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Cooperation and Competition in Climate Change Policies: Mitigation and Climate Engineering when Countries are Asymmetric

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Cooperation and Competition in Climate Change Policies: Mitigation and Climate Engineering when Countries are Asymmetric[1]

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Abstract

We study a dynamic game of climate policy design in terms of emissions and solar radiation management (SRM) involving two heterogeneous countries or group of countries. Countries emit greenhouse gasses (GHGs), and can block incoming radiation by unilateral SRM activities, thus reducing global temperature. Heterogeneity is modelled in terms of the social cost of SRM, the environmental damages due to global warming, the productivity of emissions in terms of generating private benefits, the rate of impatience, and the private cost of geoengineering. We determine the impact of asymmetry on mitigation and SRM activities, concentration of GHGs, and global temperature, and we examine whether a tradeoff actually emerges between mitigation and SRM. Our results could provide some insights into a currently emerging debate regarding mitigation and SRM methods to control climate change, especially since asymmetries seem to play an important role in affecting incentives for cooperation or unilateral actions.

Keywords: Climate change, mitigation, solar radiation management, cooperation, differential game, asymmetry, feedback Nash equilibrium.

JEL Classification: Q53, Q54.

1 Introduction

Human-driven climate change due to greenhouse gas emissions is an increasingly important driver of global environmental change associated with many potentially detrimental effects. Despite serious attempts to obtain international cooperation in reducing the emissions of greenhouse gasses (GHGs), their concentration keeps increasing, suggesting that cooperation in mitigation has not been entirely successful. In fact there has been minimal political progress toward global cooperation in mitigating GHGs over the last 30 years.

Given the current path of global emissions of GHGs, their long atmospheric residence times and the relatively limited action to date to reduce future emissions, the use of geoengineering techniques has been proposed as an additional means to limit the magnitude and impact of human-induced climate change. Geoengineering is defined as a deliberate intervention in the planetary environment of a nature and scale intended to counteract anthropogenic climate change and its impacts. Geoengineering methods focus mainly on increasing the reflectivity of the Earth's surface or atmosphere, and removing GHGs from the atmosphere; they should be differentiated from actions that mitigate (reduce or prevent) anthropogenic GHG emissions such as carbon capture and storage (CCS) (Keith 2000, Ricke et al. 2008, Shepherd 2009, Secretariat of the Convention of Biological Diversity 2012).

Sunlight reflection methods, also known as solar radiation management (SRM), aim to counteract global warming by reducing the incidence and subsequent absorption of short-wave solar radiation, reflecting a proportion of it back into space. This is achieved by injecting sulfate aerosols into the atmosphere. They are expected to act rapidly once deployed at the appropriate scale, and could potentially reduce surface global temperatures within a few months or years if this were considered desirable (e.g., Crutzen 2006, Barrett 2008, Lenton and Vaughan 2009, Robock et al. 2009, Shepherd 2009, Kravitz et al. 2011). This approach mimics what occasionally occurs in nature when a powerful volcano erupts. For example, the Mount Pinatubo eruption in 1991 injected huge volumes of sulphur into the stratosphere. The particles produced in subsequent reactions cooled the planet by about 0.5°C over the next two years by reflecting sunlight back into space (e.g., Randel et al. 1995, Robock 2000, Lucht et al. 2002, Barrett 2008).

The most compelling arguments in favor of SRM methods are first that they can be used as an emergency measure to reduce the global average temperature quickly, and second that they can be used to “buy time” by slowing down the increase in temperature so that new abatement or emission reducing technologies can be developed. On the other hand, there are many arguments against SRM, suggesting that the injection of sulfate aerosols into the atmosphere will induce detrimental effects on plants due to reduced sunlight, ozone depletion, more acid depositions, and less solar radiation available for solar power systems. Furthermore there are additional concerns regarding the inability of engineering methods to adjust regional climate to desired levels. Moreover, if geoengineering is used as a substitute for GHG emission reductions, the acidification of oceans could be intensified (see for example Robock (2008), Robock et al. (2009), Jones et al. (2013)). Regarding the two main arguments in favor of using SRM - the emergency measure and buying time arguments - it has been argued recently that SRM methods may not be useful in averting global disasters, such as the disintegration of the West Antarctic ice, but it may be tempting to use them in addressing regional environmental emergencies(Barrett et al. 2014). Furthermore the buying time argument seems not to be a credible proposal, because it implies that countries will overcome free-rider incentives when SRM is available, while the same countries have been unable to overcome the same incentives at the present time when SRM is not available.

A very important feature of SRM is that, because it is very cheap to deploy, it can be unilaterally used by a country that deems it beneficial to do so. However although SRM may be beneficial for a country or group of countries, at the same time it may be harmful for other countries (e.g. by altering the monsoons or increasing ocean acidification). This characteristic suggests that when analyzing SRM incentives and activities in a multi-country framework, much attention should be given to asymmetries among countries because these asymmetries will be very important in determining both the final outcome in terms of SRM activities and also the tradeoffs between mitigation and geoengineering.

The the main contribution of this paper is to study simultaneous mitigation and SRM decisions of individual countries in both a cooperative and a competitive environment when countries are asymmetric. We model mitigation and SRM decisions in the context of cooperative and noncooperative solutions of a differential game with asymmetric players. Earlier results obtained in the context of a symmetric differential game (Manoussi and Xepapadeas 2013) suggest that the presence of geoengineering as a policy option results in a higher level of steady-state accumulation of GHGs emissions than when geoengineering is not an option.[1] This result holds at the cooperative and noncooperative solutions, with relatively stronger incentives for geoengineering at the noncooperative solutions. Higher GHGs could be compatible with lower global temperature, at least in the short run, since geoengineering increases global albedo which tends to reduce temperature. Even if geoengineering leads to a lower temperature, maintaining this temperature requires a constant flow of geoengineering. Thus, if this flow cannot be kept constant at some point in time, there will be a jump in the temperature which will be intensified since the stock of GHGs will already be high.

In this paper, we study a dynamic game of climate change policy design in terms of emissions and geoengineering efforts involving two heterogeneous countries.[2] The model we develop consists of a traditional economic benefit function along with a climate module based on a simplified energy balance climate model (EBCM). EBCMs are based on the idea of global radiative heat balance. In radiative equilibrium the rate at which solar radiation is absorbed matches the rate at which infrared radiation is emitted. The purpose of SRM as a policy instrument is to reduce global average temperature by controlling the incoming solar radiation, thus an EBCM is a useful vehicle for modelling SRM.

We seek to characterize cooperative and noncooperative mitigation (or equivalently GHGs emissions) and SRM strategies in the framework of asymmetric countries. On the modeling side we consider a world consisting of two asymmetric regions or countries with production activities that generate GHG emissions. These GHGs emissions generate private benefits (e.g. output) for each country. The stock of GHGs blocks outgoing radiation and causes temperature to increase. Geoengineering in the form of SRM blocks incoming radiation which is expected to cause a drop in temperature. This

[1] See also Moreno-Cruz and Keith (2012) and Emmerling and Tavoni (2013) for an analysis of SRM activities under uncertainty.

[2] Throughout this paper, the game with two heterogeneous countries applies equally to two heterogenous groups of countries.

drop does not, at least in the way that our model is developed, depend on the accumulated GHGs.

We analyze the problem in the context of cooperative solutions and non-cooperative solutions associated with a differential game. The analytical framework of differential games has been widely used in the analysis of environmental and resource management problems. This is because these problems are intrinsically dynamic and are characterized by optimizing forward-looking behavior and by strategic interdependence associated with the actions of economic agents. These characteristics naturally lead to the development of state-space games, in which state variables could be the stock of a pollutant accumulated in the ambient environment (e.g. phosphorous in a lake, or acid buffer stocks), the stock of GHGs, or the biomass of a resource. Control variables could be for example emissions, harvesting, R&D expenses, or abatement.[3] We follow this approach here since our problem has all the characteristics that lead to a differential game with the state variable being the stock of GHGs and the control variables being emissions and SRM effort.

In the cooperative case there is coordination between the two countries for the implementation of geoengineering and the level of emissions, as if a global social planner were acting in order to maximize the joint or global welfare. In the noncooperative case, each government chooses SRM and emissions policies noncooperatively. The noncooperative solution is analyzed in terms of feedback Nash equilibrium (FBNE) strategies. We first derive the optimal paths and the steady-state levels of GHG emissions, SRM, global average temperature and GHGs accumulation under the fully symmetric scenario, corresponding to cooperation and feedback Nash strategies. Although this scenario is unlikely to occur in practice, it serves as a useful benchmark against which outcomes corresponding to asymmetric countries can be compared.

We consider asymmetries between countries attributed to two main sources: differences in the impacts of climate change and SRM activities across countries, and differences in the prevailing economic conditions. These differences will shape countries' actions regarding emissions (or mitigation) and SRM. In particular the first source is reflected in heterogeneity between the two

[3] See for example Basar and Olsder (1999).for the theoretical foundations and the survey by Jorgensen et al. (2010) and the references therein.

countries in terms of the social cost of geoengineering, which is the harm to a country from SRM activities undertaken by one or both countries and the environmental damages from global warming. The second source refers to heterogeneity related to how productive a country's emissions are in generating private benefits,[4] the rate of impatience and the private cost of SRM.

A similar problem has been studied by Moreno-Cruz (2010) and Millar-Ball (2012). Moreno-Cruz relates changes in temperature to mitigation and SRM activities and determines optimal mitigation and SRM policies in the context of a static two-stage game with symmetric and asymmetric countries, where asymmetries focus on climatic and SRM sources. Millar-Ball (2012), using a four-stage game studies participation in international environmental agreements. A climate model is not explicitly introduced, however the paper provides tractable results regarding the attainment of a self-enforcing climate treaty with both the mitigation and the geoengineering options being available.

The model presented here is closer in structure to the Moreno-Cruz model in terms of explicit introduction of a climate model and the use of quadratic cost and damage functions. The current model is explicitly dynamic which allows for an explicit formulation of GHGs accumulation dynamics and of the link between temperature emissions and SRM using an EBCM. It also provides, through the equilibrium feedback rule, explicit policy functions which relate instruments – emissions and SRM – with the state of the system as described by the paths of GHGs and the temperature. Furthermore a dynamic model allows the study of the long-run evolution of state and control variables of interest and the potential comparison, through calibrations, with existing results. The dynamic framework can be used to study the delayed effect of mitigation on temperature relative to the more immediate effect of SRM on temperature, using time-delay dynamics, and to further examine the issue of whether action should be taken earlier than later. These issues can be regarded as areas of further research for which our dynamic framework might by useful.

We formulate the problem in terms of a linear-quadratic (LQ) differential game. The LQ formulation allows us to provide closed form solutions of the FBNE as well as meaningful numerical simulations for both the symmetric

[4]This measure can be associated for example with how productive a country is in using energy to produce output.

and the asymmetric cases. We study the asymmetric case as a problem of sensitivity analysis where the central case corresponds to the symmetric case and the different types of asymmetries are regarded as deviation scenarios from the symmetric case. This approach allows us to characterize the impact of the specific type of heterogeneity on global GHG emissions, SRM activities, global average temperature and stock of GHGs.

Our results suggest strategies, in the context of the LQ model, regarding the expected behavior of noncooperating asymmetric countries in terms of mitigation and SRM activities, and provide insights regarding the possible existence of a tradeoff between mitigation and SRM. In particular when the sources of asymmetry are climatic, there is no tradeoff between SRM and emissions when differences exist in the global cost of SRM. This tradeoff is present when differences exist in the cost of global warming. When the asymmetries are economic, in general the most productive country increases emissions with a moderate increase in SRM activities to counterbalance the global warming effects of increased emissions. When the countries differ with respect to the degree of impatience, then as expected emissions and SRM activities increase in the more impatient country and decrease in the less impatient. The final outcome of the combined action of the two countries under the above asymmetries has varying effects on global emissions, geoengineering, the steady-state stock of GHGs and the average global temperature. Thus the introduction of asymmetries provides results that could be useful in understanding individual country incentives related to mitigation and SRM.

List and Mason (2001) have studied an LQ differential game of transboundary pollution with asymmetric players. They introduce asymmetry in the intercept of marginal benefits from emissions, and the slope of marginal damages from global pollution. They consider emissions only as a choice variable and compare cooperative with FBNE outcomes. Our approach differs from theirs in that we consider two choice variables, emissions and SRM, an explicit link between choice variable and global temperature through the EBCM, and more sources of asymmetry. Furthermore we compare symmetric with asymmetric noncooperative solutions instead of comparing cooperative with asymmetric noncooperative outcomes, since we want to study the impact of asymmetry when countries might decide unilaterally, that is noncooperatively., about SRM. Our results, although in a different context than that used by List and Mason, agree with their finding in that asymmetry matters.

The rest of the paper is organized as follows. Section 2 develops an LQ dynamic game with an economic and a climate module. In section 3 we determine cooperative and noncooperative solutions under symmetry (the benchmark case). In section 4 we determine noncooperative solutions in terms of FBNE with asymmetric players and compare the symmetric with the asymmetric solutions through numerical simulations. Section 5 concludes.

2 The Model

2.1 Benefits and Costs

The world consists of two countries indexed by $i = 1, 2$. We develop our model along the lines of the standard LQ model of international pollution control analyzed by Dockner and van Long (1993), and others. Output is a function of emissions $F_i(E_i)$, where $F_i(\cdot)$ is strictly concave with $F_i(0) = 0$. Emissions contribute to the stock of GHGs denoted by $G(t)$ at time t. The evolution of GHGs emitted by both countries is described by the linear differential equation:

$$\dot{G}(t) = E_1(t) + E_2(t) - mG, \quad G(0) = G_0 \tag{1}$$

where $0 < m < 1$ is the natural decay rate of GHGs.

Individual country net private benefits, or utility net of environmental externalities, is given by $U(F_i(E_i(t))) - C_i(\zeta_i(t))$ where $C_i(\zeta_i)$ is a strictly increasing and convex function of the private cost of geoengineering or SRM activity $\zeta_i(t)$. The utility function $U(F_i(E_i(t)))$ is given by the quadratic function

$$U(F_i(E_i(t))) = A_{1i}E_i(t) - \frac{1}{2}A_{2i}E_i^2(t) \tag{2}$$

where A_{1i}, A_{2i} are parameters indicating the intercept and the slope of the private marginal benefits from emissions which are defined as $A_{1i} - A_{2i}E_i(t)$. Thus A_{1i} can be regarded as reflecting the level effect on marginal benefits, while A_{2i} as reflecting the strength of diminishing returns.

We assume a simple quadratic cost function for the private cost of geoengineering in each country,

$$C_i\left(\zeta_i\left(t\right)\right)=\frac{1}{2}\theta\zeta_i^2\left(t\right),\ \theta>0. \tag{3}$$

We also assume two types of damage functions related to climate change, which affect private utility. The first one reflects damages from the increase in the average global surface temperature because of GHGs emissions. This damage function is represented as usual by a convex, quadratic in our case, function,

$$\Omega_T\left(T\right)=\frac{1}{2}c_{iT}T^2, \Omega_T\left(0\right)=0, \left(\Omega_T\left(T\right)\right)'>0, \left(\Omega_T\left(T\right)\right)''>0, \tag{4}$$

where $c_{iT}T$ is the marginal damage cost from a temperature increase for each country.

The second is the social damage function associated with SRM effects, such as for example ocean acidification, increased acid depositions or change in precipitation patterns.[5] Assume that country i undertakes SRM activities ζ_i, which will generate total global social damages in both countries $\frac{1}{2}\sum_{j=1}^{2}\left(c_{j\zeta}\zeta_i^2\right)$. Thus global damages from geoengineering when both countries undertake SRM efforts will be:

$$\begin{aligned}\Omega_\zeta\left(\boldsymbol{\zeta}\right) &= \frac{1}{2}\sum_{i=1}^{2}\left(\sum_{j=1}^{2}c_{j\zeta}\zeta_i^2\right) \qquad (5)\\ \text{thus, } \Omega_\zeta\left(\mathbf{0}\right) &= 0, \left(\Omega_\zeta\left(\boldsymbol{\zeta}\right)\right)'>0, \left(\Omega_\zeta\left(\boldsymbol{\zeta}\right)\right)''>0,\end{aligned}$$

where $c_{j\zeta}\zeta_i$, is the social marginal damage cost incurred by country $j=1,2$ from the geoengineering undertaken in country $i=1,2$.[6]

[5] As mentioned in the introduction, the use of geoengineering methods could intensify ocean acidification. Although the natural absorption of CO_2 by the world's oceans helps to mitigate the climatic effects of anthropogenic emissions of CO_2, it is believed that since geoengineering will cause an increase in GHG emissions, the resulting decrease in pH will have negative consequences, primarily for oceanic calcifying organisms, and so there will be an impact on marine environments. For a discussion of damage functions related to climate change, see Weitzman (2010).

[6] A situation can be envisioned in which SRM generates extra benefits to a country, in addition to those accruing from a decrease in average global temperature, due for example to favorable change in regional climatic conditions. In this case $c_{i\zeta}$ could take negative values.

2.2 Emissions, SRM and the global temperature

We model climate by a simplified "homogeneous-earth" EBCM (see for example North 1975a, 1975b, 1981, North et al. 1979, Coakley 1979, Coakley and Wielicki 1979).[7] This approach describes the relation between outgoing infrared radiation $I(t)$ at time t, and the average global surface temperature $T(t)$ (measured in degrees Celsius) at time t. The infrared radiation flux to space $I(t)$ can be represented as a linear function of the surface temperature $T(t)$ by the empirical formula:

$$I(t) = A + BT(t) \tag{6}$$

where A, B are constants used to relate outgoing infrared radiation to the corresponding surface temperature.

In our model the change in the average global surface temperature $T(t)$ is determined by the sum of the absorbed solar heating (T_0), the reduction of incoming radiation due to the aggregate SRM effort (T_1) and the increase in the surface temperature due to the emissions of GHGs (T_2) which block outgoing radiation,

$$\dot{T} = T_0 + T_1 + T_2 \tag{7}$$

$$T_0 = \frac{-(A+BT) + Sq(1-\alpha)}{B},\ T_1 = -\frac{\phi}{B}\sum_{i=1}^{2}\zeta_i,\ T_2 = \frac{\psi}{B}\ln\left(1+\frac{G}{G_0}\right). \tag{8}$$

The term $(A+BT)$ reflects outgoing radiation; S is the mean annual distribution of radiation; q is the solar constant that includes all types of solar radiation, not just the visible light; α is the average albedo of the planet; the function $\varphi(\zeta) = \frac{\phi}{B}\sum_{i=1}^{2}\zeta_i$ is the reduction in solar radiation due to aggregate geoengineering $\left(\sum_{i=1}^{2}\zeta_i\right)$; $\phi > 0$ is the sensitivity of incoming radiation to geoengineering in reducing the average global temperature;[8] ψ is a measure of climate's sensitivity; and G, G_0 denote the GHGs, where G is the current accumulation of GHGs and G_0 is the preindustrial accumulation

[7] A homogeneous-earth model is a "zero-dimensional" model since it does not contain spatial dimensions but only the temporal dimension. For the use of one-dimensional and two-dimensional EBCMs in climate change economics, see Brock et al. (2013, 2014).

[8] SRM can be regarded as increasing the global albedo, since it blocks incoming radiation. We use a sensitivity function which is linear in aggregate SRM instead of a nonlinear function in order to simplify the exposition.

of GHGs.

We substitute T_0, T_1, T_2 into (10) to obtain:[9]

$$\dot{T} = \frac{-(A + BT) + Sq(1-\alpha)}{B} - \frac{\phi}{B}\sum_{i=1}^{2}\zeta_i + \frac{\psi}{B}\ln\left(1 + \frac{G}{G_0}\right). \qquad (9)$$

From equation (9) we have that: a) the average global temperature increases when current accumulation of GHGs is above the preindustrial level because GHGs block outgoing radiation, and b) the average global temperature decreases when SRM activities manage to reduce incoming radiation.

We assume, following evidence indicating that there is a fast and a slow response of global warming to external forcing with the slow component being relatively small (Held et al. 2010), that the average global temperature T converges fast to a steady state relative to the accumulation of GHGs, (G) (see also Brock et al. 2014). Then this 'quasi steady state' for T can be used to express T as a function of G, as follows:

$$\dot{T} = 0 \Longrightarrow \frac{-(A + BT) + Sq(1-\alpha)}{B} - \frac{\phi}{B}\sum_{i=1}^{2}\zeta_i + \eta(G - G_0) = 0$$

$$T = \frac{-A + Sq(1-\alpha) - \phi\sum_{i=1}^{2}\zeta_i + \eta(G - G_0)}{B}. \qquad (10)$$

To simplify the exposition we replace the term $\frac{\psi}{B}\ln\left(1 + \frac{G}{G_0}\right)$ in (9) with its linear approximation around G_0, in this case $\eta = \frac{\psi}{2BG_0}$.

The global welfare function that could be maximized by some "global social planner" is the unweighted discounted life time utility in each country minus the private cost of geoengineering and the social damages related to the increase in global temperature and to geoengineering. Thus a coopera-

[9] At this stage we do not consider the transportation of heat across the globe, which is a standard assumption of the EBCM developed by North (e.g., North 1975a, 1975b, 1981; North et al. 1979). Thus we study a homogeneous-earth, zero-dimensional model. This allows us to obtain tractable results regarding the mitigation/geoengineering tradeoff. The analysis of the mitigation/geoengineering tradeoff in the context of a one-dimensional spatial model is an area for further research.

tive case is equivalent to having a social planner solving:

$$W = \max_{E_i,\zeta_i} \int_0^\infty e^{-\rho_i t} \left\{ \sum_{i=1}^{2} \left[U\left(F_i\left(E_i\left(t\right)\right)\right) - C_i\left(\zeta_i\left(t\right)\right) - \frac{1}{2}c_{iT}T^2 - \frac{1}{2}\sum_{j=1}^{2} c_{j\zeta}\zeta_i^2 \right] \right\} dt$$
$$\text{subject to (1) and (10).}$$

Thus the problem of the social planner is to maximize the joint welfare of both countries by choosing paths for emissions $E_i(t)$ and geoengineering $\zeta_i(t)$ subject to the constraints of the accumulation of GHGs and of the average global temperature. A noncooperative case corresponds to a differential game where each country chooses paths for emissions and SRM to maximize own welfare subject to the climate constraints.

3 Symmetry: The Benchmark Model

As a reference scenario we look at the symmetric outcomes for the cooperative and noncooperative solutions. In this context we determine the steady-state level of emissions, geoengineering, GHGs accumulation and average global temperature under cooperation and noncooperation between the two countries.

3.1 Approximations and calibrations

In this section we calibrate the critical parameters of our model in order to provide analytically tractable results regarding the optimal level of emissions and geoengineering in a symmetric-cooperative game. We use approximations for the rest of the parameters of our model.

A possible parameterization is shown in table 1. Values for the parameters S, α, q have been taken from North (1975a, 1975b, 1981), values for the parameters $\rho, G_0, m, \theta, A_1, A_2$ have been taken from Athanassoglou and Xepapadeas (2012), while values for the parameters ϕ, ψ have been taken from Wigley et al. (2005). For θ, which basically represents the private cost for the implementation of geoengineering methods, we use an estimation of the annual cost following McClellan et al. (2012).[10] The rest of

[10]McClellan et al. (2012) perform an engineering cost analysis of systems capable of delivering 1–5 million metric tonnes (Mt) of albedo modification material to altitudes of 18–30 km. They compare the cost of aircraft and airships to the cost of survey rockets,

model parameters reflecting marginal damages, c_ζ, c_T, and empirical coefficients A, B were calibrated so that the benchmark symmetric cooperative solution results in an optimally controlled steady-state carbon stock of approximately 965GtC (453ppmCO_2). According to prevailing climate science, this is consistent with a 2°C warming stabilization target. These values are summarized in table 1.

Table 1: Calibration parameters

Parameter	Description	Value	Unit
c_T	slope of social marginal damage cost from an increase in T	22.183	$10^9\$/(\text{GtC})^2$
c_ζ	slope of social marginal damage cost from SRM	3.0	$10^9\$/(\text{GtC})^2$
S	mean annual distribution of radiation	1.0	Scalar
α	average albedo of the planet	0.23	Scalar
ϕ	sensitivity of incoming radiation to SRM in decreasing T	0.00303	°C
ρ	pure rate of time preference	0.03	Scalar
G_0	preindustrial GHGs accumulation	590.0	GtC
A	empirical coefficient	253.324	Wm^{-2}
q	solar constant	340.0	Wm^{-2}
B	empirical coefficient	0.64585	$\text{W}\left(m^{-2}\right)\left(^\circ\text{C}^{-1}\right)$
m	natural decay rate of GHGs	0.0083	Scalar
θ	slope of marginal cost from SRM	0.008	$10^9\$/(\text{GtC})^2$
A_1	intercept of marginal benefit from emissions	224.26	\$/tC
A_2	slope of marginal benefit from emissions	1.9212	$10^9\$/(\text{GtC})^2$
ψ	measure of climate's sensitivity	5.35	$^\circ\text{CWm}^{-2}$
$\eta = \frac{\psi}{2BG_0}$	measure of climate's sensitivity (linearization)	2.3863×10^{-3}	$^\circ\text{CWm}^{-2}$

guns, and suspended gas and slurry pipes for the delivery of stratospheric aerosol geoengineering at middle and high altitudes. They conclude that the most cost effective way to deliver material to the stratosphere at million tonnes per year is through the use of existing aircraft or new aircraft designed for the geoengineering mission.

3.1.1 The symmetric cooperative solution

The global social planner's problem is to maximize the joint welfare of both countries by choosing paths for emissions $E_i(t)$ and geoengineering $\zeta_i(t)$, subject to the constraints of the accumulation of GHGs and of the average global temperature.

To solve for the cooperative game we formulate the LQ optimal-control problem

$$\begin{aligned} W &= \max_{E_i,\zeta_i} \int_0^\infty e^{-\rho_i t} \left\{ \sum_{i=1}^{2} \left[U\left(F_i\left(E_i\left(t\right)\right)\right) - C_i\left(\zeta_i\left(t\right)\right) - \frac{1}{2} c_{iT} T^2 - \frac{1}{2} \sum_{j=1}^{2} c_{i\zeta} \zeta_j^2 \right] \right\} dt \ , \ i = 1, 2 \\ & s.t. \ \ \dot{G}\left(t\right) = E_1\left(t\right) + E_2\left(t\right) - mG \ , \ T = \frac{-A + Sq\left(1-\alpha\right) - \phi \sum_{i=1}^{2} \zeta_i + \eta\left(G - G_0\right)}{B}. \end{aligned}$$

Given the LQ structure of the problem, a quadratic value function,

$$V\left(G\right) = -\frac{1}{2}\kappa G^2 - \lambda G - \mu,$$

with first derivative

$$V_G = -\kappa G - \lambda$$

is considered. Imposing symmetry so that $A_{1i} = A_1$, $A_{2i} = A_2$, $c_{iT} = c_T$, $c_{i\zeta} = c_\zeta$, $\rho_i = \rho$, the equilibrium must satisfy the Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned} \rho V\left(G\right) = \max_{E_i,\zeta_i} &\left\{ \sum_{i=1}^{2} \left[A_1 E_i - \frac{1}{2} A_2 E_i^2 - \frac{1}{2}\theta \zeta_i^2 - \left(\frac{1}{2} c_T\right) \times \right.\right. \\ &\left(\frac{-A + Sq\left(1-\alpha\right) - \phi \sum_{i=1}^{2} \zeta_i + \eta\left(G - G_0\right)}{B}\right)^2 - \frac{1}{2} c_\zeta \sum_{j=1}^{2} \zeta_j^2 + V_G\left(E_1 + E_2 - mG\right). \end{aligned}$$

Optimality implies that

$$\begin{aligned} E_i^* &= \frac{A_1 - \kappa G - \lambda}{A_2} \ , \ \ E_i^* = E_j^*, \ i, j = 1, 2 \\ \zeta_i^* &= \frac{2\phi c_T \left(-A + Sq\left(1-\alpha\right) + \eta\left(G - G_0\right)\right)}{B^2\left(2c_\zeta + \theta\right) + 4\phi^2 c_T}, \ \zeta_i^* = \zeta_j^*, \end{aligned}$$

where E_i^*, ζ_i^* are, respectively, the optimal cooperative emissions and geoengineering efforts for each country in a feedback form. These are the policy functions for the cooperative solution.

The symmetric-cooperative solution determines the levels of long-run GHGs stock and of the average global temperature, through the optimal policy for emissions and geoengineering. Using the numerical values for the parameters of table 1, we can define the steady-state level of emissions, geoengineering, GHGs stock and temperature in the symmetric-cooperative game as

$$\begin{aligned} E_i^* &= 4.00676,\ \zeta_i^* = 0.595871 \\ G^* &= 965.484,\ T^* = 17.1996 \end{aligned}$$

respectively. Considering the current average global temperature to be around 15°C the long-run temperature of 17.2°C obtained by the model suggests that the model is consistent with the cooperative 2°C stabilization target. To make the solution clear and to make possible comparisons of the symmetric case with the asymmetric and the noncooperative cases, we determine the numerical optimal time paths for emissions and SRM, GHGs stock and global average temperature, which converge to their respective steady-state levels, as:[11]

$$\begin{aligned} E_i^*(t) &= 4.00676 + 0.0131107e^{-0.0624t}, \zeta_i^*(t) = 0.595871 - 0.000182303e^{-0.0624t} \\ G_i^*(t) &= 965.484 - 0.484272e^{-0.0624t}, T^*(t) = 17.1996 - 0.00526212e^{-0.0624t}. \end{aligned}$$

3.1.2 Symmetric noncooperative solutions: The feedback Nash equilibrium

In this section we analyze the noncooperative symmetric game and characterize its equilibrium outcome. We assume that each country follows feedback strategies regarding the level of emissions and geoengineering efforts. Feedback strategies are associated with the concept of FBNE which is a strong time-consistent noncooperative equilibrium solution (Basar 1989). The FBNE for the LQ climate change game can be obtained as the solution to the dynamic programming representation of the non-cooperative dynamic game.

[11] Graphs of the optimal paths are available in Appendix A1.

The value function for each country is

$$V_i(G) = -\frac{1}{2}\kappa_i G^2 - \lambda_i G - \mu_i.$$

We impose symmetry so that $A_{1i} = A_1$, $A_{2i} = A_2$, $c_{iT} = c_T$, $c_{i\zeta} = c_\zeta$, $\rho_i = \rho$, and the corresponding HJB for each country is

$$\rho V_i(G) = \max_{E_i,\zeta_i} \left\{ A_1 E_i - \frac{1}{2}A_2 E_i^2 - \frac{1}{2}\theta\zeta_i^2 \right.$$
$$\left. -\frac{1}{2}c_T \left(\frac{-A + Sq(1-\alpha) - \phi\sum_{i=1}^{2}\zeta_i + \eta(G - G_0)}{B} \right)^2 - \frac{1}{2}c_\zeta \sum_{j=1}^{2} \zeta_j^2 + V_{iG}(E_1 + E_2 - mG) \right\}.$$

Optimality implies that

$$\begin{aligned} E_i^* &= \frac{A_1 - \kappa_i G - \lambda_i}{A_2}, \quad E_i^* = E_j^*, \; j \neq i, \; \kappa_i = \kappa_j, \; \lambda_i = \lambda_j \\ \zeta_i^* &= \frac{\phi c_T \left(-A + Sq(1-\alpha) + \eta(G - G_0) - \phi\zeta_j\right)}{B^2(c_\zeta + \theta) + \phi^2 c_T}, \; \zeta_i^* = \zeta_j^*, \; j = 1, 2, \end{aligned}$$

where E_i^*, ζ_i^* are the optimal noncooperative emissions and geoengineering efforts for each country in a feedback form, the policy functions. It is clear that both emissions and geoengineering efforts are in a linear feedback form and depend on the current stock of GHGs, G. The slope of the emission feedback rule is negative, while the slope of the geoengineering feedback rule is positive. This means that one country expects the other country to reduce emissions and to increase geoengineering efforts when the stock of GHGs increases.

The symmetric-noncooperative solution determines the levels of steady-state long-run GHGs stock and of the average global temperature, through the optimal policy for emissions and geoengineering. For the full solution of the problem the parameters of the value function are obtained as usual by substituting the optimal controls into the HJB equation and then equating coefficients of the same power.

Using the numerical values for the parameters of table 1, the steady-state level of emissions, SRM, GHGs stock and average global temperature in the symmetric FBNE are shown below, with the percentage increase relative to the cooperative solution in parentheses:

$$\begin{aligned} E_i^* &= 9.89\ (147\%),\ \zeta_i^* = 1.12804\ (89\%) \\ G^* &= 2383.13\ (147\%), T^* = 32.6039\ (89.6\%). \end{aligned}$$

It is interesting to note that at the FBNE, steady-state emissions increase by 147% and geoengineering increases by 89%. Thus the presence of geoengineering provides an incentive for relatively more emissions. This is to be expected since more emissions are in principle desirable because benefits will increase, while the cost of increased emissions, in terms of global warming, is counterbalanced by SRM. This results in the increase in the steady-state GHGs in the FBNE. To make the solutions clear, the FBNE time paths for emissions and geoengineering conversing to the FBNE steady states are shown below:[12]

$$\begin{aligned} E_i^*(t) &= 9.89 + 21.4364e^{-0.0385t}, \zeta_i^*(t) = 1.12804 - 0.533143e^{-0.0385t} \\ G_i^*(t) &= 2383.13 - 1418.13e^{-0.0385t}, T^*(t) = 32.6039 - 15.4095e^{-0.0385t}. \end{aligned}$$

4 Asymmetric Countries and Noncooperative Solutions

In this section we demonstrate that our climate change game admits solution when heterogeneity between the two countries is introduced. We assume that heterogeneity is reflected in the values of the parameters specifying the benefit and the damage function for each country. This is a natural way to introduce heterogeneity, since we expect countries to differ with respect either to their production structure or to the damages that they might suffer from climate change or geoengineering activities. Thus we introduce heterogeneity by considering as the source of asymmetry between the two countries: (i) differences in the level of the social cost of geoengineering (c_ζ), (ii) differences in the impact of climate change on each country (c_T), (iii) differences in the intercept A_{1i} and the slope A_{2i} of the marginal benefit function, and (iv) differences in the level of the rate of time preference (ρ), with differences in ρ reflecting differences in the degree of impatience between the two countries. We consider the impact for each potential source

[12] Graphs are provided in A2.

of heterogeneity alone, except for one case where we combine economic with climatic damage asymmetries. That is, we combine heterogeneity in A_{1i} and c_ζ in an attempt to explore the attitudes of a relatively more productive country under varying geoengineering social costs.

Our objective is to examine how each source of asymmetry will affect each country's decision about the optimal levels of emissions and SRM, and how this decision will affect the environment in terms of the steady-state level of GHGs and global average temperature. The benchmark for comparisons will be the symmetric non-cooperative optimal (FBNE) level of emissions, geoengineering, GHGs stock and global temperature.[13] In the two-country asymmetric model, an FBNE must satisfy the HJB equations

$$\begin{aligned}\rho_i V_i(G) \;=\; \max_{E_i,\zeta_i} \Bigg\{ & A_{1i}E_i - \frac{1}{2}A_{2i}E_i^2 - \frac{1}{2}\theta\zeta_i^2 - \frac{1}{2}c_{iT}\left(\frac{-A + Sq(1-\alpha) - \phi\sum_{i=1}^{2}\zeta_i + \eta(G-G_0)}{B}\right)^2 \\ & -\frac{1}{2}c_{i\zeta}\sum_{j=1}^{2}\zeta_j^2 + V_{iG}(E_1+E_2-mG)\Bigg\} \quad , \; i=1,2. \end{aligned} \tag{11}$$

Each country will take as given the emissions and geoengineering level of the other country and will solve the optimal problem for its own level of emissions and geoengineering following feedback rules which will determine emissions and SRM as time stationary functions of the current concentration of GHGs. In particular given the LQ structure of the problem, we consider two quadratic value functions $V_i(G)$, $i=1,2$. For each country we have

$$V_i(G) = -\frac{1}{2}\kappa_i G^2 - \lambda_i G - \mu_i \;, \quad i=1,2$$

with first derivatives

$$V_{iG} = -\kappa_i G - \lambda_i \;, \quad i=1,2.$$

[13] List and Mason's (2001) sources of asymmetry correspond to the intercept of marginal benefits from emissions (the A_{1i} in terms of our model), and the slope of marginal damages from global pollution (or c_T).

Thus the HJB equation for country $i = 1$ is:

$$\rho_1 V_1(G) = \max_{E_1,\zeta_1}\left\{A_{11}E_i - \frac{1}{2}A_{21}E_1^2 - \frac{1}{2}\theta\zeta_1^2 - \frac{1}{2}c_T\left(\frac{-A + Sq(1-\alpha) - \phi\sum_{i=1}^2 \zeta_i + \eta(G - G_0)}{B}\right)^2 - \frac{1}{2}c_{1\zeta}\sum_{j=1}^2 \zeta_j^2 + V_{1G}(E_1 + E_2 - mG)\right\}. \quad (12)$$

Optimality implies that

$$E_1 = \frac{A_{11} - \kappa_1 G - \lambda_1}{A_{21}} \quad (13)$$

$$\zeta_1 = \frac{\phi c_T(-A + Sq(1-\alpha) + \eta(G - G_0) - \phi\zeta_2)}{B^2(c_{1\zeta} + \theta) + \phi^2 c_T}. \quad (14)$$

Then the HJB satisfies

$$\rho_1\left(-\frac{1}{2}\kappa_1 G^2 - \lambda_1 G - \mu_1\right) = A_{11}\frac{A_{11} - \kappa_1 G^* - \lambda_1}{A_{21}} - \frac{1}{2}A_{21}\left(\frac{A_{11} - \kappa_1 G^* - \lambda_1}{A_{21}}\right)^2$$
$$-\frac{1}{2}\theta\left(\frac{\phi c_T(-A + Sq(1-\alpha) + \eta(G - G_0) - \phi\zeta_2)}{B^2 c_{1\zeta} + \phi^2 c_T}\right)^2$$
$$-\frac{1}{2}c_T\left(\frac{-A + Sq(1-\alpha) - \phi\sum_{i=1}^2 \zeta_i + \eta(G - G_0)}{B}\right)^2$$
$$-\frac{1}{2}c_{1\zeta}\sum_{j=1}^2 \zeta_j^2 + (-\kappa_1 G - \lambda_1)\left(\frac{A_{11} - \kappa_1 G^* - \lambda_1}{A_{21}} + E_2 - mG\right).$$

In a similar way the HJB equation for country $i = 2$ is

$$\rho_2 V_2(G) = \max_{E_2,\zeta_2}\left\{A_{12}E_2 - \frac{1}{2}A_{22}E_2^2 - \frac{1}{2}\theta\zeta_2^2 - \frac{1}{2}c_T\left(\frac{-A + Sq(1-\alpha) - \phi\sum_{i=1}^2 \zeta_i + \eta(G - G_0)}{B}\right)^2 - \frac{1}{2}c_{2\zeta}\sum_{i=1}^2 \zeta_j^2 + V_{2G}(E_1 + E_2 - mG)\right\}. \quad (15)$$

Optimal emissions and geoengineering, and the equation that the HJB equation satisfies, are determined in the same way as for $i = 1$.

When the feedback rules for each country are replaced in the corresponding HJB equation, then the parameters of the value functions are obtained as usual by equating coefficients of the same power. However, in the asymmetric game, the parameters of the value function for one country, say i, will depend on the emissions E_j and the geoengineering activity ζ_j of the other country. This means that in general

$$E_i = \frac{A_{1i} - \kappa_i \left(E_j, \zeta_j, c_\zeta, c_T, \rho_1, \rho_2\right) G - \lambda_i \left(E_j, \zeta_j, c_\zeta, c_T, \rho_1, \rho_2\right)}{A_{2i}}, \; i, j = 1, 2, i \neq j. \tag{16}$$

The system of equations (16) can be interpreted as the linear best response feedback rule of each country given the stock G. To obtain the equilibrium feedback rules for this asymmetric differential game, first we solve for the Nash equilibrium values for geoengineering ($j = 1, 2$) by solving (14) and its counterpart for ζ_2 simultaneously, second by replacing ζ_j ($j = 1, 2$) into (16) with the appropriate Nash equilibrium value ζ_j^* we obtain the best response feedback rules, and finally by solving for the Nash equilibrium values E_i^*, $i = 1, 2$ in (16) we obtain the linear feedback rules $E_i^* = \frac{A_{i1} - \hat{\kappa}_i G - \hat{\lambda}_i}{A_{i2}}, i = 1, 2$ where $\hat{}$ stands for the calculated parameter of the value function. In case of multiple solutions for the parameters of the value function, we choose those that ensure stable dynamics for the stock of GHGs. By replacing the optimal feedback rules in the GHGs dynamics we obtain the steady states and the stable paths for GHGs and global average temperature, along with emissions and geoengineering for each country.

Having obtained the solution, using numerical simulations we examine the impact of heterogeneity by means of sensitivity analysis with respect to the sources of asymmetry discussed above. The values of the parameters used in the calibration of the symmetric cooperative problem are used as the central values for the sensitivity analysis. In particular we consider two different scenarios for heterogeneity.

In the first, which can be regarded as a case of "symmetric heterogeneity", we consider an increase of 20%, 40%, 70% or 90% in the value of the specific parameter for country 1 with a corresponding decrease in the same parameter for country 2. These scenarios will be denoted by $\pm 20\%, \pm 40\%, \pm 70\%, \pm 90\%$.

In the second scenario, which is the case of "asymmetric heterogeneity," we consider a given change, positive or negative in the parameter for country 1, and an opposite but different change in value in the same parameter for country 2. In particular we consider the following changes for the second scenario, with the first number in each pair denoting country 1. For example $(+20\%\& - 90\%)$, $(+20\%\& - 70\%)$, means that the specific parameter increases 20% from its central value for country 1, and the same parameter is reduced by 90% in one run and by 70% in another run for country 2.[14] Using this type of approach we hope to capture the effect of both symmetric and asymmetric differences between the two countries.

4.1 Heterogeneity in the social cost of geoengineering (c_ζ)

We assume first that the implementation of geoengineering has a different impact on each country in terms of the social cost of geoengineering, which is the cost with which each society will be burdened due to the implementation of geoengineering by itself and/or the other country.

The central value of the social marginal damage cost from geoengineering is set at 3 $\left(10^9\$/(GtC)^2\right)$ and symmetric and asymmetric deviations as described above are considered. The results are shown in table 2. For the symmetric deviations we observe that both countries increase emissions, but most importantly country 2 (column $i = 2$), which experiences the lower social geoengineering costs, increases geoengineering a lot more than the reductions of country 1, which experiences the higher geoengineering social costs. The net increase in geoengineering increases the steady-state stock of GHGs. In the case of asymmetric deviation the pattern is very similar. We need a very large increase in the social cost of geoengineering, +90% or +70% in country 2 relative to a corresponding decrease in country 1 (−20%), in order to have a relatively larger reduction in the geoengineering from country 2. In general increased emissions and SRM lead to an increase in the majority of cases of the steady-state stock of GHGs, while there is a corresponding moderate decrease in global average temperature. Thus the difference in the social cost of geoengineering might lead to lower global temperature but higher stock of GHGs relative to the symmetric case.

[14] We run the following scenarios of asymmetric heterogeneity: $+20\%\ \&\ -90\%; +20\%\ \&\ -70\%; +40\%\ \&\ -70\%; +40\%\ \&\ -90\%; +20\%\ \&\ -40\%$ $; -20\%\ \&\ +90\%; -20\%\ \&\ +70\%; -40\%\ \&\ +70\%; -40\%\ \&\ +90\%; -20\%\ \&\ +40\%.$

Table 2: Changes (%) in steady-state values due to asymmetries in c_ζ

% change in c_ζ		$\Delta E_1 = \Delta E_2$	$\Delta\zeta_1$	$\Delta\zeta_2$	ΔT	ΔG
$i=1$	$i=2$					
+20	−20	0.0015	−16.6298	24.9167	−0.00031	0.0017
+90	−90	0.1518	−47.3095	876.4900	−0.0141	0.1519
+40	−40	0.00698	−28.5176	66.3709	−0.0009	0.0071
+70	−70	0.0348	−41.1139	231.2670	−0.0034	0.0348
+20	−90	0.1573	−16.6419	876.4810	−0.0147	0.1565
+20	−70	0.0392	−16.6327	231.2660	−0.0037	0.0394
+40	−70	0.0371	−28.5196	231.2660	−0.0037	0.0373
+40	−90	0.1552	−28.5274	876.4810	−0.0144	0.1552
+20	−40	0.0091	−16.6304	66.3700	−0.0009	0.0092
−20	+90	−0.0040	24.9176	−47.3019	0.0003	−0.0038
−20	+70	−0.0029	24.9176	−41.1118	0.0	−0.0029
−40	+70	0.0047	66.3709	−41.1123	−0.0006	0.0046
−40	+90	0.0035	66.3709	−47.3022	−0.0006	0.0038
−20	+40	−0.0006	24.9167	−28.5171	0.0	−0.0004

4.2 Heterogeneity in environmental damages due to global warming (c_T)

We assume that climate change in terms of increased global average temperature has a different impact on each country expressed in differences in the term c_T among the two countries. For the purposes of the sensitivity analysis, the central value for c_T was set at 22.183 $\left(10^9\$/(GtC)^2\right)$, which is the value used in the cooperative solution, and the symmetric and asymmetric deviations described above are considered. The results are shown in table 3. In the case of symmetric deviations the behavior of each county is also symmetric and in the opposite direction, so that the final outcome on GHGs and average global temperature is zero. In the case of asymmetric deviations, the country that experiences the smaller damages increases emissions relatively more and reduces geoengineering relatively less than the emission reductions and geoengineering increases in the country that experiences higher damages (e.g., see rows $+20\%, -90\%$; $+40\%, -90\%$). The country that experiences the higher damages increases geoengineering more and reduces emissions more than the geoengineering reductions and the emissions increases undertaken by the country experiencing lower damages (e.g., see rows $-20\%, +90\%$; $-40\%, +90\%$). A strong reduction in damages in one of the countries will eventually lead to a substantial increase in the steady-state GHGs and global average temperature. The country that experiences relatively low damages from global warming will tend to increase emissions and reduce geoengineering. If the reduction in damages in one country dominates the increase in the other, steady-state GHGs and global temperature will increase.

Table 3: Changes (%) in steady-state values due to asymmetries in c_T

% change in c_T		ΔE_1	ΔE_2	$\Delta \zeta_1$	$\Delta \zeta_2$	ΔT	ΔG
$i=1$	$i=2$						
+20	−20	−216.055	216.055	20.0002	−20.0	0.0	0.0
+90	−90	−972.247	972.245	90.0004	−90.0	0.0	0.0
+40	−40	−756.192	756.192	756.192	−70.0	0.0	0.0
+70	−70	−432.110	432.110	40.0004	−40.0	0.0	0.0
+20	−90	−807.98	922.922	74.7934	−85.4338	45.661	57.470
+20	−70	−590.064	662.690	54.6213	−61.3445	28.851	36.313
+40	−70	−666.828	705.895	61.7274	−65.3440	15.519	19.534
+40	−90	−868.454	941.082	80.3917	−87.1148	28.851	36.347
+20	−40	−343.582	368.346	31.8047	−34.0975	9.837	12.382
−20	+90	422.311	−482.388	−39.0930	44.6544	−23.866	−30.038
−20	+70	374.162	−420.214	−34.6358	38.8984	−18.295	−23.026
−40	+70	508.876	−538.689	−47.1061	49.8661	−11.844	−14.906
−40	+90	550.691	−596.742	−50.9769	55.2401	−18.295	−23.026
−20	+40	287.096	−307.788	−26.5762	28.4919	−8.220	−10.346

4.3 Heterogeneity in the intercept of marginal benefits from emissions (A_1)

We assume that each country has a different intercept of the marginal private benefits from its own emissions. This means that the marginal benefit function of one country is uniformly above the other, suggesting that for this country emissions are more productive in terms of private benefits. We perform sensitivity analysis by setting that central value for A_1 at 224.26 ($\$/tC$) (Athanassoglou and Xepapadeas 2012). The results are shown in table 4. In the case of symmetric deviations, the behavior of each county is also symmetric and in the opposite direction, so that the final outcome on GHGs and average global temperature is zero. In the case of asymmetric deviations, if the increase in emission productivity dominates (e.g., see rows $-20\%, +90\%$; $-40\%, +90\%$), then there is an overall increase in emissions, geoengineering, steady-state GHGs and global temperature. We have the opposite result when the reduction in emission productivity dominates.

Thus, as expected, the country with the higher marginal benefits from production/emissions will raise its emissions relative to the symmetric game and the country with low marginal benefits from production will reduce the emissions. Another result is that both countries seem to reduce geoengineering efforts by the same proportion relative to the symmetric game when the reduction in productivity dominates (e.g. row $+20\%, -90\%$) and increase geoengineering effort when the increase in productivity dominates (e.g. row $-20\%, +90\%$). This behavior could be explained by the fact that the parameter of asymmetry between the two countries affects emissions only, and thus both countries follow the same policy for their geoengineering efforts despite the difference in benefits from emissions. This result suggests that, in the context of the noncooperative solution for the LQ model, an upward shift in the marginal benefits from emissions of a country will eventually lead to relatively higher stocks of GHGs and global temperature.

Table 4: Changes (%) in steady-state values due to asymmetries in A_1

% change in A_1		ΔE_1	ΔE_2	$\Delta\zeta_1 = \Delta\zeta_2$	ΔT	ΔG
$i=1$	$i=1$					
+20	−20	236.055	−236.055	0.0	0.0	0.0
+90	−90	1062.24	−1062.25	0.0	0.0	0.0
+40	−40	472.110	−472.11	0.0	0.0	0.0
+70	−70	826.192	−826.192	0.0	0.0	0.0
+20	−90	606.030	−692.272	−34.248	−34.248	−43.121
+20	−70	500.323	−561.924	−24.463	−24.463	−30.801
+40	−70	630.670	−667.631	−14.678	−14.678	−18.480
+40	−90	736.378	−797.979	−24.463	−24.463	−30.801
+20	−40	341.762	−366.402	−9.7851	−9.785	−12.320
−20	+90	−606.030	692.272	34.248	34.248	43.121
−20	+70	−500.323	561.924	24.463	24.463	30.800
−40	+70	−630.670	667.631	14.678	14.678	18.480
−40	+90	−736.377	797.979	24.463	24.463	30.801
−20	+40	−341.762	366.402	9.7851	9.785	12.320

4.4 Heterogeneity in the slope of marginal benefits from emissions (A_2)

We assume that each country has a different slope of the marginal benefits from emissions A_2. Given the quadratic structure of the benefit function from emissions, the country with the higher slope is characterized by stronger diminishing returns in the generation of benefits from emissions. We perform sensitivity analysis using A_2 1.9212 $10^9\$/(GtC)^2$ as the central value for the parameter (see Athanassoglou and Xepapadeas 2012). The results are shown in table 5. It should be noted that a reduction in A_2 indicates weaker diminishing returns, while an increase in A_2 indicates stronger diminishing returns. The results suggest that when weak diminishing returns dominate (rows $+20\%, -90\%$ until $+20\%, -40\%$), then global emissions, geoengineering, steady-state GHGs and global temperature increase. On the other hand when strong diminishing returns dominate (rows $-20\%, +90\%$; $-20\%, +70\%$), the outcome is reversed. Thus an overall weakening of diminishing returns can be related to an increase in global emissions, geoengineering, steady-state GHGs and global temperature at the noncooperative solution.

Table 5: Changes (%) in steady-state values due to asymmetries in A_2

% change in A_2		ΔE_1	ΔE_2	$\Delta\zeta_1 = \Delta\zeta_2$	ΔT	ΔG
$i=1$	$i=2$					
+20	−20	−19.6646	20.5035	0.3333	0.333	0.419
+90	−90	−89.0764	107.549	7.3357	7.335	9.236
+40	−40	−38.9809	42.3782	1.3492	1.348	1.699
+70	−70	−68.3828	79.1638	4.2818	4.281	5.391
+20	−90	−83.1841	101.791	7.3889	7.3888	9.303
+20	−70	−57.7041	69.183	4.5583	4.558	5.739
+40	−70	−62.7382	73.8888	4.4280	4.427	5.575
+40	−90	−85.4296	103.985	7.3685	7.368	9.278
+20	−40	−31.9118	36.1759	1.6932	1.693	2.132
−20	+90	38.9151	−41.5096	−1.0301	−1.031	−1.297
−20	+70	34.7624	−36.5825	−0.7238	−0.723	−0.910
−40	+70	49.5915	−47.203	0.9485	0.948	1.194
−40	+90	53.4095	−51.555	0.7367	0.736	0.927
−20	+40	27.0313	−27.4105	−0.1507	−0.151	−0.189

4.5 Heterogeneity in the rate of impatience (ρ)

We assume that each country discounts future net benefits at a different rate, which implies a different degree of impatience between the two countries, with current net benefits being the undiscounted lifetime utility in each country minus the private cost of geoengineering and the social damages related to the increase in global temperature and to geoengineering. We associate discount rates ρ_1 and ρ_2 with the HJB equation for each country and solve the asymmetric differential game in the manner described earlier in section 4.

For the purposes of the sensitivity analysis, we consider a central value of 0.03, which was used in the symmetric game, for the discount rate and then consider deviations from this central value. We assume that country 1 will have a constantly lower rate of time preference than country 2. Thus country 2 is more impatient than country 1. The results are shown in table 6. As expected, the more impatient country (the high ρ country) increases emissions and the less impatient country (low ρ country) reduces emissions. It is important to note that the final outcome for the global environment depends on both the difference between the discount rates - the impatience gap - and the level of impatience in each country. In the first seven rows of table 6 the final outcome is a reduction in both GHGs and global temperature, since the behavior of the less impatient country dominates. In the last two rows of table 6, where country 2 is impatient and country's 1 impatience is not very small, the behavior of the impatient country dominates, resulting in an increase in both the steady-state GHGs and global average temperature.

Table 6: Changes in steady-state values-due to asymmetries in ρ

ρ		ΔE_1	ΔE_2	$\Delta\zeta_1 = \Delta\zeta_2$	ΔT	ΔG
$i=1$	$i=2$					
0.005	0.05	−656.189	544.936	−44.1825	−44.1809	−55.626
0.005	0.075	−785.695	684.924	−40.0177	−40.0179	−50.385
0.005	0.1	−861.51	766.877	−37.5806	−37.5808	−47.316
0.01	0.05	−514.625	440.448	−29.4574	−29.4575	−37.087
0.01	0.075	−658.889	600.78	−23.0766	−23.0767	−29.055
0.01	0.1	−745.34	696.858	−19.2529	−19.2529	−24.240
0.02	0.05	−291.035	275.416	−6.20279	−6.20325	−7.809
0.02	0.075	−450.796	462.693	4.72412	4.72459	5.949
0.02	0.1	−550.243	579.269	11.5271	11.5265	14.5133

4.6 Combined economic and climatic damage heterogeneity (A_1) and (c_ζ)

As a last case of heterogeneity among the two countries, a combination of differences in the social cost of geoengineering c_ζ and in the intercept of private marginal benefits from emissions A_1 is considered. In particular we examine the case where at the same time (i) A_1 increases for country 1 relative to the symmetric case, while for country 2 it remains at the level of the symmetric case, and (ii) c_ζ decreases for country 2 relative to the symmetric case, while it remains at the level of the symmetric case for country 1. The results are presented in table 7. Both countries increase geoengineering, while global emissions increase since the increase in emissions from the country with the higher A_1 dominates the decrease in emissions from the country with the relatively lower A_1. As a result both steady-state GHGs and global average temperature increase. Thus in a world where one country is relatively more productive in terms of emissions and the other country suffers relatively less damages from geoengineering, the noncooperative outcome points towards a high stock of GHGs and global average temperature.

Table 7: Changes (%) in steady-state values due to asymmetries in A_1 and c_ζ

% change A_1	% change c_ζ	ΔE_1	ΔE_2	$\Delta\zeta_1$	$\Delta\zeta_2$	ΔT	ΔG
$i=1$	$i=2$						
+20	−20	130.353	−105.702	9.78511	37.1396	9.78472	12.325
+40	−20	260.701	−211.408	19.5702	49.3626	19.5697	24.646
+70	−20	456.223	−369.969	34.2479	67.698	34.2474	43.127
+90	−20	586.571	−475.675	44.033	79.9218	44.0328	55.447
+20	−40	130.361	−105.694	9.78423	82.6496	9.7838	24.655
+40	−40	260.71	−211.399	19.5693	98.9291	19.5688	24.655
+70	−40	456.234	−369.959	34.247	123.348	34.2465	43.137
+90	−40	586.582	−475.664	44.0321	139.628	44.0315	55.459
+20	−70	130.394	−105.661	9.78068	263.679	9.78073	12.367
+40	−70	260.746	−211.364	19.5658	296.094	19.5655	24.691
+70	−70	456.274	−369.918	34.2426	344.716	34.2428	43.178
+90	−70	586.626	−475.622	44.0277	377.132	44.0276	55.502
+20	−90	130.524	−105.531	9.76916	972.028	9.76877	12.497
+40	−90	260.887	−211.222	19.5525	1067.58	19.5526	24.832
+70	−90	456.432	−369.759	34.2284	1210.9	34.2281	43.336
+90	−90	586.796	−475.451	44.0117	1306.46	44.0119	55.672

5 Conclusions

This paper analyzed a stylized climate change model represented as an LQ differential game, where countries may act predominantly noncooperatively. in affecting climate change through their emissions which generate private benefits, mitigation and SRM or geoengineering. Mitigation and geoengineering are beneficial in terms of reducing global warming, but geoengineering has private costs and 'public bad' characteristics, since SRM activities of one country may have harmful effects on another country. The novel element is that countries are asymmetric with respect to critical parameters such as the cost of global warming, the social cost of geoengineering, the private marginal benefits from emissions and the degree of impatience. To provide a benchmark for comparisons we first obtain a solution of the symmetric model and then compare the asymmetric solution with the symmetric solution in terms of individual emissions, individual SRM activities, the steady-state stock of GHGs and the average global temperature. It should be noted that given the very low private costs of SRM, any variation in this cost, other things being equal, does not provide any substantial changes in each country's behavior.[15] Thus we excluded the private cost of SRM from our sensitivity analysis. Our results provide some insights regarding whether and under what conditions tradeoffs exist between mitigation and SRM.

Thus, a country that is expected to have a substantially relatively lower social cost due to its own as well as the other country's geoengineering activities is expected to increase both geoengineering and emissions. Geoengineering increases because its social cost to the country is small and country emissions also increase because they generate private benefits, while their costs in terms of global warming can be, at least partly, counterbalanced by low cost geoengineering. Thus there is no tradeoff between emissions and SRM activities. The result is an increase in the steady-state stock of GHGs and a moderate decrease in the average global temperature.

When the asymmetry is in the cost of global warming to each country, the country with the lower costs substantially increases emissions and reduces geoengineering. This is because emissions generate private benefits but have a very low cost in terms of global warming. If the asymmetry is substantial, the behavior of the low cost country dominates and the result is an increase in both the steady-state stock of GHGs and the average global temperature. The opposite result holds when the behavior of the high cost country dominates. In this case we have emissions reduction and SRM increase. Thus a tradeoff between emissions and SRM takes place at the noncooperative solution if the asymmetry is in the cost of global warming to each country.

When the private marginal benefits from emission are uniformly higher in one country, which means that this country is more productive in generating benefits from emissions, then the more productive country increases emissions as expected. If the productivity gap is substantial in favor of the more productive country, then both the steady-state stock of GHGs and the average global temperature increase.

When countries differ in the degree of diminishing returns in private ben-

[15] See Appendix A.4.

efits generated from emissions, then the country with substantially weaker diminishing returns increases emissions. The behavior is similar to the case above where private marginal benefits from emissions are uniformly higher in one country. Again, when the diminishing returns gap is substantial, both the steady-state stock of GHGs and the average global temperature increase.

It should be noted that in the context of our noncooperative model, higher productivity in generating private benefits from emissions in one country does not lead to lower emissions. Lower emissions might have been expected since the same level of benefits could have been obtained with lower emissions and at the same time the global environment could have been improved. This is not the case however; private benefits from more productive emissions outweigh any gains in social benefits that reduced emissions, due to higher productivity, might have generated since social benefits have public good characteristics. Furthermore, the effect of productivity gaps on geoengineering behavior is not substantial. It seems that when the more productive country dominates, there is a moderate increase in SRM activities. This is explained by the fact that when emissions increase a lot as a result of the productivity gap, countries might increase SRM activities to counterbalance the global warming effects of increased emissions.

Finally if one country is more productive in generating private benefits from emissions and the other country suffers relatively less social cost from SRM, the final outcome will be an increase in global emissions, SRM activities, the steady-state stock of GHGs and the average global temperature.

Thus, we examined two major types of asymmetries among countries. The first relates to climatic damages, that is damages to a country from global SRM, or from an increase in the global temperature. Focusing on the situation where the asymmetry is substantial, the results suggest that in the first case both SRM activities and emission increase, while in the second case we have the opposite result - mitigation (i.e. emission reduction) and SRM increases. These results could but comparable to those obtained by Moreno-Cruz (2010) in the context of a different model, and emphasize that when we have asymmetries regarding damages from climate change the source of the damage – global temperature, or global SRM – matters. Furthermore these results point to the need for developing spatial models of climate and the economy in order to associate local changes in temperature and SRM effects with local damages.

The second type relates to asymmetries in the economy and in particular the benefits from emissions, which may reflect technological or developmental gaps among countries. The results here indicate that the more productive country will cause an increase in both emissions and SRM, again breaking down the tradeoff between mitigation and geoengineering. Furthermore, the combination of economic asymmetries with climatic asymmetries in a certain way could lead to a situation where emissions, SRM, GHGs and global temperature increase. Finally higher rates of impatience for both countries lead as expected to increase in emissions, SRM, GHGs and global temperature.

Given that in the real world asymmetries among countries in climatic damages, economic conditions and impatience rates are expected to be present and most likely substantial, this type of conceptual framework might be useful in climate economics.

It should be noted that the most striking results are obtained when countries deviate asymmetrically from the benchmark (the symmetric solution). If the deviations are symmetric in the sense, for example, that the amount by which the cost of global warming in one country is below the benchmark is the same in absolute value as the amount by which the cost of global warming is higher than the benchmark in the other, then the symmetric and the asymmetric solutions do not differ much. Under asymmetric deviations, however, we have substantial deviations from the benchmark solution.

Another result is that in the noncooperative solution, a tradeoff between emissions and SRM appears only in certain cases. In fact, in cases that are likely to be encountered in the real world, a country has an incentive to increase emissions in order to capture private benefits and increase SRM in order to counterbalance the effects of increased emissions on global temperature. These effects become stronger the less the social cost of SRM to the country, the higher the private productivity of emissions and the lower the private cost of SRM. In a context of substantial asymmetries, these incentives may determine the steady-state global average temperature and the stock of GHGs.

This work suggests several interesting avenues for future research. A more complete treatment of the issues presented here would extend the basic model to incorporate other geoengineering methods such as carbon capture and storage, and most importantly adaptation, in addition to mitigation and SRM, as alternative policy options against climate change. If the LQ structure is kept, then the problem can be solved. If nonlinearities are introduced, then FBNE solutions can be obtained in principle by numerical methods, although this is not a easy task. Introduction of uncertainty - especially as deep structural uncertainty - including characteristics such as model uncertainty, ambiguity aversion, robust control methods, or regime shifts - is also a very important area for further research.

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Appendix

A.1: Optimal paths under cooperation, symmetric solution

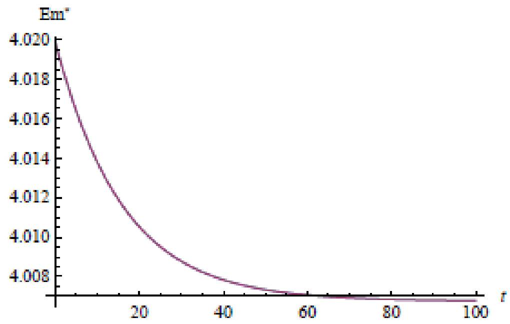


Figure 1

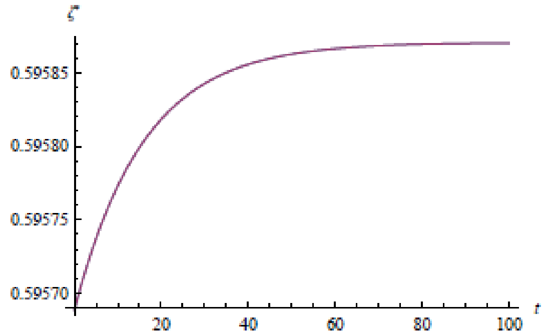


Figure 2

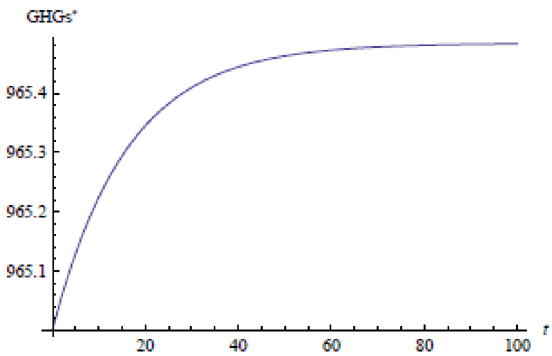


Figure 3

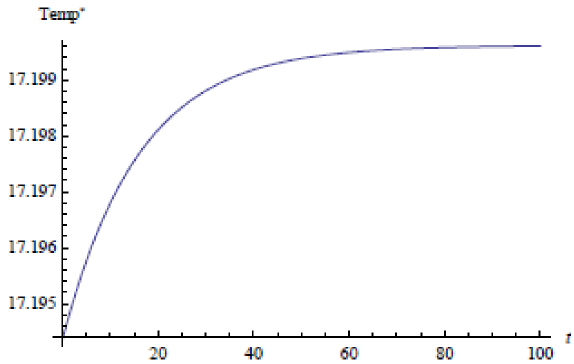


Figure 4

Figures 1 - 4: Equilibrium time paths for emissions, SRM, stock of GHGs and global average temperature under cooperation.

A.2: Equilibrium paths at the FBNE, symmetric solution

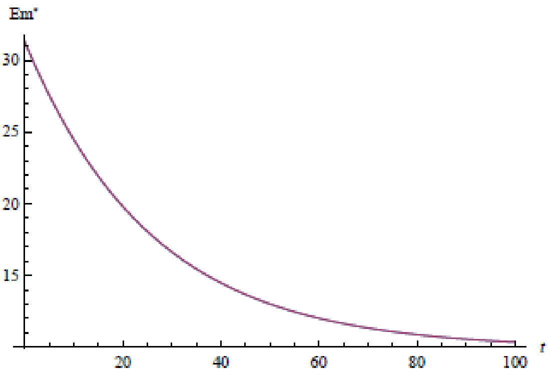


Figure 5

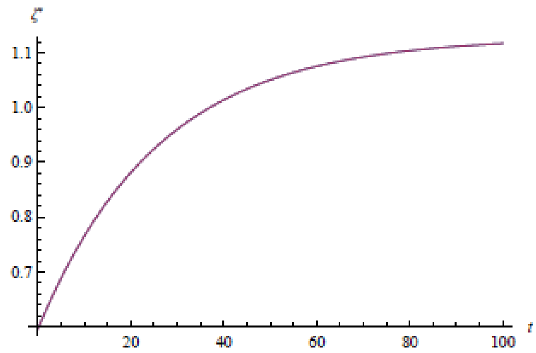


Figure 6

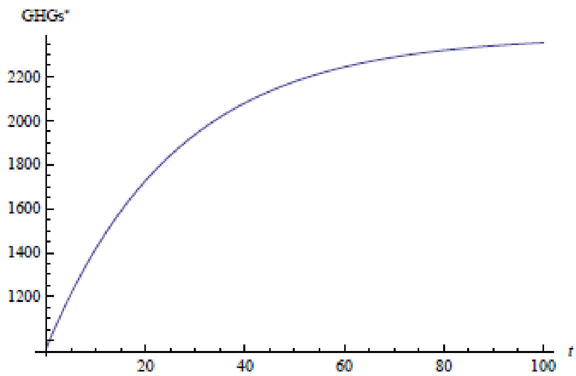


Figure 7

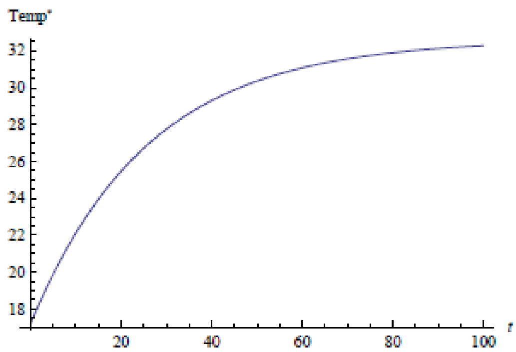


Figure 8

Figures 5 - 8: Equilibrium time paths for emissions, SRM, stock of GHGs and global average temperature at the FBNE.

A.3: Time paths of FBNE under asymmetry

For country $i = 1 \rightarrow$ - - - - - - - - -

For country $i = 2 \rightarrow$ **- - - - -**

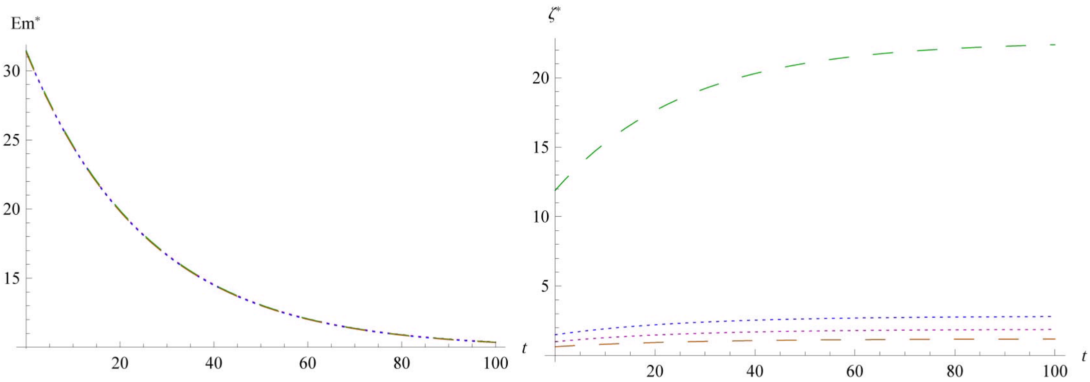
In country $i = 1$: Increase 20% $\rightarrow$ Red Dashed Line

Decrease 20% $\rightarrow$ Blue Dashed Line

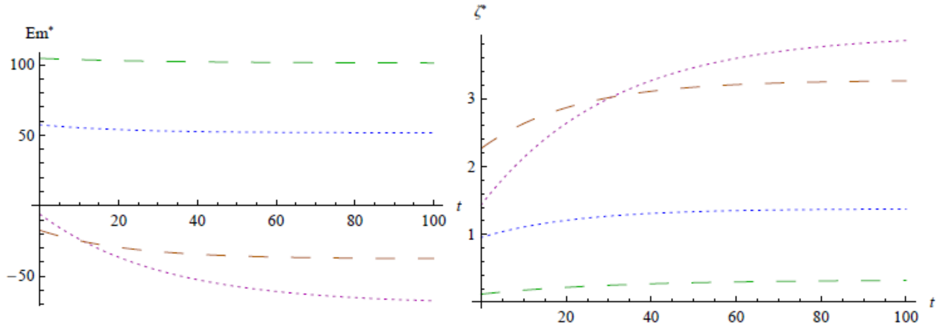
In country $i = 2$: Decrease 90% $\rightarrow$ Green Dashed Line

Increase 90% $\rightarrow$ Orange Dashed Line

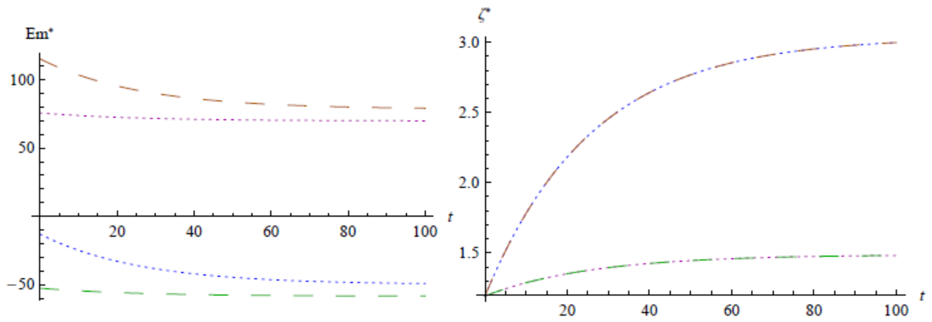
Heterogeneity in social cost of geoengineering (c_ζ)



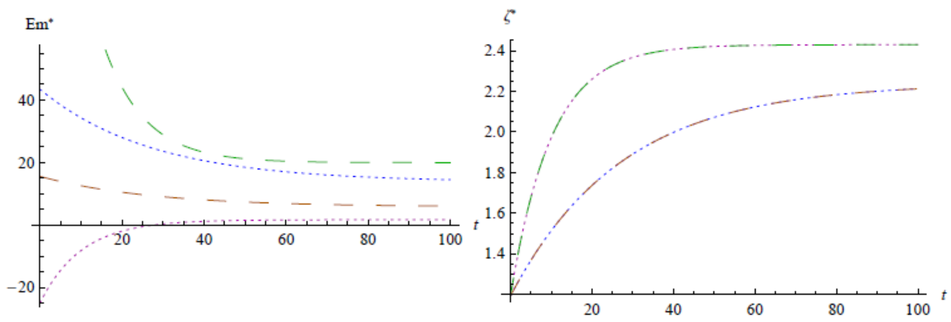
Heterogeneity in environmental damages due to global warming (c_T)



Heterogeneity in the intercept of marginal benefits from emissions (A_1)



Heterogeneity in the slope of marginal benefits from emissions (A_2)

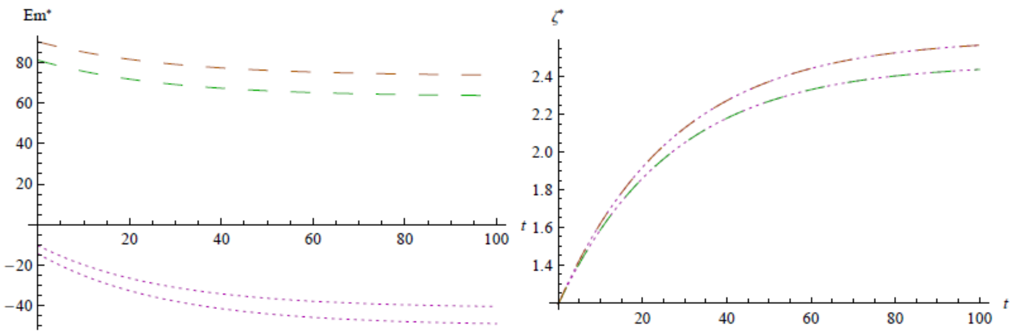


Heterogeneity in rate of time preference (ρ)

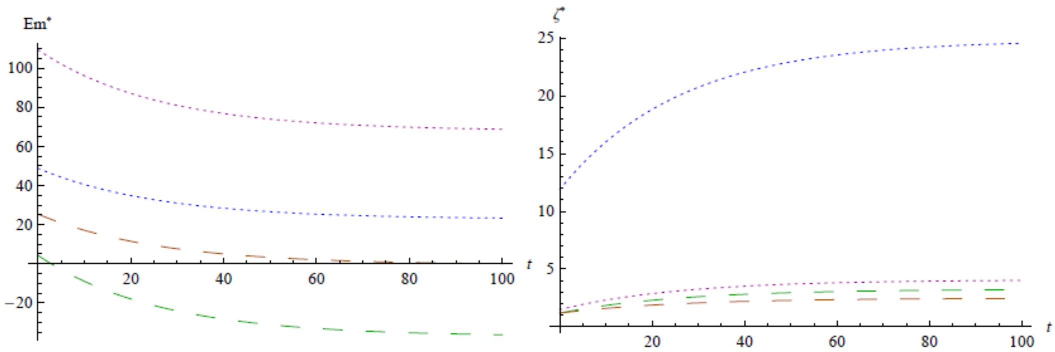
In country i = 1 : ρ = 0.02 ⟶ Red Dashed Line

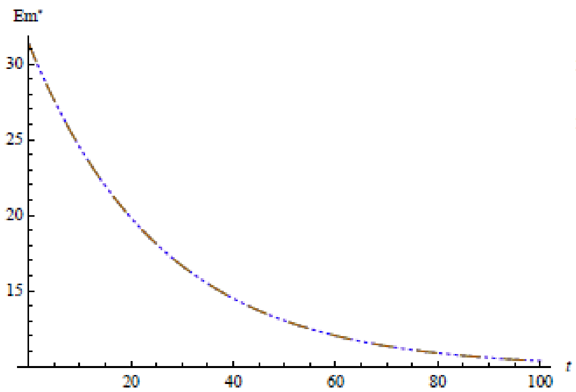
In country i = 2 : ρ = 0.075 ⟶ Green Dashed Line

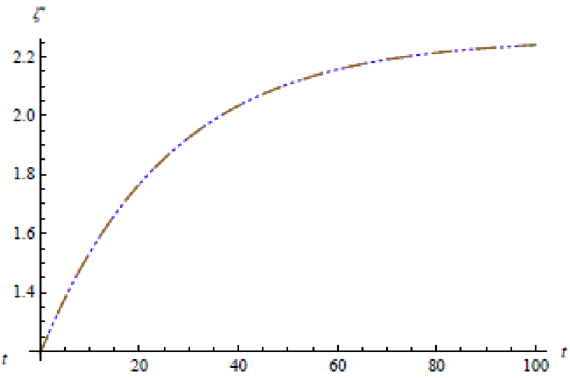
ρ = 0.1 ⟶ Orange Dashed Line



Combination of heterogeneity in the intercept of marginal benefits from emissions (A_1) and in social cost of geoengineering (c_ζ)







A.4: Heterogeneity in the private cost of SRM (θ)

Deviations are: $\pm 20\%, \pm 40\%, \pm 70\%, \pm 90\%$ *or* $+20\%, +40\%, +70\%, +90\%$ *or* $-20\%, -40\%, -70\%, -90\%$ from the symmetric steady-state level. Due to the relatively small private cost of SRM, there is no sensitivity to this parameter.

Changes (%) in steady-state values due to asymmetries in θ

$\Delta(\theta)\,\%$		$\Delta E_1 = \Delta E_2$	$\Delta\zeta_1$	$\Delta\zeta_2$	ΔT	ΔG
$i=1$	$i=2$					
+20	−20	0	−0.0531896	0.0531896	0	0
+90	−90	0	−0.238467	0.24024	0	0
+40	−40	0	−0.106379	0.106379	0	0
+70	−70	0	−0.186164	0.186164	0	0
+20	−90	0	−0.0531896	0.24024	0	0
+20	−70	0	−0.0531896	0.186164	0	0
+40	−70	0	−0.106379	0.186164	0	0
+40	−90	0	−0.106379	0.24024	0	0
+20	−40	0	−0.0531896	0.106379	0	0
−20	+90	0	0.0531896	−0.238467	0	0
−20	+70	0	0.0531896	−0.186164	0	0
−40	+70	0	0.106379	−0.186164	0	0
−40	+90	0	0.106379	−0.238467	0	0
−20	+40	0	0.0531896	0.0531896	0	0