feedback is increased even more, and the ice lines move towards the Poles, generating

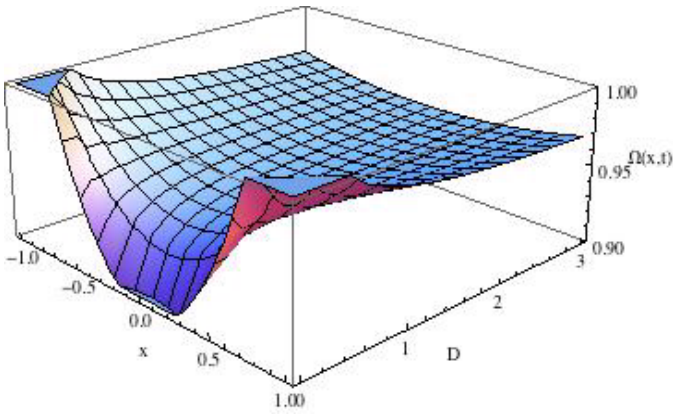

Figure 4: Damages and thermal transport

In the one-dimensional climate model we can define climate response functions (CRF) in the context of e.g. Mendelsohn and Schlesinger, 1999 and Mendelsohn et al., 2000 which determine the changes in temperature and damages at latitude x and time t, resulting from an exogenous change in the atmospheric concentration $M(t)$. To obtain a CRF, the steady state equations of the two-mode approximation ODEs (17)-(18) obtained for $\frac{dT_0}{dt} = 0$, $\frac{dT_2}{dt} = 0$ can be used to define the CRF in two stages. First for a change in $M(t)$, determine the change in $T_0(x,t)$ and $T_2(x,t)$ by using standard comparative statics to obtain $\frac{dT_0}{dM}, \frac{dT_2}{dM}$ and then determine the change in $\Omega\left(\hat{T}(x,t)\right) \equiv \Omega\left(T_0(t) + T_2(t) P_2(x)\right)$ by calculating the total differential of $\Omega\left(\hat{T}(x,t)\right)$. A potentially useful new element of this paper is that our spatial model allows us to predict in a relatively straightforward way the full geographical distribution of changes in damages across latitudes.[18]

4 An Economic Energy Balance Climate Model

We couple now the climate model with a model of the economy.

---

potential damages to lower latitudes. All these potentially important feedbacks can be modelled in the context of the one-dimensional spatial model. Since we do not have information about these feedbacks which would allow an approximate calibration, this task is left for further research.

[18] Our approach does not provide a sectoral breakdown of damages due to climate change as in Robert Mendelsohn's work, but this can be accommodated by appropriate disaggregation of the damage function to reflect sectoral damages. This requires multisector modeling which is beyond the scope of this paper and is left for further research.

4.1 Potential world output and damages from climate change

Output at each location of our economy is produced according to a standard neoclassical production function which is assumed to be of the Cobb Douglas form with constant returns to scale and exponentially growing total factor productivity (TFP), or

$$\begin{aligned} &Y(t,x) = \mathbb{A}(x,t)\Omega(T(x,t))F(K(x,t),L(x,t),q(x,t)) \\ &= \mathsf{e}^{at}\mathbb{A}(x,0)\Omega(T(x,t))K(x,t)^{\alpha_K}L(x,t)^{\alpha_L}q(x,t)^{\alpha_q} \\ &= \mathsf{e}^{(a+n\alpha_L)t}\mathbb{A}(x,0)L(x,0)^{\alpha_L}\Omega(T(x,t))K(x,t)^{\alpha_K}q(x,t)^{\alpha_q} \\ &\equiv \mathsf{e}^{(a+n\alpha_L)t}\Psi(x,T(x,t))K(x,t)^{\alpha_K}q(x,t)^{\alpha_q} \end{aligned} \tag{20}$$

where $K(x,t), L(x,t), q(x,t)$ denote capital, labour and fossil fuels respectively used at latitude (location) x and time t; $\mathbb{A}(x,t)$ is TFP growth; n is population growth; and $\Omega(T(x,t)$ are damages to output due to climate change at latitude x and time t as a function of temperature at the same latitude, with $\frac{\partial\Omega(T(x,t))}{\partial T} < 0$.

In this economy we define by $F_{total}(K(t), q(t), \{T(x,t)\}_{x=-1}^{x=1}; t)$ the "*potential world GDP at date t*". This concept represents the maximum output that the whole world can produce given total world capital $K(t)$ available and total world fossil fuel $q(t)$ used, for a given distribution of temperature $T(x,t)$ across the globe, with labor growing at a constant rate n, and treated as realistically immobile.[19] Thus F_{total} can be regarded as a natural baseline under ideal world conditions where there are no barriers to capital, and fossil fuel flows to their most productive uses across latitudes.[20] We abuse notation and write $F_{total}(K(t), q(t), \{T(x,t)\}_{x=-1}^{x=1}; x, t) = F_{total}(K(t), q(t), T; t)$. The overall resource constraint for the economy can then be defined as:

$$C(t) + \dot{K}(t) + \delta K(t) = F_{total}(K(t), q(t), T; t) \tag{21}$$

where total consumption, capital and fossil fuel are defined over all latitudes as $\eta(t) = \int_X \eta(x,t)\,dx$, $\eta = C, K, q$ respectively, for all $x, x' \in X = [-1, 1]$. As shown in Appendix 3, potential world GDP can be defined as:

$$K(x,t) = [\Psi(x, T(x,t;D))^{1/\alpha_L} / \int_X \Psi(x', T(x',t)^{1/a_L} dx']K(t) \tag{22}$$

$$q(x,t) = [\Psi(x, T(x,t;D))^{1/\alpha_L} / \int_X \Psi(x', T(x',t)^{1/a_L} dx']q(t) \tag{23}$$

$$F_{total}(K(t), q(t), T; t) = \left[\mathsf{e}^{(a+\alpha_L n)t}K(t)^{\alpha_K}q(t)^{\alpha_q}\right] J(t;D) \tag{24}$$

[19] Labor immobility at a global scale could be regarded as a reasonable approximation given restrictions on labor mobility relative to capital and fossil fuel mobility.

[20] This notion can be regarded as similar to the notions of "potential GDP", "potential output", etc. used by macroeconomists.

where

$$J\left(x,t;D\right) = \frac{\Psi(x,T(x,t))^{1/\alpha_L}}{\left[\int_X \Psi(x',T(x',t)^{1/a_L}dx'\right]^{a_K+a_q}} \tag{25}$$

$$J\left(\left\{T(x,t)\right\}_{x=-1}^{x=1}\right) = J(t;D) \equiv \int_X J\left(x,t;D\right)dx \tag{26}$$

define damages at a specific location x and global damages respectively.

The Cobb-Douglas specification allows the "separation" of the climate damage effects on production across latitudes, since the "index" $J(t;D)$, which depends on the thermal transport coefficient D, multiplies a production function that is independent of x. Thus population growth and technical change affect the "macrogrowth component" $\mathsf{e}^{(a+\alpha_L n)t}K(t)^{\alpha_K}q(t)^{\alpha_q}$, while changes in the size of D have a direct effect on the "climate component". The combination of the macrogrowth and the climate components determines the potential world input. This separability property allows for more tractable analytical and numerical work regarding the importance of the spatial dimension in the economic-climate model.

From the consumer side, the idea of working at the global scale suggests a welfare optimization problem that can be interpreted as the maximization of the welfare of an "aggregate dynastic consumer family" subject to an aggregate production function. This problem is defined and presented in Appendix 4.

4.2 Global welfare maximization

We analyze the welfare maximization problem of a social planner in the context of the coupled EBCM-growth model. Allowing for per capita damages in utility due to climate change given by $\Omega_C(T(x,t)$, with $\partial\Omega_C(T(x,t))/\partial T > 0$, the economic part of this problem is defined in terms of the potential world GDP and a Ramsey-like form for the aggregate dynastic consumer family as:

$$\max \int_0^\infty \mathsf{e}^{-\rho t}\int_X \upsilon\left(x\right)L(x,t)\left[U\left(\frac{C(x,t)}{L(x,t)}\right)-\Omega_C(T(x,t))\right]dxdt \tag{27}$$

subject to (21), (3), (7), the total consumption and total fossil fuel constraints, along with the appropriate initial conditions, where $\upsilon\left(x\right)$ are exogenously given nonnegative welfare weights. Varying the weights, all the Pareto efficient allocations can be traced. Assuming zero extraction cost for the fossil fuels,[21] the current value Hamiltonian for this problem can be written as:

[21] This simplifying assumption does not affect the validity of our results.

$$
\begin{aligned}
\mathcal{H} \;=\; & \int_X \upsilon(x)\, L(x,t)\left[U\left(\frac{C(x,t)}{L(x,t)}\right) - \Omega_C(T(x,t))\right] dx + \\
& \lambda_K(t)\left[F_{total}(K(t), q(t), T; t) - C(t) - \delta K(t)\right] \\
& -\mu_R(t)\, q(t) + \lambda_M(t)\left[\int_X \beta q(x,t)\, dx - mM(t)\right] \\
& +\lambda_T(t,x)\left[\frac{1}{B}\left[QS(x)\alpha(x, T(x,t)) - (A + BT(x,t))\right.\right. \\
& \left.\left. +\xi \ln\left(1 + \frac{M(t)}{M_0}\right) + DB\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T(x,t)}{\partial x}\right]\right]\right. \\
& \left. +\mu_C(t)\left[C(t) - \int_X C(x,t)\, dx\right] + \mu_q(t)\left[q(t) - \int_X q(x,t) dx\right].\right.
\end{aligned}
\tag{28}
$$

In this problem the state and controls are $\mathbf{v} = (K(t), R(t), M(t), T(t,x))$, $\mathbf{u} = (C(t), C(x,t), q(t), q(x,t))$, respectively, $x \in X = [-1, 1]$.

The maximum principle implies for the controls:[22]

$$
C(t), C(x,t) \;:\; \lambda_K(t) = \mu_C(t) = \upsilon(x)\, U'\left(\frac{C(x,t)}{L(x,t)}\right) \tag{29}
$$

$$
q(t) \;:\; \lambda_K(t)\, F'_{total,q} = \mu_R(t) - \mu_q(t) \tag{30}
$$

$$
q(x,t) \;:\; \lambda_M(t)\,\beta = \mu_q(t) \tag{31}
$$

$$
\text{or } F'_{total,q} \;=\; \frac{\mu_R(t) - \lambda_M(t)\,\beta}{\lambda_K(t)}. \tag{32}
$$

For equal welfare weights, (29) implies that per capita consumption should be equated across locations. For the costates we have:

$$
\dot{\lambda}_K(t) = \left[\rho + \delta - F'_{total,K}(K(t), q(t), T; t)\right]\lambda_K(t) \tag{33}
$$

$$
\dot{\mu}_R(t) = \rho \mu_R(t) \tag{34}
$$

$$
\dot{\lambda}_M(t) = (\rho + m)\,\lambda_M(t) - \frac{\xi}{B\left(1 + \frac{M(t)}{M_0}\right)}\int_X \lambda_T(t,x)\, dx \tag{35}
$$

$$
\begin{aligned}
& \dot{\lambda}_T(t,x) = (\rho + 1)\,\lambda_T(t,x) + \upsilon(x)\, L(t,x)\,\Omega'_{c,T}(T(t,x)) \\
& -\lambda_K(t)\, F'_{total,T}(K(t), q(t), T; t) - \\
& QS(x)\frac{\lambda_T(t,x)}{B}\frac{\partial \alpha(x, T(x,t))}{\partial T} - D\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial \lambda_T(x,t)}{\partial x}\right].
\end{aligned}
\tag{36}
$$

[22] Since problem (27) is nonautonomous, we assume that the discount rate is sufficiently high and that the functions of the problem satisfy the growth conditions required to apply the Pontryagin maximum principle (Malysh, 2008). To ease notation, sometimes we denote derivatives by the subscript for the relevant variable and a $(')$.

The last term on the RHS of (36) is obtained by applying the maximum principle.[23] A solution of the welfare maximization problem, provided it exists and satisfies the desirable stability properties, will determine the optimal temporal and latitudinal paths for the states, the controls and the costates. Denoting optimality by $(^*)$, these paths can be written as:

$$\begin{aligned}
&\{K^*(t;D), K^*(t,x;D)\, R^*(t;D), M^*(t;D), T^*(t,x;D)\}_{x=-1}^{x=1} \\
&\{C^*(t;D), C^*(x,t;D), q^*(t;D), q^*(x,t;D)\}_{x=-1}^{x=1} \\
&\{\lambda_K^*(t;D), \lambda_M^*(t;D), \mu_R^*(t;D), \lambda_T^*(t,x;D)\}_{x=-1}^{x=1}. \qquad (37)
\end{aligned}$$

Substituting these paths into (25) and (26) will determine the optimal damages from climate change on a global or a location basis.

4.3 Competitive Equilibrium with Fossil Fuel Taxes

To study the optimal taxation problem we consider a global market economy with each latitude x considered as a country. In each country the representative consumer maximizes utility subject to a permanent income constraint by considering as parametric damages due to climate change, and the representative firm maximizes profits by considering as parametric fossil fuel world prices and taxes on fossil fuel use. World fossil fuel firms maximize profits by considering as parametric taxes on their profits.

4.3.1 Consumers

Consumers at latitude (or country) x are a "dynastic family" that takes $\Omega_C(T(x,t)) = \bar{\Omega}_C$ as parametric beyond their control, and can borrow and lend on world bond markets at the rate $r(t)$. Assume that after-tax profits from fuel firms $\pi_{FF}(t)$ are redistributed lump sum to latitude x consumers in the fraction $s_{FF}(x,t)$ and proceeds from fuel taxes $Tax(t)$ are redistributed lump sum to latitude x consumers in the fraction $s_{Tax}(x,t)$.[24] Set $\Gamma(t) = \int_{s=0}^{t} r(s)ds$ and impose the "solvency" constraints

$$B(x,t)\mathsf{e}^{-\Gamma(t)} \to 0, \;\; K(x,t)\mathsf{e}^{-\Gamma(t)} \to 0 \;\; , \; t \to \infty \qquad (38)$$

for bonds $B(x,t)$ held at location x and time t, with $B(x,0) = 0$, and capital $K(x,t)$. The consumer's budget constraint can be written in present value form as:

[23] In the derivation of the conditions of the maximum principle, we need to differentiate by parts twice with respect to x, in order to express the derivatives of T with respect to x in terms of derivatives of λ_T with respect to x. The detailed argument is presented in Appendix 5.

[24] In baseline analysis using Arrow Debreu private ownership economies, it is standard to assume perfect markets (borrowing and lending with no frictions, defaults, etc.) with profits and taxes redistributed lump sum to consumers.

$$\int_{t=0}^{\infty} \mathsf{e}^{-\Gamma(t)} p^s(t)\, C(x,t)dt = K_0(x) + \int_{t=0}^{\infty} \mathsf{e}^{-\Gamma(t)} p^s(t)\, I(x,t)dt \quad (39)$$

$$K_0(x) = K(0,x) \quad (40)$$

$$I(x,t) \equiv w(x,t)L(x,t) + s_{FF}(x,t)\pi_{FF}(t) + s_{Tax}(x,t)Tax(t) \quad (41)$$

where $p^s(t)$ is the spot price of the consumption good at time t. The consumer solves:

$$\max_{\{C(x,t)\}} \left\{ \int_{t=0}^{\infty} \mathsf{e}^{-\rho t} \left[L(x,t)U\left(\frac{C(x,t)}{L(x,t)}\right) - \bar{\Omega}_C \right] dt \right\} \quad (42)$$

with optimality condition:

$$U'\left(\frac{C(x,t))}{L(x,t}\right) = \Lambda(x)\mathsf{e}^{\rho t} p^C(t) \quad (43)$$

where $\Lambda(x)$ is the Lagrangian multiplier for the permanent income constraint (39) expressing the marginal utility of capitalized income at location x, and $p^C(t) = \mathsf{e}^{-\Gamma(t)} p^s(t)$. We obtain an exact correspondence between the equilibrium problem and the planner's problem with a corresponding optimality condition,

$$\upsilon(x)\, U'\left(\frac{C(x,t))}{L(x,t}\right) = \lambda_K(t;D), \quad (44)$$

by letting $p^C(t) = \mathsf{e}^{-\rho t}\lambda_K(t;D)$. The welfare weights are the reciprocal of marginal utility, or the so-called Negishi weights, $\upsilon(x) = 1/\Lambda(x)$,[25] which is the First Theorem of Welfare Economics. In terms of the Second Theorem of Welfare Economics, any solution of the planner's problem for any arbitrary nonnegative set of welfare weights across locations will satisfy the conditions for competitive equilibrium except for the budget constraint in each location. Budget constraints can be satisfied with appropriate transfers across locations. Thus a solution to the planner's problem resulting from a specific choice of welfare weights can be implemented as a competitive equilibrium with transfers across locations. The choice of zero transfers corresponds to the case of using the Negishi weights as welfare weights.[26]

[25] By the definition of the current value Hamiltonian for the planner's problem, $\lambda_K(t) \equiv \hat{\lambda}_K(t)\,\mathsf{e}^{\rho t}$ is the current value costate variable, where $\hat{\lambda}_K(t)$ is the costate variable relative to time zero.

[26] Note that per capita consumption will not be equated across latitudes unless $\Lambda(x) = \Lambda(x')$ for all x, x'. For equality of the marginal utility across latitudes we will need to assume, following the theory of the Second Welfare Theorem, that intertemporal endowment flows are adjusted so that $\Lambda(x) = \Lambda(x')$ for all x, x'.

4.3.2 Firms Producing Consumption Goods

Firms located at latitude x which produce consumption goods solve the problem

$$\begin{aligned}&\max\ p^C(t)\left[\mathbb{A}(x,t)\Omega(T(x,t))F(K(x,t),L(x,t),q(x,t))-\right. \\ &\left.(r(t)+\delta)K(x,t)-L(x,t)-(p(x,t)+\tau(x,t))\,q(x,t)\right]\end{aligned} \tag{45}$$

where $p(x,t)$ is the price paid for fossil fuels at time t, $w(x,t)$ is the wage at location x and time t,[27] and $\tau(t,x)$ is a tax on fossil fuels paid by the representative firm located at point x, which we will consider as a carbon tax, and $F(K,L,q)$ is constant returns to scale. Hence profits will be zero at each x for firms that produce consumption goods. The optimality conditions for the optimal choices for K and q imply:

$$\mathbb{A}(x,t)\Omega(T(x,t;D))F'_K(K(x,t),L(x,t),q(x,t)) = r(t)+\delta \tag{46}$$

$$\mathbb{A}(x,t)\Omega(T(x,t;D))F'_q(K(x,t),L(x,t),q(x,t)) = p(x,t)+\tau(x,t) \tag{47}$$

$$\mathbb{A}(x,t)\Omega(T(x,t;D))F'_L(K(x,t),L(x,t),q(x,t)) = w(x,t). \tag{48}$$

Thus in any decentralized problem, latitude x firms will choose demands $K(x,t)$ and $q(x,t)$ according to (46) and (47). Note that these marginal value products are equated across x's for every date t only if taxes on fossil fuels are equal across locations or $\tau(x,t)=\tau(t)$. Furthermore if fossil fuel prices are equated across locations through competition, then $p(x,t)=p(t)$.

4.3.3 Fossil fuel firms

World fossil fuel firms solve the problem

$$\max_{q(x,t)}\ \int_{t=0}^{\infty}\mathsf{e}^{-\Gamma(t)}p^C(t)\left[(p(t)q(x,t)(1-\theta(t))\right]dt, \tag{49}$$

$$\text{subject to}\ \int_{t=0}^{\infty}\int_X q(x,t)dxdt\le R_0 \tag{50}$$

where μ_0 denotes the Lagrangian multiplier on the resource constraint (50), hence μ_0 is constant in time, and $\theta(t)$ denotes profit tax on fossil fuel firms. After-tax profits are redistributed lump sum to latitude x consumers in the fraction $s_{FF}(x,t)$ and proceeds from taxes are redistributed lump sum to latitude x consumers in the fraction $s_{Tax}(x,t)$.

The FONC condition for the fossil fuel firms is:

$$p(t)(1-\theta(t))=\mu_0\mathsf{e}^{\Gamma(t)}=\left[\mathbb{A}\Omega F'_q-\tau(x,t)\right](1-\theta(t)). \tag{51}$$

[27] Wages are not equated across locations due to labour immobility.

4.4 Equilibrium

In any decentralized problem, consumption goods firms at latitude x will choose demands $K(x,t)$ and $q(x,t)$ to set

$$r(t)+\delta=\mathbb{A}\Omega F'_K,\ p(t)+\tau(x,t)=\mathbb{A}\Omega F'_q. \tag{52}$$

Market clearing requires

$$\begin{aligned}\int_X B(x,t)\,dx &= 0, \int_X K(x,t)\,dx = K(t), \int_X q(x,t)\,dx = q(t)\\ \int_X C(x,t)\,dx &= C(t), \int_X Y(x,t)\,dx = Y(t).\end{aligned}$$

Conditions (46)-(48) and (51), for a multiplier value $\bar{\mu}_0$ that exhausts the fossil fuels reserves, along with the optimality conditions for the consumer and market clearing, will determine the equilibrium temporal and latitudinal paths for C, K and q. Since firms take temperature and taxes as parametric, these paths can be written, denoting equilibrium by $(^e)$, as:

$$\{C^e(x,t;D,\tau,\theta,p), K^e(x,t;T,\tau,\theta,p), q^e(x,t;T,\tau,\theta,p)\}_{x=-1}^{x=1}. \tag{53}$$

5 Optimal Carbon Taxes

Carbon taxes will be used to correct for the climate externality. For a given set of welfare weights $\upsilon(x)$, the social planner solves the Pareto optimum problem, denoted as PO$^*(\upsilon)$. The solution produces the optimal paths $(^*)$ of (37). Implementation by competitive markets implies that each actor in the economy, i.e. consumers and firms, is faced with a tax on fossil fuels equal to the social marginal cost $\tau^*(x,t)$ of using fossil fuels at each x,t. This tax will induce the consumers and firms to produce a competitive equilibrium equal to the optimal quantities, provided that the firms' problems are concave and the consumers' problems are concave. Since we assume such concavity for the consumers and producers, implementation of the PO$^*(\upsilon)$ by $\tau^*(x,t)$ is feasible. We turn now to determining $\tau^*(x,t)$.

5.1 Spatially Uniform Optimal Carbon Taxes

Implementation of PO$^*(\upsilon)$ requires that social and private marginal products for K and q be equated. Combining the market equilibrium conditions (43, 46, 47, 51) with the welfare maximizing conditions (29) - (36) and denoting by $(^*)$ welfare maximizing paths, we obtain

$$\upsilon(x) U'\left(\frac{C^*(x,t)}{L(x,t)}\right) = \lambda_K^*(t;D), \tag{54}$$

$$\tau^*(x,t;D) = \frac{\mu_R^*(t;D) - \beta\lambda_M^*(t;D)}{\lambda_K^*(t;D)} - p(t) = \tag{55}$$

$$\frac{\mu_R^*(t;D) - \beta\lambda_M^*(t;D)}{\upsilon(x) U'\left(\frac{C^*(x,t)}{L(x,t)}\right)} - p(t) \tag{56}$$

$$p(t) = \frac{\mu_0 \mathsf{e}^{\Gamma(t)}}{1-\theta^*(t)}. \tag{57}$$

Assume that the planner can carry out without cost the necessary adjustments to intertemporal endowment flows across locations so that $\Lambda(x) = \Lambda(x') = \bar{\Lambda}$ for all x, x'. This implies that per capita consumption will be equated across latitudes, and that the Pareto optimum $\mathrm{PO}(\upsilon)$ is a competitive equilibrium for the choice of weights $\bar{\upsilon} = 1/\bar{\Lambda}$. Following the second welfare theorem, $\mathrm{PO}(\bar{\upsilon})$ can be implemented with the appropriate transfers and, from (56), the optimal spatially uniform tax is

$$\tau^*(t;D) = \frac{\mu_R^*(t;D) - \beta\lambda_M^*(t;D)}{\bar{\upsilon} U'\left(C_{\bar{\upsilon}}^*(t)/L(t)\right)} - p(t). \tag{58}$$

Alternatively the regulator can obtain a spatially uniform tax by: (i) using Negishi weights to implement a competitive equilibrium with zero transfers so that $\upsilon(x) U'\left(\frac{C^*(x,t)}{L(x,t)}\right) = 1$, or (ii) making appropriate transfers so that $\hat{\upsilon}(x) U'\left(\frac{C_{\hat{\upsilon}}^*(x,t)}{L(x,t)}\right)$ is the same across locations for any arbitrary set of welfare weights $\hat{\upsilon}(x)$.

In defining the carbon tax, the climate externality is captured by the costate variable $\lambda_M^*(t;D)$. As shown in the next section, $\lambda_M^*(t;D) < 0$; therefore as expected, when we account for the climate externality fossil fuel taxes increase. The dependence of the tax functions on the thermal transport coefficient follows from the fact that damage functions depend on D through their dependence on the temperature field and in principle can be determined by the comparative static derivative $\partial\tau^*(t;D)/\partial D$, which depends on the derivatives of the costate variable λ_M^*, λ_K^* with respect to D. In section 6 we provide simplified comparative static results.

5.2 Spatially Differentiated Optimal Carbon Taxes

The analysis above suggests that a spatially differentiated optimal carbon tax can emerge if it is not possible to equalize per capita consumption across locations, or if the planner wants to change the existing distribution with transfers but full equalization is not possible or desirable. We examine two

possible cases which provide good insights into the structure of an optimal spatially differentiated carbon tax. The first is a polar case where all locations are closed economies which have their own isolated capital markets, fossil fuel reserves and fossil fuel markets. This is not a realistic case but it helps bring out the forces that generate spatially differentiated carbon taxes. The second is an intermediate case where transfers across locations can take place but they are costly. This is a more realistic but also more complicated case.

In order to provide a clearer analysis of the spatial profile of fossil fuel taxes, we simplify climate dynamics following section 2.1 by assuming that the temperature dynamics are modelled by the two-mode approximation and that the co-albedo function is independent of the temperature field, or $\alpha(x, T(x,t)) = \alpha(x)$. From (8)-(12) it can be seen that the zero-mode depends on the concentration $M(t)$ but not on the thermal transport coefficient D, while the the second mode depends on D but not on $M(t)$.[28] Then from (8) the zero mode dynamics can be written as

$$\dot{T}_0 = -T_0 - \frac{A}{B} + \int_{x=-1}^{x=1} QS(x)\,\alpha(x)\,dx + \frac{2\xi}{B}\ln\left(1 + \frac{M(t)}{M_0}\right) \quad (59)$$

$$\dot{T}_0 = -T_0 + Z_1 \ln\left(1 + \frac{M(t)}{M_0}\right) + Z_0, \quad (60)$$

$$Z_1 = \frac{2\xi}{B}, Z_0 = -\frac{A}{B} + \int_X QS(x)\,\alpha(x)\,dx \quad (61)$$

and the temperature field can then be written as $\hat{T}(x,t) = T_0(t) + T_2(t,D)\,P_2(x)$.

5.2.1 Optimal carbon taxes in closed economies

The planner maximizes (27) subject to the resource constraint in each location, the zero mode climate constraint and the fossil fuel constraint $R_0^*(x) = \int_{t=0}^{\infty} q^*(x,t)\,dt$, where $q^*(x,t)$ is the socially optimal quantities from the solution of the planner's problem (27). Let $p^*(x,t) := p(x,t) + \tau^*(x,t)$ denote the full price or social price of fossil fuels at location x and time t, including the externality costs, where (*) indicates optimal paths at the $\mathrm{PO}^{ce}(\upsilon)$, and let $p_M^*(x,t) := \frac{\mu_R^*(x,t;D)}{\lambda_K^*(x,t;D)}$ denote the market price for fossil fuels if $\lambda_M^*(t;D) = 0$, that is, in the case where there are no human induced negative externalities from emissions. Setting $\theta(x,t) = 0$ to simplify the exposition, we obtain the following result.

Proposition 2 *The optimal full social price of fossil fuels for each closed economy across latitudes is:*

[28] This is because from (10) we have that $\int_{-1}^{1} \xi \ln\left(1 + \frac{M(t)}{M}\right) P_2(x)\,dx = 0$ since $\int_{-1}^{1} P_2(x)\,dx = 0$.

$$p^*(x,t) = p(x,t) + \tau^*(x,t) = \tag{62}$$

$$\frac{\mu_R^*(x,t;D) - \beta\lambda_M^*(t;D)}{\lambda_K^*(x,t;D)} = \frac{\mu_R^*(x,t;D) - \beta\lambda_M^*(t;D)}{\upsilon(x)\,U'\left(\frac{C^*(x,t)}{L(x,t)}\right)}. \tag{63}$$

For the proof see Appendix 6.

If the planner makes no international transfers and uses Negishi weights so that $\upsilon(x)\,U'\left(\frac{C^*(x,t)}{L(x,t)}\right) = 1$ for all x, then $p^*(x,t;D) = \mu_R^*(x,t;D) - \beta\lambda_M^*(t;D)$. In this case the part of fossil fuel social price that corresponds to the climate externality is the same across locations. The optimal full social price of fossil fuels is higher in resource poor locations.

Consider the case where $\mu_R^*(x,0;D) = \mu_R^*(x',0;D)$ for all x. This means that the social planner allocates fossil fuels equally in the initial period. Along the optimal path $\frac{\dot{\mu}_R^*(x,t;D)}{\mu_R^*(x,t;D)} = \rho$ for all x, therefore $\mu_R^*(x,t;D) = \mu_R^*(x',t;D)$ for all x and t. Then

$$\frac{p^*(x,t)}{p^*(x',t)} = \frac{\upsilon(x')\,U'\left(\frac{C^*(x',t)}{L(x',t)}\right)}{\upsilon(x)\,U'\left(\frac{C^*(x,t)}{L(x,t)}\right)}.$$

Thus if the planner does not use Negishi weights, but instead uses arbitrary weights, and potentially transfers, such that marginal social valuations *are not* equated across latitudes, i.e. $\upsilon(x')\,U'\left(\frac{C^*(x',t)}{L(x',t)}\right) \neq \upsilon(x)\,U'\left(\frac{C^*(x,t)}{L(x,t)}\right)$ for all $x' \neq x$, then the optimal full social price of fossil fuels is different across locations. In the special case where the planner uses equal welfare weights across locations, we can obtain the following result.

Proposition 3 *When welfare weights across latitudes are equal and independent of x, a latitude located at the equator $x = 0$ will pay a lower social price for fossil fuels relative to a latitude located at latitude $x \neq 0$, if* $U'\left(\frac{C^*(x,t)}{L(x,t)}\right) < U'\left(\frac{C^*(0,t)}{L(0,t)}\right)$.

Since latitudes around the equator are expected to be poorer, with relatively lower per capita consumption which implies $\frac{C^*(x,t)}{L(x,t)} > \frac{C^*(0,t)}{L(0,t)}$, these latitudes will pay a lower social price for fossil fuels relative to a richer latitude located away from the equator. For example with logarithmic utility and equal welfare weights,

$$\frac{p^*(x,t)}{p^*(0,t)} = \frac{C^*(x,t)/L(x,t)}{C^*(0,t)/L(0,t)}. \tag{64}$$

5.2.2 Optimal carbon taxes with costly international transfers

We consider again a social planner that seeks to maximize social welfare defined by (27), subject to the relevant constraints. The regulator can transfer endowments across locations. Transfers across locations are however costly. The impact of constraints not allowing private goods to be transferred freely between regions on economic policy related to climate change was first noted by Chichilnisky and Heal (1994), and has since been studied by others (e.g. Chichilnisky et al., 2000; Sandmo, 2006; Anthoff, 2011; Keen and Kotsogiannis, 2011). We extend this line of research to the one-dimensional climate model and show that under plausible assumptions the social price of fossil fuels around the equator should be lower relative to northern or southern latitudes.

This cost of transfers across latitudes is captured by a quadratic cost term which affects the planner's resource constraint, which can be written as

$$\int_X \left[C(x,t) + \dot{K}(x,t) + \delta K(x,t)\right] dx = \int_X Y\,(t,x)\,dx - \frac{C_0}{2}\Theta\,(t) \quad (65)$$

$$\Theta\,(t) = \int_X \left[y\,(t,x) - C\,(t,x)\right]^2 dx,\ \ y\,(t,x) = \quad (66)$$

$$Y\,(t,x) - \delta K\,(t,x) - u\,(t,x) \quad , \quad \dot{K}\,(t,x) = u\,(t,x)\,, \quad (67)$$

$$Y\,(t,x) = \mathbb{A}(x,t)\Omega(\hat{T}(x,t))F(K(x,t), L(x,t), q(x,t)). \quad (68)$$

The quantity $y\,(t,x)$ can be interpreted as private consumption available out of the production of location x at time t. If $C_0 = 0$, any consumption transfer across location is without cost and the planner will attain the unconstrained OP(υ). If $C_0 \to \infty$, then transfers are prohibitively costly. We call the solution to this problem the constrained Pareto optimal solution, $\mathrm{PO}^c\,(\upsilon)$. We can now obtain the following result regarding the social price for fossil fuels.

Proposition 4 *Assume that the difference between private consumption available out of local production and local private consumption is approximately constant over time, or* $\frac{d[y(x,t)-C(y,t)]}{dt} \simeq 0$. *Then the optimal spatially non-uniform full social price for fossil fuels is*

$$p^*\,(x,t) = p\,(x,t) + \hat{\tau}\,(x,t) = \frac{\mu_R^*\,(t;D) - \beta\lambda_M^*\,(t;D)}{\lambda_K^*\,(t;D)\left[1 - C_0\left[y^*\,(x,t) - C^*\,(x,t)\right]\right]} \quad (69)$$

where $(^*)$ *indicates optimal paths at the* $PO^c\,(\upsilon)$.

For proof see Appendix 7. It can easily be seen that:

$$\frac{p^*\,(x,t)}{p^*\,(0,t)} = \frac{\left[1 - C_0\left[y^*\,(0,t) - C^*\,(0,t)\right]\right]}{\left[1 - C_0\left[y^*\,(x,t) - C^*\,(x,t)\right]\right]}. \quad (70)$$

Proposition 5 *If* $[y^*(x,t) - C^*(x,t)] > [y^*(0,t) - C^*(0,t)]$, *then* $p^*(x,t) > p^*(0,t)$.

Since locations around the equator are poor relative to higher latitude locations, it is expected that $[y^*(x,t) - C^*(x,t)] > [y^*(0,t) - C^*(0,t)]$, for $x \gg 0$. Therefore this proposition suggests that these poor locations should pay a smaller social price for fossil fuel relative to rich locations, which is similar to the result obtained above. If it is further assumed that $p(x,t)$ is approximately equal across locations, the proposition implies that poor locations around the equator should pay a lower carbon tax.

Thus the spatially uniform taxes emerge as an optimal solution only under transfers across locations that equalize per capita consumption or marginal social valuations, or when Negishi welfare weights are used and distribution across latitudes does not change. Negishi weights - being the inverse of marginal utility - assign relatively larger welfare weights to locations with higher per capita consumption. For example with a logarithmic utility, Negishi weights assign a welfare weight equal to per capita consumption in each location. Thus the utility of poor locations has a relatively smaller importance, compared to the utility of rich locations, in the planner's welfare function. The RICE model adopts Negishi weights and produces spatially uniform carbon taxes keeping at the same time the regional distribution of per capita consumption invariant (e.g. Stanton, 2009). Our results, on the other hand, suggest that the spatial structure of the optimal carbon tax is sensitive to the choice of welfare weights, and deviations from the Negishi solution will result into spatially differentiated taxes. Thus when intertemporal distribution is treated as fixed or it is costly to change it, and welfare weights are not Negishi weights, poor locations could, under plausible assumptions, pay lower carbon taxes.

5.3 The Temporal Profile of Optimal Taxes

As stated in the introduction, one of the purposes of this paper is to provide insights regarding the optimal time profile for current and future mitigation. Thus we study the temporal profiles of spatially uniform optimal taxes on fossil fuels $\tau(t)$ and the profits tax $\theta(t)$ that implements the Pareto optimal solution when the necessary adjustments to intertemporal endowment flows across locations can be carried out without cost.

If we take the time derivative of (51) we obtain

$$\frac{d\left[p(t)(1-\theta^*(t))\right]/dt}{p(t)(1-\theta^*(t))} = r(t) = \mathbb{A}\Omega F'_K - \delta, \tag{71}$$

which is Hotelling's rule indicating that after-tax marginal profits increase at the rate of interest.

Let us examine the cases of profits taxes and unit fossil fuel taxes separately. We examine profit taxes by setting $\tau(t) = 0$ and fossil fuel taxes by setting $\theta(t) = 0$. From (71) the optimal profit tax function should satisfy

$$-\frac{\dot{\theta}^*(t)}{1-\theta^*(t)} = r(t) - \frac{\dot{p}(t)}{p(t)}, \tag{72}$$

while the optimal unit tax function should satisfy

$$\frac{(\dot{p}(t) - \dot{\tau}^*(t))}{(p(t) - \tau^*(t))} = r(t). \tag{73}$$

The policy ramp under the gradualist approach suggests that $\dot{\tau}^*(t) > 0, \dot{\theta}^*(t) > 0$. To examine the validity of this result in the context of our model, we seek to locate sufficient conditions so that profit tax and/or the unit tax will decline through time. In order to have a declining tax schedule through time, equation (72) implies that

$$r(t) - \frac{\dot{p}(t)}{p(t)} > 0. \tag{74}$$

Note that a declining tax schedule through time contrasts dramatically with the gradualist tax schedule which increases through time. Since we are implementing the global welfare optimum we use the optimality conditions of section 3.1 without the two-mode approximation of the temperature dynamics. We denote by (*) the global welfare optimizing paths.

Lemma 1 $\zeta(t) \equiv \int_X \lambda_T^*(t, x; D)dx < 0$, $\lambda_M^*(t; D) < 0$.

For proof see Appendix 8. The lemma states an intuitive result. If we denote by V^* the maximum value function for the welfare maximization problem, we know from optimal control results that if V^* is differentiable, $\frac{\partial V^*}{\partial T(x,t)} = \lambda_T^*(x, t; D)$. That is, $\lambda_T(x, t; D)$ can be interpreted as the shadow value of temperature at time t and latitude x. Thus $\zeta(t) \equiv \int_x \lambda_T(t, x; D) < 0$ can be interpreted as the global shadow cost of temperature at time t across all latitudes, which means that an increase in temperature across all latitudes will reduce welfare. In a similar way, $\lambda_M^*(t; D) < 0$ means that an increase in atmospheric accumulation of CO_2 at any time t will reduce welfare.

Proposition 6 *If $m < \delta$, then the optimal profit tax decreases through time, or $\dot{\theta}^*(t) < 0$. Furthermore, the optimal unit tax on fossil fuels grows at a rate less than the rate of interest, or $\frac{\dot{\tau}^*(t)}{\tau^*(t)} < r^*(t)$.*[29]

[29] For similar results in a discrete time model with full depreciation of capital in one period, see Golosov et al. (2011).

For proof see Appendix 9. We can also derive sufficient conditions for an increasing tax schedule according to the gradualist approach.

Proposition 7 *If* $m > \delta$ *and* $\lambda_M^*(m-\delta) - \left(\frac{\xi}{BM(t)}\right)\int_X \lambda_T^* dx > 0$, *then* $\dot{\theta}^*(t) > 0$ *and* $\frac{d\tau^*(t)}{dt} > r^*(t)$.

For proof see Appendix 10. Thus a gradualist tax schedule requires, in the context of our model, rapid decay of atmospheric CO_2, and a relatively small global shadow cost of temperature at time t across all latitudes. In this case profit taxes on fossil fuel firms are increasing, and unit taxes on fossil fuels increase more than the rate of interest.

6 The Impact of Thermal Transportation and Endogenous Co-albedo

Since our one-dimensional climate model introduces heat transport across latitudes and endogenous co-albedo, it is interesting to examine their impact on the economic variables of the model. The impact of changes in the heat transport coefficient D can in principle be obtained by comparative analysis of the Pareto optimal solutions. Given however the complexity of the solutions for the general models, to obtain tractable results and useful insights into these effects we consider a simplified version of the social planner's problem when no transfers are possible.

Simplifying assumptions: Assume no technical change, constant population, no fossil fuel constraint at each latitude, logarithmic utility function with no damages in utility due to temperature increase, a constant returns to scale production function at each location, and an exponential damage function associated with output $\Omega(\hat{T}(x,t)) = \exp\left(-\gamma\hat{T}(x,t)\right)$.

The no technical change, no population growth simplification allows us to perform comparative statics at the steady state, the no fossil fuel constraint implies $\mu_R(x,t) = 0$ for all x and t, while the assumption about the utility function implies that $\Omega_C = 0$.

Let

$$\begin{aligned} A_1 &= \left\{x : -1/\sqrt{3} < x < 1/\sqrt{3}\right\}\ ,\ A_2 = \left\{x : x = \pm 1/\sqrt{3}\right\} \\ A_3 &= \left\{x : 1/\sqrt{3} < x \leq 1 \text{ and } -1 \leq x < -1/\sqrt{3}\right\}. \end{aligned}$$

Thus A_1 are latitudes below $x = \pm 1/\sqrt{3}$, including the equator, while A_3 are latitudes above $x = \pm 1/\sqrt{3}$, including the Poles.

Our results can be summarized in the following proposition.

Proposition 8 *Under the simplifying assumptions above, an increase in the heat transport coefficient D will have the following effects on the steady*

state Pareto optimal solution of the planner's problem in closed economies with nonnegative welfare weights: (i) in A_1 it will reduce temperature and damages, increase per capita capital and consumption, and increase the social cost of fossil fuels, (ii) in A_2 it will have no effect, and (iii) in A_3 it will increase temperature and damages, reduce per capita capital, consumption, and the social cost of fossil fuels.

For proof see Appendix 11. In terms of damages, latitudes at $x = \pm 1/\sqrt{3}$ are not affected by changes in D, latitudes above $x = \pm 1/\sqrt{3}$ are negatively affected by an increase in D since the transport of heat from the low latitudes towards high latitudes and the Poles increases temperature and damages there, while temperature and damages are reduced at the low latitudes. Some interesting questions emerge from this proposition.

Is there any bias from ignoring heat transport or equivalently ignoring cross latitude externalities? As we have shown in this paper, ignoring the heat transport is equivalent to setting D at a sufficiently high value. So accounting for D implies a reduction in the heat transport coefficient relative to the benchmark of ignoring it. In the context of our simplified example and Proposition 9, this means that the Pareto optimal solution when heat transport is ignored will underestimate temperature and damages, overestimate per capita capital and consumption and underestimate the social price of fossil fuels at low latitudes in A_1. The opposite applies to high latitudes in A_3. Zero-dimensional IAMs, which implicitly set $D \to \infty$, with regional differentiation of damages only due to local characteristics of the economy (e.g. agriculture, services, etc.) and not due to local temperature, introduce a bias. This is because the one-dimensional model, by accounting for cross latitude externalities, determines local damages as a direct function of local temperature while zero-dimensional IAMs surpass the local temperature domain and make local damages a direct function of global temperature.

What determines the size of the heat transport coefficient D? In the EBCM literature (e.g. North, 1975 a,b; North et al., 1981), the parameter D in the energy balance equation is regarded as a free parameter divided by the heat capacity of the relevant layers of the atmosphere plus the hydrosphere. If human actions change the heat capacity of the atmosphere and the oceans, then D is expected to change with effects on damages and economic variables.[30]

This analytic result confirms our intuition that heat transport across latitudes matters for economic variables as well as climate variables. For a more realistic model, without the simplifying assumptions made above where closed form solutions are not possible, the impact of heat transport can be approximated numerically. This an area for further research.

[30] Scientific evidence suggests that ocean warming over the last 50 years is due to anthropogenic causes (e.g. Pierce et al., 2011).

Finally, it should also be noted that this simplified comparative static analysis ignores any possible positive feedbacks from the Poles towards low latitudes. If the increase in temperature around the Poles due to heat transfer triggers damage reservoirs such as ice line movements or permafrost thawing, this feedback could increase damages at low latitudes. The incorporation of these effects is undoubtedly an area for further research.

The impact of D on global damages can be explored by using (25), (26) and the two-mode approximation. If D goes to infinity, only the mode zero remains and thermal transportation does not affect damages across latitudes. Therefore one measure of how much heat transport, as reflected by damages D, matters at date t is $|J(t;\infty) - J(t;D)|$.

Endogenous co-albedo generates a *discounting function effect*. As shown in the proof of Lemma 1, if we denoted by $\int_x \lambda_T(t,x)\,dx \equiv \zeta$ the global shadow cost of temperature across latitudes, then $\dot{\zeta} = \upsilon\zeta - \Xi(t)$, where

$$\upsilon \equiv \rho + 1 - \frac{Q}{B}\left(\frac{\int_x \lambda_T(x,t;D)\,S(x)\,(\partial a/\partial T)\,dx}{\int_x \lambda_T(x,t;D)\,dx}\right).$$

Thus the impact of T on co-albedo (since $\partial a/\partial T > 0$) causes the discounting function υ to fall which will make the forward discounted costs of climate change induced by burning an extra unit of fossil fuels higher than when the co-albedo function is independent of temperature, or $\partial a/\partial T = 0$. This could be very important quantitatively, if the impact of T on $a(x,T(x,t))$ can vary by latitude as well as be quite large due to effects on types of plant growth and other determinants of co-albedo besides ice.

7 Concluding Remarks

In this paper we develop a model of climate change consisting of a one-dimensional energy balance climate model which is coupled with a model of economic growth. We believe that modeling heat transport in the coupled model is the main contribution of our paper since it allows, for first time to our knowledge, the derivation of latitude dependent temperature and damage functions, as well as optimal mitigation policies in the form of optimal carbon taxes, which are all determined endogenously through the interaction of climate spatiotemporal dynamics with optimizing forward looking economic agents.

We derive Pareto optimal solutions for a social planner who seeks to implement optimal allocations with taxes on fossil fuels and we show the links between welfare weights and international transfers across locations and the spatial structure of optimal taxes. Our results suggest that when per capita consumption across latitudes can be adjusted through costless transfers for any set of nonnegative welfare weights, so that marginal valuations across latitudes are equated, or transfers are zero and Negishi welfare weights are

used, then optimal carbon taxes are spatially homogeneous. On the other hand, when marginal valuations across latitudes are not equated, due to institutional/political constraints or the cost of transfers, optimal carbon taxes are spatially differentiated. We show that if the international transfers are costly and the planner is not constrained to using Negishi weights, then taxes on fossil fuels could be lower in relatively poorer geographical zones. The degree of geographical tax differentiation depends on the heat transport across latitudes, and the way in which the planner takes into account intertemporal distribution in choosing welfare weights. Without appropriate implementation of international transfers, and without Negishi weights that keep the existing international distribution invariant, carbon taxes should be latitude specific and their sizes should depend on the heat transfer across locations. Furthermore comparative static analysis suggests that since heat transport affects local damages and local economic variables, ignoring it - by using mean global temperature as the central state variable for climate - introduces a bias.

We also provide results indicating that if the decay of atmospheric CO_2 is lower than the depreciation of capital, then profit taxes on fossil fuel firms will decline over time and unit taxes on fossil fuels will grow at a rate less than the interest rate. These results, which can be contrasted to the gradually increasing policy ramps derived by IAM models like DICE or RICE, indicate that mitigation policies should be stronger now relative to the future. Increasing policy ramps so that mitigation is stronger in the future requires rapid decay of the atmospheric carbon dioxide, and a relatively small global shadow cost of temperature increase.

Our model is a surface EBCM where the impact of oceans is reflected in the carbon decay parameter m, but no further modeling of the deep ocean component is undertaken. Further extensions of our simple comparative methods to richer climate models (e.g. Kim and North, 1992; Wu and North, 2007) with a simple "ocean" and with simple "atmospheric layers" added and where tipping phenomena are possible may help understand results like those of Challenor et al. (2006) who found higher probabilities of extreme climate change than they expected. They suggest several reasons for their findings, including: "The most probable reason for this is the simplicity of the climate model, but the possibility exists that we might be at greater risk than we believed."

We emphasize that we are still doing what economists call a "finger exercise" in this paper where one deliberately posits an oversimplified "cartoon" model in order to illustrate forces that shape, for example, an object of interest such as a socially optimal fossil fuel tax structure over time and space that might be somewhat robust to introduction of more realism into the toy model. For example, we believe that the interaction of spatial heat transport phenomena and difficulties in implementing income transfers (or their equivalent, e.g. allocations of tradeable carbon permits) will play an impor-

tant role in determining the shape of the socially optimal tax schedule over different parts of space in more complex and more realistic models. Our simple model is useful in making this type of point under institutions when income transfers are possible but also and when they are politically infeasible, i.e. essentially impossible. However, in implementing carbon taxes that depend upon location in the real world, one must take into account the effects of potential "carbon leakage" (Elliot et al., 2010).

The one-dimensional model allows the exploration of issues which cannot be fully analyzed in conventional zero-dimensional models. In particular one-dimensional models with spatially dependent co-albedo allow the introduction of latitude dependent damage reservoirs such as endogenous ice lines and permafrost. Since reservoir damages are expected to arrive relatively early and diminish in the distant future - because the reservoir will be exhausted and sufficient adaptation will have taken place - the temporal profile of the policy ramp could be declining, enforcing the result obtained for profit taxes, or even U-shaped.[31] A U-shaped policy ramp could be explained by the fact that as high initial damages due to the reservoir start declining (as the reservoir is exhausted), giving rise to a declining policy ramp, damages from the increase in the overall temperature will dominate, causing the policy ramp to become increasing. This is another potentially interesting and important area of further research.

APPENDIX

Appendix 1: The two-mode solution

In this appendix we show how to derive the two-mode solution (8)-(12). We start with the basic PDE with temperature as the state variable which is defined using (1) as:

$$B\frac{\partial T(x,t)}{\partial t} = QS(x)\alpha(x,T(x,t)) - [(A+BT(x,t)) - h(x,t)] + DB\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T(x,t)}{\partial x}\right]. \tag{75}$$

The two-mode solution is defined as:

$$\hat{T}(x,t) = T_0(t) + T_2(t,D)P_2(x),\ P_2(x) = \frac{(3x^2-1)}{2} \tag{76}$$

then, after dropping D to ease notation

$$\frac{\partial T(x,t)}{\partial t} = \frac{dT_0(t)}{dt} + \frac{dT_2(t)}{dt}P_2(x) \tag{77}$$

$$\frac{\partial T(x,t)}{\partial x} = T_2(t)\frac{dP_2(x)}{dx} = T_2(t)3x. \tag{78}$$

[31] Judd and Lontzek (2011) have formulated a dynamic stochastic version of DICE - the SDICE - which includes stochastic tipping points possibilities. They show that this complexity affects the optimal policy results in comparison to RICE.

Substituting the above derivatives into (75) and using the definition of $h(x,t)$ we obtain:

$$B\frac{dT_0(t)}{dt} + B\frac{dT_2(t)}{dt}P_2(x) = QS(x)\alpha(x,x_s) - \tag{79}$$
$$\left[(A + B(T_0(t) + T_2(t)P_2(x))) - \xi \ln\left(1 + \frac{M(t)}{M_0}\right)\right] +$$
$$BD\frac{\partial}{\partial x}\left[(1-x^2)T_2(t)\frac{\partial P_2(x)}{\partial x}\right] \text{ , or} \tag{80}$$
$$B\frac{dT_0(t)}{dt} + B\frac{dT_2(t)}{dt}P_2(x) = QS(x,t)\alpha(x,x_s) - A - \tag{81}$$
$$BT_0(t) - BT_2(t)P_2(x) + \xi \ln\left(1 + \frac{M(t)}{M_0}\right) - 6DBT_2(t)P_2(x).$$

Use

$$\int_{-1}^{1} P_n(x)P_m(x)dx = \langle P_n(x), P_m(x)\rangle = \frac{2\delta_{nm}}{2n+1} \tag{82}$$
$$\delta_{nm} = 0 \text{ for } n \neq m, \delta_{nm} = 1 \text{ for } n = 1$$

and note that $P_0(x) = 1$, $P_2(x) = \frac{(3x^2-1)}{2}$.

Multiply (81) by $P_0(x)$ and integrate from -1 to 1 to obtain

$$B\frac{dT_0(t)}{dt} + B\frac{dT_2(t)}{dt}\langle P_0(x), P_2(x)\rangle = \int_{-1}^{1} QS(x,t)\alpha(x,\hat{T}(x,t))P_0(x)dx - A$$
$$BT_0(t) - BT_2(t)\langle P_0(x), P_2(x)\rangle + \xi\left(1 + \ln\frac{M(t)}{M_0}\right)\int_{-1}^{1} dx -$$
$$6DBT_2(t)\langle P_0(x), P_2(x)\rangle \text{ , or}$$
$$B\frac{dT_0(t)}{dt} = -A - BT_0(t) +$$
$$\int_{-1}^{1}\left[QS(x,t)\alpha(x,x_s) + \xi \ln\left(1 + \frac{M(t)}{M_0}\right)\right]dx. \tag{83}$$

Multiply (81) by $P_2(x)$ and integrate from -1 to 1 noting that $\int_{-1}^{1} P_2(x)dx =$

0, and $\langle P_2(x), P_2(x)\rangle = \frac{1}{5}$ to obtain

$$B\frac{dT_0(t)}{dt}\int_{-1}^{1} P_2(x)dx + B\frac{dT_2(t)}{dt}\langle P_2(x), P_2(x)\rangle =$$
$$\int_{-1}^{1} QS(x,t)\alpha(x,x_s)P_2(x)dx - A - BT_0(t)\int_{-1}^{1} P_2(x)dx - BT_2(t)\langle P_2(x), P_2(x)\rangle +$$
$$\xi \ln\left(1+\frac{M(t)}{M_0}\right)\int_{-1}^{1} P_2(x)dx - 6DBT_2(t)\langle P_2(x), P_2(x)\rangle \ , \text{or}$$
$$\frac{2}{5}\frac{dT_2(t)}{dt} = \left[\int_{-1}^{1} QS(x,t)\alpha(x,x_s) + \xi \ln\left(1+\frac{M(t)}{M_0}\right)\right]P_2(x)dx -$$
$$\frac{2}{5}BT_2(t) - \frac{12}{5}DBT_2(t) \ , \text{or}$$
$$B\frac{dT_2(t)}{dt} = -B(1+6D)T_2(t) +$$
$$\frac{5}{2}\left[\int_{-1}^{1} QS(x,t)\alpha(x,x_s) + \xi \ln\left(1+\frac{M(t)}{M_0}\right)\right]P_2(x)dx. \qquad (84)$$

The ODEs (83) and (84) are the ODEs of the two-mode solution. □

Appendix 2: Proof of Proposition 1

Differential equation (10) can be written as $\dot{T}_2 = -(1+6D)T_2 + (5/2B)\,\Phi(t)$. As $D \to \infty$, any steady state of (10) defined as $T_2^+ = \frac{5\Phi(t)}{2B(1+6D)} \to 0$. Furthermore, consider the ODE

$$\frac{d\bar{T}_2}{dt} = -(1+6D)\bar{T}_2 + (5/2B)\,UB. \qquad (85)$$

Since $\dot{T}_2 \leq -(1+6D)T_2 + (5/2B)\,UB$, then by Gronwall's inequality the solution of (10) will be bounded above by the solution $\bar{T}_2(t)$ of (85). This solution however goes to zero as $D \to \infty$. Therefore $T_2(t) \to 0$ as $D \to \infty$. □

Appendix 3: World GDP

The potential world GDP can be analytically defined as follows. Using $\Psi(x, T(x,t))$ from (20), and the definition of location specific damages $J(x,t;D)$ and global damages $J\left(\{T(x,t)\}_{x=-1}^{x=1}\right)$ from (25) and (26) respectively, potential world GDP, $F_{total}(K(t), q(t), T; t)$, can be computed through the following optimization problem:

$$\begin{array}{l} F_{total}(K(t), q(t), T; x.t) \\ \equiv \max\{\int_x \mathsf{e}^{(a+n\alpha_L)t}\Psi(x, T(x,t))K(x,t)^{\alpha_K} q(x,t)^{\alpha_q} dx, \\ s.t.\ \int_x K(x,t)dx \leq K(t), \int_x q(x,t)dx \leq q(t)\}. \end{array} \qquad (86)$$

The Lagrangian associated with (86) is:

$$\mathcal{L} = \int_x \mathsf{e}^{(a+n\alpha_L)t}\Psi(x,T(x,t))K(x,t)^{\alpha_K}q(x,t)^{\alpha_q})dx+ \quad (87)$$

$$\mu_K(t)\left[K(t)-\int_X K(x,t)dx\right]+\mu_q(t)\left[q(t)-\int_X q(x,t)dx\right] \quad (88)$$

which leads to

$$a_K\mathsf{e}^{(a+n\alpha_L)t}\Psi(x,T(x,t))K(x,t)^{\alpha_K-1}q(x,t)^{\alpha_q} = \mu_K(t) \quad (89)$$

$$a_q\mathsf{e}^{(a+n\alpha_L)t}\Psi(x,T(x,t))K(x,t)^{\alpha_K}q(x,t)^{\alpha_q-1} = \mu_q(t) \quad (90)$$

which means that the marginal product of capital and the marginal product of the fossil fuels are equated across latitudes for all times t, in the context of the potential world GDP notion. Furthermore, since F is Cobb-Douglas:

$$K(x,t) = [\Psi(x,T(x,t;D))^{1/\alpha_L}/\int_{x'}\Psi(x',T(x',t)^{1/a_L}dx']K(t) \quad (91)$$

$$q(x,t) = [\Psi(x,T(x,t;D))^{1/\alpha_L}/\int_{x'}\Psi(x',T(x',t)^{1/a_L}dx']q(t) \quad (92)$$

$$F_{total}(K(t),q(t),T;t) = \left[\mathsf{e}^{(a+\alpha_L n)t}K(t)^{\alpha_K}q(t)^{\alpha_q}\right]J(t;D). \quad (93)$$

□

Appendix 4: The aggregate dynastic consumer family problem

This problem can be set in the following way: Allocate $C(t)$ to solve the problem

$$\max\{\int_x(\mathsf{e}^{(-\rho+n(1-\gamma))t}L(x,0)^{1-\gamma}\frac{C(x,t)^\gamma}{\gamma})dx,\ \int_X C(x,t)dx \le C(t)\} \quad (94)$$

to obtain:

$$C(x,t) = \frac{L(x,0)}{\int_X L(x,0)dx}C(t). \quad (95)$$

Allowing for per capita damages in utility due to climate change given by $\Omega_C(T(x,t)$, with $\partial\Omega_C(T(x,t))/\partial T > 0$, the economic part of the social welfare problem in the Ramsey-like form for the "aggregate dynastic consumer family" can be written as:

$$\max\int_0^\infty \mathsf{e}^{(-\rho+n(1-\gamma))t}\frac{C(t)^\gamma}{\gamma}\int_X\left[\frac{L(x,0)}{\int_{x'}L(x',0)dx'}\right]^\gamma dxdt -$$

$$\left[\int_{t=0}^\infty \mathsf{e}^{(-\rho+n)t}\int_X L(x,0)\Omega_C(T(x,t)dx\right]dt, \quad (96)$$

subject to

$$C(t)+\dot{K}(t)+\delta K(t) = \mathsf{e}^{(a+\alpha_L n)t}K(t)^{\alpha_K}q(t)^{\alpha_q}J(T;D) \quad (97)$$

$$\int_0^\infty q(t)dt \le R_0 \quad (98)$$

with R_0 denoting the total available amount of fossil fuel on the planet. □

Appendix 5: Proof of (36)

The relevant part for the Maximum Principle derivation associated with the Hamiltonian (28) is

$$... + \int_X \lambda_T(t,x)\frac{1}{B}\left[QS(x)\alpha(x,T(x,t)) - \left[(A+BT(x,t)) - \xi \ln\left(1+\frac{M(t)}{M_0}\right)\right] + DB\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T(x,t)}{\partial x}\right]\right]dx. \tag{99}$$

Put

$$v = \frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T(x,t)}{\partial x}\right], u = \lambda_T(t,x). \tag{100}$$

Then integrating by parts we obtain,

$$\int_X \lambda_T(t,x)\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T(x,t)}{\partial x}\right]dx = uv|_{x=-1}^{x=1} - \int_{x=-1}^{x=1} vdu =$$
$$= -\int_{x=-1}^{x=1}\left[(1-x^2)\frac{\lambda_T(t,x)}{\partial x}\right]\frac{\partial T(x,t)}{\partial x}dx$$

since the term

$$uv|_{x=-1}^{x=1} = \partial/\partial x[(1-x^2)\partial\lambda_T(x,t)/\partial x]dx\Big|_{x=-1}^{x=1} = 0 \tag{101}$$

is zero because it is zero at $x=-1$ and $x=1$.

Put

$$v = T(x,t), u = (1-x^2)\frac{\lambda_T(t,x)}{\partial x} \tag{102}$$

and integrate by parts once more to obtain:

$$-\int_{x=-1}^{x=1}\left[(1-x^2)\frac{\lambda_T(t,x)}{\partial x}\right]\frac{\partial T(x,t)}{\partial x}dx = \int_{x=-1}^{x=1} T(x,t)\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial\lambda_T(t,x)}{\partial x}\right]dx. \tag{103}$$

If we take the partial derivative of the Hamiltonian (28) with respect to $T(x,t)$ for each (x,t), we will obtain (36)**.** □

Appendix 6: Proof of Proposition 2

The current value Hamiltonian for the planner's problem is:

$$H = \int_X \upsilon(x) L(x,t) \left[U\left(\frac{C(x,t)}{L(x,t)}\right) - \Omega_C(\hat{T}(x,t)) \right] dx \tag{104}$$
$$+ \int_X \lambda_K(x,t) \left[\mathbb{A}(x,t)\Omega(\hat{T}(x,t))F(K(x,t), L(x,t), q(x,t)) - C(x,t) - \delta K(x,t) \right] dx$$
$$+\lambda_M(t) \left[-mM(t) + \beta \int_X q(x,t)\, dx \right] - \mu_R(x,t)\, q(x,t)$$
$$+\lambda_{T_0}(t) \left[-T_0 + Z_1 \ln\left(1 + \frac{M(t)}{M_0}\right) + Z_0 \right]$$

with optimality conditions:

$$\upsilon(x) U'\left(\frac{C(x,t)}{L(x,t)}\right) = \lambda_K(x,t) \tag{105}$$
$$\mathbb{A}\Omega(\hat{T}(x,t))F'_q = \frac{\mu_R(x,t) - \beta\lambda_M(t)}{\lambda_K(x,t)} \tag{106}$$
$$\frac{\dot{\lambda}_K(x,t)}{\lambda_K(x,t)} = \rho + \delta - \mathbb{A}(x,t)\Omega(\hat{T}(x,t))F'_K \tag{107}$$
$$\dot{\mu}_R(x,t) = \rho\mu_R(x,t) \tag{108}$$
$$\dot{\lambda}_M(t) = (\rho + m)\lambda_M(t) - \lambda_{T_0}(t) \frac{Z_1}{1 + M(t)/M_0} \tag{109}$$
$$\dot{\lambda}_{T_0}(t) = (\rho + 1)\lambda_{T_0}(t) + \int_X \upsilon(x) L(x,t) \Omega'_{C,T_0} dx \tag{110}$$
$$- \int_X \lambda_K(x,t)\, \mathbb{A}\Omega'_{T_0} F dx.$$

We implement the solution to the planner's problem with a competitive equilibrium with spatially differentiated taxes. It can easily be seen from section 3.3 that the competitive equilibrium conditions for the closed economies are:

$$\begin{aligned} U'\left(\frac{C(x,t)}{L(x,t)}\right) &= \Lambda(x)\, \mathsf{e}^{\rho t} p^C(x,t) \\ \mathbb{A}(x,t)\Omega(\hat{T}(x,t))F'_K &= r(x,t) + \delta \\ \mathbb{A}(x,t)\Omega(\hat{T}(x,t))F'_q &= p(x,t) + \tau(x,t) \\ p(x,t)(1 - \theta(x,t)) &= \mu_0 \mathsf{e}^{-\Gamma(x,t)} \end{aligned}$$

where μ_0 is defined below condition (50). Let $p^*(x,t) = p(x,t) + \tau^*(x,t)$. $p^*(x,t)$ should be equated with the marginal social cost of fossil fuel in (106). Setting $\theta(x,t) = 0$ to simplify the exposition, the full social price of fossil fuels should satisfy:

$$p^*(x,t;D) = \frac{\mu_R^*(x,t;D) - \beta\lambda_M^*(t;D)}{\lambda_K^*(x,t;D)} = \frac{\mu_R^*(x,t;D) - \beta\lambda_M^*(t;D)}{\upsilon(x) U'\left(\frac{C^*(x,t)}{L(x,t)}\right)}. \tag{111}$$

□

Appendix 7: Proof of Proposition 4

The planner maximizes (27) subject to (65)-(68), the zero mode dynamics, the CO_2 evolution and the fossil fuel resource constraint. The Hamiltonian for the planner is:

$$\begin{aligned} H &= \int_X \left\{ \upsilon(x) L(x,t) \left[U\left(\frac{C(x,t)}{L(x,t)} \right) - \Omega_C(\hat{T}(x,t)) \right] + \lambda_K(t,x) u(x,t) \right\} dx \\ &+ \lambda_K(t) \left[\int_X [Y(t,x) - u(x,t) - C(x,t) - \delta K(x,t)] dx - \frac{C_0}{2} \Theta(t) \right] \\ &+ \lambda_M(t) \left[-mM(t) + \beta \int_X q(x,t) dx \right] \\ &+ \lambda_{T_0}(t) \left[-T_0 + Z_1 \ln\left(\frac{M(t)}{M_0} \right) + Z_0 \right] - \mu_R(t) \int_X q(x,t) dx. \end{aligned} \tag{112}$$

Optimality conditions imply:

$$\upsilon(x) U'\left(\frac{C(x,t)}{L(x,t)} \right) = \lambda_K(t) [1 - C_0 [y(x,t) - C(x,t)]] \tag{113}$$

$$\mathbb{A}\Omega F'_q = \frac{\mu_R(t) - \beta \lambda_M(t)}{\lambda_K(t) [1 - C_0 [y(x,t) - C(x,t)]]} \tag{114}$$

$$\lambda_K(x,t) = \lambda_K(t) [1 - C_0 [y(x,t) - C(x,t)]] \tag{115}$$

$$\dot{\lambda}_K(x,t) = \rho \lambda_K(x,t) - \frac{\partial H}{\partial K(x,t)} \tag{116}$$

$$\mathbb{A}\Omega F'_K = \rho + \delta - \frac{\dot{\lambda}_K(t)}{\lambda_K(t)} + C_0 \frac{d[y(x,t) - C(x,t)]/dt}{[1 - C_0 [y(x,t) - C(x,t)]]} \tag{117}$$

$$\dot{\lambda}_M(t) = (\rho + m) \lambda_M(t) - \lambda_{T_0}(t) \frac{Z_1}{M(t)} \tag{118}$$

$$\begin{aligned} \dot{\lambda}_{T_0}(t) &= (\rho + 1) \lambda_{T_0}(t) + \int_X \upsilon(x) L(x,t) \Omega'_{C,T_0} dx \\ &- \lambda_K(t) \int_X [1 - C_0 [y(x,t) - C(x,t)]] \mathbb{A}\Omega'_{T_0} F dx. \end{aligned} \tag{119}$$

Note that while $\lambda_K(t)$ is the marginal utility of social income at time t, $\lambda_K(x,t)$ is the costate - or the shadow value - for the capital stock at location x and time t. It can easily be seen that if there is no transfer cost or $C_0 = 0$, then conditions (113)-(119) are reduced to the optimality conditions of the planner who is not constrained by transfer costs.

We set $\theta(x,t) = 0$ and, by taking the time derivative of (51), we obtain:

$$\frac{dp(t)/dt}{p(t)} = r(t). \tag{120}$$

If we assume that $(d[y(x,t) - C(y,t)]/dt) \simeq 0$, then condition (117) implies that

$$\mathbb{A}\Omega F'_K = \rho + \delta - \frac{\dot{\lambda}_K(t)}{\lambda_K(t)} = r(x,t) = r(t). \tag{121}$$

This means that the rates of return of capital are approximately equal across locations. From (120) and (121) we can infer that the market price of fossil fuels across locations grows at an approximately equal rate. Given initial prices $p(x,0)$, fossil fuels prices will be determined as $p(x,t) = p(x,0)\exp(\int_{s=0}^{t} r(s)ds)$.

In this set-up the constrained Pareto optimum $\mathrm{PO}^c(\upsilon)$ can be implemented by spatially differentiated fossil fuel taxes with approximately uniform interest rates across locations. Thus the optimal spatially non-uniform full social price for fossil fuels is:

$$p^*(x,t) = p(x,t) + \hat{\tau}(x,t) = \frac{\mu_R^*(t;D) - \beta\lambda_M^*(t;D)}{\lambda_K^*(t;D)[1 - C_0[y^*(x,t) - C^*(x,t)]]} \tag{122}$$

where $(^*)$ indicates optimal paths at the $\mathrm{PO}^c(\upsilon)$.□

Appendix 8: Proof of Lemma 1

The costate variables for the welfare maximization problem satisfy (35) and (36). We know that $F'_{total,T} < 0$, since $\partial\Omega(x,T(x,t))/\partial T(x,t) < 0$ by assumption, also by assumption $\partial\Omega_C(x,T(x,t))/\partial T(x,t) > 0$ and $\partial a(x,T(x,t))/\partial T(x,t) > 0$. To show that $\lambda_M^*(x,t) < 0$, it is enough to locate sufficient conditions for $\int_X \lambda_T^*(x,t)dx < 0$. Integrate the costate equation for $T(x,t)$ with respect to x to obtain:

$$\begin{aligned}
&\frac{d}{dt}\left(\int_X \lambda_T(t,x)\,dx\right) = \\
&\left[\rho + 1 - \frac{Q}{B}\left(\frac{\int_X \lambda_T(t,x)\,S(x)\,a'_T dx}{\int_X \lambda_T(t,x)\,dx}\right)\right]\int_X \lambda_T(t,x)\,dx \\
&+ \int_X \left(\upsilon(x)\,L\Omega'_{C,T} - \lambda_K F'_{to,T}\right)dx - \int_X \frac{\partial}{\partial x}[(1-x^2)\frac{\partial\lambda_T(x,t)}{\partial x}]dx.
\end{aligned} \tag{123}$$

Note that the term

$$\int_X \frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial\lambda_T(x,t)}{\partial x}\right]dx = 0 \tag{124}$$

is zero since it is an integral of a derivative of a term from $x = -1$ to $x = +1$ and that term is zero at $x = -1$ and $x = 1$. Put $\int_X \lambda_T(t,x)\,dx \equiv \zeta(t)$ and rewrite (123) as:

$$\dot{\zeta} = \left[\rho + 1 - \frac{Q}{B}\left(\frac{\int_X \lambda_T(x,t)\,S(x)\,a'_T}{\int_X \lambda_T(x,t)\,dx}dx\right)\right]\zeta \tag{125}$$

$$+ \int_X \left(\upsilon(x)\,L\Omega'_{C,T} - \lambda_K F'_{to,T}\right)dx \tag{126}$$

or as:

$$\dot{\zeta}(t) = \phi(t)\zeta(t) + \int_X \sigma\left[\upsilon(x) L\Omega'_{C,T} - \lambda_K F'_{total,T}\right] dx \tag{127}$$

where

$$\phi(t) \equiv \rho + 1 - \frac{Q}{B}\left(\frac{\int_x \sigma\lambda_T S(x) a'_T dx}{\int_x \sigma\lambda_T dx}\right) \tag{128}$$

is a time varying discount factor. Since $F'_{to,T} < 0, \Omega'_{C,T} > 0$ by assumption, we see that $\sigma\int_x \lambda_T(t,x)\,dx \equiv \zeta(t) < 0$ for all t by forward integration, since:

$$\zeta(t) = -\left(\exp\int_0^t \phi(s)\,ds\right)\int_0^t \left[\exp\left(-\int_0^t \phi(s)\,dt\right) Z(s)\right] ds \tag{129}$$

$$Z(t) = \int_X \left[\upsilon(x) L\Omega'_{C,T} - \lambda_K F'_{total,T}\right] dx. \tag{130}$$

From (35)

$$\dot{\lambda}_M(t) = (\rho+m)\lambda_M(t) - \frac{\xi}{B(1+M(t)/M_0)}\zeta(t) \tag{131}$$

which implies

$$\lambda_M(t) = \mathsf{e}^{(\rho+m)t}\int_0^t \mathsf{e}^{-(\rho+m)s}\frac{\xi\zeta(s)}{B(1+M(t)/M_0)} ds.$$

Thus solving (131) forward for each t shows that λ_M is a forward integral of negative quantities for each t, therefore $\lambda_M(x,t) < 0$ for each t.□

Appendix 9: Proof of Proposition 6

Set $\tau(t) = 0$. For a decreasing $\theta(t)$, (74) should hold. At the global social welfare maximizing path, after omitting $(x,t;D)$ to ease notation, we have that

$$p^*(t) = (F^*_{total})'_q = \frac{\mu^*_R - \beta\lambda^*_M}{\lambda^*_K} \tag{132}$$

$$r^* = \rho - \frac{\dot{\lambda}^*_K}{\lambda^*_K}, \tag{133}$$

then

$$r^* - \frac{\dot{p}^*}{p^*} = \rho - \frac{\dot{\lambda}^*_K}{\lambda^*_K} - \frac{d[(\mu^*_R - \beta\lambda^*_M)/\lambda^*_K]/dt}{[(\mu^*_R - \beta\lambda^*_M)/\lambda^*_K]}, \tag{134}$$

or using the optimality conditions for the costate variables

$$r^* - \frac{\dot{p}^*}{p^*} = \tag{135}$$

$$\frac{\beta\lambda^*_M(m-\delta) - (\beta\xi/B(1+M(t)/M_0))\int_X \lambda^*_T dx}{(\mu^*_R - \beta\lambda^*_M)}.$$

To show that $\theta^*(t)$ decreases through time we need to show that the numerator and the denominator of (135) are both positive. If the decay of atmospheric carbon dioxide is slow so that $m-\delta<0$, then by lemma 1 the numerator is positive. From the Kuhn-Tucker conditions μ_R^* is non-negative and $\lambda_M^*<0$ by lemma 1. Therefore $\dot{\theta}^*(t)<0$. To examine the time path of the optimal unit tax we use (73) to obtain

$$\dot{\tau}^*(t)=\dot{p}^*(t)-r^*(t)p^*(t)+r^*(t)\tau^*(t). \tag{136}$$

We want to locate sufficient conditions for $\dot{p}^*(t)-r^*(t)p^*(t)<0$. But this is true if and only if $r^*-\dot{p}^*/p^*>0$ which was our previous result. Therefore $\dot{\tau}^*(t)/\tau^*(t)<r^*(t)$.

Thus, we have essentially produced sufficient conditions for rapid ramp-up of profit taxes and for unit taxes to rise at a rate less than the net of depreciation rate of return $r^*(t)$ on capital.□

Appendix 10: Proof of Proposition 7

The first part of the proposition follows directly from lemma 1 and Proposition 6. For the second part note that when $\dot{\theta}^*(t)>0$, $r^*-\dot{p}^*/p^*<0$ and $\dot{p}^*(t)-r^*(t)p^*(t)>0$. □

Appendix 11: Proof of Proposition 8

The optimality conditions for the planner are:

$$\upsilon(x)\,U'\left(\frac{C(x,t)}{L(x,t)}\right)=\lambda_K(x,t) \tag{137}$$

$$\mathbb{A}\Omega(\hat{T}(x,t))F_q'=\frac{\mu_R(x,t)-\beta\lambda_M(t)}{\lambda_K(x,t)} \tag{138}$$

$$\frac{\dot{\lambda}_K(x,t)}{\lambda_K(x,t)}=\rho+\delta-\mathbb{A}(x,t)\Omega(\hat{T}(x,t))F_K' \tag{139}$$

$$\dot{\mu}_R(x,t)=\rho\mu_R(x,t) \tag{140}$$

$$\dot{\lambda}_M(t)=(\rho+m)\lambda_M(t)-\lambda_{T_0}(t)\frac{Z_1}{1+M(t)/M_0} \tag{141}$$

$$\dot{\lambda}_{T_0}(t)=(\rho+1)\lambda_{T_0}(t)+\int_X\upsilon(x)L(x,t)\Omega_{C,T_0}'dx \tag{142}$$

$$-\int_X\lambda_K(x,t)\,\mathbb{A}\Omega_{T_0}'Fdx.$$

To ease notation we write $\mathbb{A}\Omega(\hat{T}(x,t))=\mathbb{B}(x)$. Let $\bar{k}(x)=\frac{K(x)}{L(x)}$, $\bar{q}(x)=\frac{q(x)}{L(x)}$, $\bar{c}(x)=\frac{C(x)}{L(x)}$ denote per capita quantities, thus output per capita at each location is $\mathbb{B}(x)\,\bar{k}(x)^{\alpha_K}\,\bar{q}(x)^{\alpha_q}$, while utility is $\ln(\bar{c}(x))$.

We obtain the steady state as follows. Set $\dot{\lambda}_K(x,t)=0$ in (107) and divide by (106), noting that $\mu_R(x,t)=0$ for all x,t due to the simplifying

assumptions, to obtain

$$\frac{\alpha_K}{\alpha_q}\frac{\bar{q}(x)}{\bar{k}(x)} = \frac{(\rho+\delta)\lambda_K}{-\beta\lambda_M} \tag{143}$$

$$\bar{q}(x) = \frac{(\rho+\delta)\lambda_K}{-\beta\lambda_M}\frac{\alpha_q}{\alpha_K}\bar{k}(x). \tag{144}$$

From (105)

$$\lambda_K = \frac{\upsilon(x)}{\bar{c}(x)}\ ,\ \bar{c}(x) = \mathbb{B}(x)\bar{k}(x)^{\alpha_K}\bar{q}(x)^{\alpha_q} - \delta\bar{k}(x). \tag{145}$$

Substituting λ_K into (143) we obtain

$$\bar{q}(x) = \frac{(\rho+\delta)}{-\beta\lambda_M}\frac{\alpha_q}{\alpha_K}\frac{\upsilon(x)}{\mathbb{B}(x)\bar{k}(x)^{\alpha_K-1}\bar{q}(x)^{\alpha_q} - \delta}.$$

But from (107) at the steady state

$$\mathbb{B}(x)\bar{k}(x)^{\alpha_K-1}\bar{q}(x)^{\alpha_q} = \frac{\rho+\delta}{\alpha_K}.$$

Therefore

$$\bar{q}(x) = \frac{(\rho+\delta)}{-\beta\lambda_M}\frac{\upsilon(x)\alpha_q}{[\rho+(1-\alpha_K)\delta]}. \tag{146}$$

From (3) the steady state stock of CO_2 is:

$$M = \frac{\beta}{m}\int_{-1}^{1} L(x)\bar{q}(x)\,dx = \frac{\beta}{m}\int_X L(x)\left[\frac{(\rho+\delta)}{-\beta\lambda_M}\frac{\upsilon(x)\alpha_q}{[\rho+(1-\alpha_K)\delta]}\right]dx. \tag{147}$$

Using $\Omega(\hat{T}(x,t)) = \exp\left(-\gamma\hat{T}(x,t)\right)$, so that $\Omega'_{T_0} = -\gamma\Omega(\hat{T}(x,t))$ and (109) - (110), we obtain at the steady state:

$$\lambda_M = \frac{\lambda_{T_0}}{(\rho+m)}\frac{Z_1}{1+M/M_0} \tag{148}$$

$$\lambda_{T_0} = \frac{-\gamma}{(1+\rho)}\int_X \frac{\upsilon(x)}{\bar{c}(x)}\left[\mathbb{B}(x)\bar{k}(x)^{\alpha_K}\bar{q}(x)^{\alpha_q}\right]dx. \tag{149}$$

Using $\bar{c}(x) = \bar{k}(x)\left[\mathbb{B}(x)\bar{k}(x)^{\alpha_K-1}\bar{q}(x)^{\alpha_q} - \delta\right]$ and $\alpha_k\mathbb{B}(x)\bar{k}(x)^{\alpha_K-1}\bar{q}(x)^{\alpha_q} = \rho+\delta$ from (107) and substituting into (149) we obtain the steady state value of λ_{T_0} as

$$\lambda^*_{T_0} = -\gamma\int_X \frac{\upsilon(x)(\rho+\delta)}{[\rho+(1-\delta)\alpha_K]}dx = -\gamma\Gamma_0 < 0 \tag{150}$$

$$\Gamma_0 = \frac{(\rho+\delta)}{[\rho+(1-\delta)\alpha_K]}\int_X \upsilon(x)\,dx. \tag{151}$$

Then

$$\lambda_M = \frac{-\gamma\Gamma_0}{(\rho+m)}\frac{Z_1}{1+M/M_0} < 0. \tag{152}$$

Since λ_M does not depend on x, using (152) into (147) we can obtain

$$M = -\frac{1}{m\lambda_M}\Gamma_1\ ,\ \Gamma_1 = \int_X L(x)\left[\frac{\upsilon(x)\,\alpha_q\,(\rho+\delta)}{[\rho+(1-\alpha_K)\,\delta]}\right]dx. \tag{153}$$

From (152) and (153) we obtain the steady state values M^* and λ_M^*, which are both independent of D. Combining results the steady state values for the rest of the variables are:

$$T_0^* = Z_1 \ln\left(1+\frac{M^*}{M_0}\right) + Z_0 \tag{154}$$

$$\hat{T}^*(x) = Z_0 + Z_1 \ln\left(1+\frac{M^*}{M_0}\right) - \frac{Z_2}{1+6D}P_2(x) \tag{155}$$

$$Z_1, Z_2 > 0,\ P_2(x) = \frac{\left(3x^2-1\right)}{2} \tag{156}$$

$$\bar{q}^*(x) = \frac{(\rho+\delta)}{-\beta\lambda_M^*}\frac{\upsilon(x)\,\alpha_q}{[\rho+(1-\alpha_K)\,\delta]} \tag{157}$$

$$\bar{k}^*(x) = \left(\frac{\rho+\delta}{\alpha_K}\right)^{\frac{1}{\alpha_K-1}}\left[\mathbb{A}\Omega\left(\hat{T}(x)\right)\right]^{\frac{1}{1-\alpha_K}}\left[\bar{q}^*(x)\right]^{\frac{1}{1-\alpha_K}} = \tag{158}$$

$$\Gamma_2\left[\Omega\left(\hat{T}^*(x)\right)\right]^{\frac{1}{1-\alpha_K}} = \Gamma_2 \exp\left(\frac{\gamma\hat{T}^*(x)}{1-\alpha_K}\right)\ , \tag{159}$$

$$\Gamma_2 = \left(\frac{\rho+\delta}{\alpha_K}\right)^{\frac{1}{\alpha_K-1}}\left[\mathbb{A}\bar{q}^*(x)\right]^{\frac{1}{1-\alpha_K}} \tag{160}$$

$$\bar{c}^*(x) = \mathbb{A}\Omega\left(\hat{T}(x)\right)\left[\bar{k}^*(x)\right]^{\alpha_K}\left[\bar{q}^*(x)\right]^{\alpha_q} - \delta\bar{k}^*(x) \tag{161}$$

$$\lambda_K^*(x) = \frac{\upsilon(x)}{\bar{c}^*(x)}. \tag{162}$$

This indicates that heat transport D affects the steady state values of per capita capital and consumption at each location as well as the shadow value of capital through their dependence on damages $\Omega\left(\hat{T}^*(x)\right)$.

The impact of D on damages at a latitude x is determined as

$$\frac{\partial\Omega(x)}{\partial D} = \frac{\partial\Omega}{\partial\hat{T}}\frac{\partial\hat{T}^*(x)}{\partial D} \tag{163}$$

with $\frac{\partial\Omega}{\partial\hat{T}} < 0$, since damages reduce output, and an increase in temperature will reduce Ω which will in turn reduce output. Thus the impact of heat transport on damages at a given latitude depends on the sign of the derivative $\frac{\partial\hat{T}^*(x)}{\partial D}$.

From (155)

$$\frac{\partial \hat{T}^{*}(x)}{\partial D}=\frac{3Z_{2}\left(3x^{2}-1\right)}{\left(1+6D\right)^{2}}\begin{cases}=0 & for \quad x=\pm 1/\sqrt{3}\\ <0 & for \quad -1/\sqrt{3}<x<1/\sqrt{3}\\ >0 & for \quad \begin{cases}1/\sqrt{3}<x\leq 1\\ -1\leq x<-1/\sqrt{3}\end{cases}\end{cases} \tag{164}$$

and

$$\frac{\partial \Omega(x)}{\partial D}=\frac{\partial \Omega}{\partial \hat{T}}\frac{\partial \hat{T}^{*}(x)}{\partial D}\begin{cases}=0 & for \quad x=\pm 1/\sqrt{3}\\ >0 & for \quad -1/\sqrt{3}<x<1/\sqrt{3}\text{damage reduction}\\ <0 & for \quad \begin{cases}1/\sqrt{3}<x\leq 1\\ -1\leq x<-1/\sqrt{3}\end{cases}\text{ damage increase}\end{cases}. \tag{165}$$

From (158) and (159)

$$\frac{\partial \bar{k}^{*}(x)}{\partial D}=\frac{1}{1-\alpha_{K}}\Gamma_{2}\Omega^{\frac{\alpha_{K}}{1-\alpha_{K}}}\frac{\partial \Omega}{\partial \hat{T}}\frac{\partial \hat{T}^{*}(x)}{\partial D}\begin{cases}=0 & for \quad x=\pm 1/\sqrt{3}\\ >0 & for \quad -1/\sqrt{3}<x<1/\sqrt{3}\\ <0 & for \quad \begin{cases}1/\sqrt{3}<x\leq 1\\ -1\leq x<-1/\sqrt{3}\end{cases}\end{cases}. \tag{166}$$

From (161) and (162)

$$\begin{aligned}&\frac{\partial \bar{c}^{*}(x)}{\partial D}=\mathbb{A}\frac{\partial \Omega}{\partial \hat{T}}\frac{\partial \hat{T}^{*}(x)}{\partial D}\left[\bar{k}^{*}(x)\right]^{\alpha_{K}}\left[\bar{q}^{*}(x)\right]^{\alpha_{q}} \\ &+\left(\alpha_{K}\mathbb{A}\Omega\left[\bar{k}^{*}(x)\right]^{\alpha_{K}-1}\left[\bar{q}^{*}(x)\right]^{\alpha_{q}}-\delta\right)\frac{\partial \bar{k}^{*}(x)}{\partial D}\end{aligned} \tag{167}$$

$$\begin{cases}=0 & for \quad x=\pm 1/\sqrt{3}\\ >0 & for \quad -1/\sqrt{3}<x<1/\sqrt{3}\\ <0 & for \quad \begin{cases}1/\sqrt{3}<x\leq 1\\ -1\leq x<-1/\sqrt{3}\end{cases}\end{cases}$$

since $\left(\alpha_{K}\mathbb{A}\lneqq\left[\bar{k}^{*}(x)\right]^{\alpha_{K}-1}\left[\bar{q}^{*}(x)\right]^{\alpha_{q}}-\delta\right)>0$ at the steady state due to (107).

$$\frac{\partial \lambda_{K}^{*}(x)}{\partial D}=\frac{-\upsilon(x)\frac{\partial \bar{c}^{*}(x)}{\partial D}}{\left[\bar{c}^{*}(x)\right]^{2}}\begin{cases}=0 & for \quad x=\pm 1/\sqrt{3}\\ <0 & for \quad -1/\sqrt{3}<x<1/\sqrt{3}\\ >0 & for \quad \begin{cases}1/\sqrt{3}<x\leq 1\\ -1\leq x<-1/\sqrt{3}\end{cases}\end{cases}. \tag{168}$$

Furthermore from (62) the impact of D on the full social price of fossil fuels at the steady state is:

$$\frac{\partial p^{*}(x)}{\partial D}=\frac{\beta\lambda_{M}^{*}\frac{\partial \lambda_{K}^{*}(x)}{\partial D}}{\left[\lambda_{K}^{*}(x)\right]^{2}}\begin{cases}=0 & for \quad x=\pm 1/\sqrt{3}\\ <0 & for \quad -1/\sqrt{3}<x<1/\sqrt{3}\\ >0 & for \quad \begin{cases}1/\sqrt{3}<x\leq 1\\ -1\leq x<-1/\sqrt{3}\end{cases}\end{cases}.\ \square \tag{169}$$

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