DEPARTMENT OF INTERNATIONAL AND EUROPEAN ECONOMIC STUDIES

ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

Climate Change Policy under Spatially Structured Ambiguity: Hot Spots and the Precautionary Principle

Anastasios Xepapadeas

Athanasios Yannacopoulos

Working Paper Series

13-32

September 2013

Climate Change Policy under Spatially Structured Ambiguity: Hot Spots and the Precautionary Principle

Anastasios Xepapadeas and Athanasios Yannacopoulos[1], [2]

Abstract

In view of the ambiguities and the deep uncertainty associated with climate change, we study the features of climate change policies that account for spatially structured ambiguity. Ambiguity related to the evolution of the natural system is introduced into a coupled economy-climate model with explicit spatial structure due to heat transport across the globe. We seek to answer questions about how spatial robust regulation regarding climate policies can be formulated; what the potential links of this regulation to the weak and strong version of the precautionary principle (PP) are; and how insights about whether it is costly to follow a PP can be obtained. We also study the emergence of hot spots, which are locations where local deep uncertainty may cause robust regulation to break down for the whole spatial domain, or the weak PP to be costly.

Keywords: Ambiguity, Climate change, space, maxmin expected utility, robust control regulation, hot spots, precautionary principle

JEL Classification: Q54, Q58, D81, R11

1 INTRODUCTION

Ambiguity (or deep uncertainty can) be regarded as a situation where a decision maker does not formulate decisions based on a single probability model but on a set of probability models. Gilboa and Schmeilder (1989) extended decision making under uncertainty by incorporating ambiguity and by moving away from the framework of expected utility maximization. They adopted a maxmin expected utility framework by arguing that when the underlying uncertainty of the system is not well understood and the decision maker faces a set of prior probability density functions associated with the phenomenon, it is sensible - and axiomatically compelling - to optimize over the worst-case outcome (i.e., the worst-case prior) that may conceivably come to pass. Doing so guards against potentially devastating losses in any possible state of the world, and thus adds an element of robustness to the decision-making process.

Motivated by concerns about model misspecification in macroeconomics, Hansen and Sargent (2001a,b, 2008, 2012) and Hansen et al. (2006) extended Gilboa and Schmeidler's insights into dynamic optimization problems, thus introducing the concept of robust control to economic environments. A decision maker characterized by robust preferences takes into account the possibility that the model used to design regulation, call it benchmark or approximating model $\mathbb{P}$, may not be the correct one but only an approximation of the correct one. Other possible models, say $\mathbb{Q}_1, \ldots, \mathbb{Q}_J$, which surround $\mathbb{P}$, should also be taken into account with the relative differences among these models measured by an entropy measure. Hansen and Sargent (2003) characterize robust control as a theory "... [that] instructs decision makers to investigate the fragility of decision rules by conducting worst-case analyses," and suggest that this type of model uncertainty can be related to ambiguity or deep uncertainty so that robust control can be interpreted as a recursive version of maxmin expected utility theory.

Climate change is another area where ambiguity and concerns about model misspecification are present and significant. As Weitzman (2009) points out, the high structural uncertainty over the physics of environmental phenomena makes the assignment of precise probabilistic model structure untenable, while there is high sensitivity of model outputs to alternative modeling assumptions such as the functional form of the chosen damage function and the value of the social discount rate (e.g., Stern 2006, Weitzman 2010). Thus robust control approaches fit very well with climate change problems, as well as with more general environmental and resource economics problems, given the deep uncertainties associated with these issues.[3] For example a specific density function for climate sensitivity from the set of densities reported by Meinshausen et al. (2009) can be regarded as the benchmark model, but other possible densities should be taken into account when designing regulation. One of these densities that corresponds to the least favorable outcome regarding climate change impacts can be associated with the concept of the worst case.

The situation where a single model - or a unique prior - is sufficient for analyzing the phenomenon and formulating decision rules can be identified as the case of pure risk or measurable uncertainty where the decision maker is able to assign probabilities to outcomes. On the other hand the situation where the decision maker operates in the realm of many models - or multiple priors - is the case of ambiguity or deep uncertainty. Under ambiguity the decision maker does not have the ability to determine a precise probability structure for the physical or the economic model, or to put it differently, to measure uncertainty using a single probability model.[4]

The inability to measure uncertainty can be viewed as associating decision making and regulation under ambiguity with the concept of a precautionary principle (PP).[5] Different formulations and versions of the PP can be found in the literature. Sunstein (2002-2003, 2007) discusses two versions of the PP: the weak PP where "lack of decisive evidence of harm should not be a ground for refusing to regulate"; and the strong PP, suggesting that when "potential adverse effects are not fully understood, the activities should not proceed." Sunstein regards the weak PP as sensible but the strong PP as a paralyzing principle. In the context of climate change in this paper we associate the weak PP with regulation under ambiguity and the strong PP with regulation break down.

Apart from ambiguity, climate change - although a global phenomenon - is characterized by a strong spatial dimension, since its impacts are expected to vary profoundly among geographical locations in terms of temperature and damages. Transport of heat from the equator toward the Poles induces a spatially non-uniform distribution of the surface temperature across the globe. Then the interactions between the spatially non-uniform temperature distribution and the spatially non-uniform economic characteristics ultimately shape the spatial distribution of temperature and damages.

In recent decades the main driving force in the economics of climate change has been integrated assessment models (IAMs), such as the DICE, RICE models (see, for example, Nordhaus 2007, 2010). Some of these models (e.g., the RICE model) provide a spatial distribution of damages in which the relatively higher damages from climate change are concentrated in the zones around the equator. However this model, as well as other IAMs, does not account for the natural mechanism - heat transfer - which induces temperature distribution across the globe.

In the terminology of climate science, IAMs with no spatial dimension are zero-dimensional models. Energy balance climate models (EBCMs) on the other hand - one- or two-dimensional - include heat transport across latitudes or across latitudes and longitudes (e.g., North 1975, North et al. 1981, Wu and North 2007) and induce a spatial structure which conforms to reality. One- and two-dimensional coupled economic-climate models have recently been developed (Brock, Engstrom and Xepapadeas forthcoming). Among their most striking results are the generation of distributions of temperature, fossil fuel use, and damages across latitudes and time, which are derived from a social planner's optimization problem, as well as the characterization of spatially differentiated climate policy in the form of optimal carbon taxes.

When ambiguity is introduced into a spatial economic-climate model, it seems reasonable to assume that ambiguity will have some spatial structure. The emergence of this spatial structure can be associated with the fact that even if the approximating model of the regulator is the same for each location, locations could differ in terms of the worst-case model due to differences in the climate change physics across these locations. These differences cause the regulator to have different misspecification concerns for different locations and thus cause ambiguity to acquire a spatial structure. For the approximating model $\mathbb{P}$ and models $\mathbb{Q}$ surrounding it, this means that the local entropy balls containing the local $\mathbb{P}$s differ from location to location reflecting spatial differences in misspecification concerns and in worst-cases.

In this context we develop a one-dimensional spatial economic-climate model with specific climate policy instruments and localized ambiguity with explicit spatial structure. The purpose is to obtain insights regarding spatial robust control regulation which can be associated with a spatially structured PP, the possible emergence of spatial hot spots, and the associated implications in formulating climate change policies. Hot spots, as we use the term, are locations where deep uncertainty could cause regulation to break down for the whole spatial domain, or could imply that regulation under weak precaution is very costly. Thus a closely related issue that can be addressed within this framework is: how costly might being precautious be?

The rest of the paper presents these ideas in greater detail, while a technical appendix at the end provides a formal description of the model.

2 CLIMATE CHANGE POLICIES

A general framework for climate change policies should consider three main types of policies that can affect climate change and its impacts:

(1) Mitigation that involves reduction in the flow of emissions of greenhouse gases (GHGs) and consequently the stock of accumulated GHGs in the atmosphere. A reduced stock of GHGs allows a larger flux of outgoing infrared radiation and thus less radiation is "trapped." This is expected to reduce pressures for temperature to increase.

(2) Adaptation that involves policies to cope with the detrimental im-

pacts of climate change which cannot be avoided. The aim is to anticipate and adapt to the impacts in order to minimize their costs which may extend from the local to the international level. Adaptation is both a matter of need, as climate change is most likely unavoidable, and a matter of equity, as its impacts falls disproportionately on those least able to bear them. Therefore activities that use scarce resources to prevent damages from climate change can be considered adaptation.

(3) Geoengineering that involves methods that prevent GHGs from entering the atmosphere through carbon capture and storage (CCS) or carbon capture and sequestration, or methods of solar radiation management (SLR) that block incoming solar radiation by shading for example the earth from the sun through the spreading of reflective particles (e.g., Schelling 1996, Robock 2008, Shepherd 2009).

These policies can be considered as defining the foundation of a regulatory framework which affects the evolution of temperature and, through adaptation, reduces damages when increases in temperature become inevitable even after mitigation or geoengineering methods are applied.

The basic structure of the coupled economic climate system which includes climate change policies is presented in figure 1, which describes a climate module modelled by: an EBCM; an economic module, which is based on a standard neoclassical growth model; and their interactions. In this model climate change (i.e. increase in temperature) damages aggregate output and possibly reduces utility from consumption, while the economy generates emissions that increase the stock of GHGs and temperature.

[Figure 1]

Regulation can affect climate change and associated damages through a possible combination of mitigation, adaptation, and geoengineering. These policies are, however, costly in terms of output and may lead to further damages as for example in the case of SLR that involves pumping sulphur dioxide into the stratosphere. In this paper we do not study the whole coupled system but we discuss two important and interrelated aspects of the coupled system: spatial aspects and deep uncertainty in relation to regulatory policies.

The spatial aspects of climate change are related to natural or economic forces that shape the distribution of temperature and damages across locations. As shown in figure 1, the spatial structure of our model is induced by two main factors: the transport of heat from the equator toward the Poles which produces a spatially non-uniform distribution of the surface temperature across the globe; and (ii) economic-related forces which determine the damages that a regional (or local) economy is expected to suffer from a given increase in the local temperature in terms of individuals' utility and global production. These damages depend primarily on the production characteristics (e.g., agriculture vs services) or local natural characteristics (e.g., proximity to the sea and elevation) and have been estimated at a regional level by IAMs (e.g., the RICE model). On the other hand, calibrated temperature and damage distributions in the space-time domain when heat transport across the globe takes place have been derived by Brock, Engström and Xepapadeas (forthcoming).

Ambiguity in our model is associated with the evolution of the natural system and its impacts, and in particular with the effectiveness of climate change policies (mitigation, geoengineering) in affecting the rate of change of temperature across the globe, the effectiveness of adaptation in restricting damages due to climate change, and the damages created by the policies themselves.[6] The regulator (or a social planner) has concerns about model misspecification and is not able to assign a unique probability model to stochastic factors affecting the dynamics of climate change and the damages that climate change may cause, as shown in figure 1. Given the spatial structure of the model it is reasonable to assume that misspecification concerns acquire local characteristics and may differ from location to location.

Since climate change policies are expected to affect the rate of change and the distribution of temperature, as well as the distribution of damages across the globe, a regulatory framework based on global averages might not be efficient relative to regulation that depends explicitly on local characteristics. Potentially important questions in this context could be how spatial robust regulation regarding climate policies can be formulated, what the potential links of this regulation to the PP are, and how some insights about whether it is costly to follow a PP can be obtained.

3 SPATIALLY STRUCTURED AMBIGUITY, PRECAUTION AND CLIMATE CHANGE

Ambiguity and concerns about model misspecification underlying natural systems can be manifested in many probability models. The decision maker cannot choose one of them to define expected utility, but the emergence of the worst-case model could lead to severe damages or irreversible change. To prevent these damages, which are not clearly demonstrated since the decision maker does not know that the worst-case model will prevail, precaution might be desirable in designing specific policy rules, which implies that the decision rule should take into account the worst-case scenario. The maxmin expected utility could be used as a conceptual framework for designing good or robust management rules which will work reasonably well given the multiplicity of the possible models.[7] The worst case which is one of many possible models that may prevail, cannot be demonstrated clearly; therefore robust control can be regarded as adhering to a precautionary behavior under conditions of deep uncertainty and ambiguity aversion.

Being robust and precautious in policy design under ambiguity can be relevant and potentially desirable, for example in the current discussion about whether to take strong action now or have a gradual response regarding policies to address climate change, given the uncertainties associated with the issue. However, being robust and precautious could also be costly in the sense of Sunstein's (2002-2003) paralyzing situation where potential benefits are foregone due to inaction, or costly stringent regulation is called for. In such a case, a policy maker should address the relation between deep uncertainty and the structure or the limits of regulation, given a measure of the "severity" of deep uncertainty.

Assume that the regulator has a benchmark or approximate model $\mathbb{P}$ surrounded by other possible models, say $\mathbb{Q}_1, \ldots, \mathbb{Q}_J$, with the difference between $\mathbb{P}$ and $\mathbb{Q}$s measured by relative entropy. The worst-case model that the decision maker is willing to consider, given the existing knowledge and information, is the one differing the most from $\mathbb{P}$ in terms of entropy. Thus the size of ambiguity can be regarded as the length of the radius H of the entropy ball that surrounds $\mathbb{P}$.

A fundamental parameter in robust control problems is the weight, or the penalty parameter or the robustness parameter, assigned by the regulator to the possibility that the chosen probability model might not be the correct one. Equivalently the penalty parameter can be related to a measure of "how far" the worst-case climate sensitivity density can be from the benchmark sensitivity. Given a benchmark probability model in the climate change problem, the regulator can in principle approximate - given the existing knowledge - the deviation between the benchmark and the worst-case model, and determine the extra constraint that deep uncertainty imposes on the regulatory processes. The impact of this extra constraint on regulation can be associated with both the weak and the strong versions of the PP.

Regulation designed subject to the constraint that an appropriately defined worst case may emerge can be associated with the weak version of the PP. On the other hand, if the deviation between the benchmark and the worst-case distribution exceeds a threshold, then robust control regulation is not possible because the impact of the worst-case distribution is so large that regulation using the maxmin expected utility criterion is meaningless. This is because the worst case is so far from the benchmark case, i.e. H is so large, that maximization over the worst case is not possible. This breakdown can be viewed as a situation where an adversarial agent chooses the worst case, trying to minimize the regulator's objective, while the regulator is trying to maximize over this minimizing choice. Breakdown means that the adversarial agent can choose a worst case which is so "bad" that it will create a very large loss for the regulator or, put differently, it will push the regulator's objective to minus infinity. In such a case, any maximization on the regulator's part would be meaningless. Regulation breakdown due to deep uncertainty can be associated with the strong version of the PP and suggests actions such as acquiring more information that might reduce the entropy ball, thus allowing regulation in the spirit of the weak PP, or completely changing the regulatory model.

When the spatial dimension of the climate change problem is introduced, deep uncertainty acquires a spatial structure. In such a case, concerns and ambiguity about climate change, and the distribution of its impacts across the globe, introduce deviations between the local benchmark model and the worst case for the specific location. In this case there will be a benchmark model P_n of change in temperature at each $n = 1, ..., N$ location and a set of possible models $\mathbb{Q}_n = (Q_{1n}, \ldots, Q_{Jn})$. It will be reasonable to assume that even if the benchmark model is the same across locations, the entropy ball surrounding each benchmark model need not be the same. More precisely, the radius of the entropy ball will be different across locations, i.e. $H_n \neq H_m$, $n \neq m$, $n, m = 1, ..., N$. This observation suggests a spatial structure to ambiguity, which is induced by differences in the "deepness" of uncertainty across locations and by spatial interactions of the natural and the economic systems. The spatially structured ambiguity is shown in figure 2.

[Figure 2]

Due to the local interactions, regulation under the local worst-case constraint in a specific location will affect regulation in other locations operating under their own local worst-case constraint. Thus spatially structured ambiguity is expected to induce spatially dependent robust regulation in terms of mitigation, adaptation and geoengineering, which will reflect the structure of ambiguity. This type of regulation can be associated with a localized weak version of the PP. A spatially dependent climate policy emerges therefore in the context of an economic EBCM with spatially structured ambiguity. In terms of policy design this is a departure from the spatially uniform policies mostly suggested by the IAMs. The rigorous formulation of optimal spatially robust climate policies is not an easy task given the complexity of the model. A first attempt to address this issue is presented in the appendix.

3.1 How Costly is the Weak PP?

Because the constraint imposed by the worst-case model should be accounted for, robust control regulation (or regulation by following a weak PP) is different from regulation under risk, which is the case of measurable uncertainty where it is accepted that the regulator trusts the benchmark model.[8] Therefore, one way of answering the question of how costly it is to follow a weak PP is to compare robust control regulation in climate change under deep uncertainty, with regulation under risk which might be regarded as the "benchmark regulation." A way of performing this comparison is by comparing the optimized value of the regulator's objective under robust control regulation with the corresponding optimized value of the objective under benchmark regulation (see appendix, section 5.4 for details). Optimized objective in the case of climate change means the maximized global discounted value of utility less damages from climate change under the regulatory scheme (see figure 1). If the optimized objective under robust control is less than the optimized objective under benchmark regulation, the difference between the two maximized objectives can be interpreted as the cost of following the weak PP.

3.2 Spatially Structured Ambiguity and Hot Spots

What are the features of spatially robust climate policies that might be of interest to regulators? Recent results on the robust control of spatiotemporal economic systems (Brock, Xepapadeas and Yannacopoulos 2012, 2013) suggest that deep uncertainty in certain locations might have a very important impact on the regulation for the whole spatial domain. This is because, given the spatial structure of ambiguity in terms of worst-case models, the regulator designs the robust rules, not only with respect to the spatial characteristics of the problem in a specific location or the average characteristics of the whole spatial domain, but also with respect to the degree of the regulator's ambiguity - the radius of the entropy ball - for each specific location.

This observation allows us to identify locations, referred to as *spatial hot spots,* which are classified into two types (see appendix for details):

> *Hot Spot Type A* : Locations where robust control breaks down for the whole spatial domain.
>
> *Hot Spot Type B* : Locations where robust control is very costly as a function of the degree of the regulator's ambiguities across all sites, relative to standard regulation under pure risk.

A type A hot spot is a location where the deviation between the benchmark and the worst-case model exceeds a threshold, which causes the regulation for the whole spatial domain to break down. This is because mistrust of the benchmark model for this specific location is so large that it makes regulation meaningless in the sense that the worst case for this specific location will push the regulator's objective to minus infinity. Since this location is linked to the rest of the locations in the spatial domain, and regulation should be designed for the whole domain, the severe ambiguities of the hot spot are "transmitted" to the rest of the domain, thus making regulation impossible. Thus an A hot spot can be associated with the strong PP.

A type B hot spot is a location where, because of ambiguity, the maximized value of the regulator's objective under robust control is substantially lower, relative to regulation under pure risk. This means that for a given level of precaution, defined by the worst-case choice in each location, regulation for the whole spatial domain is costly due to deep uncertainties in a specific location. This could happen if precaution induces costly robust policies relative to benchmark regulation, while the expected savings in terms of damages are not sufficiently large. The emergence of a B hot spot implies that the mistrust of the benchmark model and worst-case considerations in a specific location create an interesting trade-off between the weak PP and the cost it implies.

The two types of hot spots and the associated domains for the weak and strong PP are shown in figure 3.

[Figure 3]

A reversal of the type B may also be possible. This occurs if the value of the regulator's maximized objective under robust control is relatively higher than the corresponding value when the regulator is using the benchmark model. This could happen if the policies adopted are more costly under robust control relative to benchmark regulation, but they generate relatively larger benefits in terms of expected damage savings. A reversal of the type B may be associated with the concept of optimal precaution which is the level of precaution defined by the worst-case choice that maximizes the regulator's objective.

4 CONCLUDING REMARKS

This paper discusses issues which arise in the process of regulating a coupled economic climate system when (i) there is ambiguity and concerns about model misspecification (or deep uncertainty) associated with the mechanisms of the natural system, and (ii) there are spatial interactions between the natural and the economic systems. We explore the implications of deep uncertainty and spatial interactions on climate change policies and link them to the PP. Under deep uncertainty climate change policies can be regarded as precautionary.

The main result is that the combination of deep uncertainty and spatial interactions induces spatially structured ambiguity which is an important characteristic for designing climate change policies. This spatial structure may cause certain locations to emerge as spatial hot spots. The existence of hot spots introduces a potentially important relationship between local interactions and global regulation. It has recently been argued (e.g., Haldane 2009) that increased interconnectedness among networks has made various networks - such as ecological networks, power grid networks, transportation networks, and financial networks - more unstable. This interconnectedness and the potential instabilities induced can be associated with the hot spots discussed in this paper and the impact of local properties on global regulation.

In terms of climate change policies, given the existence of deep uncertainties associated with various components of the system and the spatial interrelations between the natural and the economic systems, these observations give rise to a large number of questions. For example, how large a cost are we willing to incur in order to be precautious? Should we advocate uniform or spatially differentiated carbon taxes or other mitigation policies? How will deep uncertainties associated with the impact of solar radiation management methods affect policies based on solar radiation management? Is it likely that deep uncertainty in a specific location will cause regulation using a specific instrument to break down globally? What is the proper response in this case: do we immunize the whole system with respect to the specific location - if this is feasible - or do we look for a qualitatively different policy framework?

The general framework of spatial robust control regulation described here could provide insights and ways of formulating informed answers to these questions.

5 APPENDIX

This appendix provides a technical description of the coupled economy-climate model presented in figure 1 and the associated robust control problem. It extends Brock, Engström and Xepapadeas (forthcoming) by introducing: (i) spatially structured ambiguity, and (ii) geoengineering and adaptation expenses as additional climate change policy instruments.

5.1 Temperature and GHGs Dynamics

We develop a one-dimensional EBCM model with human inputs. We assume that the surface (sea level temperature) T depends upon location ϕ, $\phi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and time $t \geq 0$. We use $x = \sin(\phi) \in [-1, 1]$, and $x = \pm 1$ corresponds to the North and South Pole while $x = 0$ corresponds to the Equator. We refer to x as latitude, and denote by $T(x,t)$ the surface temperature in °C at latitude x at time t. The temperature is affected by heat transfer due to thermal diffusion and the solar energy input in the atmosphere, and by human actions (GHGs emissions and geoengineering). Following Wu and North (2007) the basic energy balance equation with human input added can be written in terms of a partial differential equation (PDE) connecting the temporal and spatial rates of change of the temperature, $\partial T(x,t)/\partial t$, with the various processes, which is of the form

$$
\begin{aligned}
C_c \frac{\partial T(x,t)}{\partial t} &= D \frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T(x,t)}{\partial x}\right] - [A + B\,T(x,t)] + Q\,S(x)\alpha(x) \\
&\quad -\psi(Z) + g(M(t)), \qquad (1)
\end{aligned}
$$

with initial condition $T(x,0) = T_0(x)$, and a boundary condition stating that the flux at the boundary vanishes, i.e., T must be such that:

$$
\sqrt{1-x^2}\frac{\partial T(x,t)}{\partial x} = 0 \ \text{ for } \ x = \pm 1 \ \text{ for all } \ t \geq 0.
$$

The terms on the first line of the right hand side of (1) correspond to non-human sources that affect the temperature dynamics, while the second line collects all the human-related sources. The term $D\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T(x,t)}{\partial x}\right]$ is the effect of thermal diffusion effects with D a heat transport coefficient; $-[A + B\,T(x,t)]$ is the rate of outgoing infrared radiation to space with A and B empirical coefficients; $Q\,S(x)\alpha(x)$ models absorption effects from solar energy; $S(x)$ is the mean annual distribution of solar radiation energy; $\alpha(x)$ is the co-albedo; and C_c is the effective heat capacity per unit area of earth atmospheric system.

The term $-\psi(Z(t))$ models the reduction in incoming solar radiation due to geoengineering activities of total scale $Z(t) = \int_{-1}^{1} z(x,t)\,dx$, where $z(x,t)$ denotes local geoengineering. The global concentration $M(t)$ of GHGs at time t reduces outgoing radiation thus increasing temperature. The term $g(M)$ models the effect that accumulated GHGs have on the reduction of the outgoing radiation. We assume that $g\left(M\left(t\right)\right)=\xi\ln\left(\frac{M(t)}{M_0}\right)$ where M_0 denotes the preindustrial concentration of GHGs, and ξ is a temperature-forcing parameter. GHGs emissions are assumed proportional to the amount $q(x,t)$ of fossil fuel used in production process and $M\left(t\right)$ evolves according to:

$$\frac{d}{dt}M(t)=\beta Q(t)-\delta_m M\left(t\right), \tag{2}$$

where $Q(t):=\int_{-1}^{1}q\left(x,t\right)dx$ is the global quantity of fossil fuel used, β is independent of x and t, and δ_m is a natural decay rate for the GHGs.

To facilitate the exposition and numerical analysis we obtain a finite dimensional model by discretization of the infinite dimensional dynamical system described above. This is done by approximating the continuous space $[-1,1]$ by a one-dimensional discrete finite lattice with N points $x_n\in X=\{x_1,...,x_N\}$, $n=1,...N$, with $x_1=-1$ and $x_N=1$. We approximate the function $T(x)$ by a vector $T=(T_1,\cdots,T_N)\in\mathbb{R}^N$, where $T_i\simeq T(x_i)$, $i=1,\cdots,N$. By approximating the spatial derivatives with finite differences and choosing appropriate boundary conditions, the PDE (1) is transformed to a system of coupled ODEs in $\mathbb{R}^N$, of the form

$$C_eT_i'=D(a_{i,i+1}T_{i+1}+a_{i,i}T_i+a_{i,i-1}T_{i-1})-[A+BT_i]+QS_ia_i-\psi(Z)+g(M),$$

$\forall\ i=1,\cdots,N$ where $T_i'=\frac{dT_i}{dt}, a_{i,j}\ j=i,i\pm1$ (the nearest neighbors of the site i) are real numbers chosen so as to obtain the best possible approximation for the second derivative, and

$$Z=\sum_{i=1}^{N}z_i,$$
$$M'=\beta\sum_{i=1}^{N}q_i-\delta_M M,$$

where $z_i(t)=z(x_i,t)$ and similarly for all the other functions.

The discretized system can then be written in compact form, using $\mathbb{I}_N$, the identity matrix in $\mathbb{R}^N$, and the vector $\mathbf{1}_N=(1,\cdots,1)\in\mathbb{R}^{1\times N}$, as

$$T'=\mathsf{A}_dT-\mathsf{A}_rT-\mathsf{B}_z(Z)+\mathsf{B}_e(M)+F, \tag{3}$$

where $T=(T_1,\cdots,T_N)^{tr}$, A_d is the diffusion matrix and corresponds to the discretization of the diffusion operator, $\mathsf{A}_r = B\mathbb{I}_N T$, $\mathsf{B}_z(Z) = \psi(Z)\mathbf{1}_N^{tr}$ is the geoengineering term which models the effects of global geoengineering on temperature, $\mathsf{B}_e(M) = g(M)\mathbf{1}_N^{tr}$ is the term modeling the effect of GHGs on climate and $F = (-A + QS_1a_1, \cdots, -A + QS_Na_N)^{tr}$, all properly scaled by C_e. We use the vectors $z = (z_1, \cdots, z_N)^{tr}$ and $q = (q_1, \cdots, q_N)^{tr}$ and$\mathbf{1}_N$, to express $\sum_{i=1}^N z_i = \mathbf{1}_N z = \mathbf{1}_N^{tr} \cdot z$ and similarly,

$$M' = \beta \mathbf{1}_N q - \delta_m M. \tag{4}$$

We end up with the controlled dynamical system (3), (4) in $\mathbb{R}^N \times \mathbb{R}$; $(T, M) \in \mathbb{R}^N \times \mathbb{R}$ being the state variable and $(z, q) \in \mathbb{R}^N \times \mathbb{R}^N$ being the control variables.

5.2 The Global Economy

We assume a representative household at location i having preferences described by the utility function

$$U\left(c_i\left(t\right)/\ell_i\right) = \frac{\left[c_i\left(t\right)/\ell_i\right]^{1-\upsilon} - 1}{1-\upsilon} \tag{5}$$

where $c_i\left(t\right)$ and $\ell_i\left(t\right)$ are consumption and the size of the representative household (equal to population) at time t for location i, respectively. Labor, supplied inelastically, is equal to population which is assumed constant to simplify the model.

Production takes place at each location i, according to a production function

$$y_i\left(t\right) = \Omega\left(T_i\left(t\right), \phi_i\left(t\right), Z\left(t\right)\right) F\left(k_i\left(t\right), \ell_i, q_i\left(t\right)\right) \tag{6}$$

where $k_i\left(t\right), \ell_i\left(t\right), q_i\left(t\right)$ denote capital, labor and fossil fuels respectively used at point i, time t. F is a standard Cobb-Douglas, which is multiplied by a damage function Ω modelling the effects of climate change on the economy. Local damages depend on the local temperature T_i, local adaptation expenses ϕ_i that mitigate damages, and global geoengineering activities Z.[9] It is assumed that:

$$\frac{\partial \Omega}{\partial T_i} < 0, \ \ \frac{\partial \Omega}{\partial \phi_i} > 0, \ \ \frac{\partial \Omega}{\partial Z} < 0.$$

As shown in Brock Engström and Xepapadeas (forthcoming), for the global economy, the potential world GDP can be defined as the maximum output that can be produced with fixed and immobile labor, given total capital $K(t) = \sum_{i=1}^{N} k_i(t)$ available and total fossil fuel $Q(t) = \sum_{i=1}^{N} q_i(t)$ used, for a given distribution of temperature $T(t) = (T_1(t), \cdots, T_N(t))$, or

$$\hat{Y}(t) = \Omega\left(\{T_i(t), \phi_i(t)\}_{i=1}^{N}, Z(t)\right) F(K, Q). \tag{7}$$

The global budget constraint is:

$$K'(t) = \hat{Y}(t) - [C(t) + Z(t) + \Phi(t) + \delta K(t)] \tag{8}$$

where $C(t) = \sum_{i=1}^{N} c_i(t), c = (c_1, ..., c_N), Z(t)$ denote global consumption and geoengineering expenses respectively, δ is the depreciation rate, and $\Phi(t) = \sum_{i=1}^{N} \phi_i(t), \phi = (\phi_1, ..., \phi_N)$ denote global adaptation expenses.

Under certainty a social planner will choose paths for (c, q, z, ϕ) to maximize global discounted utility

$$J = \sum_{i=1}^{N} \int_0^{\infty} e^{-\rho t} \omega_i \ell_i U(c_i(t)/\ell_i)\, dt \tag{9}$$

subject to (3),(4),(8), and initial and boundary conditions where $\omega = (\omega_1, \cdots, \omega_N)$ are welfare weights associated with the utility of consumption of each location.

5.3 Spatially Structured Uncertainty

Uncertainty is associated with temperature dynamics, GHGs accumulation, and damages. We assume that the social planner does not formulate decisions regarding the paths for (c, q, z, ϕ) based on a single probability model but on a set of probability models for each location $i = 1, ..., N$. Thus the planner has concerns about model misspecification and is willing, for each location, to consider for local temperature dynamics models $\mathbb{Q}_i$ which are within an appropriately defined entropy ball centered at the benchmark model $\mathbb{P}_i$. The entropy constraints can be written as:

$$\mathcal{Q} = \{\mathbb{Q} : \mathcal{H}(\mathbb{Q}_i \mid \mathbb{P}_i) \leq H_i, \;\; i = 1, \cdots, N+2\} \tag{10}$$

where H_i is the radius of the entropy ball indicating the planner's mistrust in the benchmark model, and $i = N+1, N+2$ correspond to the global entropy constraints for GHGs and capital stock dynamics. If $H_i = 0$ the planner trusts the i-th benchmark model and has no concerns about model misspecification. Since in general $H_i \neq H_j$ for $i \neq j$ $i, j = 1, ...N$, ambiguity acquires a spatial structure since the planner has different degrees of model mistrust across locations. If $H_i > H_j$ the planner trusts the benchmark model in location i less relative to location j.

Following Hansen and Sargent, misspecification concerns can be modelled as drift distortions of a multivariate Wiener process associated with the multiple stochastic factors affecting temperature, GHGs and capital stock dynamics, or

$$
\begin{aligned}
dT &= (\mathsf{A}_d T - \mathsf{A}_r T - \mathsf{B}_z(\mathbf{1}_N z) + \mathsf{B}_e(M) + F)\,dt + \mathsf{C}^{(T)} \upsilon dt + \mathsf{C}^{(T)} dw && (11)\\
dM &= (\beta \mathbf{1}_N q - \delta_m M)\,dt + \mathsf{C}^{(M)} + \upsilon \mathsf{C}^{(M)} dt + \mathsf{C}^{(M)} dw && (12)\\
dK &= (\Omega(T, \phi, \mathbf{1}_N z) F(K, \mathbf{1}_N q) - \delta K - [\mathbf{1}_N c + \mathbf{1}_N z + \mathbf{1}_N \phi])\,dt + && (13)\\
&\quad \upsilon \mathsf{C}^{(K)} dt + \mathsf{C}^{(K)} dw
\end{aligned}
$$

where υ is the vector of distortions of the benchmark models, w is a vector Wiener process $w = (w_1, \cdots, w_J)^{tr}$ associated with J sources of uncertainty with joint distribution $N(0, \mathbb{I}_J\, t)$ where $\mathbb{I}_J$ is the $J \times J$ identity matrix, and $\mathsf{C} = \left(\mathsf{C}^{(T)}, \mathsf{C}^{(M)}, \mathsf{C}^{(K)}\right)^{tr}$, $\mathsf{C}^{(T)} = \left[c_{ij}^T\right] \in \mathbb{R}^{N \times J}$, $\mathsf{C}^{(M)} = \left[c_{1j}^M\right] \in \mathbb{R}^{1 \times J}$ and $\mathsf{C}^{(K)} = \left[c_{1j}^K\right] \in \mathbb{R}^{1 \times J}$. The stochastic shock affecting the temperature dynamics in the i-th location can written as $\sum_{j=1}^J c_{ij}^T dw_j$ and $\mathsf{C}^{(T)}$ can be interpreted as a spatial autocorrelation matrix, while $\sum_{j=1}^J c_{ij}^T \upsilon_j$ is the corresponding drift distortion. Matrices $\mathsf{C}^{(M)}$, and $\mathsf{C}^{(K)}$ have a similar interpretation

In this set-up the multiplier robust control problem can be written as

$$
\max_{(c,q,z,\phi)} \min_{\upsilon} \mathbb{E}_Q \left[\int_0^\infty \mathrm{e}^{-\rho t} \sum_{i=1}^N \omega_i \ell_i U\left(\frac{c_i(t)}{\ell_i}\right) - \sum_{i=1}^{N+2} \frac{\theta_i}{2} \left(\sum_{j=1}^J c_{ij} \upsilon_j \right)^2 \right]
$$
$$
\text{subject to (11),(12),(13)} \qquad (14)
$$

where the adversarial agent chooses distortions υ to minimize the planner's objective. The parameters θ, the robustness parameters, can be regarded as the Lagrangean multipliers associated with the entropic constraints (10). The first N multipliers $\theta_1, \cdots, \theta_N$ correspond to the local entropic constraints for the temperature, whereas θ_{N+1} and θ_{N+2} correspond to global entropic constraints for global GHGs concentration and global capital stock accumulation respectively.

Remark *The exact value of the Lagrange multipliers $\theta_i \geq 0$ depends on the radius of the local entropy balls, i.e., on the value of H_i. Since $\theta_i^2 > C\,\frac{1}{H_i}$, the limit $\theta_i \to \infty$ corresponds to $H_i \to 0$, which is the case where the planner trusts the benchmark model and has no concerns about model misspecification. We call this limit the risk limit, in the sense that there is noise present but the benchmark model P is trusted. The opposite limit $\theta_i \to 0$ corresponds to the case where $H_i \to \infty$, therefore the planner has very little trust in the benchmark model and allows for very large model misspecification. We call this limit the deep uncertainty limit.*

The stochastic differential game (14) can be solved by using the associated Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation. This is expressed in terms of the generator operator which is a second order differential operator $\mathcal{L}$, acting on the value function $V = V(T, M, K)$. $V : \mathbb{R}^N \times \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ of the game. It can be expressed as $\mathcal{L} = \mathcal{L}^{(T)} + \mathcal{L}^{(M)} + \mathcal{L}^{(K)} + \mathcal{L}_n$ where

$$
\begin{aligned}
\mathcal{L}^{(T)}V &= (\mathsf{A}_d T - \mathsf{A}_r T - \mathsf{B}_z(\mathbf{1}_N z) + \mathsf{B}_e(M) + F + \mathsf{C}^{(T)}\upsilon) \cdot \mathsf{D}_T V \\
\mathcal{L}^{(M)}V &= (\beta \mathbf{1}_N q - \delta_m M + \mathsf{C}^{(M)}\upsilon)\mathsf{D}_M V \\
\mathcal{L}^{(K)}V &= \Big(\Omega(T, \phi, \mathbf{1}_N z)\, F(K, \mathbf{1}_N q) - \delta K - [\mathbf{1}_N c + \mathbf{1}_N z + \mathbf{1}_N \phi] + \mathsf{C}^{(K)}\upsilon\Big)\, \mathsf{D}_K V \\
\mathcal{L}_n &= \frac{1}{2} Tr\left(\mathsf{C}\mathsf{C}^{(tr)}\mathsf{D}^2 V\right),
\end{aligned}
$$

$\mathsf{D}V^{tr} = (\mathsf{D}_T V, \mathsf{D}_M V, \mathsf{D}_K V)$ is the gradient of V with respect to (T, M, K) (e.g., $\mathsf{D}_M V = \frac{\partial V}{\partial M}$ and similarly for $\mathsf{D}_K V$), and $\mathsf{D}^2 V \in \mathbb{R}^{(N+2)\times(N+2)}$ is the Hessian matrix, consisting of all the second derivatives of V with respect to (T, M, K). Since the variance of the system dynamics does not depend on the controls, and the decisions regarding (c, q, z, ϕ) and υ separate, the time protocol regarding maximization and minimization decisions does not matter, so the $\min, \max$ operators can be interchanged. This means that the robust control static game has a Nash equilibrium, which is provided by the solution of the HJBI equation which is of the form (Fleming and Souganidis

1989)

$$\rho V - H(V, DV, D^2V) = 0 \tag{15}$$
$$H(V, DV, D^2V) = \max_{(c,q,z,\phi)} \min_{\upsilon} \left[\sum_{i=1}^{N} \omega_i \ell_i U(\frac{c_i}{\ell_i}) - \sum_{i=1}^{N+2} \frac{\theta_i}{2} \left(\sum_{j=1}^{J} c_{ij} \upsilon_j \right)^2 + \right.$$
$$\left. \mathcal{L}^{(T)}V + \mathcal{L}^{(M)}V + \mathcal{L}^{(K)}V + \mathcal{L}_n V \right].$$

Feedback controls for (c, q, z, ϕ) and υ are obtained as functions of DV by performing the optimization in (15). Substituting the feedback controls into (15) we obtain the relevant HJBI equation as:

$$\mathsf{F}(V, DV, D^2V) := \rho V - H_d\,(DV) - \frac{1}{2} Tr\left(\mathsf{C}\mathsf{C}^{tr} D^2 V\right) = 0. \tag{16}$$

5.4 Solvability of the HJBI equation, Viscosity Solutions and Hot Spot Formation

The solvability of the robust control problem depends on the solvability of the related HJBI equation (16). The solution of (16) will be used to obtain the optimizers (c, ϕ, q, z), υ which are all defined in terms of $\mathsf{D}V$, and obtain therefore the robust feedback control policy. By inserting the feedback rules into the state equation, we obtain the optimum path for the controlled system.

Since our problem does not have a linear-quadratic structure, the solution of (16) is not an easy task. To address the issue we use the concept of viscosity solutions (e.g., Bardi and Capuzzo-Dolcetta 2008), which are continuous but not necessarily differentiable functions that solve the HJBI equation in a weak sense. This approach can prove very useful in addressing robust control problems without linear-quadratic structure, which are exactly the problems associated with climate change. Let $x = (T, M, K)$.

Definition (Viscosity solutions of HJBI equation)

1. $v \in C(\mathbb{R}^{N+2})$ is a viscosity subsolution of (16) if for any test function $\varphi \in C^2(\mathbb{R}^{N+2})$ such that x is a local maximum of $v - \varphi$,

   $\mathsf{F}(x, v(x), \mathsf{D}\varphi(x), \mathsf{D}^2\varphi(x)) \leq 0.$

2. $v \in C(\mathbb{R}^{N+2})$ is a viscosity supersolution of (16) if for any test function $\varphi \in C^2(\mathbb{R}^{N+2})$ such that x is a local minimum of $v - \varphi$,

$\mathsf{F}(v(x), \mathsf{D}\varphi(x), \mathsf{D}^2\varphi(x)) \geq 0.$

3. $v \in C(\mathbb{R}^{N+2})$ is a viscosity solution of (16) if it is both a viscosity subsolution and a viscosity supersolution.

Regarding the solvability of the problem, we consider the finite horizon version, i.e., $t \in [0, T]$ for large T and treat a parabolic version of the HJBI equation of the form

$$\frac{\partial V}{\partial t} + \mathsf{F}_1(\mathsf{D}V) + \mathsf{F}_2(\mathsf{D}^2 V) = 0.$$

It can be shown that if the controls (c, ϕ, q, z) are allowed to take values in a compact subset of $\mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R}^N$, then under certain regularity assumptions there exists a $T^* \in [0, T]$, such that equation (16) admits a unique viscosity solution, such that $|V(t, x)| \leq C(1 + |x|^2)$. Furthermore it can be proved that the value of the game is the viscosity solution of the relevant HJBI equation. The derivatives of the viscosity solution v can be used to construct satisfactory approximate feedback controls. The optimal state of the system can then be calculated using a forward integration of the state equation.[10]

Following Da Lio and Ley (2006), the condition for existence of a supersolution would be of the general form $r - C_0 - \frac{C_0}{\theta} e^{rT^*} > 0$ $(r < \rho)$, where C_0 is a constant. When $\theta = (\theta_1, ..., \theta_N, \theta_{N+1}, \theta_{N+2}) \to 0$, the last term which is negative dominates, and this condition cannot hold at all. This means that the robustness parameter θ plays an important role in the loss of solutions for the system. The next theorem whose proof follows along the lines of Felmer, Quaas, and Sirakov (2013) states that the HJBI equation (16) does not have a solution (even in the viscosity sense) in the limit as $\theta \to 0$.

Theorem 1 (The $\theta \to 0$ limit) *Equation (16) does not have a solution in the limit as $\theta \to 0$ in the classical or in the viscosity sense.*

This breakdown of solutions at the deep uncertainty limit, as $\theta \to 0$, induces type A hot spots. In fact, the breakdown can occur even when just one of the θ_i tends to zero, as is indicated by exact results in the linear-quadratic case (see Brock Xepapadeas and Yannacopoulos 2012, 2013). This can be shown for general problems, by a proper modification of arguments along the lines of Felmer, Quaas, and Sirakov (2013) which essentially boil down to the nonexistence of a positive solution for ODEs of the form $\rho u + \frac{1}{\theta_i}|u'|^2 - u'' = 0$ for $\theta_i \to 0$. We call this breakdown of solutions in the deep uncertainty limit a type A hot spot. This means breakdown of the solution for the whole system because there is "too much" uncertainty for just one site which propagates to the other sites through spatial interactions.

As shown in Athanassoglou and Xepapadeas (2012) for a linear-quadratic problem, solving the HJBI equation for a given robustness parameter θ is equivalent to finding a robust policy for all probability models having relative entropy less than the worst-case model, and allows us to estimate the deviation between the benchmark and the worst case. This implies that if the actual deviation between the worst case and the benchmark case can be inferred from existing knowledge, then by repetitive solving of the model for different values of θ, a value θ^0 that corresponds to the actual deviation can be calculated. This will be the 'correct value' of the robustness parameter.

Therefore by combining solutions of (16) for vectors of robustness parameters θ and existing knowledge about possible deviations between the benchmark and the worst case, the robustness parameters can be calibrated. In this context two types of *hot spots* can be defined:

Type A hot spot. Assume that the realistic deviations between the benchmark and the worst-case model imply low values $\theta_i^0 \in \theta$ such the HJBI equation (16) does not have a solution. This is a type A hot spot which means that misspecification concerns for a location cause regulation to break down. Thus local ambiguity breaks down regulation globally, and this can be associated with a strong PP.

Type B hot spot. Assume that the HJBI equation (16) has a solution, either classical or viscosity, for realistic deviations between the benchmark and the worst case. This can be associated with a weak PP and robust control regulation is feasible. The value function in this case will be a function of the states of the system and the robustness parameters θ. At the risk limit $(\theta_1, ..., \theta_N, \theta_{N+1}, \theta_{N+2}) \to \infty$, and there is complete trust in the benchmark model, with no entropic constraints. Let

$$\left(c(t)^U, q(t)^U, z(t)^U, \phi(t)^U; T(t)^U, K(t)^U, M(t)^U\right), \tag{17}$$

$$\left(c(t)^R, q(t)^R, z(t)^R, \phi(t)^R; T(t)^R, K(t)^R, M(t)^R\right) \tag{18}$$

denote, for all i, the time paths for the control and the state variables that correspond to the solution of (14) and the risk limit case respectively.

Define by

$$W^U\left(c^U,\theta^U\right)=\mathbb{E}_{\mathbb{Q}_{\upsilon^*}}\left[\int_0^\infty e^{-\rho t}\left\{\sum_{n=1}^N\omega_n\left[\ell_i U\left(\frac{c^U(t)}{\ell_i}\right)\right]\right\}dt\right] \tag{19}$$

the welfare measure for the planner where $c_i^U(t), n=1,...N$ is the Nash equilibrium consumption path for (14), and by

$$W^R\left(c^R\right)=\mathbb{E}_{\mathbb{P}}\left[\int_0^\infty \mathrm{e}^{-\rho t}\left\{\sum_{n=1}^N\omega_n\left[\ell_i U\left(\frac{c_n^R(t)}{\ell_i}\right)\right]\right\}dt\right] \tag{20}$$

the welfare measure for the regulator in the risk limit case. Then if $\Delta W = W^\infty - W^U > 0$, this difference can be interpreted as the cost of being precautious. If $\frac{\partial \Delta W}{\partial \theta_i}$ is high for some locations, these locations can be characterized as type B hot spots. On the other hand if $\Delta W < 0$, precaution is desirable and one may even discuss the optimal level of precaution in the sense that robustness parameters $\left(\theta_1^*,...,\theta_N^*,\theta_{N+1}^*,\theta_{N+2}^*\right)$ may exist such that they maximize the difference $W^U - W^R$, under the constraint that these parameters correspond to realistic deviations between the benchmark and the worst case.

Notes

[1] Anastasios Xepapadeas, Professor, Athens University of Economics and Business, Department of International and European Economic Studies, xepapad@aueb.gr.

Athanasios Yannacopoulos, Professor, Athens University of Economics and Business, Department of Statistics, ayannaco@aueb.gr.

[2] Many of the issues discussed here have emerged as a result of joint work with William Brock whom we would like to thank for valuable comments and suggestions on this paper. An earlier version of this paper was presented at the conference on “Developing Regulatory Policy in the Context of Deep Uncertainty: Legal, Economic, and Natural Science Perspectives,” University of Chicago, April 2013. We would like to thank the participants for comments and suggestions. This research has been co-financed by the European Union (ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the NSRF - Research Funding Program: Excellence – Athens University of Economics and Business “Spatiotemporal Dynamics in Economics.”

[3] Issues of regulation under ambiguity have been studied using two main approaches: smooth ambiguity and robust control. Smooth ambiguity (Klibanoff, Marinacci and Mukerji 2005), parameterizes uncertainty or ambiguity aversion in terms of preferences and nests the worst-case, corresponding to robust control, as a limit of absolute ambiguity aversion. The approach has been used in climate change issues (e.g., Millner, Dietz, and Heal 2010), but questions regarding the calibration of the regulator’s ambiguity aversion remain open. Robust control methods have been applied to climate change by Athanassoglou and Xepapadeas (2012).

[4] When the decision maker lacks adequate information to assign probabilities to events, we are in the realm of uncertainty as introduced by Frank Knight (1921).

[5] For example Weisbach (2012) studies whether environmental taxes should be precautionary.

[6] Geoengineering in the form of SLR is very likely to create environmental damages such as ocean acidification or acid depositions.

[7] Approaches such as minimax regret, or H_∞ regulation, can also be considered. We follow the maxmin criterion suggested by robust control, since it clearly defines a regulatory framework that can incorporate spatially structured ambiguity.

[8] Confidence in the benchmark model means that $H_n = 0$ for all n.

[9] The local damage function depends on global geoengineering activities to allow for negative externalities at i due to other regions’ geoengineering activities.

[10] There is a well established literature on numerical methods for the calculation of viscosity solutions of fully nonlinear elliptic and parabolic equations of the general type of the HJBI equation obtained here (e.g., Souganidis 1985, Nikolopoulos and Yannacopoulos 2010).

References

[1] Athanassoglou, S., and A. Xepapadeas. 2012. Pollution Control with Uncertain Stock Dynamics: When and How To Be Precautious. *Journal of Environmental Economics and Management*. 62:304-320.

[2] Bardi, M., and I. Capuzzo-Dolcetta. 2088. *Optimal Control and Viscosity Solutions of Hamilton-Jacobi-Bellman Equations*. Boston: Birkhauser.

[3] Brock, W., G. Engström, and A. Xepapadeas. Forthcoming. Spatial Climate-Economic Models in the Design of Optimal Climate Policies across Locations. *European Economic Review*.

[4] Brock, W., A. Xepapadeas, and A. Yannacopoulos. 2012. Robust Control and Hot Spots in Spatiotemporal Economic Systems, Working Paper 12-23. DIEES, Athens University of Economics and Business.

[5] Brock, W., A. Xepapadeas, and A. Yannacopoulos. 2013. Robust Control of a Spatially Distributed Commercial Fishery, in *Dynamic Optimization in Environmental Economics,* edited by Vladimir Veliov. Springer, in press.

[6] Da Lio, F., and O. Ley. 2006. Uniqueness Results for Second-order Bellman Isaacs Equations under Quadratic Growth Assumptions. *SIAM Journal on Control and Optimization*. 45:74-106.

[7] Felmer, P., A. Quaas, and B. Sirakov. 2013. Solvability of Nonlinear Elliptic Equations with Gradient Terms. *Journal of Differential Equations.* 254:4327-4346.

[8] Fleming, W. and P. Souganidis. 1989. On the Existence of Value Function of Two-player, Zero Sum Stochastic Differential Games. *Indiana University Mathematics Journal.* 3:293–314.

[9] Gilboa, I., and D. Schmeidler. 1989. Maxmin Expected Utility with Non-unique Prior. *Journal of Mathematical Economics.* 18:141–153.

[10] Haldane, A. 2009. Rethinking the Financial network, Speech delivered at the Financial Student Association, Unpublished Manuscript. Amsterdam, April.

[11] Hansen, L., and T. Sargent. 2001a. Robust Control and Model Uncertainty. *American Economic Review*. 91:60–66.

[12] Hansen, L., and T. Sargent. 2001b. Acknowledging Misspecification in Macroeconomic Theory. *Review of Economic Dynamics*. 4:519-5351.

[13] Hansen, L., and T. Sargent. 2003. Robust Control of Forward-looking Models. *Journal of Monetary Economics*. 50:581-604.

[14] Hansen, L., and T. Sargent. 2008. *Robustness*, Princeton and Oxford: Princeton University Press.

[15] Hansen, L., and T. Sargent. 2012. Three Types of Ambiguity. *Journal of Monetary Economics*. 59:442-445.

[16] Hansen, L., T. Sargent, G. Turhumambetova, and N. Williams. 2006. Robust Control and Model Misspecification. *Journal of Economic Theory.* 128:45–90.

[17] Klibanoff, P., M. Marinacci, and S. Mukerji. 2005. A Smooth Model of Decision Making under Ambiguity. *Econometrica* 73:1849–1892.

[18] Knight, F. 1921. *Risk, Uncertainty, and Profit*, Houghton Mifflin, USA.

[19] Meinshausen, M., N.I Meinshausen, W. Hare, S. Raper, K. Frieler, R. Knutti, D. Frame, and M. Allen. 2009. Greenhouse-Gas Emission Targets for Limiting Global Warming to 2°C, *Nature* 458:1158-1163.

[20] Millner, A., S. Dietz, and G. Heal. 2010. Ambiguity and Climate Policy, NBER Working Paper No. w16050.

[21] Nikolopoulos, C. and A. Yannacopoulos. 2010. A Model for Optimal Stopping in Advertisement. *Nonlinear Analysis: Real World Applications*.11:1229-1242.

[22] Nordhaus, W. D. 2007. *A Question of Balance*. New Haven & London: Yale University Press.

[23] Nordhaus, W. D. 2010. Economic Aspects of Global Warming in a Post-Copenhagen Environment. *Proceedings of the National Academy of Sciences of the United States of America* 107:11721-11726.

[24] North, G. 1975. Theory of Energy-Balance Climate Models. *Journal of Atmospheric Sciences* 32:2033–43.

[25] North, G., R. Cahalan, and J. Coakley. 1981. Energy Balance Climate Models. *Reviews of Geophysics and Space Physics* 19:91–121.

[26] Robock. A. 2008. 20 Reasons why Geoengineering May be a Bad Idea. *Bulletin of the Atomic Scientists.* 64:14-18.

[27] Schelling, T. 1996. The Economic Diplomacy of Geoengineering. *Climate Change.* 33:303–307.

[28] Shepherd, J.G. 2009. *Geoengineering the Climate: Science, Governance and Uncertainty.* London: The Royal Society.

[29] Souganidis, P. 1985. Approximation Schemes for Viscosity Solutions of Hamilton-Jacobi Equations. *Journal of Differential Equations.* 59:1-43.

[30] Stern, N. 2006. *The Economics of Climate Change: The Stern Review.* Cambridge: Cambridge University Press.

[31] Sunstein, C. 2002-2003. The Paralyzing, Principle. *Regulation.* Winter 2002-2003:32-37.

[32] Sunstein, C. 2007. *Worst-Case Scenarios.* Cambridge Massachusetts: Harvard University Press.

[33] Weisbach, D. 2012. Should Environmental Taxes Be Precautionary? Working Paper No.12-06. The Center for Robust Decision Making on Climate and Energy.

[34] Weitzman, M. 2009. On Modeling and Interpreting the Economics of Catastrophic Climate Change. *Review of Economics and Statistics.* 91:1–19.

[35] Weitzman, M. 2010. What is the “Damages Function” for Global Warming and what Difference might it Make? *Climate Change Economics.* 1:57–69.

[36] Wu, W., and G. North. 2007. Thermal Decay Modes of a 2-D Energy Balance Climate Model. *Tellus* A 59:618–26.

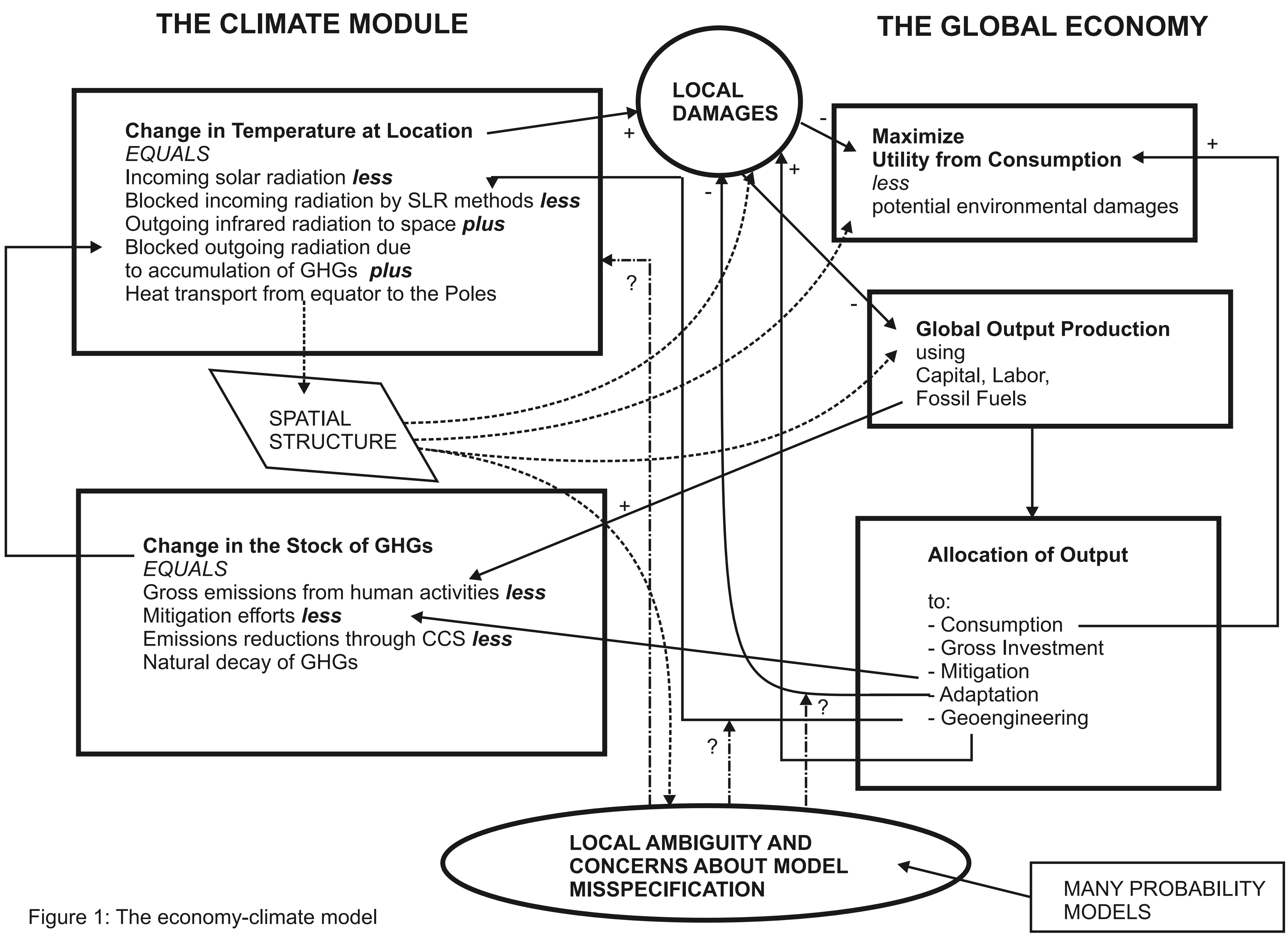

Figure 1: The economy-climate model

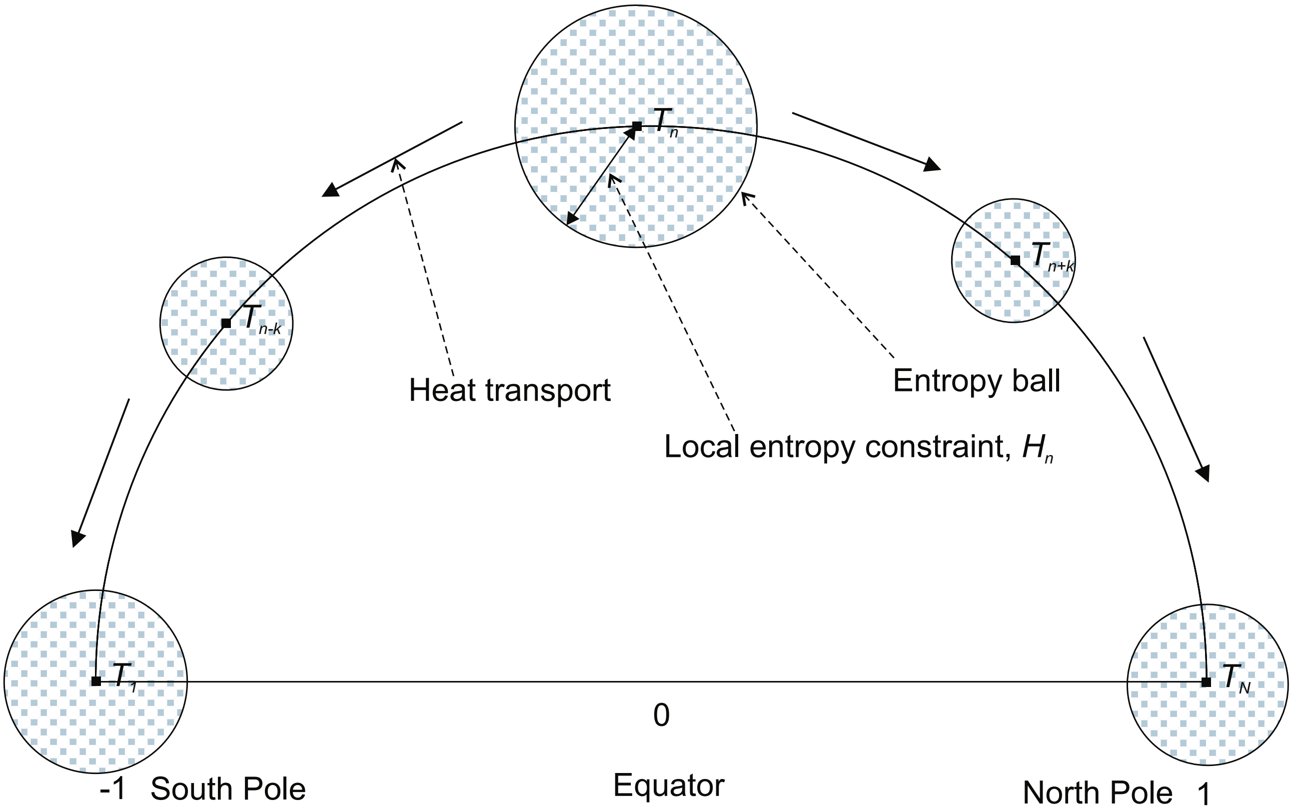

Figure 2: Spatially structured ambiguity

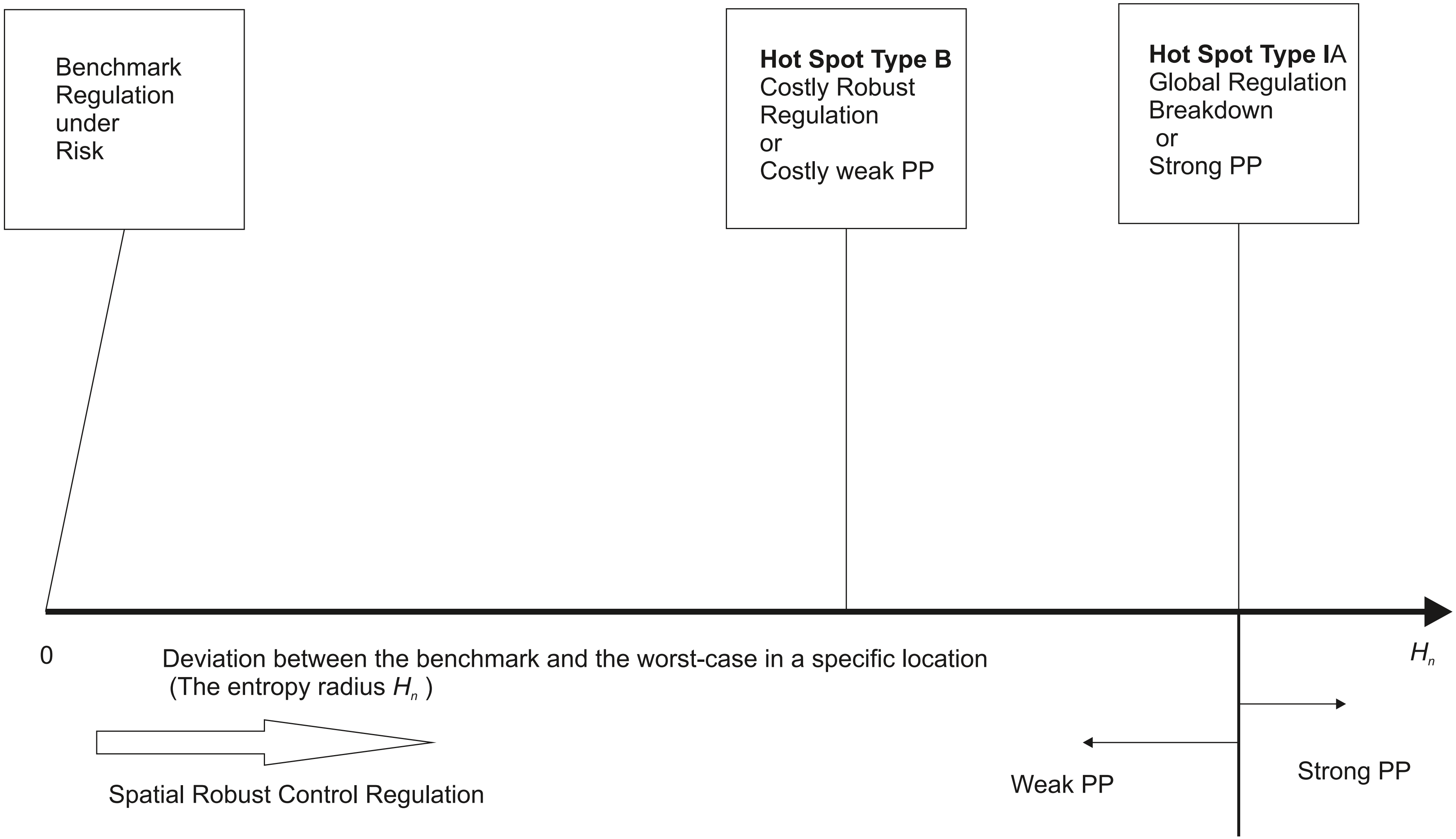

Figure 3: Emergence of Hot Spots, Weak and Strong PP