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SOLOW MEETS LOVELOCK "ECONOMIC GROWTH IN DAISYWORLD"

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Solow meets Lovelock

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1 Introduction

This paper presents the first, to our knowledge, dynamic equilibrium model of a coupled biota-climate system integrated with an economic growth model at the global scale. It introduces this coupled system to aspects of resilience theory, which may be new to both economists working on climate-economy models as well as natural scientists working on models of climate and ecosystems. In particular, we show how the resilience of the coupled system is affected by perturbations within both the human and natural system. Before proceeding further, we believe that this paper is best motivated, by providing a short historical background of model foundations and discussing the role played by the biota in climate regulation at the outset.

The idea of Earth as an intertwined system where the biosphere, and its constituents, acts as a regulatory environment, having a stabilizing effect on the climate, was originally proposed as the Gaia Hypothesis by James Lovelock (Lovelock, 1972) and later formalized in detail in a series of papers by Lovelock and Margulis (1974a,b); Margulis and Lovelock (1974). The Gaia hypothesis embraced the notion that Earth's living and nonliving components constitute a set of interactive feedback processes, reflecting a whole system, having properties and displaying phenomena not likely to be revealed by the study of each subsystem by itself. Lovelock and Margulis suggested that through the interactions between these processes the biota made the physical environment more fit for life, a clear departure from earlier scientific ideas (Schneider, 2001). A starting point for the hypothesis was the recognition that the atmospheric composition of earth was profoundly anomalous to the expected atmosphere of a typical planet interpolated between Mars

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and Venus. In particular, the simultaneous presence of carbon dioxide and methane at the present levels violated the equilibrium laws of chemistry, which differed vastly from the near equilibrium conditions of Mars or Venus (Lovelock and Margulis, 1974a). Geological records further showed that this perturbed state had also been stable over a very long period of time. Likewise, Earth's average surface temperature had also remained fairly stable although the amount of incoming solar radiation had increased vastly since the origins of life.[1] Altogether, this lead Lovelock and Margulis to propose the Gaia hypothesis of "atmospheric homeostasis by and for the biosphere", adding that both the oxidation state and the acidity of the Earth's surface are anomalous, compared with our planetary neighbors, and can therefore be tolerated by life (Lenton, 1998).[2]

The Gaia hypothesis has raised much debate in the scientific community and has given rise to two American Geophysical Union Chapman Conferences addressing the topic (Kerr, 1988; Kump, 2009). The critic of the Gaia hypothesis has been for example, that planetary self-regulation would require foresight or planning on part of unconscious organisms (Doolittle, 1981; Dawkins, 1983) or that the hypothesis would be difficult or perhaps impossible to test (Kirchner, 1989). In order to address some this critic, while still avoiding the immense complexity of real earth system dynamics where the role of the biosphere was poorly understood, Lovelock constructed a simple parable model of Earth he called the Daisyworld (Lovelock, 1983; Watson and Lovelock, 1983). The purpose of Daisyworld was to demonstrate how planetary self-regulation could emerge automatically from physically realistic feedback mechanisms between life and its environment in order to show how environmental regulation could arise within a simple Earth like model (Wood *et al.*, 2008).

Since its birth the Daisyworld model has given rise to a slew of papers which have tested its assumptions and explored its self regulating properties using e.g. alternative population models (Maddock, 1991), enhanced biodiversity (Harding and Lovelock, 1996), adaptation and competition for resources (Stöcker, 1995; Robertson and Robinson, 1998; Lenton and Lovelock, 2000; Staley, 2002), different seeding strategies (Seto and Akagi, 2005), spatial extensions (Von Bloh *et al.*, 1997; Adams *et al.*, 2003; Ackland *et al.*, 2003), hydrological modeling (Baldocchi *et al.*, 2004; Salazar and Poveda, 2009), maximum entropy production (Pujol, 2002; Ackland, 2004; Dyke, 2008), food-web dynamics (Harding, 1999; Bagdassarian *et al.*, 2007), chaotic dynamics (Zeng *et al.*, 1990a,b; Jascourt and Raymond, 1992) and much more. A recent review of the literature on Daisyworld can be

[1] Some geochemists have asserted that there is no need to invoke life to explain the maintenance of habitable conditions on Earth (Lenton, 1998). For example, they argue that abiotic, purely geochemical and geophysical feedbacks are enough to maintain a favourable climate, offering silicate-weathering negative feedback (Walker *et al.*, 1981).

[2] Stephan H. Schneider has collected a series of a articles on Gaia containing critical discussion among scientists from various disciplines in (Schneider and Boston, 1992; Schneider *et al.*, 2004). Homeostasis refers to the property of a system to regulate its internal environment maintaining a constant state when subject to external forcing.

found in Wood *et al.* (2008). Although, there has also been contradictory papers showing how life could also possibly act to destroy regulation (Keeling, 1992), the basic homeostatic property has still proven to be remarkably resistant to many kinds of alternative engineering attempts which shows that environmental regulation can arise under minimal assumptions (McDonald-Gibson *et al.*, 2008). Perhaps due to its simplicity, the model has also become widely used in teaching see e.g. Hartmann (1994); Ford (1999); Kump *et al.* (1999); McGuffie and Henderson-Sellers (2005); Pierrehumbert (2008).

In this paper we will extend the Daisyworld model in a completely novel direction by populating the planet with humans.[3] Surprisingly, despite all evidence of anthropogenic activities influencing the Earth system, this has previously not yet been explored in the Daisyworld literature. We will set the stage in the late anthropocene where human development on Daisyworld has just passed through an industrial revolution much like the one that took place on Earth at the beginning of the 19$^{\text{th}}$ century.[4] The influence of human development on the biosphere of the Daisyworld can thus be characterized using the theory of economic growth. We will use one of the most predominant and well known models of economic growth namely the Solow growth model (Solow, 1956). The Solow model has become the workhorse model of macroeconomics and it is probably safe to say that this model is currently taught to all undergraduate students of economics across the globe.[5] The model is remarkably simplistic, it cuts through many real world complications concerning for example individual tastes, abilities, incomes as well as sectoral varieties and multi-level social interactions. It consists of a single economic sector in a simple one-good economy, where little reference is made to individual decision making. Despite its simplicity the Solow model has still succeeded well in explaining several stylized facts of growth that has been observed over time in modern industrialized economies.[6]

We will connect the biosphere of Daisyworld to the Solow model following the tradition of many integrated assessment models (IAMs) of climate change see e.g. Nordhaus (1992).[7] Carbon dioxide emissions arising from production in the Solow economy are thus assumed to accumulate into the atmosphere, hence altering the energy balance of the planet. This results in changes in average planetary temperature which carries a feedback into the economy in terms of damages to aggregate production.[8] Using this

[3]Several possible model formulations of Daisyworld exists in the literature. We have chosen to extend the version analyzed by Weber (2001) as this paper provides a complete analytical solution of the model and hence serves us well as a baseline model.

[4]The Solow growth model we will be using here was designed to fit the behavior of economies that had passed through the industrial revolution (Lucas, 2002).

[5]See for example Robert J. Barro and Xavier Sala-i-Martin (2003) for a graduate level textbook or Jones (1998) undergraduate level.

[6]See (Jones, 1998) Chapter 2.

[7]See (Stanton *et al.*, 2009) for a recent review of popular IAMs in the literature.

[8]Although, we believe this to be the most straightforward method of integration many alternative connections could have been explored. As an example one could consider harvesting strategies of daisies analogous to harvesting in fishery models see e.g. Clark (1976) or let humans compete for space on the planet having a different albedo. This goes beyond the scope of this article and is left for future research.

setup, we proceed by following the typical equilibrium analysis that has become common practice in the Daisyworld literature. The homeostatic properties of the model are thus explored by considering perturbations in the amount of incoming solar radiation (solar lumonosity).[9] Understanding the response of the climate system to changes in solar lumonosity has recieved much attention in the climate literature. In particular, Lovelock recognized that the Sun is thought to have warmed by about 25% since the origin of life on Earth over 3.8 billion years ago, an increase which should have raised temperature by a much larger amount than the current prevailing conditions (Lenton, 2002), something which is not captured in simple energy balance models. Likewise, we explore the affect of policy parameters such as saving and in particular the emission control rate. An immediate finding is that the resilience of the system to perturbations is reduced when humans inhabit Daisyworld.[10] To be more precise, in our coupled Solow-Daisyworld model, the current value of solar luminosity lies closer to a specific point, where further increases in solar luminosity would cause a shift to a basin of attraction where both the black and white daisies become extinct resulting in an approximate doubling of global average temperature. Such shifts are commonly observed in many ecological systems characterized by multiple basins of attractions (Scheffer *et al.*, 2001). Similar to the original model the shift would also be followed by a hysteresis effect implying that after the bifurcation has occurred, large reductions in solar luminosity are needed in order to restore the model to its original state. We also explore issues concerning the balanced growth path. In particular how assumptions concerning exogenous growth affects our coupled Solow-Daisyworld model.

So what can we learn from studying economic growth in this parable world? In particular, quantitative researchers might feel frustrated that the two daisies lack effective counterparts to the real world. However, despite the fact that the model cannot be properly quantified, the skeptical economist might still recognize that this highly stylized Earth system model has survived almost three decades of scrutiny by many very prominent natural scientists, and it still remains on of the most recognized workhorse models where no other alternative model, similar in size, seems to be getting as much attention (Schneider *et al.*, 2004; Wood *et al.*, 2008). Further, human dependence on the biosphere today goes unrepresented in most IAMs of climate change. To our knowledge, only the IMAGE model (MNP, 2006) has included elements of the biosphere.[11] However, the IMAGE model is very large and complex making it very difficult for any single person to gain a thorough understanding of its dynamics. As was explicitly recognized in the 2001 report from the Intergovernmental Panel on Climate Change (Houghton *et al.*, 2001), simple and complex models play complementary roles in the science of climate change.

[9]Here a homeostasic equilibrium refers to an equilibrium temperature which is hardly sensitive to variations in the solar luminosity over a large range of values.

[10]Here the term resilience follows the definition of Holling (1973).

[11]By biosphere we mean the global sum of all ecosystems.

Our personal opinion is that the main contribution of the Solow-Daisyworld model developed in this paper lies both in highlighting the three-way dependence between human development, the biosphere and the climate. As pointed out in a recent in the editorial of The Economist, 26 May, 2011, reporting from the 3rd Nobel laureate symposium on global change: - *Humans have changed the way the world works. Now they have to change the way they think about it, too.*
As many high level scholars of various disciplines are recognizing that global scale problems requires an understanding of global scale interactions the analogies put forth in the Solow-Daisyworld model analyzed in this paper could help serve as an interdisciplinary teaching tool accessible, and to some parts familiar, to students within various fields such as ecology, climatology and economics. Models remain important tools in understanding complex dynamics and the Solow-Daisyworld model could thus help bridge the understanding between scholars of various disciplines. However, to gain further insights into real world dynamics, might still require a model having descriptive features corresponding to the real world (Petersen, 2004). The paper is structured as follows. Section 2 describes the Daisyworld model used in this paper and its connection to the Solow model. Section 3 analysis how Daisyworld attributes are affected by the dynamics of the Solow model. Section 4 discusses economic features. Section 5 concludes.

2 The Daisyworld

The original Daisyworld model featured an imaginary planet, similar to Earth, but with no clouds, a negligible atmosphere and a simplified biota consisting of two species of life, black and white daisies. The black and white daisies compete for open space on the planet with expansion rates determined by a single environmental variable, temperature. The two daisies differ in their respective albedo's. The black daisies have a low albedo and hence absorb more of the incoming solar radiation while the white daisies reflect most of the incoming energy back out to space. In this way the daisies modify the radiative heat budget of the planet. The feedback mechanisms of the global temperature are otherwise governed by the same geophysical laws which are usually applied in energy balance models (North *et al.*, 1981). Together this produces a highly nonlinear system which demonstrates how a planets biota could help stabilize its environment when subjected to chocks or other large variations in external variables. This feature is generally referred to as homeostasis.

In this paper we employ an alternative Daisyworld model taken directly from Weber (2001), which provides a thorough analytical derivation of the model. We follow a Daisyworld system consisting of white a_w and black a_b daisies, regulating local daisy temperatures $T_{w,b}$ as well as global temperature level T. The Daisyworld system is given by:

$$\frac{da_w}{dt} = a_w \left[(1 - a_b - a_w) \beta_w (T_w) - \gamma \right] \quad (1a)$$

$$\frac{da_b}{dt} = a_b \left[(1 - a_b - a_w) \beta_b (T_b) - \gamma \right] \quad (1b)$$

$$\beta_j (T_j) = 1 - 4 \frac{(T_j - T_{opt})^2}{(T_{max} - T_{min})^2} \ , j = b, w \quad (1c)$$

$$T_j = q(\mathcal{A} - A_j) + T, j = b, w \quad (1d)$$

$$\mathcal{A} = a_w A_w + a_b A_b + (1 - a_b - a_w) A_g \quad (1e)$$

$$\frac{dT}{dt} = SL(1 - \mathcal{A}) - (\Psi + \lambda T) \quad (1f)$$

This is slightly modified version of the original model by Watson and Lovelock (1983), the main difference being that it uses a linear form for the long-wave radiation term. Such a linear form is often applied in box models of the global heat budget. The error introduced by this is small, because the temperature range achieved by the model is small compared to the absolute temperature (North *et al.*, 1981). The model results do not change in any significant manner, when a fourth-order term is used (Saunders, 1994). The base case parameter values are the same as those used by Weber (2001) and are displayed in Tab. 2. The baseline value for lumonosity L is unity but this parameter will later become one of our bifurcation parameters.

Parameter	Description	Value
Ψ	130	Long-wave radiation constant (W/m^2)
λ	2	Long-wave emission parameter ($W/m^{2\circ}C$)
S	340	Incoming solar radiation ($W/m^{2\circ}C$)
A_w	0.75	Albedo of white daisies
A_g	0.5	Albedo of uncovered ground
A_b	0.25	Albedo of black daisies
T_{min}	2.5	Minimum temperature ($^\circ C$)
T_{opt}	20	Optimal temperature ($^\circ C$)
T_{max}	37.5	Maximum temperature ($^\circ C$)
γ	0.3	Death rate
q	20	Redistribution parameter ($^\circ C$)

Table 1: The geophysical parameter values for the base case.

3 Introducing humans into the daisyworld

Human development is introduced in system as a an augmented Solow model having exogenous technological and population growth. The economy is described as follows

$$\dot{K} = sY_n - \delta K, \tag{2}$$

$$Y_n = \Omega(T)(1 - \Lambda(\mu))F(K, AN), \tag{3}$$

$$\dot{N} = nN,\ \dot{A} = gA \tag{4}$$

where, Y_n denotes output of goods and services, net of abatement and damages. As is standard in the Solow model, capital K accumulates via a fixed saving s net depreciation δ. $\Omega(T)$ represents the damages to the economy (climate damages as fraction of output) as a function of temperature. We will assume a quadratic damage function $\Omega(T) = (1+\theta T^2)^{-1}$ as is common in many climate-economy models (See e.g. Nordhaus (2007)).[12] $\Lambda(\mu)$ represents the abatement cost function (abatement costs as fraction of output), we adopt a similar function as in Nordhaus (1994) and set $\Lambda(\mu) = \phi\mu^2$, where $\mu \in [0, 1]$ represents the emission control rate. $F(K, AN)$ is a constant returns to scale production function, where technology (A) and population (N) grow at exogenous rate g and n.[13] Production in the Solow economy also generates carbon dioxide emissions in the following way:

$$E = \sigma(1 - \mu)F(K, AN) \tag{5}$$

Here, σ denotes the emission to output ratio which is assumed to improve at an exogenous rate φ so that $\sigma = \sigma_0 e^{-\varphi t}$. Emissions then affect global temperature thru an increase global carbon dioxide concentration M which is modelled in as simplified box-diffusion model (See fore example Nordhaus (1994)) which can be written as:

$$\dot{M} = \beta E - \delta_M(M - M_0) \tag{6}$$

where M is the atmospheric CO_2 mass, M_0 is the pre- industrial atmospheric CO_2 mass, β is the fraction of emissions that enters the atmosphere and δ_M represents the natural removal rate of atmospheric CO_2 over time.[14]

[12]This function is concave-convex for all $T > 0, \theta > 0$. However, for a certain temperature range, for example $T \in [0, 10]$, it remains concave for a low enough value for θ. Nordhaus (2007); Nordhaus and Boyer (2000) exploits this property as he is only concerned with a certain predicted range of temperatures, where $\Omega(T)$ remains concave (hence marginal damages are increasing with temperature over the temperature range of interest) for his calibrated value of θ. This function also works reasonably well as damage function in the Daisy-Solow world since decreasing temperature is also of interest and assumed to be bad. Hence, a temperature departing from the current level is assumed to affect the economy negatively.

[13]Throughout the rest of this paper we will for explicit calculations assume that the production function takes on the standard cobb-douglas form: $F(K, AN) = K^{\alpha}(AN)^{1-\alpha}$.

[14]For the parameter values of equation (5)-(6) we will rely on previous work by in Nordhaus (1994). He provides the following estimates $\sigma_0 = 0.52$, $\beta = 0.64$ and $\delta_M = 0.0083$, where σ and β are his

The accumulation of carbon dioxide in the atmosphere affects the global mean temperature by increasing the amount of radiative forcing. The temperature equation (1f) is thus modified to include the impact of radiative forcing via carbon dioxide emissions in the following way:

$$\dot{T} = SL(1-\mathcal{A}) - (\Psi + \lambda T) + \xi \ln \frac{M}{M_0} \tag{7}$$

$\xi \ln \frac{M}{M_0}$ captures the approximate relationship between an increase in the amount of atmospheric carbondioxide and radiative forcing. IPCC (2001) provides an estimate of $\xi = 5.35$.

Then after defining capital per effective worker $k = \frac{K}{AL}$ and transforming the model into intensive form the complete model can thus be written as:

$$\frac{da_w}{dt} = a_w \left[a_g \beta_w \left(T_w\right) - \gamma\right] \tag{8a}$$

$$\frac{da_b}{dt} = a_b \left[a_g \beta_b \left(T_b\right) - \gamma\right] \tag{8b}$$

$$\beta(T_j) = 1 - 4\frac{(T_j - T_{opt})^2}{(T_{max} - T_{min})^2} \ , \ j = b, w \tag{8c}$$

$$T_w = q(\mathcal{A} - A_w) + T \tag{8d}$$

$$T_b = q(\mathcal{A} - A_b) + T \tag{8e}$$

$$\mathcal{A} = a_w A_w + a_b A_b + a_g A_g \tag{8f}$$

$$\text{with } a_g := (1 - a_b - a_w) \tag{8g}$$

$$\frac{dM}{dt} = \beta E - \delta_M (M - M_0) \tag{8h}$$

$$\frac{dT}{dt} = SL(1-\mathcal{A}) - (\Psi + \lambda T) + \xi \ln \frac{M}{M_0} \tag{8i}$$

$$\frac{dk}{dt} = s\Omega(T)\left(1 - \psi\mu^2\right) f(k) - (\delta + n + g)k \tag{8j}$$

$$E = \sigma(1-\mu) f(k) AL \tag{8k}$$

$$\Omega(T) = (1 + \theta T^2)^{-1} \tag{8l}$$

where $f(k)$ denotes production per effective worker.[15] In the following we will refer to this system as the Daisy Solow system (DS) as opposed to the Daisy original (DO) given by system 1. The Solow model introduced above was done in a way consistent with

own empirical estimates and δ_M corresponds to a 120 year initial e-folding time of CO_2 estimated by the Intergovernmental Panel on Climate Change Watson *et al.* (1990). It should be noted that this parametrization has been heaviliy critized (see e.g. Schultz and Kasting (1997)), however for the current modelling excerise they should serve us well. Finally, the inclusion M_0 in equation (6) implies that if emissions remain zero for all time periods the equilibrium level of carbon dioxide should eventually stabalize around pre-industrial levels.

[15] This transformation exploits the constant returns to scale property of the aggregate production function.

the standard macroeconomic notion of a balanced growth path, meaning that aggregate capital, consumption and production all grow at a constant rate $(g + n) > 0$ while the capital output ratio K/Y remains constant. However, in doing so we have also rendered the entire system non-autonomous.[16] In the original Solow model, a necessary prerequisite for the model to converge to a balanced growth path is that the production function takes on a labor augmenting form i.e. $F(K, AL)$ (Acemoglu, 2007).[17] As formalized in Proposition 1 this is not enough to ensure the existence of a balanced growth path in the economy supported by Daisyworld.

Proposition 1 (unbounded exogenous dynamics) *Inclusion of unbounded exogenous dynamics in the production function implies that the system* (8) *does not have a steady-state equilibrium solution and therefore the economy does not approach a balanced growth path unless emissions are abated at a rate larger or equal to population plus technological growth rates* $(g + n)$. □

The proof can be found in appendix B.
Brock and Taylor (2010) develop a model they refer to as the Green Solow model. In this model they define the sustainable growth rate as the amount of abatement that exactly offsets emissions, while still sustaining a rising per capita income. As their model does not involve feedback effects from emissions onto production both sustainable and unsustainable balanced growth paths arise. This is not the case in the DS system. As becomes clear from Proposition 1 unsustainable growth is inconsistent with the notion of a balanced growth path since global temperature becomes unbounded. In order to avoid such scenarios we assume that growth is sustainable in the sense of Brock and Taylor and that the emissions to output ratio improves at a rate large enough to offset emissions due to technology and population growth i.e. we have $\varphi \geq g + n$.

In the following analysis it is thus important to recognize that the effect of introducing humans into Daisyworld is done under the assumption that economic growth is already sustainable implying that (i) new technology is emission neutral and (ii) that aggregate emissions are independent of population size. Most notably, besides the brave assumption of emission neutral technology, it is also clear that population size is strongly correlated with the aggregate amount of carbon dioxide emissions (Onozaki, 2009). In this sense our numerical analysis that follows could be considered as a lower bound in robustness framework exploring the remaining problem after population growth and dirty technologies has already been taken care of.

[16] By non-autonomous we mean that the dynamical system depends explicitly on time.

[17] An assumption which also makes the Solow model autonomous in the transformed variable k.

4 Equilibrium analysis

As in Weber (2001) we will do an equilibrium and local stability analysis for the system (8). The analysis is primarily done using the a the Matcont software for bifurcation analysis in Matlab.[18] In appendix we have also derived some analytical results in terms of comparitive static results. Weber (2001) derives an analytical solution to the system (1) and shows that it possesses four equilibria for the present state of lumonosity ($L = 1$), where one is stable and three are unstable. This result remains unchanged in our augmeted system (8). We proceed by comparing how the results found in our augmented system compares to those of Weber (2001) and Solow (1956). Figure 1 plots the global mean temperature as a function of solar lumonosity for the original model of Weber (2001) and the Solow augmented model.[19] This plot is typical of the Daisyworld literature. It shows how the feedback mechanisms inherent in daisy system counteracts the effect of changes in solar lumonosity on temperature compared to a dead planet. This property gives support to the Gaia argument and shows how an active biosphere could have regulated global temperature keeping it stable despite the 25% increase in solar lumonosity since the origins of life (Lenton, 2002). As previously mentioned this property is referred to as homeostasis which is a robust feature of all Daisyworld models of this type (Weber, 2001). In both graphs plotted in figure 1 the thick dashed (blue) line which is fairly inelastic to changes in the solar luminosity, represents stable equilibria of the system for different values of solar lumonosity L. Along this line both daisies are alive (i.e. $\{a_w,\ a_b\} > 0$) and hence covering a land area larger than zero. This shows that when both daisies are alive they actively help regulate the global temperature. The vertical upward sloping line, from the left lower hand corner to the right upper hand corner, shows the temperature affect without the regulating capacity of the daisies i.e. along this line both daisies are dead so that $\{a_w,\ a_b\} = 0$. Now, imagine that the system is initially in the state where luminosity is unity ($L = 1$). The plot shows how the system responds to changes in solar lumonosity. At, $L = 1$ the system has only one entirely stable equilibrium where both daisies cover positive amounts of land. This equilibrium lies on the thick blue dashed line in the left and right figure. The small differences in temperatures between the two systems shows for this value of solar luminosity the regulatory capacity of the daisies. However, what can not be seen from figure 1 is that the balance between the black and white daisies has been disturbed in the DS system compared to DO system. In DO the daisies cover approximately 33% of available land each whereas in DS white daisies cover approximately 50% while the black cover only 16%. As we now move to the right (left), increasing (decreasing) lumonosity, along the dashed line the system bifurcates at the branch point (BP) where the black (white) daisies become

[18] See (Dhooge *et al.*, 2003). The starting equilibria was calculated using the OCMAT toolbox (See Grass *et al.* (2008) ch. 7).

[19] Note that the left plot of figure 1 is identical to figure 1 of Weber (2001).

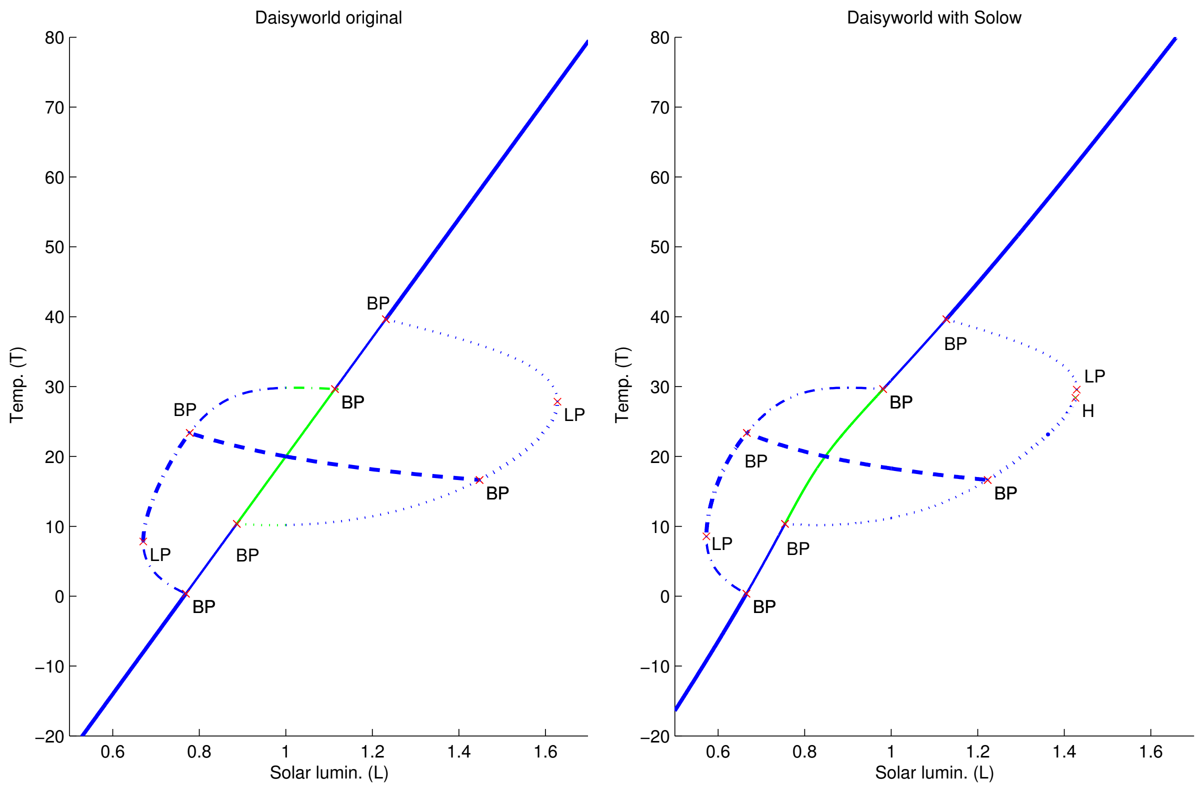

Figure 1: Equlibrium temperature as a function of solar lumonosity. No daisies (solid line), only black daisies (dash-dot line), white daisies only (dotted line). Thick lines are complete stability. Blue thin lines have 4 negative eigenvalues. Green lines have 3 negative eigenvalues. BP denotes branching points, indicating a change in the sign of an eigenvalue. LP are the limit points or saddle-node bifurcations of the system. H denotes a Hopf bifurcation.

extinct. At this stage the equilibrium has coincided with the unstable equilibrium on the lower (upper) branch. For higher (lower) values of lumonosity the temperature the elasticity has now increased. Further decreases (increases) in solar luminosity thus causes larger variations in temperature up to a limit point (LP) or saddle-node bifurcation where the equilibrium disappears and system moves to the only stable equilibrium where both daisies are extinct.[20]

After this shift in the point of attraction it can be seen from the figure that a crossing of this threshold implies that the we get a hysteresis effect. By this we mean that after crossing the threshold given by the limit points (LP's) of the figure, decreasing (increasing) the luminosity does not imply that the system moves back to its previous branch. Now luminosity must be decreased (increased) until it reaches the branching point (BP) where the equilibrium loses stability and shifts back to the positive daisy

[20]From here on the term limit point and saddle-node bifurcation will be used interchangeably. A definition can be found in Kuznetsov (1998).

population state.

4.1 Capital

We have now looked at the global temperature variation. So what happens to capital? Figure 2 plots the corresponding bifurcation diagram for the DS system but with capital

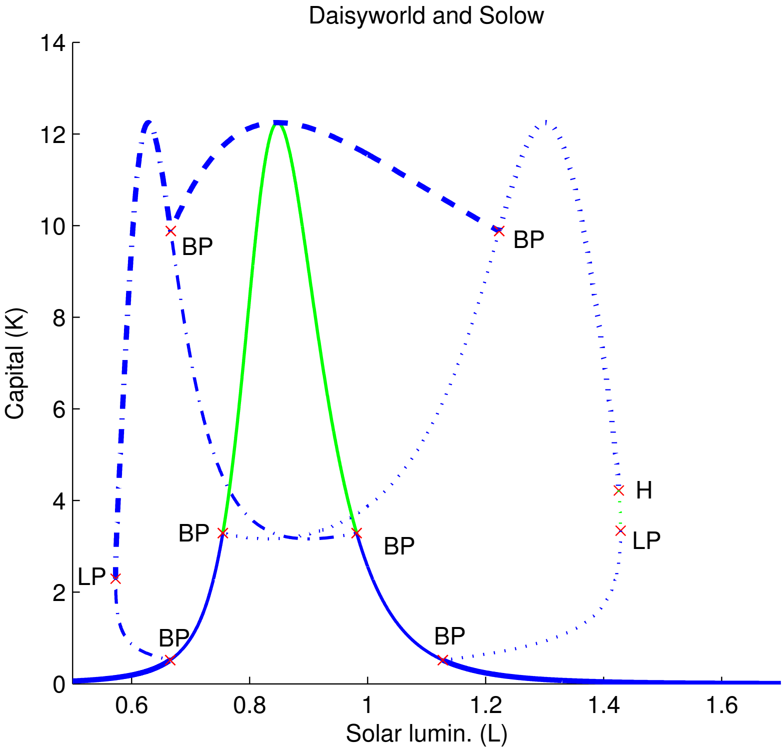

Figure 2: Equilibrium capital as a function of solar lumonosity. No daisies (solid line), only black daisies (dash-dot line), white daisies only (dotted line). Thick lines are complete stability. Blue thin lines have 4 negative eigenvalues. Green lines have 3 negative eigenvalues. BP denotes branching points, indicating a change in the sign of an eigenvalue. LP are the limit points of the system. H denotes a Hopf bifurcation.

as a function of solar lumonosity. From this graph we can really see the positive effect of daisies on the capital stock. The difference is large between a system without daisies which follows the green line and the system with daisies along the dashed thick lines. At the point $L = 1$ the equilibrium with a positive coverage of daisies (lying on the dashed line) has almost three times the capital stock compared to the point on the green line where the two daisies are extinct. This shows how the biosphere supports a higher capital stock and hence production than a world without a biosphere. Further, the figure also illustrates how the homeostatic properties of the daisies make the equilibrium capital stock is fairly robust to changes in solar luminosity compared to the case with zero daisies. Hence an economy supported by the regulatory power of the biota is not as sensitive to external forcings.

Combing the Daisyworld system with the Solow model thus illustrates how the economy might depend on the ability of nature to regulate the earth system in a way which makes it robust to external forcings.

5 Resilience and the distance to the closest saddle-node bifurcation point

Resilience refers to the capacity of a social-ecological system to undergo disturbances without collapsing into a qualitatively different state that is controlled by a completely different set of processes.[21] The concept of resilience is widely used in a great variety of interdisciplinary work concerned with the interactions between people and nature. An underlying assumption of resilience analysis is that social-ecological systems generally contain thresholds and can exhibit hysteretic and irreversible behavior making it important to identify and understand their underlying processes (Walker *et al.*, 2002). However as pointed out in Carpenter *et al.* (2001) the concept of resilience, typically has multiple levels of meaning, ranging from the metaphorical to the very specific.

In Daisyworld, resilience can be thought of as the ability of the present state of the model to maintain its qualitative features when subject to perturbations in the parameter space such as the effect of changes in solar lumonosity. In figure 1 we saw how the resilience of global temperature in Daisworld is affected after being coupled to the Solow model. Here, resilience is measured as the distance from the point of origin, i.e. the present solar luminosity ($L = 1$), to the closest of the two limit points. If we denote L_0 as the solar luminosity at our point of origin and L_* as the closest limit point, here $L_* \approx 1.42$, the distance between these two points can be thought of as a measure of the resilience of temperature w.r.t the amount of incoming solar radiation. This distance to bifurcation point approach to evaluating resilience has previously been applied as a measure of resilience in purely ecological systems (See Ludwig *et al.* (1997)). This way of approaching resilience gives us a measure of how robust any particular stable equilibrium of a system is to changes in given parameters of that system.[22]

However, in a complex system such as DS there is usually not only one, but several parameters that might become subject to perturbations due to outside disturbances. Hence, an analysis looking at only one of these parameters at a time will miss important scale effects that might be inherent within such a system. This line of thought has and is receiving much attention within the literature on electrical engineering, addressing phenomenon known as voltage instability which typically occurs in heavily loaded

[21]See www.resalliance.org/

[22]It is important to notice that although parameters might be assumed to be held constant within the borders of a simple model they might still be subject to external changes and could thus alternatively be seen as slow moving variables (Carpenter *et al.*, 2001).

electrical power systems (Dobson and Chiang, 1989; Dobson, 2003). Voltage instability can become a major source of uncontrollable drops in voltage power leading to electrical blackouts. When modeling electrical systems using the theory of differential equations such a collapse can then be described as the loss of stability that occurs when a stable equilibrium disappears into a saddle-node bifurcation. Assessing the robustness of such a system can thus be seen as a problem of finding the saddle-node bifurcation closest to the point at which the system is currently being operated. Depending on the number of parameters that might become subject to perturbation this can be viewed as a geometric problem of calculating distances in a multidimensional parameter space. This becomes increasingly relevant if one considers that dynamical systems often are calibrated based on statistical inference implying where each parameter estimate is an approximation of its true value. For instance, when estimating the equations of a dynamical system by ordinary least squares the law of large numbers and the central limit theorem tells us that, given certain conditions, we will obtain consistent estimates having asymptotically normal distributions. If the estimated parameters are then used as best guess estimates of some parameters of a dynamical system their corresponding variances provide us with a confidence region for the model. Moreover, if these normally distributed parameters have independent uncertainties, then a closest bifurcation can be interpreted as a most likely bifurcation (Dobson, 2003). In order to make this point clear we setup a simple example which illustrates this approach. Consider a simple system consisting of a single differential equation:

$$\frac{dx}{dt} = \hat{b}_1 - x - \hat{b}_2 x^2, \quad x \in \mathbb{R}, \ \{\hat{b}_1, \hat{b}_2\} \in \mathbb{R} \tag{9}$$

Here, $\hat{b}_1$ and $\hat{b}_2$ are assumed to be approximate estimates of their true values (b_1, b_2) drawn from a normal probability distribution with equal variances. Given these estimates the system has two equilibrium solution branches given by:

$$\hat{x}_{1,2} = -\frac{1}{2\hat{b}_2} \pm \sqrt{\left(\frac{1}{2\hat{b}_2}\right)^2 + \frac{\hat{b}_1}{\hat{b}_2}} \tag{10}$$

where a real valued solution exists given that $\left(\frac{1}{2\hat{b}_2}\right)^2 + \frac{\hat{b}_1}{\hat{b}_2} > 0$.[23] The corresponding eigenvalues of the two equilibria are $\hat{\lambda}_{1,2} = \pm\sqrt{\left(\frac{1}{2\hat{b}_2}\right)^2 + \frac{\hat{b}_1}{\hat{b}_2}}$. A saddle-node bifurcation thus occurs when the two equilibria collide and disappear. When this happens the bifurcation point is characterized by a single equilibrium having a zero eigenvalue. From (10) we see that $\left(\frac{1}{2\hat{b}_2}\right)^2 + \frac{\hat{b}_1}{\hat{b}_2}$ must equal zero at this point which further implies that $sign(\hat{b}_1) \neq sign(\hat{b}_2)$ becomes a necessary condition for a saddle-node bifurcation to oc-

[23] For the case of $\hat{b}_2 = 0$ the solution is simply $\hat{x} = \hat{b}_1$.

cur. An example of this is depicted in figure 3 in the $\hat{b}_1 - \hat{b}_2$ parameter space. The figure

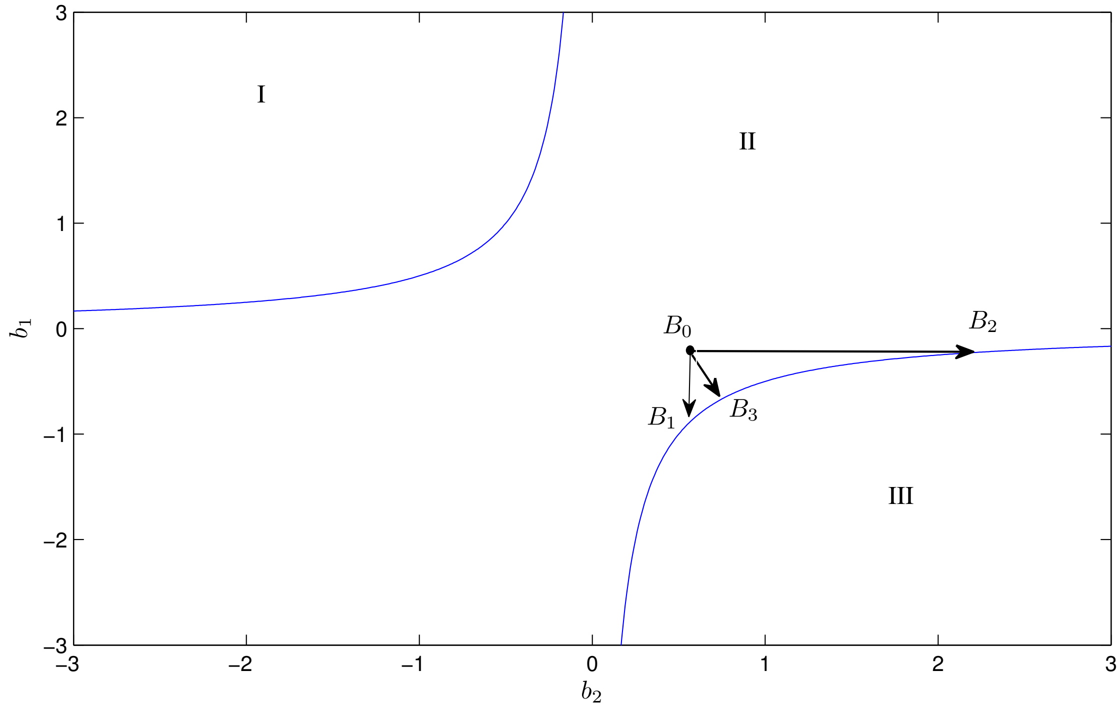

Figure 3: Parameter space of equation (9). I and III are regions where no real valued solution to (10) exist. The lines bordering to these regions are the saddle-node bifurcation curves. The arrows gives distances from the point B_0 to different bifurcation points along the curve. The resilience of the system w.r.t. perturbations in B_1 or B_2 could thus be thought of as the distance from an equilibrium point lying in region II to the closest saddle-node bifurcation.

separates three different regions within the parameter space. In area II there exists two equilibrium solutions while area I and III do not possess any real valued solutions. The lines separating these regions are bifurcation curves depicting all points where saddle-node bifurcations can occur. B_0 is assumed to the vector containing the approximate values $\hat{b}_1$ and $\hat{b}_2$ while B_1, B_2 and B_3 denote points on the bifurcation curve. given above the shortest distance approach gives the best estimate of the most likely bifurcation. However, whether this applies or not will of course depend upon what is assumed regarding these uncertainties.

How can resilience be measured given the above arguments? Typically, a system as the one above constitutes a vast simplification trying to capture general features underlying complex processes, existing in the real world. Therefore an assessment of the systems resilience should challenge the assumption that $\hat{b}_1$ and $\hat{b}_2$ are both constants. Here we have assumed that these approximations are normally distributed estimates with equal variances. In this case measuring resilience of the system becomes a problem of calculating the geometrical distance function to the closest bifurcation point present in the system.[24] This is shown in figure 9. Consider the approximated best guess estimated point B_0 in figure 9. As can be seen from the figure, when only observing changes in the parameter $\hat{b}_2$, the resilience of the system seems in fairly good shape since the distance from our best-guess approximation B_0 is relatively far from the saddle-node bifurcation occurring at B_3. However, when considering also bifurcations in the parameter $\hat{b}_1$ one notices that the resilience of the system is not as in good shape since the distance B_0-B_2 is relatively shorter. Further, due to the concave shape of the bifurcation curve surrounding region III, we notice that the closest distance to a bifurcation point lies in between these horizontal and vertical vectors and is given by the vector going from B_0 to B_3. If we denote $\Sigma \in \mathbb{R}^2$ as the set of parameter values lying on one of the saddle-node bifurcation curves, $\tilde{B} \in \Sigma$ as a point on this curve, this implies that the resilience measure of the system or alternatively the closest bifurcation point w.r.t changes in B_0 thus becomes the solution to the problem of minimizing the distance $D(\tilde{B}) = \|B_0 - \tilde{B}\|$ s.t. $\tilde{B} \in \Sigma$, where $\| \cdot \|$ denotes the vector length/norm. However in a more complex situation the probability distributions of the above parameters need not be normal and the most likely bifurcation will thus be an increasingly complex multidimensional stochastic problem.

Coming back to the Solow-Daisyworld model figure 4 applies these principles of resilience w.r.t. changes in not only in L but also in ξ. It has been deeply acknowledged within the field of atmospheric science that the sensitivity of the global temperature to a doubling of the amount of carbon dioxide in the atmosphere is highly uncertain (Roe and Baker, 2007).[25] Hence, based on the discussion above this calls for a bifurcation analysis also in the climate sensitivity parameter ξ in order to get an acceptable measurement of resilience in the system. Figure 4 depicts the parameter space L-ξ in the Solow-Daisy model. Here, v_1 and v_2 are vectors of equal length starting at the best guess estimates of these parameters (the initial point). The (blue) dashed circle denotes all points having equal length to these vectors. The (red) dash-dot line denotes the saddle-node bifurcation surface of the system. Although, its hard to see from the figure the bifurcation surface is slightly concave implying that the points located in between the vectors v_1 and v_2 lie closer to the initial point. This implies that a measure of resilience of the system w.r.t only solar luminosity would lead us to assume that the system is more resilient than if resilience was considered w.r.t both solar lumonosity and climate sensitivity.

Although the difference here is almost neglegible in comparison to the earlier example this still serves to illustrate a point that if bifurcations in the multi dimensional parameter space are ignored this might give an unsatisfactory measure of resilience implying that

[24]This can also give rise to so called codim 2 bifurcations (Kuznetsov, 1998).

[25]In Roe and Baker (2007) the distribution of the climate sensitivity parameter follows from a transformation of a normal feedback parameter. Using this probability distribution in the DS system is beyond the scope of the present paper and instead it is simply assumed to be normally distributed with the similar variance as the solar luminosity.

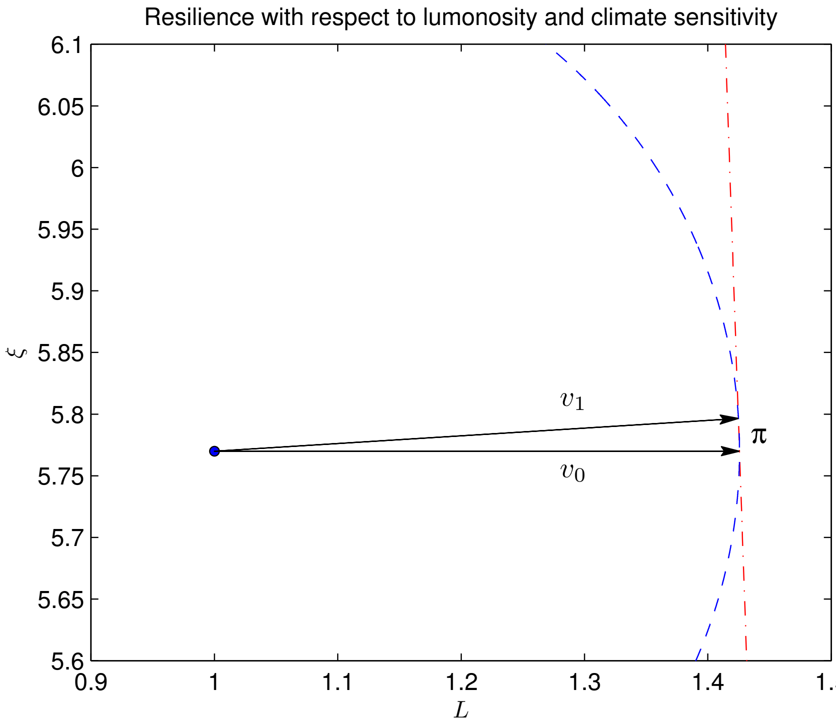

Figure 4: Resilience with respect to the solar luminosity and climate sensitivity. The (red) dash-dot line denotes the saddle-node bifurcation surface. The blue dashed line is a circle depicting the length of the vectors v_1 and v_2.

thresholds or tipping points might be closer than expected.

6 Conclusions

This paper has explored several aspects of human biosphere interdependence, self-regulation and resilience within a climate-biota model coupled to an economic growth model. In particular, the model developed in this paper stresses human dependence upon the biosphere but also shows how human development can threaten not only the biosphere per se but also human existence in itself. This is a point which was also made clear at the summit of the 3rd Nobel Laureate symposium held in Stockholm in Maj 2011. At the symposium a document stressing the dependence and influence that human species have on the biosphere was signed by several of the former laureates and influential thinkers attending the meeting. Among other things the following statement was made:

> *Humans are now the most significant driver of global change, propelling the planet into a new geological epoch, the Anthropocene. We can no longer exclude the possibility that our collective actions will trigger tipping points, risking abrupt and irreversible consequences for human communities and ecological systems.* (Stockholm Memorandum, 2011)

Although the model developed here is fictive in nature, it still captures the interdependence between humans and the biosphere which is currently not being modeled in many models of the climate and the economy. The Daisy-Solow model is thus an attempt to explore what possible aspects that might be missing in current models that do not include these types of feedbacks. The direction for future research is clearly to move towards a model with elements that better portrays real world entities or processes than that of the two daisies.

A Appendix

The analytical solution can be characterized as follows:

A steady-state equilibrium of the system (8) consist of a fixed point $[\hat{a}_w, \hat{a}_b, \hat{T}, \hat{k}, \hat{M}] \in \mathbb{R}^5$ where $\frac{da_w}{dt} = \frac{da_b}{dt} = \frac{dT}{dt} = \frac{dM}{dt} = \frac{dk}{dt} = 0$ for all $t \in [0, \infty)$. Hence, for the two daisies it implies that either:

$$a_w = 0 \quad \text{or} \quad (1 - a_w - a_b)\beta(T_w) - \gamma = 0 \tag{11}$$

and

$$a_b = 0 \quad \text{or} \quad (1 - a_w - a_b)\beta(T_b) - \gamma = 0 \tag{12}$$

must hold.

A.1 Case 1 ($(1-a_w-a_b)\beta(T_w)-\gamma = 0$ and $(1-a_w-a_b)\beta(T_b)-\gamma = 0$)

This implies that $\beta(T_w) = \beta(T_b) \Rightarrow T_w^2 = T_b^2 + 2T_{opt}(T_w - T_b)$. From condition (8d) and (8e) we derive that $T_w - T_b = q(A_w - A_b)$ and we can thus write:

$$\begin{aligned} T_w^2 &= (T_w - q(A_w - A_b))^2 + 2T_{opt}q(A_w - A_b) \\ T_b^2 &= (T_b - q(A_b - A_w))^2 + 2T_{opt}q(A_b - A_w) \end{aligned}$$

from this we can thus derive the equilibrium local temperatures $\hat{T}_w$ and $\hat{T}_b$:

$$\hat{T}_w = T_{opt} + \frac{q}{2}(A_w - A_b) \tag{13}$$

$$\hat{T}_b = T_{opt} - \frac{q}{2}(A_w - A_b) \tag{14}$$

from these we also directly obtain $\hat{a}_g$ and $\hat{\beta}$.

$$\hat{a}_g = \frac{\gamma}{1 + k_d(\frac{q}{2}(A_w - A_b))^2}$$
$$\text{with } k_d = 4/(T_{max} - T_{min})^2$$

implying that from (8g) the albedo can be written as:

$$\mathcal{A} = a_w A_w + (1 - \hat{a}_g - a_w)A_b + \hat{a}_g A_g = (1 - \hat{a}_g)A_b + \hat{a}_g A_g + (A_w - A_b)a_w \tag{15}$$

By substituting (13) and (14) back into (8d) and (8e) and we get an explicit expression for the relationship between temperature and planetary albedo:

$$T - T_{opt} = q\left(\frac{A_b + A_w}{2} - \mathcal{A}\right) \tag{16}$$

rewrite (8i):

$$\frac{dT}{dt} = SL\left(1 - \frac{A_b + A_w}{2}\right) + SL\left(\frac{A_b + A_w}{2} - \mathcal{A}\right) - (\Psi + \lambda T) + \xi \ln \frac{M}{M_0} \tag{17}$$

substitute (16) into (17) we get:

$$\frac{dT}{dt} = SL\left(1 - \frac{A_b + A_w}{2}\right) + \frac{SL}{q}(T - T_{opt}) - (\Psi + \lambda T) + \xi \ln \frac{M}{M_0} \tag{18}$$

By setting $\frac{dT}{dt} = 0$ we can now derive the following expression for global temperature T as a function of carbon dioxide:

$$T = \left(\frac{SL}{q} - \lambda\right)^{-1}\left(\Psi - SL\left(1 - \frac{A_b + A_w}{2}\right) + \frac{SL}{q}T_{opt} - \xi \ln \frac{M}{M_0}\right) \tag{19}$$

From this we see that in the two daisy regime, an increase in carbon dioxide M decreases temperature. This thus works in the same direction as an increase in solar luminosity L. Next, we take on the cobb-douglas form form the production function, $f(k) = k^{\alpha}$, and after setting $\frac{dk}{dt} = 0$ of (8j) we can solve for k:

$$k = \left(\frac{s}{\tilde{\delta}}\Omega(T)(1 - \psi\mu^2)\right)^{\frac{1}{1-\alpha}} \tag{20}$$

where $\tilde{\delta} = \delta + n + g$ set $\frac{dM}{dt} = 0$ of (8h) and using (8k) we can write M as a function of temperature:

$$M = M_0 + \frac{\beta}{\delta_M}\sigma(1 - \mu)\left(\frac{s}{\tilde{\delta}}\Omega(T)(1 - \psi\mu^2)\right)^{\frac{\alpha}{1-\alpha}} AN \tag{21}$$

Assuming that the growth rate in the emission to output ratio $\sigma(t)$ exactly offsets increased emissions due to population and technological growth $\varphi = g + n$ we can thus normalize and set $A = N = 1$ and write the equilibrium temperature $\hat{T}$ as:

$$\hat{T} = \left(\frac{SL}{q} - \lambda\right)^{-1}\left(\Psi - SL\left(1 - \frac{A_b + A_w}{2}\right) + \frac{SL}{q}T_{opt} - \xi \ln\left(\frac{M(\hat{T})}{M_0}\right)\right) \tag{22}$$

Equation (22) is a fixed point problem that can be solved using numerical methods.[26]

[26] See for example Judd (1998).

B Proof of proposition 1

PROOF From (18) and (21) we have can write:

$$\frac{dT}{dt} = SL\left(1 - \frac{A_b + A_w}{2} - \frac{T_{opt}}{q}\right) - \Psi + \left(\frac{SL}{q} - \lambda\right)T$$
$$+ \xi \ln\left(1 + \frac{\beta}{\delta_M M_0}\sigma(1-\mu)\left(\frac{s}{\delta}\frac{(1-\psi\mu^2)}{1+\theta T^2}\right)^{\frac{\alpha}{1-\alpha}} AN\right) \tag{23}$$

A steady-state equilibrium is defined as a fixed point $[\hat{a}_w, \hat{a}_b, \hat{T}, \hat{k}, \hat{M}] \in \mathbb{R}^5$ of (8), where $\frac{da_w}{dt} = \frac{da_b}{dt} = \frac{dT}{dt} = \frac{dM}{dt} = \frac{dk}{dt} = 0$ for all $t \in [0, \infty)$. Hence this requires that there exists a fixed $\hat{T} \in \mathbb{R}$ that solves (23) for $\frac{d\hat{T}}{dt} = 0$ for all $t \in [0, \infty)$. Assume that growth is unsustainable i.e. we have $\varphi < g + n$.

Then, since $\lim_{t\to\infty} \sigma AN = \infty$ then for any finite $\hat{T}$, the limit as $t \to \infty$ of the last term in logarithm (i.e. $\xi \ln(\cdot)$) will be infinity, which violates the condition that $\frac{dT}{dt}$ must equal zero in steady-state. From this it follows that neither capital K or net output Y_n grow at constant rates since both $\frac{dK/dt}{K}$ and $\frac{dY_n/dt}{Y_n}$ depend on T. Hence the economy supported by Daisyworld does not converge to a balanced growth path unless $\varphi \geq g + n$.

C Comparitive statics

Since this equilibrium turns out to be stable we can thus do some comparitive statics and have a look at how the equilibrium temperature changes with respect to the policy parameters s and μ. First define $\hat{k} \equiv \left(\frac{s}{\delta}\Omega(\hat{T})(1-\psi\mu^2)\right)^{\frac{\alpha}{1-\alpha}}$

C.1 Comparitive statics w.r.t. the saving rate s

We start with comparitive statics on $\hat{T}(s)$ w.r.t. s which can be derived from (22):

$$\frac{dT\hat{(s)}}{ds} - \left(\frac{SL}{q} - \lambda\right)^{-1} \xi \frac{M_0}{M(\hat{T}(s))} \frac{\partial M(\hat{T}(s), s)}{\partial s} = 0 \tag{24}$$

where:

$$\frac{\partial M(\hat{T}(s), s)}{\partial s} = \frac{\beta}{\delta_M}\sigma(1-\mu)\frac{\partial \hat{k}}{\partial s}\frac{(1-\psi\mu^2)}{\delta}\left(\Omega(\hat{T}) + s\frac{\partial \Omega(\hat{T})}{\partial \hat{T}}\frac{\partial \hat{T}}{\partial s}\right) \tag{25}$$

and hence:

$$\frac{d\hat{T}(s)}{ds} = \frac{\left(\frac{SL}{q} - \lambda\right)^{-1} \xi \frac{M_0}{M(\hat{T}(s))} \frac{\beta}{\delta_M} \sigma(1-\mu) \frac{\partial \hat{k}}{\partial s} \frac{(1-\psi\mu^2)}{\delta} \Omega(\hat{T}(s))}{1 - \left(\frac{SL}{q} - \lambda\right)^{-1} \xi \frac{M_0}{M(\hat{T}(s))} \frac{\beta}{\delta_M} \sigma(1-\mu) \frac{\partial \hat{k}}{\partial s} \frac{(1-\psi\mu^2)}{\delta} s \frac{\partial \Omega(\hat{T}(s))}{\partial \hat{T}}}$$

where $\frac{\partial \hat{k}}{\partial s} > 0$. For $\hat{T} < 0$ the sign of $\frac{d\hat{T}(s)}{ds}$ is ambiguous and will depend among other things the emission control rate μ. However, if $\hat{T} > 0$ then $\frac{\partial \Omega(\hat{T})}{\partial \hat{T}} < 0$ which implies that $\frac{d\hat{T}(s)}{ds} > 0$ and an increased saving rate will thus imply an increase in the equilibrium temperature $\hat{T}$.

C.2 Comparitive statics w.r.t. the emission control rate μ

From (22) we can write:

$$\frac{dT(\hat{\mu})}{d\mu} - \left(\frac{SL}{q} - \lambda\right)^{-1} \xi \frac{M_0}{M(\hat{T}(\mu))} \frac{\partial M(\hat{T}(\mu), \mu)}{\partial \mu} = 0 \tag{26}$$

where:

$$\frac{\partial M(\hat{T}(\mu), \mu)}{\partial \mu} = \frac{\beta \sigma A}{\delta_M} \left(-\hat{k} + (1-\mu) \frac{\partial \hat{k}}{\partial \mu} \frac{s}{\tilde{\delta}} \left(\frac{\partial \Omega(\hat{T})}{\partial \hat{T}} \frac{\partial \hat{T}}{\partial \mu} (1 - \psi_1 \mu^{\psi_2}) - \psi_2 \psi_1 \mu^{\psi_2 - 1} \Omega(T) \right) \right)$$

and hence:

$$\frac{dT(\hat{\mu})}{d\mu} = \frac{\left(\frac{SL}{q} - \lambda\right)^{-1} \xi \frac{M_0}{M(\hat{T}(\mu))} \frac{\beta\sigma}{\delta_M} \left(-\hat{k} - (1-\mu) \frac{\partial \hat{k}}{\partial \mu} \frac{s}{\tilde{\delta}} \psi_2 \psi_1 \mu^{\psi_2 - 1} \Omega(T) \right)}{1 - \left(\frac{SL}{q} - \lambda\right)^{-1} \xi \frac{M_0}{M(\hat{T}(\mu))} \frac{\beta\sigma}{\delta_M} \left((1-\mu) \frac{\partial \hat{k}}{\partial \mu} \frac{s}{\tilde{\delta}} \left(\frac{\partial \Omega(\hat{T})}{\partial \hat{T}} (1 - \psi_1 \mu^{\psi_2}) \right) \right)} \tag{27}$$

where $\frac{\partial \hat{k}}{\partial \mu} < 0$. Here the sign of $\frac{dT(\hat{\mu})}{d\mu}$ depends on the magnitude of the chosen model parameters.

References

Acemoglu D., 2007, Introduction to modern economic growth, Levine's bibliography, UCLA Department of Economics.

Ackland G.J., 2004, Maximization principles and daisyworld, *Journal of Theoretical Biology*, vol. 227(1), pp. 121 – 128.

Ackland G.J., Clark M.A. and Lenton T.M., 2003, Catastrophic desert formation in Daisyworld, *Journal of Theoretical Biology*, vol. 223(1), pp. 39 – 44.

Adams B., Carr J., Lenton T. and White A., 2003, One-dimensional Daisyworld: spatial interactions and pattern formation, *Journal of Theoretical Biology*, vol. 223, p. 505513.

Bagdassarian C.K., Dunham A.E., Brown C.G. and Rauscher D., 2007, Biodiversity maintenance in food webs with regulatory environmental feedbacks, *Journal of Theoretical Biology*, vol. 245(4), pp. 705–714.

Baldocchi D.D., Krebs T. and Y. L.M., 2004, Wet/dry Daisyworld: a conceptual tool for quantifying the spatial scaling of heterogeneous landscapes and its impact on the subgrid variability of energy fluxes., *Tellus*, vol. 57B, p. 175188.

Brock W.a. and Taylor M.S., 2010, The Green Solow model, *Journal of Economic Growth*, vol. 15(2), pp. 127–153.

Carpenter S., Walker B., Anderies J.M. and Abel N., 2001, From Metaphor to Measurement: Resilience of What to What?, *Ecosystems*, vol. 4(8), pp. 765–781.

Clark C.W., 1976, *Mathematical Bioeconomics*, Wiley and Sons, New York, NY.

Dawkins R., 1983, *The Extended Phenotype*, Oxford Univ. Press, New York.

Dhooge A., Govaerts W., Kuznetsov Y.A., Mestrom W. and Riet A.M., 2003, MATLAB continuation software package CL MATCONT, `http://www.math.uu.nl/people/kuznet/cm/`.

Dobson I., 2003, Distance to bifurcation in multidimensional parameter space : margin sensitivity and closest bifurcations, in G. Chen, D. Hill and X. Yu, eds., *Bifurcation control, theory and application*, vol. 293, chap. 3, Springer Verlag.

Dobson I. and Chiang H.D., 1989, Towards a theory of voltage collapse in electric power systems, *Systems & Control Letters*, vol. 13(3), pp. 253–262.

Doolittle W.F., 1981, Is Nature really motherly?, *CoEvolution Quarterly*, vol. 29, p. 5863.

DYKE J., 2008, Entropy production in an energy balance Daisyworld model, in S. Bullock, J. Noble, R. Watson and M.A. Bedau, eds., *Artificial Life XI: Proceedings of the Eleventh International Conference on the Simulation and Synthesis of Living Systems*, pp. 189–196, MIT Press, Cambridge, MA.

FORD A., ed., 1999, *Modelling the Environment*, Island Press, Washington, D. C.

GRASS D., CAULKINS J., FEICHTINGER G., TRAGLER G. and BEHRENS D., 2008, *Optimal Control of Nonlinear Processes: With Applications in Drugs, Corruption, and Terror*, Springer Verlag, Berlin.

HARDING S.P., 1999, Food web complexity enhances community stability and climate regulation in a geophysiological model, *Tellus*, vol. 51B, pp. 815–b82.

HARDING S.P. and LOVELOCK J.E., 1996, Exploiter-mediated Coexistence and Frequency-Dependent Selection in a Numerical Model of Biodiversity, *Journal of Theoretical Biology*, vol. 182(2), pp. 109 – 116.

HARTMANN D.L., ed., 1994, *Global Physical Climatology*, Academic Press, San Diego, CA.

HOLLING C.S., 1973, Resilience and stability of ecological systems, *Annual Review of Ecology and Systematics*, vol. 4, pp. 1–23.

HOUGHTON J., DING Y., GRIGGS D., NOGUER M., VAN DER LINDEN P., DAI X., MASKELL K. and JOHNSON C., 2001, *Climate Change 2001: The Scientific Basis. Contribution of Working Group I to the Third Assessment Report of the Intergovernmental Panel on Climate Change.*, Cambridge University Press, Cambridge.

IPCC, 2001, *Climate Change 2001: The Third Assessment Report of the Intergovernmental Panel on Climate Change*, Cambridge University Press, Cambridge.

JASCOURT S.D. and RAYMOND W.H., 1992, Comments on Chaos in Daisyworld, *Tellus*, vol. 44B, p. 243246.

JONES C.I., 1998, *Introduction to Economic Growth*, WW Norton & Co., New York.

JUDD K.L., 1998, *Numerical Methods in Economics*, vol. 2, MIT Press Books, The MIT Press, 1 ed.

KEELING R., 1992, Mechanisms for stabilization and destabilization of a simple biosphere: catastrophe on Daisyworld, in S.H. Schneider and P.J. Boston, eds., *Scientists on Gaia*, pp. 118–120, MIT Press, Cambridge, MA.

KERR R.A., 1988, No Longer Willful, Gaia Becomes Respectable, *Science*, vol. 240(4851), pp. 393–395.

KIRCHNER J.W., 1989, The Gaia hypothesis: can it be tested?, *Reviews of Geophysics*, vol. 27, pp. 223–235.

KUMP L.R., 2009, A Second Opinion for Our Planet, *Science*, vol. 325(5940), pp. 539–540.

KUMP L.R., KASTING J.F. and CRANE R., eds., 1999, *The Earth System*, Prentice-Hall, New York.

KUZNETSOV Y.A., 1998, *Elements of applied bifurcation theory*, Springer Verlag, 2nd ed.

LENTON T., 1998, Gaia and natural selection., *Nature*, vol. 394, pp. 439–47.

LENTON T.M., 2002, Testing Gaia: The Effect of Life on Earth's Habitability and Regulation, *Climatic Change*, vol. 52(4), pp. 409–422.

LENTON T.M. and LOVELOCK J.E., 2000, Daisyworld is Darwinian: Constraints on Adaptation are Important for Planetary Self-Regulation, *Journal of Theoretical Biology*, vol. 206(1), pp. 109 – 114.

LOVELOCK J.E., 1972, Gaia as seen through the atmosphere, *Atmospheric Environment*, vol. 6, pp. 579–580.

LOVELOCK J.E., 1983, Gaia as seen through the atmosphere., in P. Westbroek and E.W. d. Jong, eds., *Biomineralization and biological metal accumulation*, pp. 15–25, Springer.

LOVELOCK J.E. and MARGULIS L., 1974a, Atmospheric homeostasis by and for the biosphere: the Gaia Hypothesis, *Tellus*, vol. 26, pp. 2–10.

LOVELOCK J.E. and MARGULIS L., 1974b, Homeostatic tendencies of the Earth's atmosphere, *Origins Life*, vol. 5, pp. 93–103.

LUCAS R.E., 2002, The Industrial Revolution: Past and Future, in R.E. Lucas, ed., *Lectures on Economic Growth*, pp. 109–190, Harvard University Press, Cambridge, MA.

LUDWIG D., WALKER B. and HOLLING C.S., 1997, Sustainability, stability and resilience, *Conservation Ecology*, vol. 1(7), `http://www.consecol.org/vol1/iss1/art7/index.html`.

MADDOCK L., 1991, Effects of Simple Environmental Feedback on Some Population Models, *Tellus*, vol. 43B, p. 331337.

MARGULIS L. and LOVELOCK J.E., 1974, Biological modulation of the Earth's atmosphere, *Icarus*, vol. 21, pp. 471–489.

MCDONALD-GIBSON J., DYKE J., PAOLO E.D. and HARVEY I., 2008, Environmental regulation can arise under minimal assumptions, *Journal of Theoretical Biology*, vol. 251(4), pp. 653 – 666.

MCGUFFIE K. and HENDERSON-SELLERS A., 2005, *A Climate Modelling Primer*, John Wiley & Sons, 3rd ed.

MNP, 2006, Integrated modelling of global environmental change., in A. Bouwman, T. Kram and K.K. Goldewijk, eds., *An overview of IMAGE 2.4. Netherlands Environmental Assessment Agency (MNP)*, Bilthoven, The Netherlands.

NORDHAUS W. and BOYER J., 2000, *Warming the World: Economic Models of Global Warming*, MIT Press.

NORDHAUS W.D., 1992, An optimal transition path for controlling greenhouse gases, *Science (New York, N.Y.)*, vol. 258(5086), pp. 1315–9.

NORDHAUS W.D., 1994, *Managing the Global Commons: the economics of the greenhouse effect*, MIT Press, Cambridge, MA.

NORDHAUS W.D., 2007, A Question of Balance: Weighing the Options on Global Warming Policies, Tech. rep., Yale University.

NORTH G.R., CAHALAN R.F. and COAKLEY J.A., 1981, Energy Balance Climate Models, *Reviews of Geophysics and Space Physics*, vol. 19(1), pp. 91–121.

ONOZAKI K., 2009, Population Is a Critical Factor for Global Carbon Dioxide Increase, *Journal of Health Science*, vol. 55(1), pp. 125–127.

PETERSEN A.C., 2004, Models and Geophysiological Hypotheses, in S.H. Schneider, J.R. Miller, E. Crist and P.J. Boston, eds., *Scientists debate gaia: the next century*, MIT Press, Cambridge, MA.

PIERREHUMBERT R.T., 2008, *Principles of Planetary Climate*, Cambridge University Press, UK.

PUJOL T., 2002, The Consequence of Maximum Thermodynamic Efficiency in Daisyworld, *Journal of Theoretical Biology*, vol. 217(1), pp. 53 – 60.

ROBERT J. BARRO AND XAVIER SALA-I-MARTIN, 2003, *Economic Growth*, MIT Press, Cambridge, MA.

Robertson D. and Robinson J., 1998, Darwinian Daisyworld, *Journal of Theoretical Biology*, vol. 195, p. 129134.

Roe G.H. and Baker M.B., 2007, Why is climate sensitivity so unpredictable?, *Science*, p. 318: 629632.

Salazar J.F. and Poveda G., 2009, Role of a simplified hydrological cycle and clouds in regulating the climatebiota system of daisyworld, *Tellus B*, vol. 61(2), pp. 483–497.

Saunders P.T., 1994, Evolution without natural selection: further implications of the daisyworld parable, *Journal of Theoretical Biology*, vol. 166, pp. 365–373.

Scheffer M., Carpenter S., Foley J.A., Folke C. and Walker B., 2001, Catastrophic shifts in ecosystems, *Nature*, vol. 413, pp. 591–596.

Schneider S.H., 2001, A goddess of earth or the imagination of a man?, *Science*, vol. 291(5510), pp. 1906–1907.

Schneider S.H. and Boston P.J., eds., 1992, *Scientists on Gaia*, MIT Press, Cambridge, MA.

Schneider S.H., Miller J.R., Crist E. and Boston P.J., eds., 2004, *Scientists debate gaia: the next century*, MIT Press, Cambridge, MA.

Schultz P.a. and Kasting J.F., 1997, Optimal reductions in CO2 emissions., *Energy policy*, vol. 25(5), pp. 491–500.

Seto M. and Akagi T., 2005, Daisyworld inhabited with daisies incorporating a seed size/number trade-off: the mechanism of negative feedback on selection from a standpoint of the competition theory, *Journal of Theoretical Biology*, vol. 234(2), p. 167172.

Solow R.M., 1956, A Contribution to the Theory of Economic Growth, *Quarterly Journal of. Economics*, vol. 70, pp. 65–94.

Staley M., 2002, Darwinian selection leads to gaia, *Journal of Theoretical Biology*, vol. 218(1), pp. 35 – 46.

Stanton E.A., Ackerman F. and "Kartha S., 2009, Inside the integrated assessment models: Four issues in climate economics, *Climate and Development*, vol. 1, pp. 166–184.

Stöcker S., 1995, Regarding mutations in daisyworld models, *Journal of Theoretical Biology*, vol. 175(4), pp. 495 – 501.

STOCKHOLM MEMORANDUM, 2011, The Stockholm Memorandum - Tipping the Scales towards Sustainability, in *3rd Nobel Laureate Symposium on Global Sustainability*, Stockholm, Sweden.

VON BLOH W., BLOCK A. and SCHELLNHUBER H., 1997, Self-stabilization of the biosphere under global change: a tutorial geophysical approach, *Tellus*, vol. 49B, p. 249262.

WALKER B., CARPENTER S., ANDERIES J., ABEL N., CUMMING G., JANSSEN M., LEBEL L., NORBERG J., PETERSON G.D. and PRITCHARD R., 2002, Resilience Management in Social-ecological Systems: a Working Hypothesis for a Participatory Approach, *Conservation Ecology*, vol. 6(1).

WALKER J.C.G., HAYS P.B. and KASTING J.F., 1981, A negative feedback mechanism for the long-term stabilization of Earth's surface temperature, *Journal of Geophysical Research*, vol. 86, pp. 9776–9782.

WATSON A.J. and LOVELOCK J.E., 1983, Biological homeostasis of the global environment: the parable of Daisyworld, *Tellus*, vol. 35B, pp. 284–289.

WATSON R.T., RODHE H., OESCHGER H. and SIEGENTHALER U., 1990, Greenhouse gases and aerosols, in J. Houghton, T. Jenkins and J. Ephraums, eds., *Climate Change, The IPCC Scientific Assessment*, pp. 1–40, Cambridge University Press, Cambridge.

WEBER S.L., 2001, On homeostasis in daisyworld, *Climate Change*, vol. 48, pp. 465–485.

WOOD A.J., ACKLAND G.J., DYKE J.G., WILLIAMS H.T.P. and LENTON T.M., 2008, Daisyworld : A Review, *Reviews of Geophysics*, vol. 46, pp. 1–23.

ZENG X., PIELKE R.A. and EYKHOLT R., 1990a, Chaos in Daisyworld, *Tellus*, vol. 42B, p. 309318.

ZENG X., PIELKE R.A. and EYKHOLT R., 1990b, Reply to Jascourt and Raymond, *Tellus*, vol. 44B, p. 247 248.