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Climate Change Policy under Polar Amplification

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Abstract

Polar amplification is an established scientific fact which has been associated with the surface albedo feedback and to heat and moisture transport from the Equator to the Poles. In this paper we unify a two-box climate model, which allows for heat and moisture transport from the southern region to the northern region, with an economic model of welfare optimization. Our main contribution is to show that by ignoring spatial heat and moisture transport and the resulting polar amplification, the regulator may overestimate or underestimate the tax on GHG emissions. The direction of bias depending on the relations between marginal damages from temperature increase in each region. We also determine the welfare cost when a regulator mistakenly ignores polar amplification. Finally we show the adjustments necessary to the market discount rate due to transport phenomena as well as how our two-box model can be extended to Ramsey-type optimal growth models. Numerical simulations confirm our theoretical results.

Keywords: Polar amplification, spatial heat and moisture transport, optimal policy, emission taxes, market discount rate

JEL Classification: Q54, Q58

1 Introduction

In a recent contribution, Dietz and Stern (2015) pointed out that "it is important to stress that the science of climate change was running years ahead of the economics (something that arguably remains the case today in understanding the impacts of climate change)." A well-established fact in the science of climate change is that when the climate cools or warms, high latitude regions tend to exaggerate the changes seen at lower latitudes. This effect is called polar amplification.[1] Polar amplification has been associated with the surface albedo feedback (SAF), by which global warming leads to snow and ice melt and thus greater absorbtion of solar energy, but recent research[2] suggests that significant polar amplification may also emerge as a result of atmospheric heat transport, even without SAF.

Polar amplification and spatial heat transport across the globe are parts of the science of climate change that have been largely ignored by the economics of climate science. The purpose of this paper is to introduce polar amplification and spatial heat transport into an economic model of climate change and explore the impacts on the design of climate policy from ignoring these factors, when in reality they are present and affect the evolution of climate.

Alexeev and Jackson (2012, 2013) develop a useful two-box model that presents mechanisms of heat transport, polar magnification, and ice line movement effects due to outside forcing along with a simple treatment of moisture transport. The two boxes represent the higher latitudes in box 2 (30°N to 90°N) and the lower latitudes in box 1 (0°N to 30°N).[3] The Alexeev

[1] As Langen and Alexeev (2007) point out, polar amplification is seen in model projections of future climate (e.g. Holland and Bitz 2003, ACIA 2004) and, in fact, in the very earliest simple model of CO2-induced climate change (Arrhenius 1896). Polar amplification is found in proxy-records of both deep past warm periods (e.g., Zachos et al. 2001) and of the more recent cold glacials (e.g., Masson-Delmotte et al. 2006).

[2] See for example Langen and Alexeev (2007) and Alexeev and Jackson (2012) and the references there in. Winton (2006) and Alexeev and Jackson (2012) compares the strength of ice-line feedback effects, i.e. Surface Albedo Feedback (SAF), to heat and moisture transport effects upon polar amplification. They argue that heat and moisture transport effects, independent of SAF effects, contribute importantly to polar amplification. The simple two-box model makes it easy to compute the Polar Amplification Factor which is the ratio of temperature change in the high latitude box to the global average temperature change for the whole planet.

[3] Brock and Xepapadeas (2015) use a more realistic energy balance model because it models the Earth by a continuum of latitudes and considers heat transport across latitudes, i.e., it has a “continuum of boxes” with heat transport across each. However,

and Jackson (2012) and Langen and Alexeev (2007) two-box models allow us to treat heat transport and also to use elementary mathematics at the price of simplifications. Their two-box models are useful as a quick way of making the following points.

First, if we denote the temperature anomaly, i.e. the change in temperature relative to a given benchmark temperature, in each box or region by T_1 and T_2 respectively, the relaxation time of the box anomaly temperature gradient, $T_1 - T_2$, is faster than the relaxation time of the box anomaly global mean temperature, $(T_1 + T_2)/2$ (Langen and Alexeev 2007, equation (23). Thus we should look out for a faster response to forcing of polar amplification than global mean temperature in more complicated models like Brock and Xepapadeas (2015). This difference is economically relevant for damages related to temperature differences across different latitudes in contrast to damages related to the planetary global average temperature.[4]

Therefore polar amplification apart from its importance for climate science, also is important for the economics of climate change. In particular, polar amplification causes loss of Arctic sea ice which in turn has consequences for melting land ice along with other effects. There is growing evidence suggesting rapid Arctic warming relative to the Northern hemisphere mid-latitudes. This phenomenon has been called Arctic amplification and is expected to increase the frequency of extreme weather events (Francis and Vavrus 2014). Melting land ice associated with a potential meltdown of Greenland and West Antarctica ice sheets due to polar amplification might cause serious global sea level rise. It is estimated that the Greenland ice sheet holds an equivalent of 7 metres of global sea level rise, while the West Antarctica ice sheet holds the potential for up to 3.5 metres of global sea level rise (see Lenton et al. 2008).[5] On the other hand, the loss of Arctic sea ice due to the Arctic amplification may generate economic benefits by making possible the exploitation of natural resources and fossil fuel reserves which are not accessible now because of the sea ice. Another source of damages caused by polar amplification relates to the thawing of permafrost.

more advanced mathematics is required for this analysis.

[4]Brock and Xepapadeas (2015, equations (19) and (20)) show that the response of the difference is indeed faster than the response of global mean temperature.

[5]In the discussion about tipping points it has been stressed that the time scale of melting of the Greenland ice sheet is much longer than Arctic sea ice melting. However the Antarctic ice sheet could melt very fast once it gets started, but it will need an increase of 5°C of surface temperature to cause a serious destabilization.

Permafrost or permafrost soil is soil at or below the freezing point of water (0°C or 32° F) for two or more years. Permafrost regions occupy approximately 22.79 million square kilometers (about 24 percent of the exposed land surface) of the Northern Hemisphere (Zhang et al. (2003)). Permafrost occurs as far north as 84° N in northern Greenland, and as far south as 26° N in the Himalayas, but most permafrost in the Northern Hemisphere occurs between latitudes of 60° N and 68° N. (North of 67° N, permafrost declines sharply, as the exposed land surface gives way to the Arctic Ocean.) Recent work investigating the permafrost carbon pool size estimates that 1400-1700 Gt of carbon is stored in permafrost soils worldwide. This large carbon pool represents more carbon than currently exists in all living things and twice as much carbon as exists in the atmosphere (Tarnocai et al. (2009)). Thawing of permafrost caused by polar amplification is expected to bring widespread changes in ecosystems, increase erosion, harm subsistence livelihoods, and damage buildings, roads, and other infrastructure. Loss of permafrost will also cause release of greenhouse gases with global effects. Issues, therefore, such as melting of land ice or thawing of permafrost suggest that polar amplification might be an important factor in the effort to design efficient climate policies. [6]

In this context, the two-box models help to focus our attention on economic cross effects of temperature increases in the higher latitudes upon the lower latitudes, as well as the economic effects of temperature increases for each latitude. We shall see that the sign of the derivative of total energy use and, hence, emissions, w.r.t. the rate of spatial transport of heat energy from the lower to the higher latitudes, depends upon the difference between the marginal damages caused by temperature increase at the high latitudes and the temperature increase at the lower latitudes. Furthermore it is easy to see the economics interacting with climate science in the two-box model to illustrate the importance of taking into account heat and moisture transport from the lower latitudes to the higher latitudes. For example, we show that neglect of transport effects leads to overstating (understating) how big carbon taxes should be if marginal damages from one degree temperature increases are smaller (larger) at the higher latitudes compared to the lower latitudes.

[6] Melting of land ice and permafrost thawing are reletaed to the concept of damage reservoirs. (see Brock et al. 2014a)

With hindsight this insight into climate economics is quite clear but the two box model does a nice job of helping us to see it. To put it another way, if humans could move heat energy to where it does the least damage, then carbon taxes would be lower compared to a world where this ability to move heat energy around at will was absent. Since the real climate system moves heat energy from the lower latitudes to the higher latitudes, the direction of transport is fixed by the climate system. This directionality of heat energy and moisture transport interacting with the pattern of relative marginal damages from temperature increases across latitudes determines the bias in optimal carbon taxes. As we show below, neglecting what climate science knows about heat and moisture transport in Integrated Assessment Modeling in climate economics can, theoretically, lead to serious biases in recommended carbon taxes, in estimates of welfare effects from climate change. While we are able to use theory to isolate potential directions and strengths of these biases, and to make plausible qualitative statements about their potential sizes, serious calibration and computational work is needed to get quantitative estimates. That is beyond the scope of this article.

Third, and most important, we shall see that all relevant quantities are functions of the optimal carbon tax rate, $\tau \equiv -\lambda\left(\lambda_{T_1}+\lambda_{T_2}\right)$, where λ is the global mean average temperature increase per unit of cumulative emissions, and $-\lambda_{T_i}, i=1,2$ are the shadow prices of a unit of extra emissions for latitude belts $i=1,2$ where lower latitude belts are indexed by lower numbers. We create a "how much spatial heat transport matters index" by taking the ratio of the value of τ when there is spatial heat transport to the value of τ when spatial transport is zero. We shall see below that "space matters" when the high latitude share of marginal damages deviates from $\frac{1}{2}$.

Thus the main contribution of this paper, apart from introducing a more realistic climate model to the economic modeling of climate change, is to show that by using this model, economic policy which does not account for heat and moisture transport will be incirrect unless the shares of high to low latitude damages are the same. Since this damage structure is rather restrictive, while heat and moisture transport are real phenomena, our approach provides insights into the ways that economic policy for climate change should be corrected so that it is founded on solid climate science. Furthermore, we show how the welfare cost of incorrect policy can be calculated and how heat and moisture transport affects discount rates used in the Cost

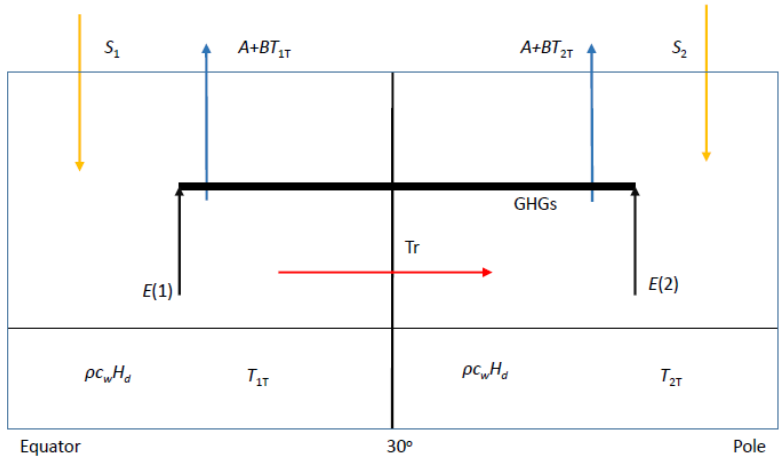

Figure 1: The two-box energy balance model

Benefit Analysis of projects in low and high latitudes. Although in our main analytical framework we are abstracting away from the problem of optimal capital accumulation, we show in the last section how our analysis can be extended to Ramsey-type optimal growth models.

2 A two-box energy balance model with anthropogenic emissions of greenhouse gasses

The two-box energy balance model introduced by Langen and Alexeev (2007) and Alexeev and Jackson (2012) consists of a single hemisphere with two boxes or regions divided by the 30th latitude, which yields similar surface area of the two boxes. Following Langen and Alexeev (2007), the two-box model is presented below.

In figure 1, T_{xT} , $x = 1, 2$ is the surface temperature in each box, with 1 denoting the lower latitude and 2 the higher latitude. This temperature is defined as the sum of equilibrium, or baseline, average temperature in each box (T_{b1}, T_{b2}) when anthropogenic forcing through emissions of GHGs is zero, plus the temperature anomaly (T_1, T_2) . Thus $T_x = T_{xT} - T_{xb}$. By the definition of the boxes (or regions), the baseline average yearly temperatures (T_{b1}, T_{b2}) satisfy the inequality $T_{1b} > T_{2b}$. The downwelling short wave radiation in each region is denoted by S_x, the outgoing longwave radiation by $A + BT_x$, the heat transport from box 1 to box 2 by Tr and the stock of greenhouse gases created by anthropogenic emissions by GHG. This stock traps part of the outgoing longwave radiation. In the two-box model the ocean mixed layer has a depth of H_d, density ρ_d, and heating capacity c_w, thus we denote by $H = \pi a_e^2 \rho_d c_w H_d$ the heat capacity in each of the boxes. Assuming no anthropogenic forcing, the evolution of the surface temperature in each box is:

$$\dot{T}_{1T} = \frac{1}{H}\left(S_1 - A - BT_{1T} - Tr\right) \tag{1}$$

$$\dot{T}_{2T} = \frac{1}{H}\left(S_2 - A - BT_{2T} + Tr\right). \tag{2}$$

The meridional heat transport is defined in terms of the temperature anomaly as:

$$Tr = \bar{T}r + \gamma_1 \left(T_1 - T_2\right) + \gamma_2 T_1. \tag{3}$$

In (3) the first term is the equilibrium heat transport, the second term captures the increase in transport due to increasing baroclinicity,[7] while the third term captures the effect of an increased moisture supply and thus greater latent heat transport with increased low- to mid-latitude temperatures. In the dynamical system (1)-(2), we use the parametrization of Alexeev et al. (2005) and add anthropogenic forcing as in Alexeev and Jackson (2012). The anthropogenic forcing is assumed to be $\Delta f\,(t) = \lambda E\,(t)$ for all dates t following Matthews et al. (2009) and MacDougal and Friedlingstein (2015), where λ is their cumulative carbon response parameter and $E\,(t) = E\,(1,t) + E\,(2,t)$ is global GHGs emissions at date t. Emissions can also be interpreted by appropriate choice of units as fossil fuel use. Global emissions are defined as the sum of emissions in box 1,$E\,(1,t)$, and box 2, $E\,(2,t)$, Under these assumptions the dynamical system (1)-(2) can be expressed in terms of the evolution of the temperature anomaly in each box as:

[7]In meteorology a baroclinic atmosphere is one for which the density depends on both the temperature and the pressure. In a barotropic atmosphere, on the other hand, the density depends only on the pressure. In atmospheric terms, the barotropic zones of the Earth are generally found in the central latitudes, or tropics, whereas the baroclinic areas are generally found in the mid-latitude/polar regions.

$$\dot{T}_1 = \frac{1}{H}\left[\left(-B-\gamma_1-\gamma_2\right)T_1+\gamma_1 T_2+\Delta f\right],\ T_1(0)=0 \quad (4)$$
$$\dot{T}_2 = \frac{1}{H}\left[\left(\gamma_1+\gamma_2\right)T_1+\left(-B-\gamma_1\right)T_2+\Delta f\right],\ T_2(0)=0 \quad (5)$$
$$\Delta f = \lambda E(t)\ ,\ E(t)=E(1,t)+E(2,t)\,. \quad (6)$$

It can easily be seen from (4)-(6) that when $\gamma_2 = 0$ the steady state temperature anomaly between low and high latitudes is the same, that the ratio between low latitude warming and high latitude warming is one. On the other hand, in a steady state where $\gamma_2 > 0$, the ratio is greater than one. Thus the term $\gamma_2 T_1$ in (3) breaks symmetry.

3 Social Welfare Optimization under Polar Amplification

To study optimal climate policy in the context of the two-box climate model described above, we consider a simple welfare maximization problem with logarithmic utility, where world welfare is expressed by the sum of welfare in each region and is given by:

$$\int_{t=0}^{\infty} \mathrm{e}^{-\rho t}\left[\sum_{x=1}^{x=2} v(x)\,L(x,t)\ln\left[y(x,t)\,E(x,t)^{\alpha}\,\mathrm{e}^{-\phi(x,T_b+T)}\right]\right]dt, \quad (7)$$

where $y(x,t)\,E(x,t)^{\alpha}$, $0<\alpha<1$, $E(x,t)$, $T_{bi}(x,t)$, $T_i(x,t)$ $L(x,t)$ are output per capita, fossil fuel input or emissions of GHGs, baseline temperature, temperature anomaly and fully employed population in each region x at date t, respectively. The term $\mathrm{e}^{-\phi(x,T_b+T)}$, $T_b+T=(T_{b1}+T_1, T_{b2}+T_2)$ reflects damages to output per capita in region $x=1,2$ from an increase in the temperature anomaly in either region, since polar amplification in region 2 might generate damages to region 1. We assume that $y(x,t)$, $L(x,t)$ are exogenously given. That is, we are abstracting away from the problem of optimally accumulating capital inputs and other inputs in order to focus sharply on optimal fossil fuel taxes. In this context $y(x,t)$ could be interpreted as the component of a Cobb-Douglas production function that embodies all other inputs along with technical change that evolve exogenously. Finally, $v(x)$ represents welfare weights associated with box (or region) x.

Assuming that each region has its own fossil fuels reserves, denoted by $R_0(x)$, the resource constraint for each region becomes:

$$\int_{t=0}^{\infty} E(x,t)\,dt \leq R_0(x) \ \ , \ x = 1,2. \tag{8}$$

The welfare optimization problem is, therefore, to choose the fossil fuel (or GHG emissions) path to maximize (7) subject to (4)-(6) and (8). To simplify and allow study of the property of optimal steady states, we assume that $L(x,t) = L(x)$, $y(x,t) = y(x)$, $x = 1,2$ for all dates.[8] Dropping the term $v(x)L(x,t)\ln y(x,t)$, which does not affect optimality conditions, the current value Hamiltonian function for the welfare maximization problem becomes:

$$\begin{aligned} H \ &= \ \sum_{x=1}^{x=2} \left\{ v(x)\,L(x)\left[\alpha \ln E(x,t) - \phi(x, T_b + T)\right] - \lambda_{R_0^x} E(x,t) \right\} + \quad (9) \\ & \lambda_{T_1} \frac{1}{H}\left[(-B - \gamma_1 - \gamma_2)\,T_1 + \gamma_1 T_2 + \lambda\left[E(1,t) + E(2,t)\right]\right] + \\ & \lambda_{T_2} \frac{1}{H}\left[(\gamma_1 + \gamma_2)\,T_1 + (-B - \gamma_1)\,T_2 + \lambda\left[E(1,t) + E(2,t)\right]\right] \\ & T_b = (T_{b1}, T_{b2}) \ \ , \ T = (T_1, T_2)\,. \end{aligned}$$

The following first order necessary Conditions (FONC) for the optimal choice of fossil fuel (or emissions) use can be obtained by differentiating the Hamiltonian w.r.t. $E(x,t)$,

$$\frac{\alpha v(x)\,L(x)}{E(x,t)} = \frac{-\lambda\left(\sum_{i=1}^{2} \lambda_{T_i}(t)\right)}{H} + \lambda_{R_0^x}(t) \quad , \text{ or} \tag{10}$$

$$E(x,t) = \frac{-\alpha v(x)\,L(x)\,H}{\left[-\lambda\left(\sum_{i=1}^{2} \lambda_{T_i}(t)\right) + \lambda_{R_0^X}(t)\right]} \ \ , \ x = 1,2. \tag{11}$$

If we assume that both regions share the total initial fossil fuel reserves, then resource constraints (8) should be replaced by the single constraint

$$\int_{t=0}^{\infty} E(t)\,dt = \int_{t=0}^{\infty} \sum_{x=1}^{x=2} E(x,t)\,dt \leq R_0 \ , R_0 = R(1) + R(2)\,. \tag{12}$$

[8] We could also have assumed that L and y grow exponentially and have their growth rates absorbed into the utility discount rate.

Then the multipliers $\lambda_{R_0^x}(t)$ should be replaced by the single multiplier $\lambda_{R_0}(t)$ in the FONC (10).

It can be seen from (10) that the externality tax associated with anthropogenic emissions of GHGs is:

$$\tau(t) = \frac{-\lambda\left(\sum_{i=1}^{2}\lambda_{T_i}(t)\right)}{H}. \tag{13}$$

Note that the externality tax is likely to increase as the cumulative carbon response parameter, λ, of Matthews et al. (2009) increases and the heat capacity decreases H. Of course we must take into account changes in these parameters upon the temperature co-states in order to get the total effect on the externality tax. Under our simplifying assumptions to be stated below, the shadow prices of emissions turn out to be constants over time. Thus τ is a useful "sufficient parameter" for all the quantities that are policy-relevant. That is, the emissions at each set of latitude belts, $x = 1, 2$, the optimal "price" path of reserves, $R_0(x)$, and optimal welfare are all functions of τ, as we shall see below.

Furthermore if fossil fuel reserves plus anticipated new discoveries in each region are infinite, then $\lambda_{R_0^x}(t) = 0$ for all dates t, and $x = 1, 2$ or $\lambda_{R_0}(t) = 0$ for all dates t, if we consider the case in which the two regions share infinite reserves. If the reserves are finite, then their shadow price λ_{R_0} rises at the rate ρ over time. When the initial reserve plus anticipated new discoveries is finite, the initial value $\lambda_{R_0}(0)$ is set by the resource constraints,

$$\int_{t=0}^{\infty} E(x,t)\,dt = R_0(x) \ \ , \ x = 1, 2, \ \text{or} \ \int_{t=0}^{\infty}\sum_{x=1}^{x=2} E(x,t)\,dt = R_0. \tag{14}$$

In order to obtain some straightforward insights about the interaction of climate and economics in the simplest possible setting, we restrict ourselves to the case in which $\frac{\partial\phi(x,T_b+T)}{\partial T_i}$ is constant for all $x = 1, 2$ and $i = 1, 2$.

Assumption 1: *Define marginal damage cost of temperature increase in box $x = 1, 2$ by*

$$d_i = d_{1i} + d_{2i} = \sum_{x=1}^{x=2} v(x)\,L(x)\,\frac{\partial\phi(x,T_b+T)}{\partial T_i} \ , i = 1, 2. \tag{15}$$

where

$$v(1)\,L(1)\phi(1,T_b+T) = d_{11}(T_{b1}+T_1)+d_{12}(T_{b2}+T_2) \quad (16)$$

$$v(2)\,L(2)\phi(2,T_b+T) = d_{21}(T_{b1}+T_1)+d_{22}(T_{b2}+T_2) \quad (17)$$

We assume d_i, $i=1,2$ are constants at all dates.

In Assumption 1 the parameters (d_{12}, d_{21}) capture the cross effects from an increase of the temperature anomaly in one region on the damages of the other region. In particular, d_{12} captures the effects of polar amplification in region 2 on damages in region 1. Thus $d_1 = d_{11}+d_{21}$ is the aggregate impact (i.e. the impact on both regions) from a temperature increase in region 1, while $d_2 = (d_{12}+d_{22})$ is the aggregate impact from a temperature increase in region 2. If we assume that the polar amplification effects on region 1 are sufficiently strong, and d_{21} is negligible, then $d_2 > d_1$ might reflect strong polar amplification effects.

The optimality conditions for co-state equations of the climate dynamics of (9) imply

$$\frac{d\lambda_{T_1}}{dt} = \left[\rho+\frac{(B+\gamma_1+\gamma_2)}{H}\right]\lambda_{T_1} - \left[\frac{(\gamma_1+\gamma_2)}{H}\right]\lambda_{T_2}+d_1 \quad (18)$$

$$\frac{d\lambda_{T_2}}{dt} = -\left(\frac{\gamma_1}{H}\right)\lambda_{T_1} + \left[\rho+\frac{(B+\gamma_1)}{H}\right]\lambda_{T_2}+d_2, \quad (19)$$

and the forward solutions of (18)-(19) are constants by Assumption 1. The evolution of the co-states can be described by the linear dynamical system

$$\dot{\boldsymbol{\lambda}}_T = A\boldsymbol{\lambda}_T+\mathbf{d} \quad (20)$$

$$A = \begin{pmatrix} \rho+\frac{(B+\gamma_1+\gamma_2)}{H} & \frac{-(\gamma_1+\gamma_2)}{H} \\ -\left(\frac{\gamma_1}{H}\right) & \rho+\frac{(B+\gamma_1)}{H} \end{pmatrix},\ \boldsymbol{\lambda}_T = \begin{pmatrix} \lambda_{T_1} \\ \lambda_{T_2} \end{pmatrix},\ \mathbf{d} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix},$$

with terminal conditions at infinity determined by the steady state of the Hamiltonian system associated with (9). System (20), along with temperature dynamics (4)-(6) in which emissions in each region are given by the optimal emissions (11), constitute this Hamiltonian system which determines optimal paths for the temperature anomalies $(T_1(t), T_2(t))$, the associated costate variables or shadow values $(\lambda_{T_1}(t), \lambda_{T_2}(t))$, the optimal fossil fuel (or emission) path $E(x,t)$, and the corresponding steady states.

From the steady state values for the costates can easily be obtained as

$$\lambda_{T_1} = -\left(\beta d_1 + \frac{\gamma_1 d_2}{H} + \frac{\gamma_2 d_2}{H}\right)/\Gamma \quad (21)$$

$$\lambda_{T_2} = -\left(\beta d_2 + \frac{\gamma_2 d_2}{H} + \frac{\gamma_1 d_1}{H}\right)/\Gamma \quad (22)$$

$$\beta = \rho + \frac{(B+\gamma_1)}{H}. \quad (23)$$

$$\Gamma = \beta^2 - \left(\frac{\gamma_1}{H}\right)^2 + \beta\frac{\gamma_2}{H} - \left(\frac{\gamma_1\gamma_2}{H}\right). \quad (24)$$

4 Heat Transport and Climate Policy

The results of the welfare optimization problem can be used to explore the impact of heat transport and polar amplification on climate policy. In particular we are interested in calculating the error made if the planner mistakenly ignores heat transfer Tr in computing optimal carbon taxes. To calculate this error we compute the solution by the planner who acts as if $Tr = 0$, but it is present in the actual climate. The planner mistakenly replaces (18)-(19) with

$$\frac{d\hat{\lambda}_{T_1}}{dt} = \left[\rho + \frac{B}{H}\right]\hat{\lambda}_{T_1} + d_1 \quad (25)$$

$$\frac{d\hat{\lambda}_{T_2}}{dt} = \left[\rho + \frac{B}{H}\right]\hat{\lambda}_{T_2} + d_2, \quad (26)$$

with the steady-state externality tax defined by

$$\hat{\tau} = \frac{-\lambda\left(\hat{\lambda}_{T_1} + \hat{\lambda}_{T_2}\right)}{H} \quad (27)$$

$$\hat{\lambda}_{T_1} = \frac{-d_1}{\rho + B/H}\ ,\ \hat{\lambda}_{T_2} = \frac{-d_2}{\rho + B/H}. \quad (28)$$

The planner's incorrect tax rate may be compared with the correct steady-state tax rate

$$\tau = \frac{-\lambda\left(\lambda_{T_1} + \lambda_{T_2}\right)}{H}, \quad (29)$$

with $(\lambda_{T_1}, \lambda_{T_2})$ given by (21)-(24).

From this point on, due to notational clutter, we set $H = 1$. As we can see from the above formulae, setting $H = 1$ just amounts to absorbing H into the parameters $\lambda, B, \gamma_1, \gamma_2$ because it always enters as a ratio.

It is convenient to write the correct tax rate $\tau(\gamma_1, \gamma_2)$ as a function of heat and moisture transport parameters (γ_1, γ_2) as follows, using (21)-(24):

$$\begin{aligned} \tau(\gamma_1, \gamma_2) &= -\lambda\left(\left(\lambda_{T_1} + \lambda_{T_2}\right)\right) \\ &= \frac{\lambda\left[(\rho + B + 2\gamma_1)(d_1 + d_2) + 2\gamma_2 d_2\right]}{(\rho + B + \gamma_1)^2 - \gamma_1^2 + (\rho + B + \gamma_1)\gamma_2 - \gamma_1\gamma_2} \\ &= \frac{\lambda\left[(\rho + B + 2\gamma_1)(d_1 + d_2) + 2\gamma_2 d_2\right]}{(\rho + B)(\rho + B + 2\gamma_1 + \gamma_2)}. \end{aligned} \tag{30}$$

Since the incorrect tax can be written as $\hat{\tau} = \tau(\gamma_1, 0)$, i.e. the optimal tax rate is the same as the planner's optimal tax rate unless $\gamma_2 > 0$. The ratio of the planner's incorrect choice of "optimal" tax rate and the true tax rate is

$$\frac{\hat{\tau}}{\tau(\gamma_1, \gamma_2)} = \frac{\tau(\gamma_1, 0)}{\tau(\gamma_1, \gamma_2)} = \frac{(d_1 + d_2)(\rho + B + 2\gamma_1 + \gamma_2)}{(\rho + B + 2\gamma_1)(d_1 + d_2) + 2\gamma_2 d_2}. \tag{31}$$

It is informative to compute relative error in setting tax rates when γ_2 goes to infinity. Using L'Hospital's rule we obtain

$$\frac{\tau(\gamma_1, \infty)}{\tau(\gamma_1, 0)} = \frac{2d_2}{d_1 + d_2} \tag{32}$$

We see from (32) that the correct tax rate can be as much as twice the tax rate with no polar amplification due to heat and moisture transport (i.e. when $\gamma_2 = 0$) when the share of region 2's damages $\frac{d_2}{d_1+d_2}$, is one. We sum up our discussion at this point in Proposition 1 below.

Proposition 1 *The planner who mistakenly ignores spatial heat transport taxes carbon too little, i.e., when* $\frac{\tau(\gamma_1,0)}{\tau(\gamma_1,\gamma_2)} < 1$ *if and only if,* $(d_1 + d_2)\gamma_2 < 2\gamma_2 d_2$*. It taxes carbon too much if and only if* $(d_1 + d_2)\gamma_2 > 2\gamma_2 d_2$*. Since the damage share of the damage contributions from the warming of the high latitudes is* $s_2 \equiv \frac{d_2}{d_1+d_2}$*, the direction of bias in carbon taxes in this model from ignoring spatial heat transport is described by a very simple relation between the damage shares and the two basic parameters of heat transport.*

Proof. The computations above in equations (31) and (32) show that

$$\frac{\hat{\tau}}{\tau(\gamma_1, \gamma_2)} = \frac{\tau(\gamma_1, 0)}{\tau(\gamma_1, \gamma_2)} \lesseqgtr 1 \; iff \; (d_1 + d_2)\gamma_2 \lesseqgtr 2\gamma_2 d_2 \tag{33}$$

■

The key role of $\gamma_2 > 0$ in the above conclusions warrants some discussion. As Langen and Alexeev (2007) and Alexeev and Jackson (2012) stress, $\gamma_2 > 0$ captures aspects of moisture transport in addition to aspects of heat transport. A more elaborate model that includes both heat and moisture transport is that of Fanning and Weaver (1996). The Langen and Alexeev (2007) and Alexeev and Jackson (2012) models can be usefully viewed as abstractions that capture aspects of the more complicated Fanning and Weaver (1996) model which, in turn, is a drastic simplification of the more realistic Weaver et al. (2001) model.

We believe the analytical clarity in showing us how the ratio depends upon marginal damages for each of the two regions of latitude belts as well as the two basic parameters of heat and moisture transport counterbalances the cost of abstracting away from more realistic features of damages and heat and moisture transport dynamics.

4.1 A numerical simulation

In order to obtain some more insights into results obtained above, we proceed with a simple numerical exercise of the Hamiltonian system associated with (9). To calibrate the model we adopt benchmark estimates from the literature. In particular, following Langen and Alexeev (2007), we set $B = 0.1$ PW/K (1 PW is 10^{15}W), $\gamma_1 = \gamma_2 = 0.15$ PW/K. For the heat capacity $H = \pi a_e^2 \rho_d c_w H_d$ we use the condition $\tau_s = H/B$, where, as in Langen and Alexeev (2007), $\tau_s = (5.5 \times 100 \text{ months})/12$ and $B = 0.1$ PW/K which implies that $H = 4.58$ (PW year)/K. For the value of the Matthews et al. (2009) cumulative carbon response parameter λ, we consider that about 287.5 petagrams (PG) of cumulative emissions yield about 0.8°C increase in global mean temperature and set $0.8 = \lambda \times (0.287 \text{ teratons C})$. Thus $\lambda = 2.787$. Regarding the damage parameter in $\mathrm{e}^{-\phi(x,T)}$, we follow Brock et al. (2013) and set the damage parameter of an exponential damage function to 0.01. However we caution that the value of .01 was set for a global, non spatial model. This value is considered to provide a decent approximation to the quadratic damage function used in Nordhaus (2007) (e.g. a temperature increase of 4°C corresponds to ≈ 5% loss of output). There is a considerable literature suggesting that the poorest and most vulnerable groups will disproportionately experience the negative effects of climate change and that such changes are likely to impact significantly on developing world countries, where natural-resource dependency is high (see for example Thomas and Twyman 2005). In our set up region 1 which corresponds to latitudes from 0°N to 30°N includes mainly developing world countries. Thus we expect a relatively high d_{11} and also a high d_{12} under polar amplification. Parameter d_{11} will tend to increase the value of $d_1 = d_{11} + d_{21}$ while d_{12} will tend to increase the value of $d_2 = d_{12}+d_{22}$. In the absence of more specific information about the relative sizes of these parameters, we adopt two alternative assumptions for marginal damages. In the first $(d_1, d_2) = (0.014, 0.008)$, while in the second $(d_1, d_2) = (0.008, 0.014)$ which reflects strong polar amplification effects. This allows us to explore the impact of reversing the ranking of damages in each region and numerically verify proposition 1. For the rest of the parameters we use $L(1) = L(2) = 0.5$ since evidence suggests that 50% of the global population lives above 27°, $\alpha = 0.05$, $\rho = 0.02$. Finally at the first stage of the simulation we assume equal welfare weights between the two regions, $v(1) = v(2) = 1$.

The simulation proceeds as follows. Using $(d_1, d_2) = (0.014, 0.008)$ and assuming infinite fossil fuel reserves, we define the Hamiltonian system,

$$\frac{dT_1}{dt} = \frac{1}{H}\left[(-B-\gamma_1-\gamma_2)T_1 + \gamma_1 T_2 + \Delta f\right],\ T_1(0) = 0 \quad (34)$$

$$\frac{dT_2}{dt} = \frac{1}{H}\left[(\gamma_1+\gamma_2)T_1 + (-B-\gamma_1)T_2 + \Delta f\right],\ T_2(0) = 0 \quad (35)$$

$$\frac{d\lambda_{T_1}}{dt} = \left[\rho + \frac{(B+\gamma_1+\gamma_2)}{H}\right]\lambda_{T_1} - \left[\frac{(\gamma_1+\gamma_2)}{H}\right]\lambda_{T_2} + d_1 \quad (36)$$

$$\frac{d\lambda_{T_2}}{dt} = -\left(\frac{\gamma_1}{H}\right)\lambda_{T_1} + \left[\rho + \frac{(B+\gamma_1)}{H}\right]\lambda_{T_2} + d_2 \quad (37)$$

$$\Delta f = \lambda E(t)\ ,\ E(t) = E(1,t) + E(2,t) \quad (38)$$

$$E(x,t) = \frac{-\alpha v(x) L(x) H}{-\lambda\left(\sum_{i=1}^{2}\lambda_{T_i}(t)\right)}, \quad (39)$$

and compute the steady state $\mathbf{z} = \left(\bar{T}_1, \bar{T}_2, \bar{\lambda}_{T_1}, \bar{\lambda}_{T_1}\right)$, which for our parametrization is $\mathbf{z} = (3.38262, 5.91959, -0.267593, -0.224763)$. The steady state has the saddle point property with eigenvalues $\mathbf{e} = (0.140087, -0.120087, 0.0418341, -0.0218341)$. To obtain insights into the optimal paths for the state and the costate variables, we solve the linear approximation of the Hamiltonian system around the steady state. Setting the constants associated with positive eigenvalues equal to zero and using initial and steady state values for the state

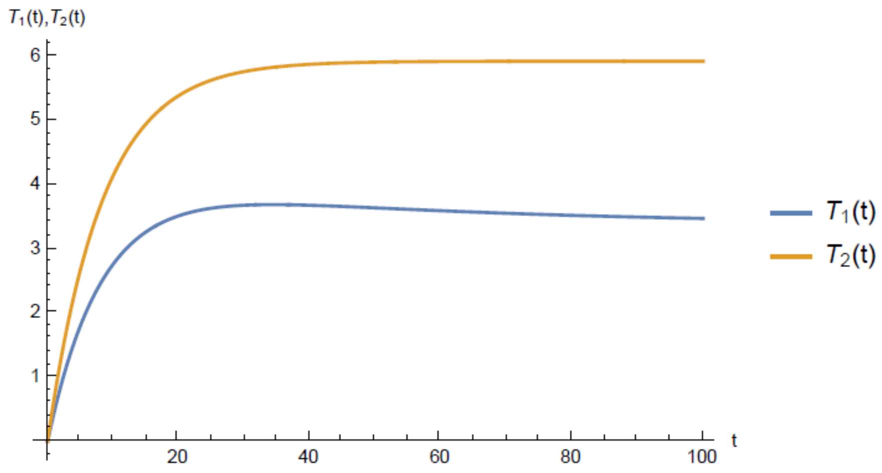

Figure 2: Paths for the temperature anomalies

variable, we compute the remaining constants and the initial values for the costates. We obtain the paths for temperature anomalies and the corresponding costate variables. The paths for the temperature anomalies are shown in Figure 2, where polar amplification under the optimal policy is clear.

Figure 3 presents the externality tax when the planner takes explicitly into account heat and moisture transport. The figure shows that the optimal policy ramp is not gradual but requires a high externality tax at the beginning which declines and eventually converges to its steady state. Since $s_2 \equiv \frac{d_2}{d_1+d_2} < 1/2$ at the steady state, $\hat{\tau} > \tau$.

Figure 4 presents the corresponding optimal path for emissions.

When we reverse the order of marginal damages, i.e., $(d_1, d_2) = (0.008, 0.014)$, to allow for strong polar amplification effects the qualitative characteristics of the solution are the same. In this case the steady state is $\mathbf{z} = (5.95424, 10.4199, -0.258294, -0.301124)$. Figure 5 presents the path for the optimal externality tax $\tau(t)$ along with the corresponding externality tax $\hat{\tau}$. Since $s_2 > 1/2$ at the steady state, $\hat{\tau} < \tau$. Thus ignoring polar amplification causes undertaxing.

Finally we consider the case in which $(d_1, d_2) = (0.014, 0.008)$ but $(v(1), v(2)) = (1.25, 1)$. That is, the welfare weight attached to region 1 is 25% higher than the weight attached to region 2. The qualitative characteristics of the solution are the same but the optimal fossil fuel paths are different now. In this

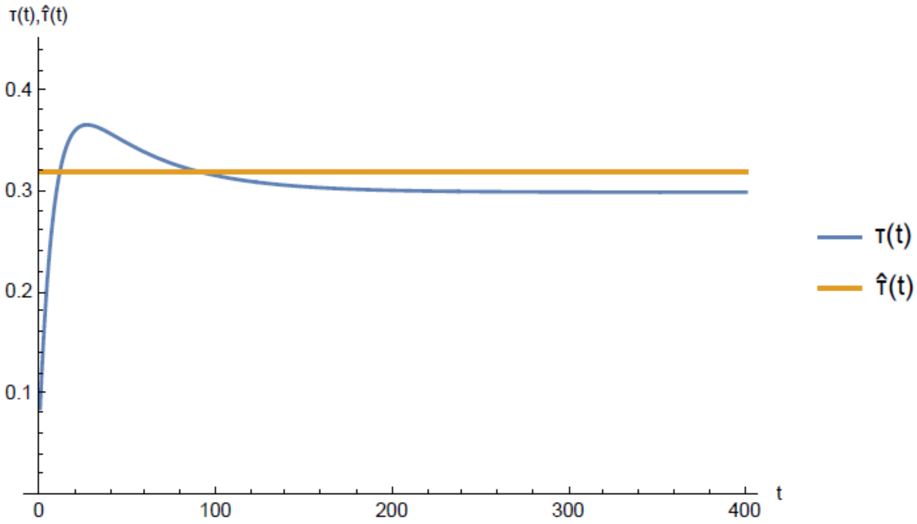

Figure 3: The externality tax $(d_1, d_2) = (0.014, 0.008)$

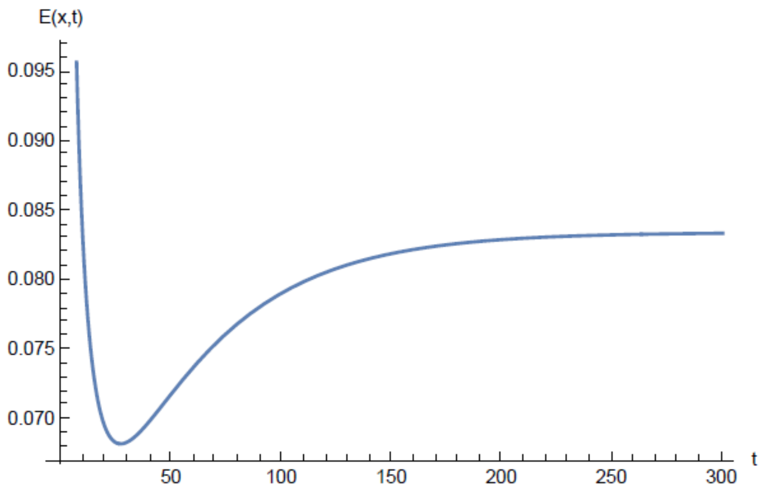

Figure 4: Optimal emission path

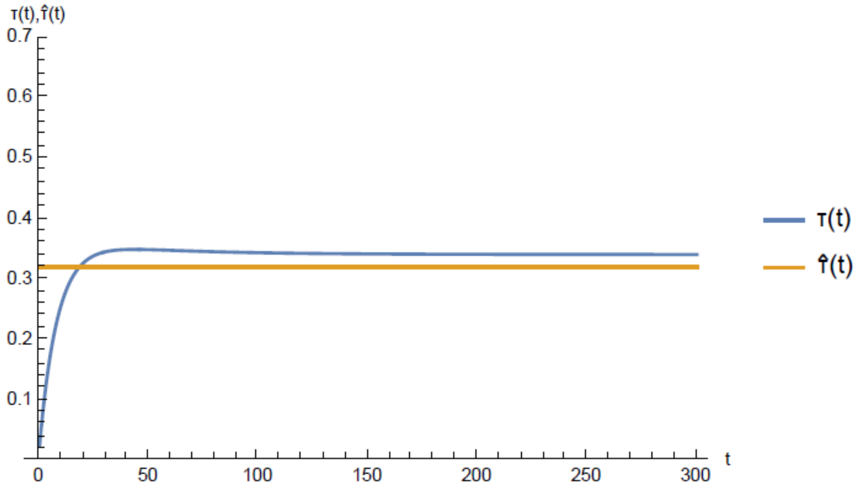

Figure 5: The externality tax $(d_1, d_2) = (0.008, 0.014)$

case fossil fuel use is higher in region 1 as shown in figure 6.

4.2 Polar Amplification and Adaptation Policy

We augment the above model by considering the possibility of mitigation of industrial emissions through abatement. Let $A(x,t)$ denote abatement expenses undertaken in each region in order to reduce damages from global warming. We assume that the cost of adaptation $A(x,t)$ can be expressed as a function of output, $Y(x,t) = y(x,t)\,E(x,t)^a$, as $\xi(A(x,t))\,Y(x,t)$. So output after adaptation is $[1-\xi(A(x,t))]\,Y(x,t)$. In order to obtain tractable results we consider a linear function for $\xi(\mu) = A(x,t)\,\theta(x)$, where $\theta(x)$ is a region specific parameter of adaptation cost. Damages after adaptation are given by $\exp[-\phi(x)[(T_b + T(x,t)) - b(x)A(x,t))]]$ where $b(x)$ captures the effectiveness of adaptation in region $x = 1, 2$. We extend Assumption 1 for the case of adaptation to

Assumption 1$'$: *Define marginal damage cost of temperature increase*

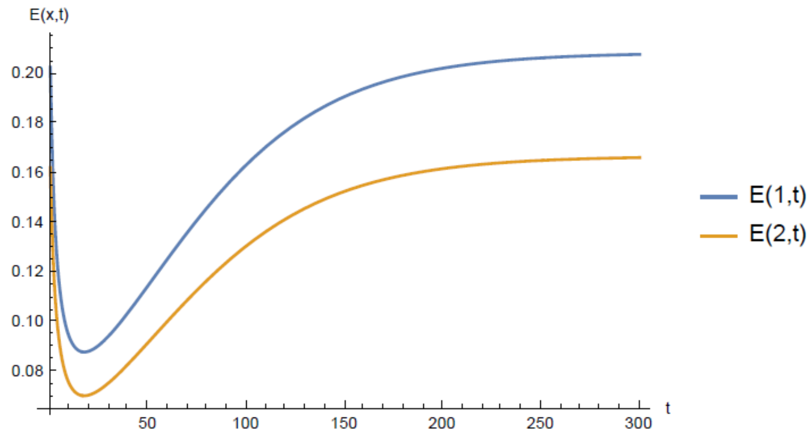

Figure 6: Optimal emission paths $v(1) > v(2)$

d_i and marginal damage savings from abatement b_x in region $i, x = 1, 2$ by

$$d_i = \sum_{x=1}^{x=2} v(x) L(x) \frac{\partial [\phi(x) [(T_b + T(x,t)) - b(x) A(x,t))]]}{\partial T_i}, i = 1, 2 \quad (40)$$

$$b_x = \frac{\partial [\phi(x) [(T_b + T(x,t)) - b(x) A(x,t))]]}{\partial A(x,t)}, x = 1, 2 \quad (41)$$

Under these assumptions the relevant Hamiltonian function can be written as:

$$\begin{aligned} H = & \sum_{x=1}^{x=2} \left\{ v(x) L(x) [\ln \{[1 - A(x,t)\theta(x)] E(x,t)^a\} - \phi(x, T_b + T, A(x))]) - \lambda_{R_0^x} E(x,t) \right\} + \\ & \lambda_{T_1} \frac{1}{H} [(-B - \gamma_1 - \gamma_2) T_1 + \gamma_1 T_2 + \lambda [E(1,t) + E(2,t)]] + \\ & \lambda_{T_2} \frac{1}{H} [(\gamma_1 + \gamma_2) T_1 + (-B - \gamma_1) T_2 + \lambda [E(1,t) + E(2,t)]] \\ & T_b = (T_{b1}, T_{b2}) \ , \ T = (T_1, T_2) \\ & \phi(x, T_b + T, A(x)) = \phi(x) [(T_b + T(x,t)) - b(x) A(x,t))] . \end{aligned} \quad (42)$$

In this formulation the planner chooses optimal fossil fuel use, $E(x,t)$, and adaptation expenditure, $A(x,t)$. Optimality conditions for fossil use and adaptation imply that

$$\frac{\alpha v(x) L(x)}{E(x,t)} = \frac{-\lambda[\lambda_{T_1} + \lambda_{T_2}]}{H} + \lambda_{R_0^x}(t) = 0 \quad (43)$$

$$\frac{\theta(x)}{1 - A(x)\theta(x)} = b_x, \text{ or } A(x) = \frac{b_x - \theta(x)}{b_x \theta(x)}, \; x = 1, 2. \quad (44)$$

Condition (44) implies that as long as $b_x - \theta(x) > 0$, the corresponding region will undertake adaptation. Since adaptation expenditure does not affect temperature dynamics and the damage function is linear in adaptation and separable between temperature and adaptation, the optimal paths and steady states for the temperature anomaly fossil fuel use and optimal taxes remain the same as in the case in which adaptation was not available. In this simple model emissions and adaptation are independent. Adaptation expenditure will affect temperature dynamics, emissions and taxes, if it affects the costate variables through a damage function that allows for a link between adaptation and temperature.

5 Welfare Cost of Ignoring Heat and Moisture Transport

While the computation of qualitative effects of spatial transport on optimal emissions tax rates analyzed above is useful, it does not tell us much about the economic importance of taking spatial heat and moisture into account, i.e. we need to compute the impact on economic welfare measures. We turn to this task now.

Suppose a planner mistakenly believes that heat and moisture transport is not present, i.e. $(\gamma_1, \gamma_2) = (0, 0)$ but the true dynamics are $\gamma_1 > 0, \gamma_2 > 0$. How big is the error in welfare units and how big is the error in energy use and emissions taxes? We formulate a conceptual framework and study these questions here.

Consider the social welfare optimization problem

$$V\left[(\gamma_1,\gamma_2)\mid(\gamma_1,\gamma_2)\right]\equiv \tag{45}$$
$$\max\int_{t=0}^{\infty}\mathrm{e}^{-(\rho-\eta)t}\left[\sum_{x=1}^{x=2}v\left(x\right)L_0\left(x\right)\ln\left[y\left(x,t\right)E\left(x,t\right)^{\alpha}\mathrm{e}^{-\phi(x)(T)}\right]\right]dt$$
subject to
$$\int_{t=0}^{\infty}\sum_{x=1}^{x=2}E\left(x,t\right)dt\leq\sum_{x=1}^{x=2}R_0x)\equiv R_0 \tag{46}$$
$$\dot{T}_1=\left[\left(-B-\gamma_1-\gamma_2\right)T_1+\gamma_1T_2+\lambda E\left(t\right)\right],\ T_1\left(0\right)=0 \tag{47}$$
$$\dot{T}_2=\left[\left(\gamma_1+\gamma_2\right)T_1+\left(-B-\gamma_1\right)T_2+\lambda E\left(t\right)\right],\ T_2\left(0\right)=0 \tag{48}$$
$$E\left(t\right)=E\left(1,t\right)+E\left(2,4\right). \tag{49}$$

Here, as in Section 2, $T\left(t\right)=T_b+T\left(t\right)$ and we have assumed that population grows at the same rate η in each region, so that $L\left(x,t\right)=L_0\left(x\right)\mathrm{e}^{\eta t}$. We have also denoted the optimal value by $V\left[(\gamma_1,\gamma_2)\mid(\gamma_1,\gamma_2)\right]$ when the planner believes the transport parameters are (γ_1,γ_2) and the true transport parameters are, (γ_1,γ_2). We denote the value when the planner believes the transport parameters $(a_1.a_2)\neq(\gamma_1,\gamma_2)$ but the true transport parameters are (γ_1,γ_2) by $V\left[(a_1.a_2)\mid(\gamma_1,\gamma_2)\right]$.

By construction we have

$$V\left[(a_1.a_2)\mid(\gamma_1,\gamma_2)\right]\leq V\left[(\gamma_1,\gamma_2)\mid(\gamma_1,\gamma_2)\right], \tag{50}$$

for all $(a_1.a_2)\geq(0,0)$. Hence we may use the relative error measure,

$$\psi=\frac{V\left[(\gamma_1,\gamma_2)\mid(\gamma_1,\gamma_2)\right]-V\left[(a_1.a_2)\mid(\gamma_1,\gamma_2)\right]}{V\left[(\gamma_1,\gamma_2)\mid(\gamma_1,\gamma_2)\right]}\geq 0, \tag{51}$$

as an economic measure of the error made by a planner who believes $(a_1.a_2)$ when the true parameters are (γ_1,γ_2).

We use similar notation for total emissions at date t, $E\left[(a_1.a_2)\mid(\gamma_1,\gamma_2)\right]$ and the temperature anomalies $T_1\left[(a_1.a_2)\mid(\gamma_1,\gamma_2)\right]$, $T_2\left[(a_1.a_2)\mid(\gamma_1,\gamma_2)\right]$ for the planner who believes the transport parameters are $(a_1.a_2)$ but the true parameters are (γ_1,γ_2).

The computational procedure can borrow a lot of material from Section 3, once we recognize that the beliefs of the planner determine the co-state variables for the two temperature anomalies,[9]

$$\begin{aligned}\dot{\lambda}_{T_1} &= [\rho - \eta + (B + a_1 + a_2)]\,\lambda_{T_1} - [(a_1 + a_2)]\,\lambda_{T_2} + d_1 &(52)\\ \dot{\lambda}_{T_2} &= -a_1\lambda_{T_1} + [\rho - \eta + (B + a_1)]\,\lambda_{T_2} + d_2. &(53)\end{aligned}$$

The steady-state solutions for the costate variables are given by (21)-(24) with $H = 1$ and (γ_1, γ_2) replaced by $(a_1.a_2)$. To put it another way, "beliefs" about the parameters of the temperature dynamics determine the co-state equations (52)-(53). Those beliefs determine the externality tax associated with anthropogenic emissions of GHGs

$$\tau(a_1, a_2) = -\lambda\,(\lambda_{T_1} + \lambda_{T_2}) = \frac{\lambda\,[(\rho - \eta + B + 2a_1)(d_1 + d_2) + 2a_2 d_2]}{(\rho - \eta + B)(\rho - \eta + B + 2a_1 + a_2)}. \tag{54}$$

This externality tax determines emissions according to

$$\begin{aligned}E(x, t, (a_1, a_2)) &= \frac{\alpha v(x)\,L_0(x)}{\tau(a_1, a_2) + \lambda_{R_0}(a_1, a_2)\,\mathrm{e}^{(\rho-\eta)t}} &(55)\\ \int_{t=0}^{\infty}\left[\sum_{x=1}^{x=2} E(x, t, (a_1, a_2))\right] dt &= R_0. &(56)\end{aligned}$$

The true transport parameters now determine the actual paths of the temperature anomalies,

$$\begin{aligned}\dot{T}_1 &= [(-B - \gamma_1 - \gamma_2)\,T_1 + \gamma_1 T_2 + \lambda E(t, (a_1.a_2))],\ T_1(0) = 0 &(57)\\ \dot{T}_2 &= [(\gamma_1 + \gamma_2)\,T_1 + (-B - \gamma_1)\,T_2 + \lambda E(t, (a_1.a_2))],\ T_2(0) = 0 &(58)\end{aligned}$$

Using (57)-(58) and (55), the steady-state temperature anomalies can be obtained as

$$\begin{aligned}T_1 &= \frac{\alpha\lambda\,[v(1)\,L_0(1) + v(2)\,L_0(2)]\,(B + 2\gamma_2)}{B\,(B + 2\gamma_1 + \gamma_2)\left[\tau(a_1, a_2) + \lambda_{R_0}(0)\,\mathrm{e}^{(\rho-\eta)t}\right]} &(59)\\ T_2 &= \frac{\alpha\lambda\,[v(1)\,L_0(1) + v(2)\,L_0(2)]\,[B + 2(\gamma_1 + \gamma_2)]}{B\,(B + 2\gamma_1 + \gamma_2)\left[\tau(a_1, a_2) + \lambda_{R_0}(0)\,\mathrm{e}^{(\rho-\eta)t}\right]}. &(60)\end{aligned}$$

In computing the welfare effects, it is convenient to separate out in (45)

[9] Note that we have set $H = 1$.

the term

$$\int_{t=0}^{\infty} \mathrm{e}^{-(\rho-\eta)t} \left[\sum_{x=1}^{x=2} v(x) L_0(x) \ln\left[y_0(x,t) - \phi(x) T_b(x)\right]\right] dt, \tag{61}$$

which does not vary with emissions and compute the component of welfare which does vary with emissions. Hence we focus on the variable component of welfare in equation (62) below. We denote this component, when beliefs are correct, by

$$W\left[(\gamma_1,\gamma_2) \mid (\gamma_1,\gamma_2)\right] \equiv \max \int_{t=0}^{\infty} \mathrm{e}^{-(\rho-\eta)t} \left[\sum_{x=1}^{x=2} v(x) L_0(x) \ln\left[E(x,t)^{\alpha}\, \mathrm{e}^{-\phi(x) T_x(t)}\right]\right] dt, \tag{62}$$

with analogous notation for the variable component of welfare, $W\left[(a_1,a_2) \mid (\gamma_1,\gamma_2)\right]$, when beliefs about the parameters of the temperature anomaly dynamics are (a_1,a_2) but the true parameters are (γ_1,γ_2). It is very tedious to compute $W\left[(a_1,a_2) \mid (\gamma_1,\gamma_2)\right]$ for the case of finite known reserve, R_0. We proceed as follows noting that from the FONC, optimal emissions when the tax is $\tau(a_1,a_2)$ are given by (55). In the variable welfare component (62), substitute optimal emissions (55) and steady state temperature anomalies (59)-(60). Next we gather all terms in $W\left[(\gamma_1,\gamma_2) \mid (\gamma_1,\gamma_2)\right]$ that are common to $\tau(a_1,a_2) + \lambda_{R_0}(a_1,a_2)\,\mathrm{e}^{(\rho-\eta)t}$, and separate out terms in $W\left[(\gamma_1,\gamma_2) \mid (\gamma_1,\gamma_2)\right]$ that are not. Define the following quantities:

$$w_1 = \sum_{x=1}^{x=2} v(x) L_0(x) \left[\alpha \ln\left(\alpha v(x) L_0(x)\right)\right] \tag{63}$$

$$w_2 = \sum_{x=1}^{x=2} v(x) L_0(x) \tag{64}$$

$$w_3 = \frac{\lambda\left(\sum_{x=1}^{x=2} \alpha v(x) L_0(x)\right)}{D_T} \times \times \left[v(1)\phi(1)(B+2\gamma_1) + v(2)\phi(2)(B+2\gamma_1+2\gamma_2)\right] \tag{65}$$

$$D_T = (B+\gamma_1)(B+\gamma_1+\gamma_2) - \gamma_1(\gamma_1+\gamma_2). \tag{66}$$

Using these quantities we obtain

$$W\left[(a_1,a_2)\mid(\gamma_1,\gamma_2)\right]= \tag{67}$$
$$\int_{t=0}^{\infty}\mathrm{e}^{-(\rho-\eta)t}\left[w_1-w_2\ln\left(\zeta\left(a_1,a_2\right)\right)-\frac{w_3}{\zeta\left(a_1,a_2\right)}\right]dt$$
$$\zeta\left(a_1,a_2\right)=\tau\left(a_1,a_2\right)+\lambda_{R_0}\left(a_1,a_2\right)\mathrm{e}^{(\rho-\eta)t}, \tag{68}$$

where $\lambda_{R_0}\left(a_1,a_2\right)$ solves the equation,

$$\int_{t=0}^{\infty}\left[=\sum_{x=1}^{x=2}\frac{\alpha v\left(x\right)L_0\left(x\right)}{\zeta\left(a_1,a_2\right)}\right]dt=R_0. \tag{69}$$

We can obtain some insight by computing $W\left[(a_1,a_2)\mid(\gamma_1,\gamma_2)\right]$ for the case of infinite known reserve because in this case $\lambda_{R_0}\left(a_1,a_2\right)=0$ and we can obtain steady states and compute $W\left[(a_1,a_2)\mid(\gamma_1,\gamma_2)\right]$ for these steady states. However, we know that unless $\hat{\rho}\equiv\rho-\eta=0$, optimal steady states do not typically solve a maximization problem. Hence we restrict ourselves to the study of steady states for the case $\hat{\rho}\equiv\rho-\eta=0$ and adjust the values of the weights $w_i, i=1,2,3$ accordingly.

When $\hat{\rho}\equiv\rho-\eta=0$ an optimal steady state solves the problem.

$$W\left[(a_1,a_2)\mid(\gamma_1,\gamma_2)\right]\equiv \tag{70}$$
$$\max\left\{\sum_{x=1}^{x=2}\alpha v\left(x\right)L_0\left(x\right)\ln E\left(\left(a_1,a_2\right),x\right)-\left[E\left(\left(a_1,a_2\right),1\right)+E\left(\left(a_1,a_2\right),2\right)\right]h\left(\gamma_1,\gamma_2\right)\right\},$$

where

$$d\left(x\right)\equiv v\left(x\right)L_0\left(x\right)\phi\left(x\right)\ ,\ x=1,2 \tag{71}$$
$$h\left(\gamma_1,\gamma_2\right)=\frac{\lambda}{D_T\left(\gamma_1,\gamma_2\right)}\left[d\left(1\right)\left(B+2\gamma_1\right)+d\left(2\right)\left(B+2\gamma_1+2\gamma_2\right)\right] \tag{72}$$
$$D_T\left(\gamma_1,\gamma_2\right)\equiv\left(B+\gamma_1\right)\left(B+\gamma_1+\gamma_2\right)-\gamma_1\left(\gamma_1+\gamma_2\right), \tag{73}$$

and $\{E\left(\left(a_1,a_2\right),x\right)\ ,\ x=1,2\}$ solves the problem

$$\max_{E(1),E(2)}\left\{\sum_{x=1}^{x=2}\alpha v\left(x\right)L_0\left(x\right)\ln E\left(x\right)-\left[E\left(1\right)+E\left(2\right)\right]h\left(a_1,a_2\right)\right\}. \tag{74}$$

We may now compute

$$W\left[(\gamma_1,\gamma_2) \mid (\gamma_1,\gamma_2)\right] - W\left[(a_1,a_2) \mid (\gamma_1,\gamma_2)\right] = \tag{75}$$

$$\left[\left[z - \ln(z+1)\right]\left[\sum_{x=1}^{x=2} \alpha v(x) L_0(x)\right]\right], \tag{76}$$

$$z \equiv \frac{h(\gamma_1,\gamma_2)}{h(a_1,a_2)}. \tag{77}$$

Since the function $z - (\ln z + 1)$ is strictly convex and takes a unique minimum at $z = 1$, then,

$$z - (\ln z + 1) > 0, \text{ for all } z \neq 1. \tag{78}$$

Using (72) we examine the ratio $\frac{h(\gamma_1,\gamma_2)}{h(0,a)}$ to determine how far from one it can be. Some tedious algebra yields the formula

$$\frac{h(\gamma_1,\gamma_2)}{h(0,0)} = \frac{1}{(B + 2\gamma_1 + \gamma_2)}\left[B + 2\gamma_1 s_1 + 2(\gamma_1+\gamma_2) s_2\right] \tag{79}$$

$$s_i \equiv \frac{d(i)}{d(1) + d(2)}, i = 1,2. \tag{80}$$

Note that when, $\gamma_2 = 0$ we have $\frac{h(\gamma_1,0)}{h(0,0)} = 1$. If $\gamma_2 \to \infty$, then $\frac{h(\gamma_1,\gamma_2)}{h(0,0)} \to 2s_2$. Some additional algebra shows that if $s_2 = 0$, then $\gamma_2 \to \infty$ implies $\frac{h(\gamma_1,\gamma_2)}{h(0,0)} \to 0$ and also it is always the case that $\frac{h(\gamma_1,\gamma_2)}{h(0,0)} \leq 2$. Hence the furthest from one for the ratio $\frac{h(\gamma_1,\gamma_2)}{h(0,0)}$ are the extreme points zero and two. If $z \equiv \frac{h(\gamma_1,\gamma_2)}{h(0,0)}$ in (75)-(77), we see that the "loss" is infinite when $z \to 0$.

Proposition 2 (Bounds on costs of wrong beliefs about transport parameters)

$$\begin{aligned} W\left[(\gamma_1,\gamma_2) \mid (\gamma_1,\gamma_2)\right] - W\left[(a_1,a_2) \mid (\gamma_1,\gamma_2)\right] &\to \infty, \ \textit{when } z \to 0 \\ W\left[(\gamma_1,\gamma_2) \mid (\gamma_1,\gamma_2)\right] - W\left[(a_1,a_2) \mid (\gamma_1,\gamma_2)\right] &\to [1 - \ln(2)] \sum_{x=1}^{x=2} \alpha v(x) L_0(x), \\ \textit{when } z &\to 2. \end{aligned}$$

Proof. The proof follows from the discussion above. ■

As we saw from the discussion above, when $s_2 = 0$, i.e. marginal damages at the high latitudes are zero then a planner who mistakenly believes γ_2 is zero when in reality γ_2 is very large makes a serious loss relative to planning

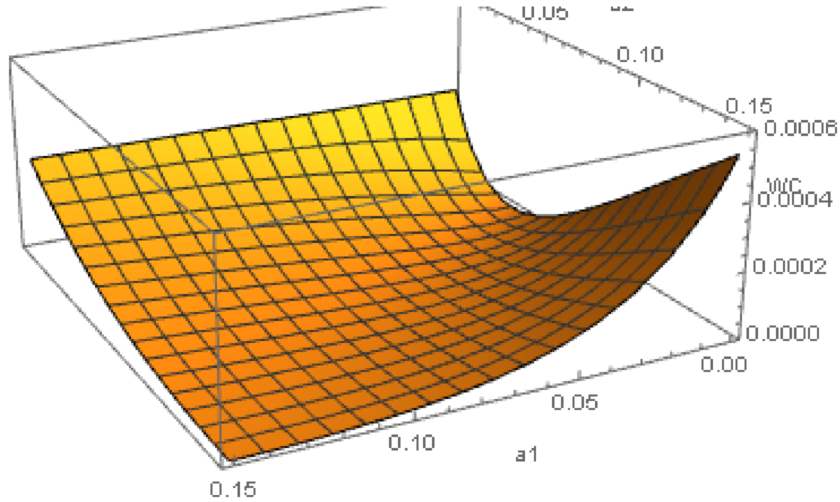

Figure 7: Welfare cost $(\gamma_1, \gamma_2) = (0.15, 0.15)$, $(d_1, d_2) = (0.014, 0.008)$

under correct beliefs. The main message from Proposition 2 is that it is very important to avoid the mistaken belief that γ_2 is small when the damage share of the high latitudes is small and the true value of γ_2 is large.

A picture of the cost caused by wrong beliefs can be obtained by using Assumption 1 and the calibration of the previous section to explicitly compute (75)-(77). The results are presented in Figures 7 and 8 for the cases in which $(\gamma_1, \gamma_2) = (0, 15, 0.15)$, $(d_1, d_2) = (0.014, 0.008)$, and $(d_1, d_2) = (0.008, 0.014)$ respectively and in Figures 9 and 10 for the cases in which $(\gamma_1, \gamma_2) = (0, 15, 0.25)$, $(d_1, d_2) = (0.014, 0.008)$, and $(d_1, d_2) = (0.008, 0.014)$ respectively. Figure 11 presents the cost for the case in which $s_2 = 0$, i.e. marginal damages at the high latitudes are zero, the true transport coefficients are $(\gamma_1, \gamma_2) = (0, 15, 0.25)$ and the planner is mistaken and believes that the true parameters are (a_1, a_2).

From the above figures it becomes clear that the maximum welfare cost occurs when transport coefficients are completely ignored, or $(a_1.a_2) = (0, 0)$. On the other hand, correct beliefs about the transport coefficients, i.e. $(\gamma_1, \gamma_2) = (a_1.a_2)$, do not imply any welfare cost. It can also be seen from the figures that the welfare cost for wrong beliefs increases with γ_2. The welfare cost also increases when $d_1 > d_2$ and transport coefficients are ignored. These simulation results confirm, therefore, our theoretical results.

We suspect that the lessons taught from Proposition 2 will carry over to

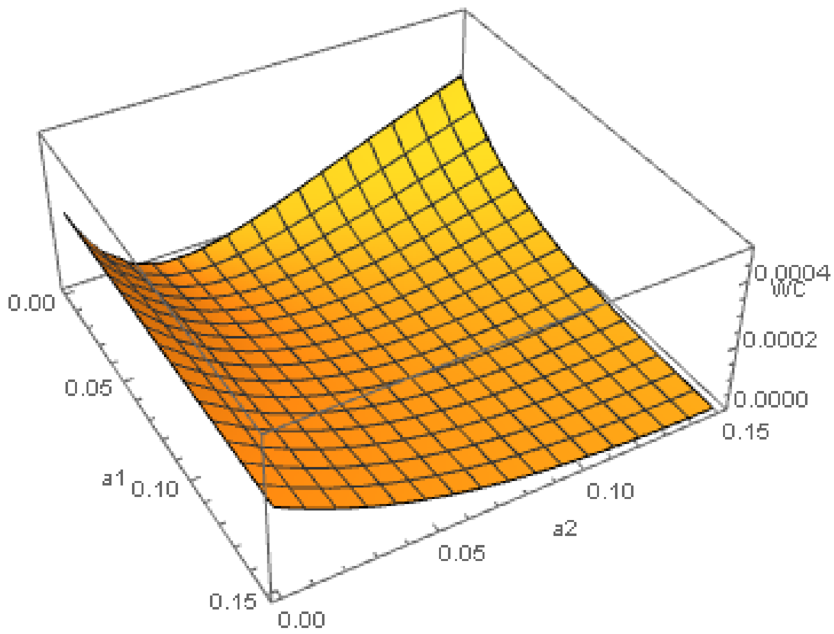

Figure 8: Welfare cost $(\gamma_1, \gamma_2) = (0.15, 0.15)$, $(d_1, d_2) = (0.008, 0.014)$

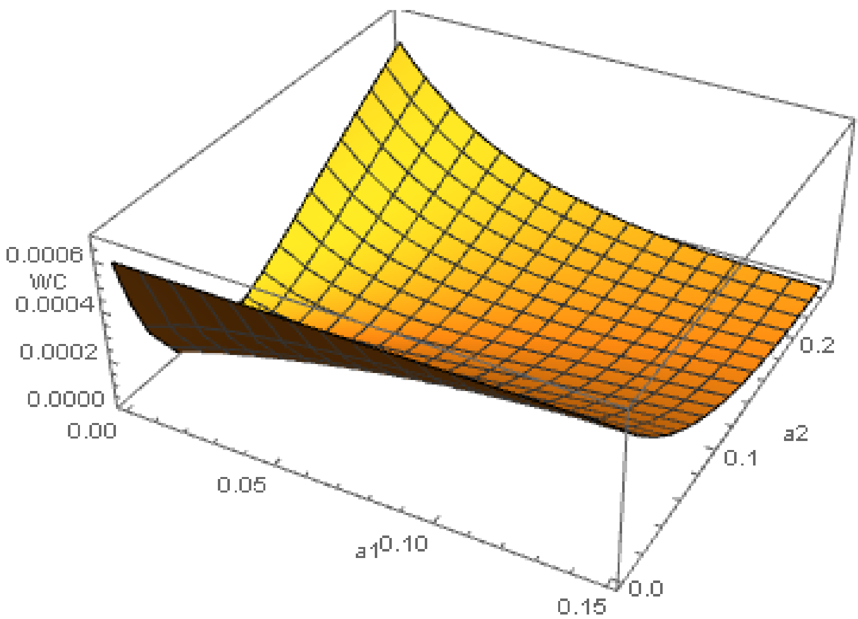

Figure 9: Welfare cost $(\gamma_1, \gamma_2) = (0.15, 0.25)$, $(d_1, d_2) = (0.014, 0.008)$

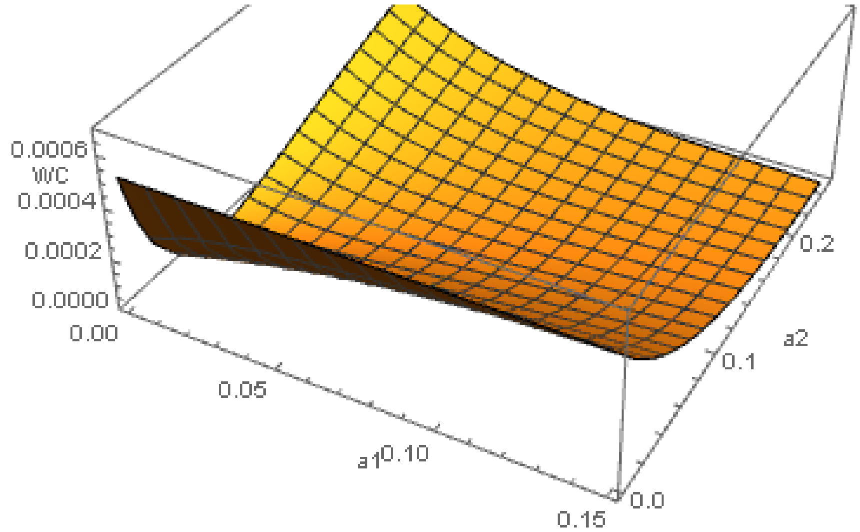

Figure 10: Welfare cost $(\gamma_1, \gamma_2) = (0.15, 0.25)$, $(d_1, d_2) = (0.008, 0.014)$

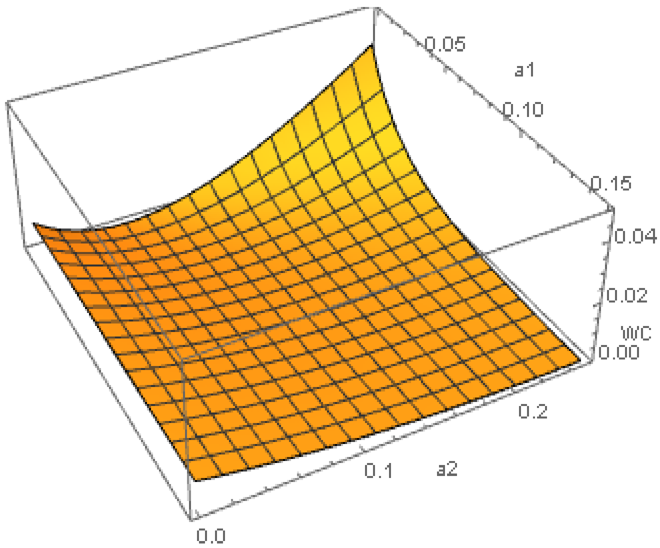

Figure 11: Welfare cost $(\gamma_1, \gamma_2) = (0.15, 0.25)$, $(d_1, d_2) = (0.014, 0.0)$

the more complicated general problems but more analysis and computational work is needed in future research. We use the remaining space in this paper to examine the potential impact of spatial heat and moisture transport on equilibrium market discount rates, discussed in the following section.

6 Impact of Spatial Heat and Moisture Transport on Market Discount Rates

There is a substantial literature on the choice of the market discount rate, or the consumption discount rate, which is appropriate for discounting future costs and benefits associated with environmental projects (e.g., Arrow et al. 1996; Weitzman 1998, 2001; Newell and Pizer 2003). In this section we are interested in determining discount rates in each of the two regions when heat and moisture transport are taken into account. The consumption discount rate can be defined by the equilibrium condition in two equivalent ways: (i) following Arrow et al. (2014) and considering a social planner who would be indifferent between \$1 received at time t and \$$\varepsilon$ received today when the marginal utility of \$$\varepsilon$ today equals the marginal utility of \$1 at time t, or (ii) following Gollier (2007) and considering a marginal investment in a zero coupon bond which leaves the marginal utility of the representative agent unchanged. Consider a general utility function $u\left(c\left(t\right),q\left(t\right)\right)$ of material goods consumption, $c\left(t\right)$, and climate quality, $q\left(t\right)$. Recall that under the equilibrium conditions above, the deterministic market discount rate is defined by

$$r\left(t\right)\equiv\rho-\frac{d}{dt}\ln\left(\frac{\partial u\left(c\left(t\right),q\left(t\right)\right)}{\partial c}\right). \tag{81}$$

Hence, in the additive separable case, i.e., if $u\left(c,q\right)=u_1\left(c\right)+u_2\left(q\right)$ as in, for example, the log utility case examined in Section 3 above, we see right away from (81) that climate quality effects in utility have no direct effect on the discount rate . Climate quality must enter through the direct effect on consumption to matter for the discount rate, although it could impact production of consumption per capita, $c\left(x,t\right)=y\left(x,t\right)E\left(x,t\right)^{\alpha}$, where $y\left(x,t\right)$ is interpreted as in (7). If climate change damages consumption so that actual consumption is $c\left(t\right)=e^{-D(T(t))}c^P\left(t\right)$ where $c^P\left(t\right)$ is potential consumption when climate is pristine, and $e^{-D(T(t))}$ is the "shrinking" factor due to damages to potential consumption from climate change, then one can get an impact on the market discount rate from this channel. Notice that if the utility function is homogeneous of degree one in (c, q), as in many popular specifications, e.g. C.E.S, then $\partial^2 u(c, q) / \partial c \partial q \geq 0$. This restriction imposes a limit on what kind of effects climate change can have on the market rate discount of if $u(c, q)$ is homogeneous of degree one.

Our main interest here is comparing market discount rates in the two regions $x = 1, 2$ and comparing the impact of spatial heat and moisture transport on market discount rates. Looking ahead and thinking about the economics before doing any computations, we can see from the definitional formula for the market discount rate the following intuitions.

First, any force that increases (decreases) the growth rate of consumption of over time is likely to increase (decrease) the market rate of discount for the simple reason that the extra utility from an extra unit of consumption at date t is smaller (larger) relative to date 0, the richer (poorer) the future at date t. For example if climate quality impacts productivity, e.g. $y(x, t) = Y(x, q(x, t), t)$, then a decline in climate quality that decreases productivity could lead to a poorer future and a decrease in the market rate of discount in region x.

Second, any effect of climate change that makes the extra utility from an extra unit of consumption at date t worth less than an extra unit of utility from an extra unit of consumption at date 0 will increase the market rate of discount. For example the force of mortality might increase due to future climate change. This effect is like an increase in ρ in (81).

Third, if we introduce adaptation by diverting some of $c(t)$ into mitigating negative effects of decreasing climate quality, e.g. hot climate regions expending consumption resources to mitigate extreme heat, then this effect impacts the market rate of discount depending upon whether this type of increasing cost of adaptation effect makes the effective marginal utility of consumption worth more (or worth less) in the future than it is worth today. Of course this effect could go the other way. For example adapting to extreme cold weather in the high latitudes may become easier which could lead to a higher market rate of discount in cold regions because "effective income" available for consumption will increase.

We work through some examples below.

Case 1: $U(c, q) = u(cq), u' > 0, u'' < 0.$

Consider the case

$$c(x,t)\,q(x,t) = y(x,t)\,E(x,t)^{a}\,\mathrm{e}^{-\phi(x)\hat{T}(x,t)} \tag{82}$$

$$q(x,t) = \mathrm{e}^{-\phi(x)\hat{T}(x,t)}\ ,\ c(x,t) = y(x,t)\,E(x,t)^{a} \tag{83}$$

$$\hat{T}(x,t) = T_b(x) + T(x,t)\,. \tag{84}$$

For this case we have

$$r(x,t) \;=\; \rho - \left(\frac{d}{dt}\frac{\partial U}{\partial c}\right)/\frac{\partial U}{\partial c} = \tag{85}$$

$$= \rho + \sigma\frac{\dot{c}(x,t)}{c(x,t)} + (1-\sigma)\,\phi(x)\,\dot{T}(x) \tag{86}$$

$$\sigma(z) \;\equiv\; \frac{-u''(z)\,z}{u'(z)}. \tag{87}$$

Here $\sigma(z)$ is the relative rate of risk aversion (RRA) and $1/\sigma(z)$ is the intertemporal elasticity of substitution (IES). Gourinchas and Parker (2002) estimate ρ where we identify our ρ with their implied ρ in the formula, $\beta = 0.940 = 1/(1+\rho)$, as around 4 to 4.5 percent because they estimate their discount factor as around .940. They estimate RRA, σ's, in the range $[0.5, 1.4]$. These numbers can be used to get some idea of the magnitudes of the economic parameters (ρ, γ) in (85) above and in what follows. Note that

$$\frac{\dot{c}(x,t)}{c(x,t)} = \frac{\dot{y}(x,t)}{y(x,t)} + a\frac{\dot{E}(x,t)}{E(x,t)}, \tag{88}$$

which determines the rate of growth of consumption in each spatial region by the corresponding growth rate of inputs and technical change other than fossil fuels in use, $(\dot{y}/y)$, plus the rate of growth of the fossil fuel use, $(\dot{E}/E)$, weighted by the the energy share in production a. The term $(\dot{E}/E)$ might be expected to be negative due to the rising shadow price of reserves. In reality, however, it may be positive for a while before turning negative, due to the inability of the world to coordinate on reducing emissions and the continual discovery of new reserves and new technologies for extracting previously un-extractable reserves. Regarding a, we have seen estimates of energy's share in U. S. output, as high as 8%, but a value around 0.05 could be a reasonable choice. World growth rates of consumption per capita and output per capita vary from high positive growth rates in China that have reached 10% to even negative growth rates in some countries. A range of

2 to 4 percent can be used to choose rough numbers. Formula (85) can thus be regarded as a Ramsey rule for the discount rate in spatial region $x = 1, 2$ adjusted by damages associated with temperature growth in each region which are weighted by 1 minus the RRA. When spatial transport phenomena are present, emissions in region 1 affect temperature in region 2 through polar amplification and affect therefore the discount rate for region 2. Since we are using the Matthews et al. (2009) modeling of the temperature anomaly response to cumulative emissions in both regions, it would be plausible to assume that $\dot{T}(x)$ is proportional to cumulative emissions, which is another way of looking at the link between emissions in region 1 and the discount rate n region 2. This discussion suggests that a natural mechanisms such a spatial transport and polar amplification may generate interactions between the discount rates used in cost benefit analysis in different geographical regions. At the present, however, we do not have reliable estimates for marginal damages per degree of regional temperature increase, or data on regional temperature increases for the regions defined by the Langen and Alexeev (2007) and Alexeev and Jackson (2012) models to produce reasonable estimates for discount rates. Obtaining these data would undoubtedly provide insights in obtaining spatially differentiated discount rates for cost benefit analysis purposes.

Case 2: $U(c, q) = u(c, q)$ concave and increasing in both variables.

In this case we have

$$r(x,t) = \rho - \left(\frac{d}{dt}\frac{\partial U}{\partial c}\right) / \frac{\partial U}{\partial c} = \tag{89}$$

$$= \rho + \sigma \frac{\dot{c}(x,t)}{c(x,t)} - q\frac{u_{cq}}{u_c}\frac{\dot{q}}{q} \tag{90}$$

$$= \rho + \sigma \frac{\dot{c}(x,t)}{c(x,t)} + q\frac{u_{cq}}{u_c}\phi(x)\dot{T}(x,t). \tag{91}$$

Here subscripts on the utility function denote partial derivatives. Notice that when $\dot{T}(x) > 0$, the direct effect on the market discount rate of the cross partial derivative, u_{cq}, is positive (negative) on $r(x,t)$ when u_{cq} is positive (negative). Since γ_2 governs the strength of polar amplification, the larger γ_2 is, the larger will be the direct effect of polar amplification on $r(2,t)$.

Recall that polar amplification implies that the direct effect on market discount rate in the high latitude region of a one degree rise in global yearly average temperature is larger for the higher than for the lower latitudes. This effect of polar amplification may have interesting implications for capital flows across the two regions that have been neglected until now due to the neglect of heat and moisture transport in most of climate economics modeling.

Finally, Moyer et al. (2014) have shown that if climate change has negative effects on growth rates as well as levels of GDP the impact can be quite dramatic. We do a crude exercise here to illustrate a potential effect of this kind.

Case 3: $y(x,t) = A(q(x,t), x, t), U(c(x,t)), q(x,t) = u(c(x,t), q(x,t))$

where $A(q(x,t), x, t)$ is a productivity index that increases as climate quality increases. In this case we have, by adapting (89) above,

$$\begin{aligned} r(x,t) &= \rho - \left(\frac{d}{dt}\frac{\partial U}{\partial c}\right) / \frac{\partial U}{\partial c} = & (92) \\ &= \rho + \sigma \frac{\dot{c}(x,t)}{c(x,t)} - \frac{q u_{cq}}{u_c}\frac{\dot{q}}{q} & (93) \\ &= \rho + \sigma \frac{\dot{c}(x,t)}{c(x,t)} + \frac{q u_{cq}}{u_c}\phi(x)\dot{T}(x,t) & (94) \\ &= \rho + \sigma \left[\left(\frac{qA_q}{A}\right)\left(-\phi(x)\dot{T}(x,t)\right)\right] + \frac{q u_{cq}}{u_c}\phi(x)\dot{T}(x,t). & (95) \end{aligned}$$

We see the Moyer et al. (2014) impact operating on the market rate of discount by having a negative direct effect on $r(x,t)$. The economic intuition for this direct effect is the increase in marginal utility of consumption in a poorer future compared to the marginal utility of consumption today. The future is expected to be poorer because the derivative $A_q > 0$ by assumption and $(\dot{q}/q)$ due to negative climate change. Note that $\frac{qA_q}{A}$ and $\frac{q u_{cq}}{u_c}$ are elasticities of A and u_q w.r.t. climate quality.

We have ignored modeling capital accumulation as in Ramsey, Cass, Koopmans type macro-growth modeling in the two box model in order to focus entirely on the effects of spatial heat and moisture transport. We turn now to some very preliminary work on extending the classical Ramsey, Cass, Koopmans model, “Ramsey model” for short, to the two-box climate dynamics setting.

7 Two-Box Ramsey Type Models

This section develops Ramsey type modeling in the context of Alexeev and Jackson (2012), and Langen and Alexeev (2007) two-region climate models. We continue to make rather drastic simplifying assumptions but we ultimately would like to be able to move towards a two region (or many regions) extension of the important work of Cai et al. (2015).

In developing the Ramsey model, we explicitly consider a Cobb-Douglas production function in each region,

$$Y(x,t) = A(x,t)\, K(x,t)^{\alpha_K}\, L(x,t)^{\alpha_L}\, E(x,t)^{\alpha_E} \ \ , \ x = 1,2, \tag{96}$$

where $K(x,t)$ is the stock of capital and $A(x,t)$ is a productivity factor. Using this production function the capital budget constraint for each region becomes

$$\dot{K}(x,t) = Y(x,t) - C(x,t) - \delta K(x,t) \ \ , \ x = 1,2. \tag{97}$$

We consider a deterministic Ramsey two-region optimization model which we will refer to as the "closed economy" problem. In this model each region is limited by its own budget constraint. The particular assumptions connected to this scenario are restrictive and perhaps not so realistic but they help to set up a benchmark model that can be compared with the other polar case in which the economy is completely open with free flows of capital, fossil fuel and consumption goods across locations. The Hamiltonian associated with this problem, with only the relevant parts of the Hamiltonian appearing is

$$H = \sum_{x=1}^{x=2} \left\{ v(x)\, L(x,t) \left[\alpha \ln C(x,t) - \phi(x, T_b + T)\right] - \lambda_{R_0^x}(t)\, E(x,t) \right\} + \tag{98}$$

$$\lambda_{T_1}(t) \frac{1}{H} \left[(-B - \gamma_1 - \gamma_2) T_1 + \gamma_1 T_2 + \lambda \left[E(1,t) + E(2,t)\right]\right] + \tag{99}$$

$$\lambda_{T_2}(t) \frac{1}{H} \left[(\gamma_1 + \gamma_2) T_1 + (-B - \gamma_1) T_2 + \lambda \left[E(1,t) + E(2,t)\right]\right] \tag{100}$$

$$\sum_{x=1}^{x=2} \lambda_K(x,t) \left[Y(x,t) - C(x,t) - \delta K(x,t)\right]. \tag{101}$$

To be able to study steady states we make the simplifying assumptions of infinite reserves, so that $\lambda_{R_0^x}(0)=0$, constant population and no productivity growth in each region so that $L(x,t)=L(x)\,, A(x,t)=A(x)$. Under these simplifying assumptions, the optimality condition for the two-region Ramsey model can be written as follows. For the controls $C(x,t), E(x,t)$:

$$\frac{\alpha v(x)\,L(x)}{C(x,t)} = \lambda_K(x,t) \text{ or } C^0(x,t)=\frac{\alpha v(x)\,L(x)}{\lambda_K(x,t)} \tag{102}$$

$$\left(\lambda_{T_1}(t)+\lambda_{T_2}(t)\right)\frac{\lambda}{H} = -\lambda_K(x,t)\,Z(x)\,\alpha_E K(x,t)^{\alpha_K}\,E(x,t)^{a_E-1} \tag{103}$$

$$E^0(x,t) = \left[\frac{-\left(\lambda_{T_1}(t)+\lambda_{T_2}(t)\right)(\lambda/H)}{\lambda_K(x,t)\,Z(x)\,\alpha_E K(x,t)^{\alpha_K}}\right]^{\frac{1}{a_E-1}} \tag{104}$$

$$Z(x) = A(x)\,L(x)^{\alpha_L}\,. \tag{105}$$

Using Assumption 1, the Hamiltonian dynamical system in the states and the costates becomes:

$$\dot{T}_1 = \frac{1}{H}\left[(-B-\gamma_1-\gamma_2)\,T_1+\gamma_1 T_2+\lambda\left[E^0(1,t)+E^0(2,t)\right]\right] \tag{106}$$

$$\dot{T}_2 = \frac{1}{H}\left[(\gamma_1+\gamma_2)\,T_1+(-B-\gamma_1)\,T_2+\lambda\left[E^0(1,t)+E^0(2,t)\right]\right] \tag{107}$$

$$\dot{K}(x,t) = Z(x)\,K(x,t)^{\alpha_K}\,E^0(x,t)^{\alpha_E}-C^0(x,t)-\delta K(x,t)\,,\ x=1,2 \tag{108}$$

$$\dot{\lambda}_{T_1} = \left[\rho+\frac{(B+\gamma_1+\gamma_2)}{H}\right]\lambda_{T_1}-\left[\frac{(\gamma_1+\gamma_2)}{H}\right]\lambda_{T_2}+d_1 \tag{109}$$

$$\dot{\lambda}_{T_2} = -\left(\frac{\gamma_1}{H}\right)\lambda_{T_1}+\left[\rho+\frac{(B+\gamma_1)}{H}\right]\lambda_{T_2}+d_2 \tag{110}$$

$$\dot{\lambda}_K(x,t) = \left[\rho+\delta-Z(x)\,\alpha_E K(x,t)^{\alpha_K-1}\,E^0(x,t)^{a_E}\right]\lambda_K(x,t)\,. \tag{111}$$

The complexity of the Hamiltonian system does not allow analytical results so we obtain some insight by resorting to simulations. We use the parameters of section 4.1 for the climate system, while for the production system we consider the following values

$$\alpha_K=0.35, a_L=0.60, a_E=0.05, A(1)=A(2)=1, \delta=0.05. \tag{112}$$

Table 1 presents steady-state values for state, costate, and control variables the steady state externality tax, which can be defined as $\tau=-\left(\lambda_{T_1}+\lambda_{T_2}\right)\frac{\lambda}{H}$, for three cases: (i) $\gamma_1=\gamma_2=0.15$, $(d_1,d_2)=(0.014,0.008)$; (ii) $\gamma_1=\gamma_2=0$, i.e. no spatial transport, $(d_1,d_2)=(0.014,0.008)$; and (iii) $\gamma_1=\gamma_2=0.15$, $(d_1,d_2)=(0.008,0.014)$.

Table 1: Steady states of the Ramsey two-box model

Case	T_1	T_2	$K(x)$	λ_{T_1}	λ_{T_2}	$\lambda_K(x)$	$C(x)$	$E(x)$	τ
(i)	2.661	4.657	0.508	−0.267	−0.225	0.866	0.076	0.037	0.300
(ii)	3.507	3.507	0.488	−0.335	−0.191	0.901	0.073	0.033	0.320
(iii)	2.451	4.289	0.471	−0.258	−0.301	0.935	0.071	0.029	0.340

$x = 1, 2$. All the steady states have the saddle point property with four negative eigenvalues.

Cases (i) and (iii) provide the optimal amplification given the parameters of the climate system and the damage parameters, while case (ii) provides the optimal steady state without any polar amplification when spatial transport phenomena are ignored. The steady-state values suggest that spatial transport matters since, if we consider the no transport case as a benchmark, accounting for heat and moisture transport results in a ±6.5% variation in the steady state tax, and in a ±12.1% variation in the steady state consumption and fossil fuel use. Thus ignoring polar amplification may result in overtaxing or under taxing fossil fuel use.

8 Conclusions

Polar amplification is an established scientific fact which has been associated with the surface albedo feedback and, by recent research, to heat and moisture transport from the Equator to the Poles. In the present paper we unify a two-box (or two-region) climate model, which allows for heat and moisture transport from the southern region to the northern region, with an economic model of welfare optimization. In the economic model a regulator chooses fossil fuel use which is equivalent to GHG emissions. Emissions induce temperature anomaly, relative to baseline temperature in the two regions, along with damages from temperature increase over the baseline.

Our main contribution is to show that by ignoring spatial heat and moisture transport and the resulting polar amplification the regulator may overestimate or underestimate the tax on GHG emissions. The direction of bias depends on the relations between marginal damages from temperature increase in each region. We also determine the welfare cost when a regulator mistakenly ignores spatial heat and moisture transport. Numerical simulations that use a plausible parametrization based on climate science confirm our theoretical results regarding taxation of GHGs.

D'Autume et al. (2015) study carbon taxation in second best frameworks as well as settings where lump sum compensatory transfers are possible and where they are not possible. They locate a set of sufficient conditions for carbon taxes to be uniform across locations, especially if lump sum compensatory transfers are available. However, under the realistic political constraints on transferring resources across countries, they find that equity concerns force carbon taxes to be lower for poorer areas. Brock et al. (2013), Brock et al. (2014b) and Brock and Xepapadeas (2015), in a model of continuous space that allows for spatial heat transport show that externality taxes, (i.e. carbon prices) should be uniform when compensatory transfers are possible, but tend to be lower in poorer areas when such transfers are not available. They also show how heat transport impacts the set of spatial carbon prices across locations. In the context of the present results it will be interesting to study the impact of heat transport and polar amplification of the potential spatial differentiation between rich and poor regions when compensatory transfers among regions are not possible.

Using our framework we also calculate the discount rate for discounting cost and benefit flows in cost-benefit analysis we show that polar amplification emission in the southern region may affect through the discount rates in the northern region. In order to produce analytic results, our economic model is simple and does not allow for capital accumulation. In the last section we show how the two-region model with polar amplification can be unified with a Ramsey type optimal growth model. Numerical simulations indicate that the steady states of the economic and the climate systems obtained with and without spatial heat and moisture transport differ from each other. This result confirms that ignoring spatial phenomena and polar amplification in climate change may result in suboptimal policies.

Further research - apart from introducing factors like human capital, R&D, or uncertainty - could study pollution externalities from fossil fuel emissions. Their effects can be modeled by introducing an extra state variable for each box together with transport across the two boxes. Some of these externalities may be as important as the climate change externalities.[10]

[10] For example, Parry et al. (2014) estimate co-benefits from the control of such pollutants for 20 major polluting countries. They show that co-benefits vary widely across the 20 countries but are substantial for all of them.

References

[1] ACIA, 2004 Impacts of a warming Arctic: Arctic climate impact assessment. Cambridge University Press, Cambridge

[2] Alexeev, V.A. and C. H. Jackson, 2012, Polar amplification: is atmospheric heat transport important?, Climate Dynamics, DOI 10.1007/s00382-012-1601-z.

[3] Alexeev V.A., and C. H. Jackson, 2013, Erratum to: Polar amplification: is atmospheric heat transport important? Climate Dynamics; 41(2). DOI:10.1007/s00382-013-1763-3 ·

[4] Alexeev, V.A., P.L. Langen, and J.R. Bates (2005), Polar amplification of surface warming on an aquaplanet in "ghost forcing" experiments without sea ice feedbacks, Climate Dynamics, DOI 10.1007/s00382-005-0018-3.

[5] Arrow, K.J., W.R. Cline, K.-G. Maler, M. Munasinghe, R. Squitieri, J.E. Stiglitz, 1996, Intertemporal equity, discounting and economic efficiency, in J.P. Bruce, H. Lee, and E.F. Haites (eds.), Climate Change 1995 - Economic and Social Dimensions of Climate Change, Cambridge University Press, Cambridge

[6] Arrow, K.J, M.L. Cropper, C. Gollier, B. Groom, G.M. Heal, R.G. Newell, W.D. Nordhaus, R.S. Pindyck, W.A. Pizer, P.R. Portney, T. Sterner, R.S.J. Tol, and M.L. Weitzman, 2014, Should governments use a declining discount rate in project analysis?, Review of Environmental Economics and Policy, doi:10.1093/reep/reu008

[7] Arrhenius S.,1896, On the influence of carbonic acid in the air upon the temperature of the ground. Phil Mag 41:237–276

[8] Brock, W.A., and A. Xepapadeas, 2015, Spatial Heat Transport, Polar Amplification and Climate Change Policy, Working Paper, https://ideas.repec.org/p/aue/wpaper/1515.html

[9] Brock, W., G. Engström, and A. Xepapadeas, 2014, Spatial climate economic models in the design of optimal climate policies across locations, European Economic Review 69: 78-103.

[10] Brock, W., G. Engström, and A. Xepapadeas, 2014a, Energy Balance Climate Models, Damage Reservoirs, and the Time Profile of Climate Change Policy, in L. Bernard and W. Semmler (Eds) The Oxford Handbook of the Macroeconomics of Global Warming, Chapter 3, Oxford Univ. Press, Oxford.

[11] Brock, W.A., G. G. Engström, D. Grass, and A. Xepapadeas, 2013, Energy balance climate models and general equilibrium optimal mitigation policies, Journal of Economic Dynamics and Control 37(12): 2371-2396.

[12] Burgess, R., D., Deschenes, D. Donaldson, and M. Greenstone, 2014, The unequal effects of weather and climate change: Evidence from mortality in India, http://www.lse.ac.uk/economics/people/ facultyPersonalPages/facultyFiles/RobinBurgess/UnequalEffectsOfWeather.

[13] Cai, Y., K.L. Judd, and T. S. Lontzek, 2015, The Social Cost of Carbon with Economic and Climate Risks, Papers 1504.06909, arXiv.org, revised Apr 2015.

[14] D'Autume, A., K. Schubert, and C. Withagen, 2015, "Should the carbon price be the same in all countries, https://hal.inria.fr/file/index/docid/654239/filename/11076.pdf, forthcoming in Journal of Public Economic Theory

[15] Fanning, A., and Weaver ,A., 1996, Anatmospheric energy moisture-balance model: climatology, interpentadal climate change and coupling to an OGCM, Journal of Geophysical Research, 101(15), 111–115.

[16] Francis, J. and S. Vavrus, 2014, Evidence for a wavier jet stream in response to rapid Arctic warming, Environmental Research Letters 10: 1-12.

[17] Dietz, S. and Stern, N., 2015, Endogenous growth, convexity of damage and climate risk: how Nordhaus' framework supports deep cuts in carbon emissions. The Economic Journal, 125 (583). pp. 547-620. ISSN 1468-0297

[18] Gollier, C., 2007, The consumption-based determinants of the term structure of discount rates, Mathematical Financial Economics 1: 81-101

[19] Gorinchas, P., and J. Parker, 2002, “Consumption over the life cycle,” Econometrica, Vol. 70, No. 1, pp. 47-89

[20] Holland M.M., and Bitz C.M., 2003, Polar amplification of climate change in coupled models. Clim Dyn 21:221–232

[21] Langen P.L. and V. A. Alexeev, 2007, Polar amplification as a preferred response in an idealized aquaplanet GCM, Climate Dynamics, DOI 10.1007/s00382-006-0221-x

[22] Lenton, T., H. Held, E. Kriegler, J. Hall, W. Lucht, S. Rahmstorf, and H.J. Schellnhuber, 2008, Tipping elements in the Earth’s climate system, PNAS 105(6): 1786-1793.

[23] MacDougal A.H, and P. Friedlingstein, 2015, The Origin and Limits of the Near Proportionality between Climate Warming and Cumulative CO2 Emissions, Journal of Climate, 28, 4217-4230

[24] Matthews, H.D., N.P. Gillett, P.A. Stott, and K. Zickfield, 2009, The proportionality of global warming to cumulative carbon emissions, Nature, "459 829-833."

[25] Masson-Delmotte V, Kageyama M, Braconnot P, Charbit S, Krinner G, Ritz C, Guilyardi E, Jouzel J, Abe-Ouchi A, Crucifix M, Gladstone RM, Hewitt CD, Kitoh A, LeGrande AN, Marti O, Merkel U, Motoi T, Ohgaito R, Otto-Bliesner B, Peltier WR, Ross I, Valdes PJ, Vettoretti G, Weber SL, Wolk F, Yu Y (2006) Past and future polar amplification of climate change: climate model intercomparisons and icecore constraints. Clim Dyn 26:513–529

[26] Moyer E.J., Woolley M.D., Glotter M.J., Matteson N.J., Weisbach D.A. (2014). Climate impacts on economic growth as drivers of uncertainty in the social cost of carbon. Journal of Legal Studies, 43(2):401:425. doi 10.1086/678140

[27] Newell, R. and W. Pizer, 2003, Discounting the benefits of climate change mitigation: how much uncertain rates increase valuations?, Journal of Environmental Economics and Management 46(1): 52-71

[28] Nordhaus, W.D., 2008, A Question of Balance: Weighing the Options on Global Warming Policies, New Haven, CT.: Yale University Press.

[29] Parry, I., C. Veung, C. and D. Heine, 2014, How Much Carbon Pricing is in Countries' Own Interests? The Critical Role of Co-Benefits, IMF Working Paper, WP/14/74

[30] Tarnocai C., Canadell J., Schuur E., Kuhry P., Mazhitova G. and Zimov S., 2009, Soil organic carbon pools in the northern circumpolar permafrost region, Global Biogeochemical Cycles, vol. 23(2), p. GB2023.

[31] Thomas, D.S., and C. Twyman, 2005, Equity and justice in climate change adaptation amongst natural-resource-dependent societies, Global Environmental Change, 15: 115-24.

[32] Weaver, A.J., M. Eby, E.C. Wiebe, C.M. Bitz, P.B. Duffy, T.L. Ewen, A.F. Fanning, M.M. Holland, A. MacFadyen, H.D. Matthews, 2001, The UVic earth system climate model: model description, climatology, and applications to past, present and future climates, Atmosphere Ocean 39(4): "361-428."

[33] Weitzman, M.L., 1998b, Why the far-distant future should be discounted at its lowest possible rate?, Journal of Environmental Economics and Management 36: 201-208.

[34] Weitzman, M.L., 2001, Gamma discounting, American Economic Review 91: 260-271

[35] Winton, M., 2006, Amplified Arctic climate change: What does surface albedo feedback have to do with it?, Geophysical Reserach Letters, Vol. 33, L03701, doi:10.1029/2005GL025244, 2006

[36] Zachos J.C., Pagani, M., Sloan L.C., Thomas E, Billups K, 2001, Trends, rhythms and aberrations in global climate 65 Ma to present. Science 292:686–693

[37] Zhang T., Barry R., Knowles K., Ling F. and Armstrong R., 2003, Distribution of seasonally and perennially frozen ground in the northern hemisphere, in Proceedings of the 8th International Conference on Permafrost, 21-25 July 2003, Zurich, Switzerland, pp. 1289{1294, AA Balkema, Lisse, the Netherlands.