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A NOVEL HYDRO - ECONOMIC – ECONOMETRIC APPROACH FOR INTEGRATED TRANSBOUNDARY WATER MANAGEMENT UNDER UNCERTAINTY

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ABSTRACT

Competitive use of transboundary waters across different countries and among different sectors can be approached as a stochastic multistage dynamic game. In this paper we use this approach to develop and apply a novel framework for optimal management of limited transboundary water resources and evaluation of different international strategies, under hydrological uncertainty. The Omo-Turkana River Basin in Africa is used as a case study application, since it faces the above challenges within the water-food-energy nexus framework. The basic mathematical model consists of the water balance (availability and demand for the different sectors), the costs of water extraction, and the social benefits from water resources. The non-cooperative and cooperative (Stackelberg "*leader–follower*") cases are solved and compared based on the future water availability. The empirical application of the model calls for sector-specific production function estimations, for which we employ nonparametric treatment of the production functions, while we extend it to allow for technical inefficiency in production and autocorrelated Total Factor Productivity, providing thus a more realistic model. For this purpose, Bayesian analysis is performed using a Sequential Monte Carlo/Particle-Filtering approach. The cooperative solution is the optimal pathway not only for both riparian countries, but for the sustainable water use of the basin too, as in future uncertainty conditions it maintains the maximum welfare option. The detail and sophistication of both the mathematical and econometric models are key elements for this novel approach, supporting robust policy recommendations towards sustainable management of transboundary resources.

Keywords: Transboundary water management; cooperation games; stochasticity; endogenous adaptation; production functions technical inefficiency; demand curve.

1. INTRODUCTION

According to the theories of Integrated Water Resources Management and Economics, transboundary river basins should be treated as a single unit to maintain the physical integrity of the system and achieve globally optimum management solutions (Qaddumi, 2008). Both theories have gone beyond the traditional approach of monitoring and measuring the spatiotemporal allocation of resources, costs, and benefits, and seek the optimal way to control and manage systems in a way that maximizes the users' welfare under environmental constraints (Gupta et al., 2016). The problem of covering competitive and increasing needs with limited (and often deteriorating) resources becomes more serious and complicated when considering the broad impacts of transboundary water decisions on the context of Water-Energy-Food (WEF) nexus. Subsequently, integrated and detailed modelling is increasingly used in the decision-making process, especially in cases where holistic approaches and cooperative management cannot be taken for granted (Uitto and Duda, 2003).

Game theory has been used to describe the actions of the countries-players (Frisvold and Caswell, 2000; Dinar and Hogarth, 2015). Kucukmehmetoglu (2012) analysed the problem of scarce water resources allocation combining game theory and Pareto frontier, using also linear programming to maximize net economic benefits. Zeng et al. (2019) proposed a hybrid game theory and mathematical programming model for solving transboundary water conflicts, by the optimal water allocation, considering water quality and quantity and the associated benefits and costs. Menga (2016) highlights the interplay between domestic and foreign policy for transboundary waters through the example of the two-level game theory of Putnam (1988). Hu et al. (2017) used the case of hydropower and water supplies within the water-energy nexus using stochastic competitive and cooperative (Nash–Cournot model) analysis. The authors suggested that future studies need to include uncertainty in hydrological processes.

Indeed, external factors (hydrological flows, climate change, human interventions, etc.) create an inherent uncertainty in such models, so its consideration for long-term planning is required (Wine, 2018). The stochastic and uncertain characteristics of water flow in transboundary waters led Degefu et al. (2017) to apply a stochastic game theoretic extension of the bankruptcy concept to transboundary water management, under water scarce and uncertain conditions. A similar analysis was performed by Janjua and Hassan (2020), who introduced the 'weighted bankruptcy' approach which favors agents with 'high agricultural productivity'. Bhaduri et al. (2011) investigated the uncertainty in a transboundary water sharing problem and evaluated the scope and sustainability for a potential cooperative agreement between countries. Jiang et al. (2019) constructed a stochastic differential game to analyze transboundary pollution control options, supporting ecological compensation agreements. They compared the non-cooperative and Stackelberg cooperative game pollution results and examined the most efficient long-term incentives. Basheer et al. (2019) used a data-scarce African basin under hydrologic uncertainty to address hydropower generation and irrigation issues. These applications however refer to allocation (resource or pollution) and do not include any economic extensions, while they focus on the one or two main sectors-drivers of water demand. Integrated hydro-economic models have been highlighted as the most promising policymaking tools for addressing the aforementioned issues (Booker et al., 2012; Alamanos et al., 2020; Wang et al., 2020). Their use in uncertain transboundary management problems (e.g. Jeuland, 2010) has been very limited (Tayia, 2019).

The lack of integrated approaches that consider hydrological and economic aspects, from the actual (game theory) perspective of transboundary water management, under uncertainty was the initiative for this study. Hydro-economic models' potential to simulate in an integrated and expandable way multistage stochastic and dynamic processes under uncertainty fits well into the concept of transboundary water management-game. The present study exploits this potential by developing a novel framework combining *hydrological* (precipitation, runoff, outflows from the upstream country, and water stock, stochastically) and *economic components* (social benefits, marginal and total costs), considering the *five sectors-drivers* of water demand and economy (mining, energy, tourism, residential, and agriculture), as well as their water demand curves through production functions and productivity.

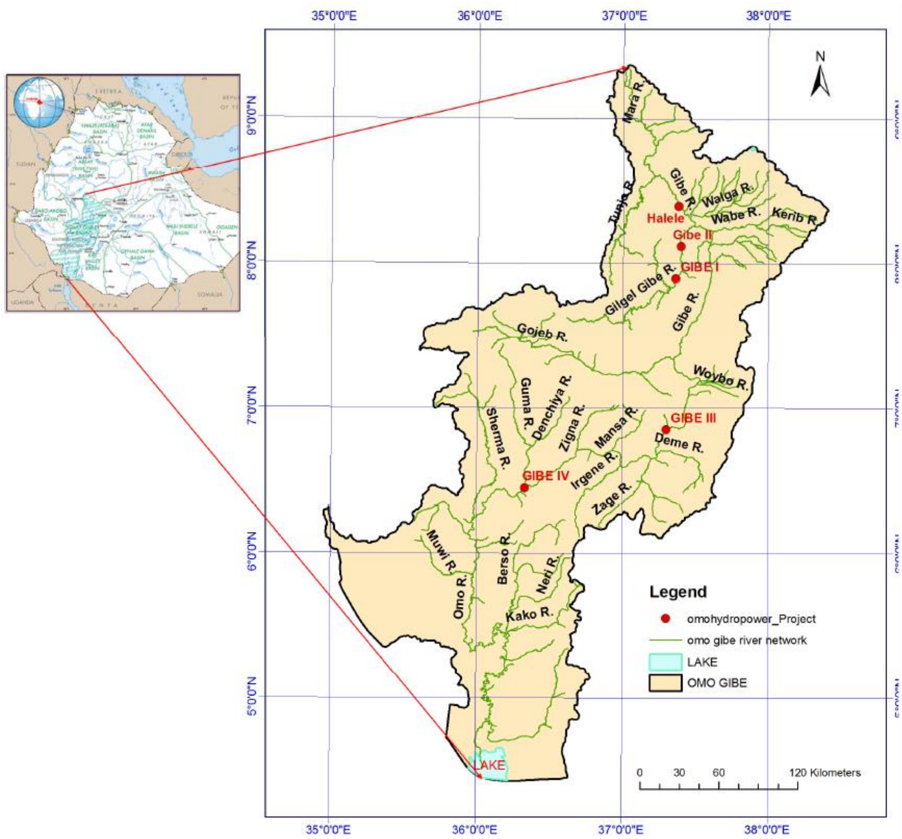
The estimation of the above relations has been one of the most challenging econometric processes: Biases, inconsistencies, and correlation among the regressors (explanatory variables, e.g. capital or labor, with the error term) often cause endogeneity problems. Traditional approaches (Olley and Pakes, 1996; Levinsohn and Petrin, 2003) lack of instruments to control for the endogenous inputs and suffer from collinearity problems (Ackerberg et al., 2015; Gandhi et al., 2017). Endogeneity problems are still a challenge for stochastic frontier models in efficiency analysis, too (Shee and Stefanou, 2014). It usually biases the commonly used tools (e.g. DEA) and Monte Carlo techniques are recently suggested to control the effects of endogeneity in efficiency analysis and estimates (von Cramon-Taubadel and Saldias, 2014; Santín and Sicilia, 2017). We present a new estimation method of sector-level productivity as an extension of the model proposed by Gandhi et al. (2017), to tackle the existing limitations, introducing technical inefficiency in production, and allowing for autocorrelation of Total Factor Productivity (TFP).

The whole framework is tested under a non-cooperative and a cooperative (Stackelberg leader–follower) game, considering the agreements (e.g. food or energy trade-offs) between the upstream and downstream countries, providing thus a direct link to WEF nexus. The aim is to provide an integrated methodology for managing sustainably transboundary waters under uncertainty. The transboundary Omo-Turkana River Basin in Africa is used as an example to showcase the framework, while highlighting the significance and impacts of proper management of scarce resources to the economic and WEF issues of the area under baseline and future scenarios. The novelties of the proposed approach are highlighted in each step as presented in the following sections, but briefly are: i) its integrated character (to our knowledge, no study has combined all the above components in a single framework), ii) the proposed way for the stochastic description of the hydrological components, iii) the connection of the follower's reaction to the leader's strategy, together with the (quantitatively tractable) optimization of their objective functions over all possible strategies of the stochastic game, iv) the multistage and dynamic consideration of all sectors, v) the realistic production function estimations, controlling-allowing for endogeneity, technical inefficiency and autocorrelated TFP.

2. STUDY AREA

The Omo-Turkana (Omo River and Lake Turkana) basin in Eastern Africa is an area of 130,860 km^2 across Ethiopia and Kenya, and small encroachments into Uganda and South Sudan (95% of the basin is in Ethiopian and Kenya). The water-land uses of the broader area are agriculture (main use, including livestock), energy production, mining, residential, and touristic. Lake Turkana receives its inflows from Omo River, which defines its levels and water quality. Turkana concentrates over 70% of Kenya's population, relying on food aid, flood retreat farming along Omo River, cattle-grazing, and fishing (Kaijage and Nyagah, 2010; Reta, 2016; Oakland Institute, 2014; Anaya 2010). A five-plant hydroelectric dam cascade is being constructed in Ethiopia (three of them- GIBE I, GIBE II, GIBE III, are already operating in Omo River) to fulfill energy demand and electricity export ambitions (Regi, 2011; Ficquet, 2015).

The case is controversial as there are studies highlighting the engineering achievement of the dams' construction, or criticising it from the ecological point of view (Ambelu et al., 2013). Hydrological studies argue that the impact on the water level of Lake Turkana is negligible (Yesuf, 2013), or dependent on the rainfall and the lake's initial level (Velpuri and Senay, 2012), while there are reported phenomena of extreme hunger in the Omo Valley, attributing it to the GIBE III reservoir which holds back the Omo River's annual floods, preventing retreat agriculture for local pastoralists, (Avery, 2013; Survival International, 2013) and around Lake Turkana where people (and ethnic groups) are already fighting over dwindling resources (Avery, 2012; Carr, 2012;2017). In any case, there are transboundary tensions and territorial conflicts/border disputes around the Lake Turkana border, in contrast with the Ethiopian agricultural and rural-factories development (Kamski, 2016; Sugar Corporation, 2019). Kenya sees the dam construction as growing poverty because of increased water scarcity; Ethiopia is concerned by land erosion, water access, increased poverty, change in livelihood, while points out the positive impact of regulating floods to provide a more constant water availability throughout the downstream (DAFNE 2019). The broader area was in the spotlight this year because of the food crisis caused from a historic locust swarm invasion[1].



[1] Locust swarm: UN warns of food crisis in Ethiopia, Kenya, Uganda, Tanzania and Somalia. (2020, February 14). Retrieved November 22, 2020, from https://www.bbc.com/news/world-africa-51501832

Figure 1: Study area (Gebresenbet, 2015).

The dams' construction allows Ethiopia to export electricity to Kenya, Sudan, and Djibouti. This agreement exists only in a form of Memorandum of Understanding (MoU), that only Kenya's Electricity Company has signed in 2006 (Eastern Electricity Highway Project – construction of a 1,000km power line from Ethiopia to Kenya), while other trade-offs refer to food production (irrigation and fishing) and tourism (DAFNE, 2019). In particular, the downstream country offers a discounted price for food exports to the upstream country, in exchange for greater transboundary water flow (and hydropower) that results in a higher water reserve accumulation and sequentially in a higher production of food (Fig.2). The environmental and social impact assessment report was approved in 2012, although it has been criticised as it was conducted after any objection could be made (Abbink, 2012). Following a World Bank loan of US$684 million (World Bank, 2012), construction began in June 2016[2]. While the 2016 agreement is not yet publicly available, it is reported that the agreement will allow Ethiopia to supply Kenya with 400 megawatts of hydropower at less than 1 US cent/kwh[3]. However, the hydropower source (or sources) that will supply this transmission line is not officially stated, although the World Bank modified an official project report specifying that power would be sourced "from Ethiopia's GIBE hydropower scheme", changing the reference to the dam in its next report instead to "Ethiopia's power grid" (AthiWater, 2018).

3. CONCEPTUAL FRAMEWORK: HYDRO-ECONOMIC MODEL

The situation described is a typical example of transboundary water management problem, where the links to the WEF nexus are expressed as agreements and social welfare for both the Upstream (h=U) and Downstream (h=D) countries. Hydrological, economic, WEF, uncertainty factors and leader-follower games can describe the general form of the problem (Fig.2).

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[2] Kenya-Ethiopia Electricity Highway. (2020, November 18). Retrieved November 22, 2020, from https://www.power-technology.com/projects/kenya-ethiopia-electricity-highway/

[3] Ethiopia, Kenya to enhance cooperation on energy sector. (n.d.). Retrieved November 22, 2020, from http://www.china.org.cn/world/Off_the_Wire/2016-06/24/content_38742095.htm

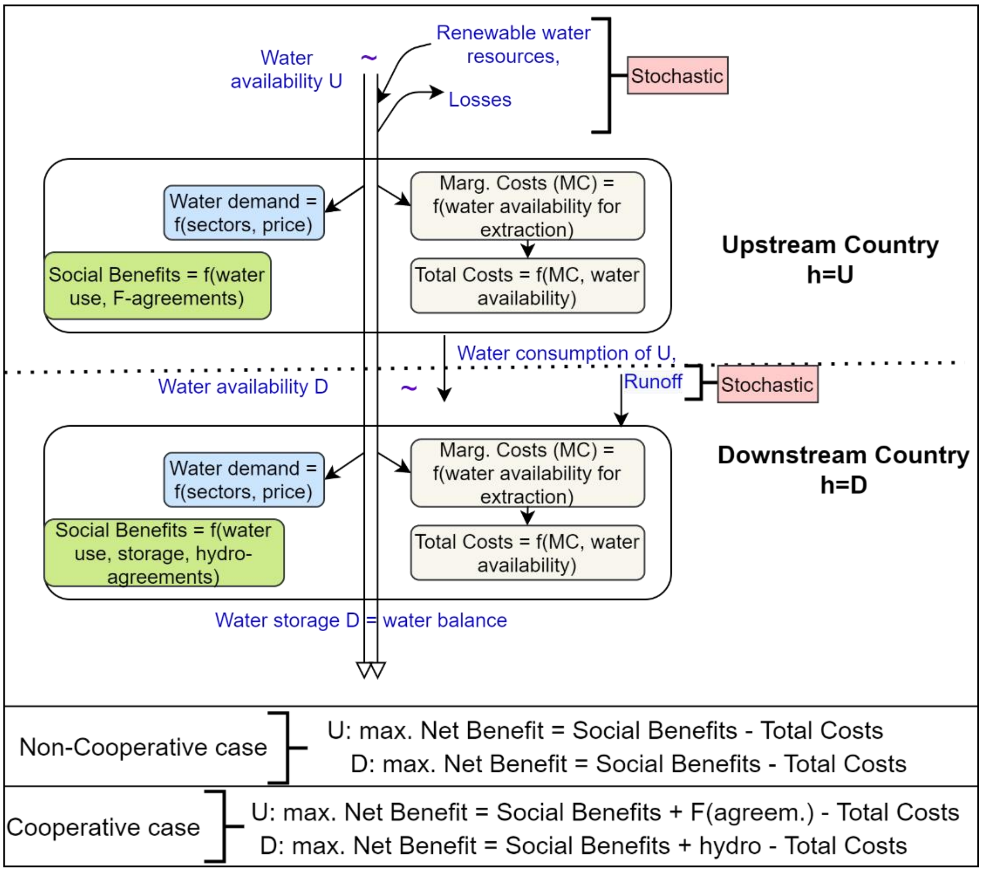

Figure 2: Conceptual flowchart with the factors considered and their brief description.

The proposed framework enables the quantitative estimation of the influence of stochastic water resources on transboundary water allocation over multiple (all the five) sectors of the economy, following a multistage dynamic cooperative game (Stackelberg "leader–follower") framework.

Deconstructing the flowchart, the proposed approach is based on the following pillars:

Water resources:

Hydrological cycle's components such as water availability, losses, and runoff, that are necessary for integrated modelling often face many data limitations and their accurate simulation is accompanied with many uncertainties. Hydrological modelling itself is not always enough for their complete and integrated simulation (Van Emmerik et al., 2014). Thus, in this framework these components are expressed stochastically, by geometric Brownian motion functions, which have been proved to simulate flows better than other deterministic models (Lefebvre, 2002), and its proportional changes describe the most natural continuous random movements. Given the different hydrological-social-future regional climate conditions that may affect the flows in the upstream and downstream countries, we provide the option (and develop the framework accordingly) to use Brownian motions with different characteristics in terms of variance between the upstream and the downstream country. Additionally, this allows to determine how the water abstraction of the riparian countries will change in the long run, considering the greater variability of water availability caused by climate change or other uncertainties. Another benefit of this approach is the ability to model the water allocation between the upstream and the downstream country, with and without any cooperation in water sharing, taking into account how uncertainty in water supply affects the water abstraction rates of the countries, and explore the underlying conditions that may influence allocation decisions. The upstream country has the upper riparian right to unilaterally divert water while the freshwater availability of the downstream one partially depends on the water usage in the upstream country.

Following Bhaduri et al. (2011), we consider at first a complete filtered probability space (Ω, $\mathcal{J}$, $\mathcal{J}_t$, P) for the stochastic water flow. Then the annual renewable water resource (mainly precipitation) due to the river basin, W_t, evolves through time according to the Geometric Brownian motion:

$$dW_t = \sigma^W W_t dz_t^W \text{ , } t \geq 0 \tag{1}$$

Where σ^W is the volatility of water flow in the upstream country, z_t^W is a standard Wiener process (standard Brownian Motion), and W_t the total freshwater utilization (see also next paragraph).

In Fig.2, the term losses refer to the natural outflows and evaporation/ evapotransiration (ET), here denoted by O_t which can be formulated by another Geometric Brownian motion:

$$dO_t = \sigma^O O_t dz_t^O \text{, } t \geq 0 \tag{2}$$

Where σ^O is the volatility of the losses and z_t^O a standard Wiener process.

The water availability in D depends on the total water consumption in U and runoff (to the Lake), denoted by R, which is expressed by a third Geometric Brownian motion as:

$$dR_t = \sigma^R R_t dz_t^R \text{, } t \geq 0 \tag{3}$$

Where σ^R is the runoff volatility and z_t^R the standard Wiener process (z_t^W, z_t^O, z_t^R are independent Wiener processes).

Water Demand:

As mentioned above, the framework provides the option to use all the involved sectors-water consumers i (here i=5), and their water use in a way that highlights the scarce character of the input resource, unlike with previous studies (as in Eq.4, for the upstream country h=U):

$$dW_{jt}^U = \left[W_t - \textstyle\sum_{i=j}^5 w_{it}^U\right] dt, \quad T_{j-1}^U \leq t < T_j^U, \quad j = 1,2,\ldots,5 \tag{4}$$

Where W_i^h is the total freshwater utilization (see Eq.1) by country U, w_{it}^U is the water utilization per sector i in U, for a specific time: T_i^h is the end of use (*exit*) time[4] of the i-th sector of U (T_0=0 and $T_5=\infty$). So, Equation (4) expresses the water stock (available resources) change in the upstream country, W_{jt}^U, for the j-th *exit* stage.

The stock of water (water storage D = water balance, as in Fig.2) in country D (i.e. in the lake), where agricultural products and fisheries are produced, is denoted by S and is actually based on the general water balance equation: *ΔS=Available – Use + Runoff – Losses*. Thus, Equation (5) is a function of the stochastic water resources and the control (water use) variables w_i^h=(w_1^h, w_2^h,…, w_5^h) per country h = U, D. For the (j,k)-th exit stage of U and D, respectively, it follows the dynamics:

[4] when an economic sector exits the market as its water demand reaches zero.

$$dS_{jkt} = \left\{W_t - \sum_{i=j}^{5} w_{it}^U - \sum_{l=k}^{5} w_{lt}^D + R_t - O_t\right\}dt, \quad T_{j-1}^U \leq t < T_j^U \tag{5}$$

with $T_{k-1}^D \leq t < T_k^D$, $j, k = 1,2,\ldots,5$ and S(0)=S_0 (initial condition).

So, the inverse demand function takes into account the water utilization w_i^h of the j-th exit stage, and the price of water p_{jt}^h which is the same for the different sectors i:

$$p_{jt}^h = \frac{a_i^h}{b_i^h} - \frac{1}{b_i^h} \cdot w_{it}^h, \quad T_{j-1}^h \leq t < T_j^h, \quad i = j,\ldots,5, \quad j = 1,2,\ldots,5 \tag{6}$$

Where $a_i^h \in \mathbb{R}$, $b_i^h > 0$ are constant sector-specific parameters that define their water demand.

The sector-specific inverse demand curves are ordered so that $a_1^h/b_1^h < a_2^h/b_2^h < \cdots < a_5^h/b_5^h$, which implies that water demand for each of the five sectors reaches zero sequentially over time as the price of water increases over time, leading to the endogenously defined exit times T_j^h, giving thus piecewise linear demand functions.

Costs:

Water abstraction from rivers may be taken directly from the flowing waters in the channel (surface water abstraction) or can be achieved through inter-basin flow transfer schemes. Thus, we may assume that the marginal extraction cost (MC) for the j-th exit stage of the upstream country is a decreasing function of the available water W^U of the form:

$$MC^U\left(W_j^U\right) = k_2^U - k_1^U W_j^U, \quad j = 1,2,\ldots,5 \tag{7}$$

Where k_1^U, $k_2^U > 0$ given constants which define the cost magnitudes.

As water becomes increasingly scarce in the economy, the government will exploit water through appropriating and purchasing a greater share of aggregate economic output, in terms of dams, pumping stations, supply infrastructure, etc. (Barbier, 2004). Given the high cost of building infrastructure and expanding supplies, this will lead to a higher marginal cost of water. Then the Total Cost (TC) function of water withdrawing w_i^U from the river per sector i=j, …,5, for the j-th exit stage of the upstream country is given by an increasing function of the water extraction variable:

$$TC^U\left(W_j^U, w_i^U\right) = (k_2^U - k_1^U W_j^U) w_i^U, \quad i = j,\ldots,5, \quad j = 1,2,\ldots,5 \tag{8}$$

On the other hand, D country extracts water from its available stock, thus for the (j,k)-th exit stage the MC of the downstream country is a decreasing function of the available water stock S_{jk}(Eq.9). Similarly, the TC function of water withdrawing w_l^D from the water stock per sector l = k, …, 5 for the (j,k)-th exit stage is given by Eq. (10).

$$MC^D\left(S_{jk}\right) = k_2^D - k_1^D S_{jk}, \quad j, k = 1,2,\ldots,5 \tag{9}$$

$$TC^D\left(S_{jk}, w_l^D\right) = (k_2^D - k_1^D S_{jk}) w_l^D, \quad l = k,\ldots,5, \quad k = 1,2,\ldots,5 \tag{10}$$

where k_1^D, $k_2^D > 0$ given constants.

Social Benefits:

The last component of Figure's 2 flowchart refers to the *Benefits*. Since consumers are deriving benefits from water, the inverse demand curve (Eq. 6) is the marginal social benefit curve. Hence, consider further the benefit of water consumption w_i^h per sector i of country h, namely social benefit (SB), as:

$$SB_i{}^h\left(w_i^h\right) = \int_0^{w_i^h} \left(\frac{a_i^h}{b_i^h} - \frac{1}{b_i^h} \cdot w_i^h\right) dw_i^h = \frac{a_i^h}{b_i^h} w_i^h - \frac{1}{2b_i^h} \cdot (w_i^h)^2 \qquad (11)$$

It is obvious that the benefit function is strictly concave for all possible values of w_i^h.

As mentioned, D country's benefits occurring from storing water, while U country receives an additional benefit in the cooperation case, from their agreement, as the net consumer surplus or economic benefit from food (agricultural product and fisheries) production. This can be described by a linear function of water stock S_{jk} per (j,k)-th exit stage:

$$F\left(S_{jk}\right) = \eta_1 S_{jk} + \eta_2\,, \quad j,k = 1,2,\ldots,5\,, \quad \eta_1 > 0,\ \eta_2 \in R \text{ (constants)} \qquad (12)$$

This relation's form describes these benefits, and allow us to use the coefficient η_1 to represent the intensity of the contribution that the water storage of the lake has to the corresponding food benefits enjoyed by the upstream country.

<u>Game:</u>

Figure 2 also shows the two game-cases we define, using an inter-sectoral Stackelberg leader (U)-follower (D) game. Bhaduri et al. (2011) used a stochastic differential Stackelberg game to produce qualitative results on the optimal transboundary water allocation between an upstream and a downstream area. The leader (U) applies its strategy first, a priori knowing that the follower (D) observes its actions and posteriori moves accordingly. In contrast to Bhaduri et al. (2011), who had to restrict the U's strategy space to quadratic functions of the state variable in order to obtain a sub-optimal qualitative solution of the problem, we maximize the leader's objective function, using the D's reaction strategy, over all possible strategies to provide an optimal solution of our stochastic game problem that is also quantitatively tractable. Assuming that both countries use Markovian perfect strategies, since all model coefficients are deterministic functions of time, a subgame perfect equilibrium and an equilibrium set of decisions dependent on previous actions are defined. These strategies are decision rules that dictate the optimal action, conditional on the current values of the state variables (e.g. water resources of U, water stock of D), that summarize the latest available information of the dynamic system. The following sections analyse the two cases of the game.

4. NON-COOPERATIVE CASE

In the case of a non-cooperative framework, where there is no agreement between the two countries regarding either water or food sharing, the benefit maximization and the impact on water balance is presented for each country (hydro-economic model).

<u>Upstream:</u> The upstream country chooses the economically potential rate of water utilization that maximizes its own net benefit (NB) per j-th exit stage:

$$NB_j{}^U = \sum_{i=j}^{5} SB_i{}^U(w_i{}^U) - \sum_{i=j}^{5} TC^U\left(W_j^U, w_i^U\right), \quad j = 1,2,\ldots,5 \qquad (13)$$

Thus, U country's maximization problem is based on its net social benefit (J_j^U) of the j-th exit stage (j=1,2,…,5), and is formulated as follows:

$$J^U = \max_{w^U} \sum_{j=1}^{5} J_j^U = \max_{w^U} \sum_{j=1}^{5} E\left\{\int_{T_{j-1}^U}^{T_j^U} e^{-rt} NB_j^U dt\right\} =$$

$$= \max_{w^U} \sum_{j=1}^{5} E\left\{\int_{T_{j-1}^U}^{T_j^U} e^{-rt} \sum_{i=j}^{5} \left[SB_i^U(w_{it}^U) - TC^U(W_{jt}^U, w_{it}^U)\right] dt\right\} = \qquad (14)$$

$$= \max_{w^U} \sum_{j=1}^{5} E\left\{\int_{T_{j-1}^U}^{T_j^U} e^{-rt} \sum_{i=j}^{5} \left[\frac{a_i^U}{b_i^U} w_{it}^U - \frac{1}{2b_i^U} \cdot (w_{it}^U)^2 + c_i^U - (k_2^U - k_1^U W_{jt}^U) w_{it}^U\right] dt\right\}$$

Which subjects to the renewable water (precipitation) in U (Eq. 1), and the water stock change in U (Eq. 4). An explicit solution of this stochastic control problem via a decoupling method for forward-backward stochastic differential equations (FBSDEs) is analytically derived in Appendix A.

<u>Downstream:</u> On the other hand, the water consumption/production of D depends on the inflow from U, and the runoff generated within the country's share of the water stock in D (Fig.2). Based on the given water availability, D maximizes its NB per exit stage (j,k) as:

$$\mathrm{NB}_{jk}^{D} = \sum_{l=k}^{5} \mathrm{SB_l}^{D}\left(\mathrm{w}_{jl}^{D}\right) - \sum_{l=k}^{5} \mathrm{TC}^{D}\left(\mathrm{S}_{jk}, \mathrm{w}_{jl}^{D}\right) \qquad (15)$$

Thus, putting together Eq.(11), and Eq.(10) in the above relation, the maximization problem of the net social benefit (J_j^D) of the j-th exit stage (j=1,2,…,5), is:

$$\mathrm{J}^{D} = \max_{w^D} \sum_{k=1}^{5} \sum_{j=1}^{5} \mathrm{J}_{jk}^{D} = \max_{w^D} \sum_{k=1}^{5} \sum_{j=1}^{5} \mathrm{E}\left\{\int_{\{T_{j-1}^U \le t < T_j^U\} \cap \{T_{k-1}^D \le t < T_k^D\}} e^{-rt} \mathrm{NB}_{jk}^{D} dt\right\}$$

$$= \max_{w^D} \sum_{k=1}^{5} \sum_{j=1}^{5} \mathrm{E}\left\{\int_{\{T_{j-1}^U \le t < T_j^U\} \cap \{T_{k-1}^D \le t < T_k^D\}} e^{-rt} \begin{bmatrix} \sum_{l=k}^{5} \mathrm{SB_l}^{D}(\mathrm{w}_{lt}^{D}) \\ - \sum_{l=k}^{5} TC^{D}(\mathrm{S}_{jkt}, \mathrm{w}_{lt}^{D}) \end{bmatrix} dt\right\} =$$

$$\max_{w^D} \sum_{k=1}^{5} \sum_{j=1}^{5} \mathrm{E}\left\{\int_{\{T_{j-1}^U \le t < T_j^U\} \cap \{T_{k-1}^D \le t < T_k^D\}} e^{-rt} \begin{bmatrix} \sum_{l=k}^{5}\left(\frac{a_l^D}{b_l^D} w_{jlt}^D - \frac{1}{2b_l^D} \cdot (w_{jlt}^D)^2\right) \\ -(k_2^D - k_1^D S_{jkt}) \sum_{l=k}^{5} w_{jlt}^D \end{bmatrix} dt\right\} \qquad (16)$$

where J_{jk}^D represents the downstream country's net social benefit of the (j,k)-th exit stage, j, k=1,2,…,5, and w_{jlt}^D=(w_{1lt}^D, w_{2lt}^D,…, w_{5lt}^D) is the sectorial water extraction vector for D. This relation subjects to the river basin annual renewable water resource Eq.(1), outflow Eq.(2), runoff Eq.(3), the upstream area water resources Eq.(4), and the stock of water (state variable) in the downstream area Eq.(5). The analytical solution of this stochastic optimization problem can be found in Appendix A.

5. COOPERATIVE CASE

In this case the agreements described earlier apply, so the formed Stackelberg game determines the inter-sector optimal water allocation between U and D countries. First, we find the solution to the follower's (D) problem of maximizing a payoff function, and then, using D's reaction strategy, we maximize the U's objective function.

Downstream: Receiving now hydropower benefits, denoted by a variable *hydro*, from U at a discount rate and given its announced intersectoral water abstraction policy w_{jkt}^U=(w_{1kt}^U, w_{2kt}^U,…, w_{5kt}^U) per (*j*,*k*)-th exit stage, the follower D is faced with an optimal water management problem as in the non-cooperative case, i.e., maximise Eq.(16) augmented by *hydro* subject to the state Eq.(2)-(6). For every *j*, *k* = 1, 2, …, 5, the (*j*,*k*)-th exit stage Hamiltonian of the system is also given by Eq.(A.14), whose necessary optimality conditions Eq.(A15,A16) result in the optimal water allocation path of Eq.(A17) and in the same FBSDEs system which will constitute a state system for the upstream country, too.

Upstream: U receives now food benefits from D as in Eq.(12), and its NB function (Fig.2) is given by:

$$\mathrm{NB}_{jk}^{U} = \sum_{i=j}^{5} \mathrm{SB_i}^{U}(\mathrm{w_{it}}^{U}) + \mathrm{F}\left(\mathrm{S_{jk}}\right) - \sum_{i=j}^{5} \mathrm{TC}^{U}\left(\mathrm{W_j^U}, \mathrm{w_i^U}\right), \quad \mathrm{j,k} = 1,2,\dots,5 \tag{17}$$

Therefore, U, anticipating the D's optimal response as analysed in the previous case, chooses the optimal water abstraction vector process $w^U = (w_1^U, w_2^U, \dots, w_5^U)$ under cooperation by solving the maximization problem:

$$\begin{aligned} J^U &= \max_{w^U} \sum_{j=1}^{5} \sum_{k=1}^{5} J_{jk}^U = \max_{w^U} \sum_{j=1}^{5} \sum_{k=1}^{5} E\left\{\int_{\left\{T_{j-1}^U \le t \le T_j^U\right\} \cap \left\{T_{k-1}^D \le t \le T_k^D\right\}} e^{-rt} NB_{jk}^U dt\right\} \\ &= \max_{w^U} \sum_{j=1}^{5} \sum_{k=1}^{5} E\left\{\int_{\left\{T_{j-1}^U \le t \le T_j^U\right\} \cap \left\{T_{k-1}^D \le t \le T_k^D\right\}} e^{-rt} \begin{bmatrix} \sum_{i=j}^{5} SB_i{}^U(w_{ikt}{}^U) + F_{jk}\left(S_{jkt}\right) \\ -\sum_{i=j}^{5} TC^U\left(W_{jt}^U, w_{ikt}^U\right) \end{bmatrix} dt\right\} = \\ &= \max_{w^U} \sum_{j=1}^{5} \sum_{k=1}^{5} E\left\{\int_{\left\{T_{j-1}^U \le t \le T_j^U\right\} \cap \left\{T_{k-1}^D \le t \le T_k^D\right\}} e^{-rt} \begin{bmatrix} \sum_{i=j}^{5} \left(\frac{a_i^U}{b_i^U} w_{it}^U - \frac{1}{2b_i^U} \cdot (w_{it}^U)^2\right) \\ +\eta_1 S_{jkt}, +\eta_2 - (k_2^U - k_1^U W_{jt}^U) \sum_{i=j}^{5} w_{it}^U \end{bmatrix} dt\right\} \end{aligned} \tag{18}$$

subject to the state equation subject to the river basin annual renewable water resource Eq(1), the upstream country water demand Eq.(4), the runoff Eq.(2), the outflow Eq.(3), and the Hamiltonian FBSDEs state system of the downstream country, Eq.(A18). In Appendix 2 one can find an explicit solution of this stochastic maximization problem.

6. ECONOMETRIC MODEL: PRODUCTION FUNCTIONS THROUGH STOCHASTIC FRONTIER ESTIMATION AND WATER DEMAND CURVES

The hydro-economic model shows how all parts of the economy – in our case the sectors (agriculture, residential, mining industry, energy production, tourism) are based on water use directly or indirectly, so are the benefits of U and D. Water is an input (as well as labour, capital, natural capital, etc.) for the production process, hence the inverse demand curves we imposed in section 3, as a way to express the input price-quantity relation. The marginal contribution of water in consumption and production of each sector, can be obtained if in Eq.(6), we collapse all variables, except of w_i, to their means (ceteris paribus). Then we will have a relation of the form $p_i = \hat{f}_i' \ (w_i)$, where $\hat{f}_i$ expresses the maximum Willingness-To-Pay (WTP) by sector i for each unit of water, in a price p_i. The integration of this curve will result the SB of each sector[5].

We propose a stochastic frontier model and a typical quadratic production function, the form of which remains unknown (Brems, 1968). Copulas are used to estimate non-parametrically the dependence between the endogenous regressors and the composed error terms directly, and thus the marginal product function of our hydro-economic model without biases. Bayesian analysis is performed using a Sequential Monte Carlo/ Particle-Filtering approach for the computations (Tsionas, 2017; Tsionas and Mamatzakis, 2018; Tsionas and Mallick, 2019, see Appendix B).

Consider the following stochastic frontier model for the production function(s):

$$y_{it} = \varphi(x_{it}, z_{it}; \beta) + v_{it} - u_{it}, i = 1, \dots, n, t = 1, \dots, T \quad (19)$$

where y_{it} is the output of sector i in time t, $\varphi()$ is an unknown functional form, z_{it} is a $p \times 1$ vector of exogenous inputs, x_{it} is a $p \times 1$ vector of endogenous inputs, β is a $d \times 1$ vectors of unknown parameters, v_{it} is a symmetric random error, u_{it} is the one-sided random disturbance representing technical inefficiency[6]. We assume that z_{it} is uncorrelated with v_{it} and u_{it} but x_{it} is allowed to be correlated with v_{it} and possibly with u_{it}. This, of course, generates an endogeneity problem. We also assume that u_{it} and v_{it} are independent and leave the form of u_{it} unrestricted. The model can be easily extended to the case of exogenous (environmental) variables are included in the distribution of technical inefficiency (e.g. Battese and Coelli,1995; Caudill et al., 1995). To address the endogeneity problem, we propose a copula function approach to determine the joint distribution of the endogenous regressors and the composed errors that effectively capture the dependency among them.

We first assume that $v_{it} \sim$ [7]$i.i.d. N(0, \sigma_v^2)$ and $u_{it} \sim i.i.d. |N(0, \sigma_u^2)|$. Then the density of $\varepsilon_{it} = v_{it} - u_{it} = y_{it} - \varphi(x_{it}, z_{it}; \beta)$ is given by:

$$g(\varepsilon_{it}) = \int_0^\infty f_v(\varepsilon_{it} + u_{it}) f_u(u_{it}) du_{it} = \frac{2}{\sigma} \varphi\left(\frac{\varepsilon_{it}}{\sigma}\right) \Phi\left(-\frac{\lambda \varepsilon_{it}}{\sigma}\right) \quad (20)$$

where $\sigma^2 = \sigma_v^2 + \sigma_u^2$, $\lambda = \sigma_u / \sigma_v$, $\varphi(\cdot)$ and $\Phi(\cdot)$ are the Probability Density Function (PDF) and cumulative distribution function of a standard normal random variable, respectively. To avoid the non-negativity restrictions we make use of the following transformation: $\bar{\lambda} = log(\lambda)$ and $\bar{\sigma}^2 = log(\sigma^2)$. Let $\theta = (\beta', \bar{\lambda}, \bar{\sigma}^2)'$ then the conditional PDF of y given x and z is:

$$f(y_{it} | x_{it}, z_{it}) = \frac{2}{\bar{\sigma}} \varphi\left(\frac{y_{it} - \varphi(x_{it}, z_{it}; \beta)}{\bar{\sigma}}\right) \Phi\left(-\frac{\bar{\lambda}}{\bar{\sigma}_v}(y - \varphi(x_{it}, z_{it}; \beta))\right) \quad (21)$$

and conditional log-likelihood is then given by:

$$log\, L(\theta) = \sum_{i=1}^n \sum_{t=0}^T log\, f(y_{it}; \theta | x, z) \quad (22)$$

From the estimated production function for each of the two countries (considering regional differences in productivity) we can easily obtain their corresponding marginal product function, which is connected with the water use (w_i^h) input variable via Eq.(23)

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[5] As analysed in section 3, the inverse demand curve (Eq. 6) is the marginal SB curve.

[6] The production function used to express the "maximum" output that can be obtained from any fixed and specific set of inputs and describes how inputs are transformed into output. As in reality, cases of reducing outputs by inefficient management (getting less output from its input than the maximum), are considered, by the concept of technically inefficiency (Shephard, 1970; Saari, 2006; 2011), as an one-sided random disturbance.

[7] Independent and Identically Distributed (probability distribution).

(see first paragraph of this section). Consequently, the derived demand curve for water of the producer is represented at equation (24):

$$Marginal\ product = \alpha + \beta \cdot w_i^h \tag{23}$$

$$w_i^h = a + b \cdot price \tag{24}$$

where α,β are water demand parameters (coefficients) of each sector and b water demand price elasticity, estimated as:

$$price\ elasticity = \frac{d(w_{it}^h)}{d(price)} \cdot \frac{price}{w_{it}^h} \tag{25}$$

6.1.Copula approach:

As mentioned, copulas will determine the joint distribution of the endogenous regressors and the composed errors that capture their dependencies (Nelsen, 2006). In this sub-section we scrutinise this concept, taking the function $\varphi()$ as given, while in sub-section 6.2 we elaborate on the dynamic latent productivity.

To this end, let $F(x_1, \dots, x_p, \varepsilon)$ be the joint distribution of $(x_1, \dots, x_p)$ and ε_i. Since the information contained in the correlation between $(x_1, \dots, x_p)$ and ε_i is also contained in its joint distribution, and if this is known to belong to a class of parametric density, then consistent estimates of the model parameters can be obtained by simply maximizing the log-likelihood function derived from $F(x_1, \dots, x_p, \varepsilon)$. Thus, there is no need for resorting to instruments nor to consistently estimate the parameters of the model. However, in practice $F(x_1, \dots, x_p, \varepsilon)$ is typically unknown. Following Park and Gupta (2012) and suggesting a copula to determine this joint density, we can capture the dependence in the joint distribution of the endogenous regressors and the composed errors. More precisely, suppose the joint distribution of $(x_1, \dots, x_p, \varepsilon)$ with joint density$f(x_1, \dots, x_p, \varepsilon)$, and let $f_j(x_j)$, $F_j(x_j)$, for $j = 1, \dots, p$, $g(\varepsilon)$ and $G(\varepsilon)$ denote the marginal density and Cumulative Distribution Function (CDF) of x_j and ε, respectively. Also, C denotes the "copula function" defined for $(\xi_1, \dots, \xi_{p+1}) \in [0,1]^{p+1}$ by:

$$C(\xi_1, \dots, \xi_{p+1}) = P(F_1(x_1) \leq \xi_1, \dots, F_p(x_p) \leq \xi_p, G(\varepsilon) \leq \xi_{p+1}) \tag{26}$$

so that the copula function is itself a CDF.

Moreover, since $F_j(x_j)$ and $G(\cdot)$ are marginal distribution functions, each component $U_j = F_j(x_j)$ and $U_\varepsilon = G(\varepsilon)$ has a uniform marginal distribution (Li and Racine, 2007)[8]. Let $c(\xi_1, \dots, \xi_p)$ denote the PDF associated with $C(\xi_1, \dots, \xi_p)$, then by Sklar's theorem (Sklar,1959), we have:

$$f(x_1, \dots, x_p, \varepsilon) = c\big(F_1(x_1), \dots, F_p(x_p), G(\varepsilon)\big) g(\varepsilon) \prod_{j=1}^{p} f_j(x_j) \tag{27}$$

Thus, Eq.(21) shows that the copula function completely characterizes the dependence structure of $(x_1, \dots, x_p, \varepsilon)$, and $c(\xi_1, \dots, \xi_p) = 1$ if and only if $(x_1, \dots, x_p, \varepsilon)$ are independent of each other. To obtain the joint density, we need to specify the copula function; here the Gaussian copula is used[9]:

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[8] Many producers use their own strategies to maximize profits. The individualistic behaviour of each can be described by modelling the marginals. Copulas can model marginals and multivariate probabilities.

[9] Other copula functions such as Frank, Placket, Clayton, and Farlie-Gumbel-Morgenstern can also be used, but here we used the Gaussian as the most generally robust and wellness of performance (Song, 2000; Danaher and Smith, 2011).

Let $\Phi_{\Sigma,p+1}$ denote a $(p+1)$-dimensional CDF with zero mean and correlation matrix Σ. Then the $(p+1)$ -dimensional CDF with correlation matrix Σ is given by:

$$C(w;\Sigma) = \Phi_{\Sigma,p+1}\big(\Phi^{-1}(U_1), \dots, \Phi^{-1}(U_p), \Phi^{-1}(U_\varepsilon)\big),$$
$$\text{where } w = (U_1, \dots, U_p, U_\varepsilon) = (F_1(x_1), \dots, F_p(x_p), G(\varepsilon)) \quad (28)$$

The copula density is:

$$c(w;\Sigma) = (det(\Sigma))^{-1/2} \times exp\left\{-\frac{1}{2}\big(\Phi^{-1}(U_1), \dots, \Phi^{-1}(U_p), \Phi^{-1}(U_\varepsilon)\big)'\big(\Sigma^{-1} - I_{p+1}\big)\big(\Phi^{-1}(U_1), \dots, \Phi^{-1}(U_p), \Phi^{-1}(U_\varepsilon)\big)\right\} \quad (29)$$

And the log-likelihood function is:

$$log\, L\,(\theta, \Sigma) = \sum_{i=1}^{n}\sum_{t=1}^{T}\Big\{ln\, c\,(F_1(x_{1,it}), \dots, F_p(x_{p,it}), G(\varepsilon_{it};\theta);\Sigma) + \sum_{j=1}^{p} ln\, f_j\,(x_{j,it}) + ln\, g\,(\varepsilon_{it};\theta)\Big\} \quad (30)$$

where $\theta = (\beta', \bar{\lambda}, \bar{\sigma}^2)'$ and the form of $c(.)$ is given in Eq.(22). Notice that the first term in the summation of Eq.(30) is derived from the copula density and reflects the dependence between endogenous variables and composed errors. In addition, since the marginal density $f_j(x_j)$ does not contain any parameters of interest, the second term in the summation of Eq.(30) can be dropped from the log-likelihood function. Finally, it is clear that if there is no endogeneity problem, Eq.(30) collapses to the log-likelihood function of the standard stochastic frontier models.

By maximizing the log-likelihood function, consistent estimates of (θ, Σ) can be obtained, and this can be done as we described by the algorithm below:

A. Estimation of $F_j\big(x_j\big), j = 1, \dots, p$; and $G(\varepsilon\,;\theta)$

Since $F_j(x_{ji})$ are unknown and we have an observed sample of $x_{ji}, j = 1, \dots, p; i = 1, \dots, n$; in the first step, we can estimate $F_j(x_{ji})$ by

$$\tilde{F}_{nj} = \frac{1}{nT+1}\sum_{i=1}^{n} 1\big(x_{j,it} \leq x_{0j}\big), \qquad j = 1, \dots, p \quad (31)$$

where $1(.)$ is an indicator function. Note that we used the rescaling factor $1/(nT+1)$ rather than $1/nT$ to avoid difficulties arising from the potential unboundedness of the $ln\, c\,\big(F_1(x_{1,it}), \dots, F_p(x_{p,it}), G(\varepsilon_{it};\theta);\Sigma\big)$ as some of the $F_j(x_j)$ tend to one. To estimate $G(\varepsilon_{it};\theta)$, note that its density $g(\varepsilon_{it}\,;\theta)$ is given in Eq.(20) and by definition, $G(\varepsilon_{it};\theta) = \int_{-\infty}^{\varepsilon_{it}} g(s;\theta)ds$, thus $G(\varepsilon\,;\theta)$ can be estimated using numerical integration, and let denotes the estimator of $G(\varepsilon\,;\theta)$.

B. Maximization of the log-likelihood function

Maximization of the log-likelihood function of Eq.(30) where $F_j(x_j)$ and $G(\varepsilon_{it}\,;\theta)$ are replaced by their estimates $\tilde{F}_j(x_j)$ and $\tilde{G}\,(\varepsilon_j;\theta)$, respectively:

$$(\hat{\theta}, \hat{\Sigma}) = \underset{\theta\in\Theta,\Sigma}{argmax} \sum_{i=1}^{n}\Big\{ln\, c\,(\tilde{F}_1(x_{1i}), \dots, \tilde{F}_p(x_{pi}), \tilde{G}(\varepsilon_i;\theta);\Sigma) + ln\, g\,(\varepsilon_i;\theta)\Big\} \quad (32)$$

C. Estimating Technical Inefficiency

Once the parameters have been estimated, the ultimate goal is to predict the technical inefficiency values (term u_i). This can be calculated based on Jondrow et al. (1982):

$$\hat{u}_{it} = \hat{E}(u_{it}|\varepsilon_{it}) = \frac{\hat{\sigma}\hat{\lambda}}{1+\hat{\lambda}^2}\left[\frac{\varphi\left(\frac{\hat{\lambda}\hat{\varepsilon}_{it}}{\hat{\sigma}}\right)}{1-\Phi\left(\frac{\hat{\lambda}\hat{\varepsilon}_{it}}{\hat{\sigma}}\right)} - \frac{\hat{\lambda}\hat{\varepsilon}_{it}}{\hat{\sigma}}\right] \quad (33)$$

where $\hat{\varepsilon}_{it} = y_{it} - \varphi(x_{it}, z_{it}; \hat{\beta})$ and $\hat{\beta}, \hat{\lambda}$ and $\hat{\sigma}^2$ are the parameter estimates obtained from the approach discussed above.

6.2.Local likelihood estimation:

The functional form $\varphi(x_{it}, z_{it}; \beta)$ was left unspecified so far. By all means, any parametric form can be used, but here we focus on non-parametric estimation by the local likelihood method. We use the simpler notation $\varphi(x_{it}; \beta)$ as the extension to the case of exogenous covariates is straightforward. Since we have a multivariate covariate, we use the method of local linear estimation. This means that all parameters of the model become functions of x, and they are denoted by $\theta(x)$. We denote the conditional density of y given x by $p(y|x) = g(y; \theta(x))$, where $\theta(x) \in \mathbb{R}^k$ is unknown and we define $q(y; \theta(x)) = log\, g\,(y; \theta(x))$. For example, a standard frontier would take the form:

$$y_{it} = m(x_{it}) + v_{it} - u_{it},$$
$$where\ v_{it}|x_{it} \sim N\big(0, \sigma_v^2(x_{it})\big), u_{it}|x_{it} \sim N\big(0, \sigma_u^2(x_{it})\big).\ Then: \quad (34)$$

$$\theta(x) = [m(x), \sigma_v^2(x), \sigma_u^2(x)]' \quad (35)$$

Our fundamental departure from the standard model is the introduction of productive performance or technical efficiency:

$$y_{it} = m(x_{it}, z_{it}) + v_{it} + \omega_{it} - u_{it} \quad (36)$$

where the productivity process is:

$$\omega_{it}|x_{it}, \omega_{i,t-1} \sim N\left(r\big(\omega_{i,t-1}, x_{it}, z_{it}\big), \sigma_\omega^2\big(\omega_{i,t-1}, x_{it}, z_{it}\big)\right) \quad (37)$$

In this specification, $r(\omega_{i,t-1}, x_{it}, z_{it})$ is a non-parametric productivity mean process, and $\sigma_\omega^2(\omega_{i,t-1}, x_{it}, z_{it})$ is the variance. For ease in notation, we omit explicit dependence on z and we continue to denote $\theta(x) \in \mathbb{R}^k$ with

$$\theta(x) = [m(x), r(\omega_{-1}, x), \sigma_v^2(x), \sigma_u^2(x), \sigma_\omega^2(\omega_{-1}, x)]' \quad (38)$$

where ω_{-1} denotes the lagged value of productivity. As productivity is latent special problems are introduced into the analysis.

There is a multivariate kernel which satisfies:

$$\int K(u)du = 1, \quad \int uu'\, K(u)du = \mu_2 I_d \quad (39)$$

To fix notation, we start with the analysis of the simpler model in Eq.(36). The conditional local linear log-likelihood is given by[10]:

$$log\, L\,(\theta_o, \Theta_1) = \sum_{i=1}^{n}\sum_{t=1}^{T} q(\,y_{it}, \theta_o + \Theta_1(x_{it} - x))K_H(x_{it} - x) \quad (40)$$

where θ_o, Θ_1 is a vector $(k \times 1)$ and matrix $(k \times d)$ respectively, H is a bandwidth matrix which is symmetric, positive definite and $K_H(u) = |H|^{-1}K(H^{-1}u)$. We choose a multivariate product kernel so that $K(u) = \prod_{j=1}^{d} K_j\,(u_j)$ in which case $\int uu' K(u)du = (\int u_1^2\, K_1(u_1)du_1)\, I_d$.

[10] We include z_{it} in the kernel functions because, in this instance, they represent important environmental variables that help in modeling heterogeneity. For ease in notation we redefine x=[x', z']'.

The local linear estimator is $\hat{\theta}(x) = \hat{\theta}_o(x)$ where $\hat{\theta}_o(x)$ and $\hat{\Theta}_1(x)$ maximize the log-likelihood $L(\theta_o, \Theta_1)$ with respect to θ_o, Θ_1. Computational details can be found in (Kumbhakar et al., 2007-Sections 3.1, 3.2) as also followed in this paper.

For the model with latent productivity ω_{it} as in Eq.(37) the likelihood function is

$$L(\theta_o, \Theta_1) = \int_{\mathbb{R}^{nT}} \{\prod_{i=1}^{n} \prod_{t=1}^{T} g(y_{it}, \omega_{it}, \theta_o + \Theta_1 (\Lambda_{it} - \Lambda)) \cdot K_H(\Lambda_{it} - \Lambda)\} d\omega \quad (41)$$

where $\Lambda_{it} = [x_{it}', \omega_{i,t-1}]'$, $\Lambda = [x', \omega_{-1}]'$, and

$$g(y, \omega; \theta(\Lambda)) = \frac{2}{\sigma(x)} \varphi\left(\frac{y_{it} - \varphi(x_{it}; \beta(x)) - \omega_{it}}{\sigma(x)}\right) \Phi\left(-\frac{\lambda(x)}{\sigma_v(x)} (y - \varphi(x_{it}; \beta(x))) - \omega_{it}\right) \cdot \frac{1}{\sigma_\omega(x, \omega_{-1})} \varphi\left(\frac{\omega_{it} - r(x_{it}, \omega_{i,t-1}; \gamma(x, \omega_{-1}))}{\sigma_\omega(x, \omega_{-1})}\right) \quad (42)$$

Moreover, $\gamma(x, \omega_{-1})$ denotes the localized parameters in the r() function of Eq.(37). For ease in notation, we define $\theta(x, \omega_{-1}) = [\beta(x)', \gamma(x, \omega_{-1})']' \in \mathbb{R}^k$. In Eq.(41) there is an nT-dimensional integral which cannot be evaluated analytically, which is obvious from the definition of Eq.(42). The computation relies in two steps:

Step 1: Integrate out $\{\omega_{it}\}$ from Eq.(41) using a Sequential Monte Carlo (SMC) algorithm (Pitt and Shephard, 1999).

Step 2: Maximize the resulting expression using numerical optimization techniques.

For reasons of computational convenience and without sacrificing generality we assume:

$$\omega_{it} = \rho\omega_{i,t-1} + \xi_{it}, \{\xi_{it}\} \sim i.i.d. N(0, \sigma_\xi^2) \quad (43)$$

We will still need the SMC algorithm in step 1 (Appendix B), for which we used 10^6 particles per likelihood evaluation, and a standard conjugate gradients algorithm for maximization. Our results were insensitive to using 105 or 107 particles per likelihood evaluation.

7. RESULTS AND DISCUSSION

7.1. Production functions and water demand functions

In this section we present a simple nonparametric estimation of the production function per sector in Ethiopia and Kenya. Human input (labour, machinery), land, and ecosystem-based inputs need to be accounted in production function estimations, which lead to the integrated hydro-economic modelling (the existence of natural capital[11] is necessary to characterise water resources in each country). For each sector involved data on Natural Capital were collected using Environmental Indices (EI) as approximations of both quality and quantity, indicatively shown in Table 1, in detail described in Appendix C.

Table 1: Factors (data) used per sector.

Sector	Factors
Agriculture	Land use (agricultural area, arable land, permanent crops, total area equipped for irrigation), forest

[11] Natural Capital is linked with its Ecosystem Services (ES), e.g. provisioning services (water, food), regulating services (flood prevention, erosion control), supporting-habitat services (biodiversity), cultural-recreational services (tourism). Based on these categories we selected the factors per sector.

	Soil erosion/degradation Agricultural production, fishery production, aquaculture production Use of pesticides /fertilizers Raw materials (biomass)
Energy	Energy for renewable resources Dam capacity
Mining	Raw materials (construction material, and total fossil fuel)
Tourism	International tourism, Expenditures Number of arrivals Terrestrial Conservation Areas
Residential Water Supply	Access to clean water
*all EIs were converted to same scale and units through normalization (log means)	

The results of the nonparametric estimation are presented below (Table 2), following the Copula function approach and production frontier analysis, described in the previous section. From the estimated production functions we can easily obtain their corresponding marginal product function, which is connected with the water use input variable, according to Eq.(23) (see also Fig.5). The estimated α,β parameters have the expected signs, which define the form of the demand curves.

Regarding the price elasticity, which is also presented in Table 2, based on Eq.(24)-(25), as expected, all sectors are exceptionally inelastic to a price change for water use (price cannot affect water use). Agriculture seems to be perfectly inelastic to any price change, which means that in both countries the demand will remain stable for any price change. This implies an extremely strong relationship between the input (w_i) and the corresponding crop output, since the producer lacks alternatives, actually depends on the scarce water resources, which is highly valued. These well-known findings that are confirmed by our results, strengthen the validity of the proposed framework.

Table 2: Parameters α, β and price elasticities per sector.

Empirical results: β parameter for each sector					
	Mining	**Energy**	**Tourism**	**Residential**	**Agriculture**
Ethiopia	-0.0010	-0.0014	-0.0012	-0.0013	-0.0000321
Kenya	-0.0011	-0.0013	-0.0010	-0.0015	-0.0000319
Empirical results: α parameter for each sector					
Ethiopia	1.80	1.73	1.48	1.65	1.48
Kenya	1.54	1.70	1.56	1.77	1.56
Empirical results: water price elasticity for each sector					
Ethiopia	-0.099	-0.131	-0.096	-0.116	-0.003
Kenya	-0.092	-0.120	-0.085	-0.143	-0.003

The respective demand curves (Eq.(24)), provide an ordering of these sectors via their demand function intercepts (Fig.3). Sequential "exits from the market" are defined by the relative importance of sector-specific demand parameter ratio a, with a=α/β. As w_i reaches zero sequentially, its price increases revealing producers' preferences for water use. At these prices, in Ethiopia, Tourism sector should exit the market first followed by Residential and Energy sectors, while in Kenya, Mining would exit the market first trailed by the Tourism sector. Moreover, mining producers in Kenya value higher the water than in Ethiopia, and that happens because Kenya relies strongly on groundwater for mining production. In both cases, in case of river/lake depletion, agriculture sector should be the last one to exit the market, since it is valuing water use more than any other sector.

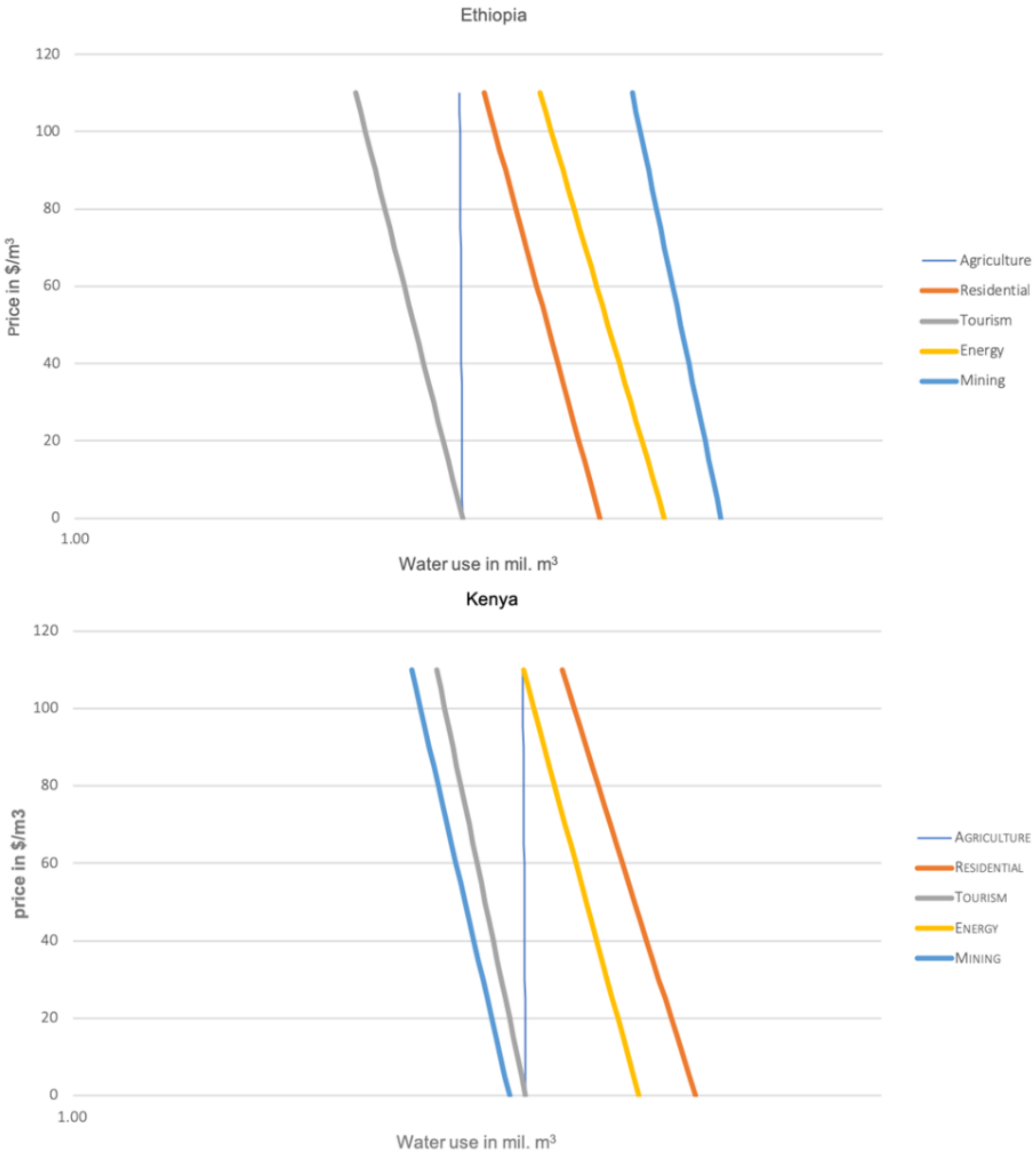

Figure 3: Water demand curves per sector for each country.

The water price elasticity (Eq.25,b) sampling distributions tend not to vary significantly between the two countries (Fig.4). Except of the Ethiopian residential sector's distribution which seems like a normal distribution, the others slightly diverge from the normal distribution at their tails, showing disorders during extreme cases. None of these means is the mode of the distribution as well, although the chasm between those values is not notable. In economic terms, the elasticities for water demand in each sector do not deviate remarkably, letting so similar behavioural patterns to be observed in each sector across the two countries.

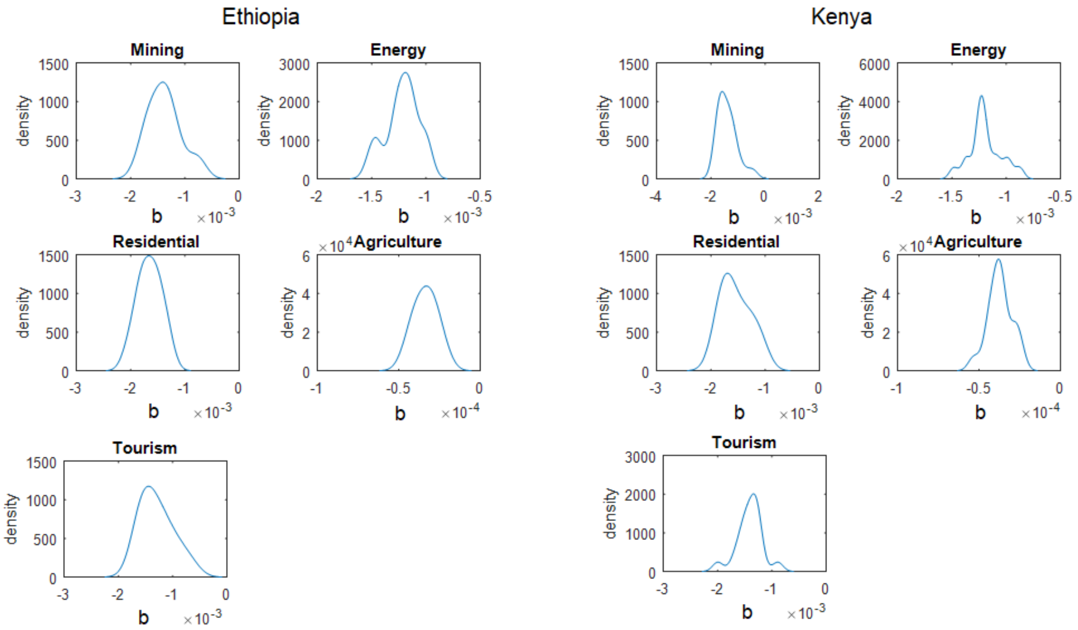

Figure 4: Sampling distributions of water price elasticities by sector for both countries.

The second parameter of the inverse demand curve is the constant term (α), which is responsible for the starting point of the demand curve, revealing the stakeholders' WTP per sector. Figure 5 shows the distributions of constant terms of the inverse demand functions and interestingly we can see that in most cases the WTP for water use in energy sector is greater than the corresponding one in agriculture and tourism, which implies greater profitability in energy sector. Additionally, in terms of WTP, mining sector in Ethiopia, which follows a leptokurtic distribution seems to be the most stable one. The technical inefficiency parameter u_{it} (Eq.(19),(33)) shows how (in)efficiently the water-input is transformed into production output (Fig.5). Mining and Residential sectors in Ethiopia follow exactly the same distribution with a positive skew to the right. Energy and Tourism in both countries, u_{it} has two district peaks (bimodal distribution), which indicates that in these sectors there are two groups of producers: some of them achieve to maximize their outputs given their inputs, while some others do not with technical inefficiency taking greater values than the former group. However, it is noteworthy that Energy sector is more technically efficient compared with Tourism, since the lowest peak of Tourism is as great at the biggest one of Energy sector.

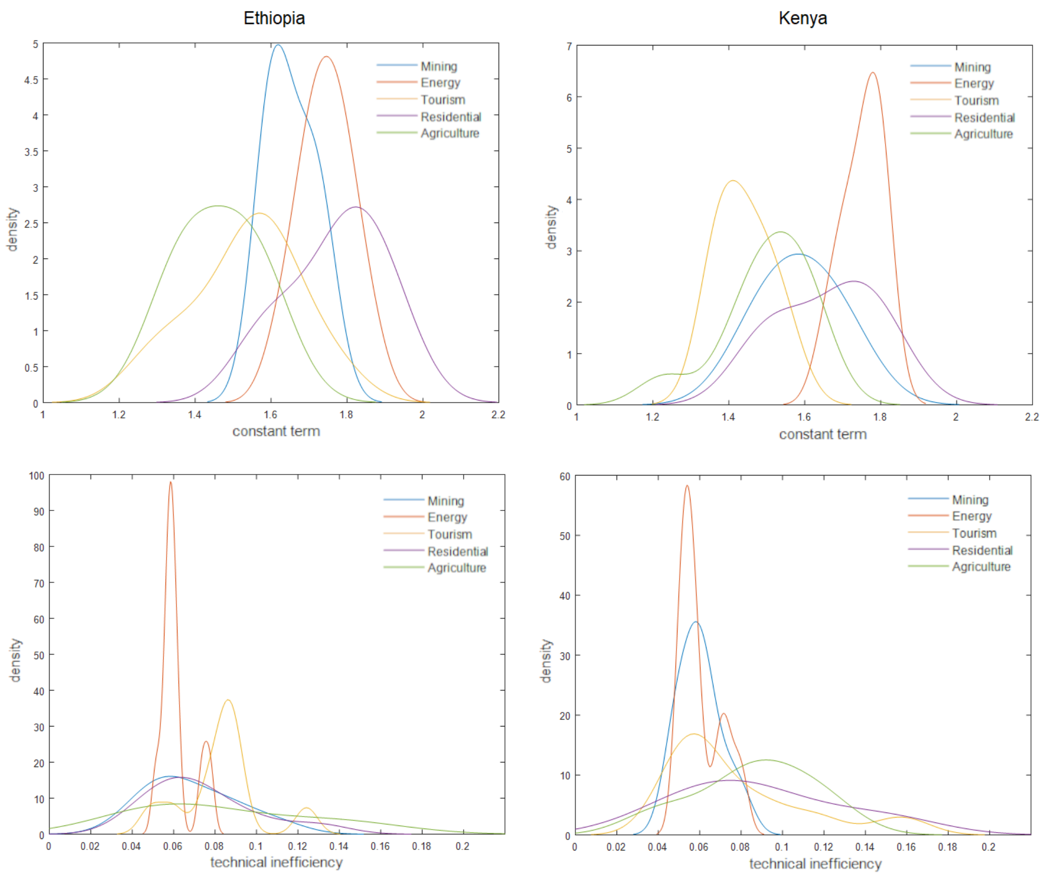

Figure 5: Sampling distributions of constant terms (up) and technical inefficiency (down) by sector for each country.

All the above 'clues' that can be derived from the two graphs, justify the proposed framework in terms of selecting a multi-sectorial approach, and introducing the term of technical inefficiency. Those novel elements give a significantly added value compared with the more 'narrowed' approaches so far.

7.2.Games under uncertainty

Historical hydrological data of the basin (e.g. precipitation, runoff of the Omo River to Lake Turkana, and evaporation/ ET), can be used to estimate their corresponding historical volatilities, σ, as in Eq.(1)-(3), and storage of the lake, as in Eq.(5), while pumping costs per country can be used to represent water tariffs (detailed data and parameters of the solved models can be found in Appendix C, Table C.2). Subsequently, the stochastic optimization hydro-economic model, for both game cases can be solved with the described decoupling method for linear FBSDEs (section 3). For the sake of scale consistency, the optimal water abstraction and the resulting NB are presented via the percentage of the water availability inside the river basin over the total water availability of each of the two countries.

Regarding the game, both players have two available strategies:

- myopic (the country follows short-term water exploitation, without considering the benefits coming from the natural resource sustainable use, i.e. from the river for U and from the lake for D): A myopic strategy amounts to the depletion of the resource that is owned as a common property. In the myopic equilibrium, the marginal benefit of the water use equals current marginal extraction cost, ignoring the water scarcity rents (conventional user costs) that represent instantaneous benefit of foregoing water extraction currently as a means of reducing future extraction costs. Analytically, the NB function is maximized without taking into account the constraint imposed by the resource (state) equation.

- non-myopic (consider natural resource and long-term plan – preservation benefits). In a non-myopic strategy, the marginal benefit of the water use equals current marginal extraction cost plus marginal user cost (as defined above). Analytically, the NB function is maximized subject to the constraint imposed by the resource (state) equation.

Non-cooperative case:

The optimal scenario would be a Non-myopic–Non-myopic combination, where the lake runs out of water after 33 years, while the worst-case scenario in environmental terms is realised when both countries follow a myopic strategy, where the Lake Depletion Time (LDT) is 15 years, accompanied by lack of trust, institutions bridging the limited disposable information, or a limited technical support (Table 3).

Table 3: non-cooperative case for myopic and non-myopic combinations.

Downstream (Kenya)- D	**Upstream (Ethiopia)- U**	
	Myopic	Non-Myopic
Myopic	$NB_u = \$1.5191 \cdot 10^9$ $NB_D = \$2.8429 \cdot 10^7$ $LDT = 15.49$ years	$NB_u = \$1.4635 \cdot 10^9$ $NB_D = \$5.747 \cdot 10^7$ $LDT = 23.62$ years
Non-Myopic	$NB_u = \$1.5188 \cdot 10^9$ $NB_D = \$1.5141 \cdot 10^7$ $LDT = 22.85$ years	$NB_u = \$1.4637 \cdot 10^9$ $NB_D = \$2.2543 \cdot 10^7$ $LDT = 33.35$ years

Although Kenya on average seems to gain more at the myopic case, the total losses of that strategy surpass the gains, as for fifteen more years it could have an average net benefit equal to $\$2.2543 \cdot 10^7$, while from the myopic perspective it is zero. So, if Kenya (D) controls its water use over time (non-myopic), it can increase its total benefits from \$743,919,000 to \$1,321,810,000 no matter what Ethiopia decides, while in the myopic equilibrium it gains only \$342,435,000. At the same time, Ethiopia (U) has every time higher NBs in the non-myopic strategy. However, Ethiopia's negative externalities to Kenya in the event of both following the myopic strategy can be seen at the LDT (in half of the time compared to the non-myopic strategies).

NB values represent the average value of the economy as long as there is water. Ethiopia's benefit curves are the average of a 200-year period, where there is no sector exit, while Kenya's benefit curves are the average of 15- to 33-year period, until the point, where first all sectors leave, and the lake depletes. Hence, in myopic-myopic combination, the 16th year in Kenya is characterized by zero SB and costs, while all the demand for goods and services is met by imports.

The water use of all sectors in both countries (Fig.6) is characterised by increased rates and faster depletion in the myopic-myopic case, compared to the non-myopic–non-myopic one. Kenya's water use becomes zero at the LDT (15.5 years for the myopic-

myopic and 33.4 years for the non-myopic–non-myopic case). Ethiopia's time horizon is 200 years, to indicate the lack of limitations on water reserves of Omo River.

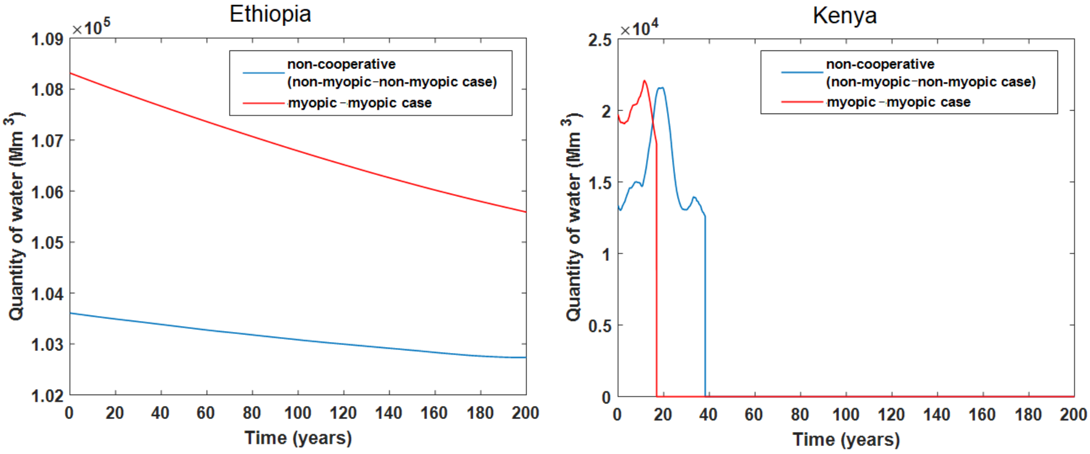

Figure 6: Total water use for the two extreme strategy-combinations.

Cooperative case:

In this case that the players benefit from their goods' exchange, NBs are higher for both[12]. So, the most crucial concept is relative efficiency. After a three-case numerical exploration of η_1 coefficient of Eq.(12), for a number of periods, it seems that the lake does not deplete under the cooperative case. This very promising outcome is important for both countries, because since they trade, there is interest in the sustainable development of the neighbours. Table 4 presents the indicative results of the solutions in terms of maximized NBs and lake depletion times.

Table 4: Cooperative case: a numerical simulation of different n values to optimize NB.

Cooperative Case: Optimal $(\eta_1 = \mathbf{0.7})$	**Cooperative Case: Optimal** $(\eta_1 = \mathbf{0.8})$	**Cooperative Case: Optimal** $(\eta_1 = \mathbf{0.9})$
$NB_u = \$24.075 \cdot 10^9$ $NB_D = \$3.8182 \cdot 10^7$ $LDT = \text{Never}$	$NB_u = \$30.992 \cdot 10^9$ $NB_D = \$4.0333 \cdot 10^7$ $LDT = \text{Never}$	$NB_u = \$39.74 \cdot 10^9$ $NB_D = \$4.0388 \cdot 10^7$ $LDT = \text{Never}$

Apparently, for all possible outcomes given the preferences of Ethiopia, NB are outstandingly greater than the non-cooperative case (Fig.7 – indicatively for the least possible rate of η_1), not to mention the sustainability of the lake (LDT=never). Thus, indisputably the cooperative is the best strategy, and the more beneficial for both players as η_1 increases.

[12] As Ricardo showed 200 years ago, even if e.g. Ethiopia, can produce all goods and services cheaply than Kenya, they can still trade under conditions where both get benefited.

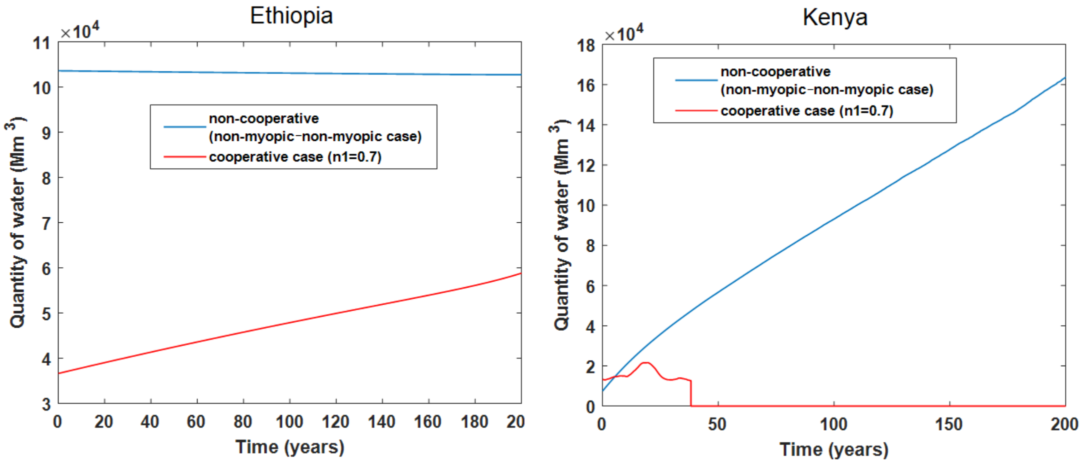

Figure 7: Total water use in for the best non-cooperative case versus the cooperative case (for the lower η_1).

In this graph, Ethiopia realises the upcoming benefits coming from giving up a considerable amount of water in exchange of food supply produced by the downstream country. In response (reaction), Kenya significantly increases its water use over the years, to increase production. Moreover, the total water use of both countries in the cooperative case is less compared to the non-cooperative (Kenya's peak in the 20^{th} year is seven times less than Ethiopia's maximum use).

Uncertainty effects:

As analysed, the impact of altering the volatility of the hydrological variables, will affect both water stocks and NBs. The comparative results for the non-cooperative and cooperative cases, are presented in Table 4, for the maximum observed historical changes.

Table 4: Hydrological variability impacts on NBs and LDT.

Hydrological changes	**Non-Cooperative Case**	**Cooperative Case ($\eta_1 = 0.7$)**	**Comments**
Increase of outflow volatility $\sigma_O = 0.3$	$NB_u = \$1.639 \cdot 10^9$ $NB_d = \$2.6384 \cdot 10^7$ $LDT = 33.27$	$NB_u = \$24.269 \cdot 10^9$ $NB_d = \$3.8216 \cdot 10^7$ $LDT =$ Never	Significantly impacts Kenya's water stock (the levels of the lake vary, increasing the chances of droughts/floods). Its NBs in the cooperative case are slightly higher compared to the non-cooperative. The most significant change is on Ethiopia's NBs.
Increase of precipitation volatility $\sigma_W = 0.3$	$NB_u = \$1.7031 \cdot 10^9$ $NB_d = \$1.3903 \cdot 10^7$ $LDT = 18.94$	$NB_u = \$4.4280 \cdot 10^9$ $NB_d = \$2.8968 \cdot 10^7$ $LDT = 99.57$	Different impact for U,D: Ethiopia gains more when it does not trade (more water = more consumption, less water = consumes as it needs, limiting runoff to Kenya). Kenya depends on the trades, so it adjusts its water use.
Decrease of runoff variability $\sigma_R = 0.1$	$NB_u = \$1.6390 \cdot 10^9$ $NB_d = \$2.5545 \cdot 10^7$ $LDT = 31.60$	$NB_u = \$\,24.091 \cdot 10^9$ $NB_d = \$\,3.4510 \cdot 10^7$ $LDT =$ Never	Here a decrease in the runoff to the lake is considered (by 0.331 compared to the BAU) imposing a sharpened water scarcity in the future. The results are similar to the first (outflow) uncertainty case, regarding Ethiopia, while Kenya adjusts its water use, but it can be sustained only under cooperation.
*the three types of hydrological uncertainty are presented as independent cases to show which one affects more the NBs and the water stocks (sensitivity), however combinations can be also explored.			

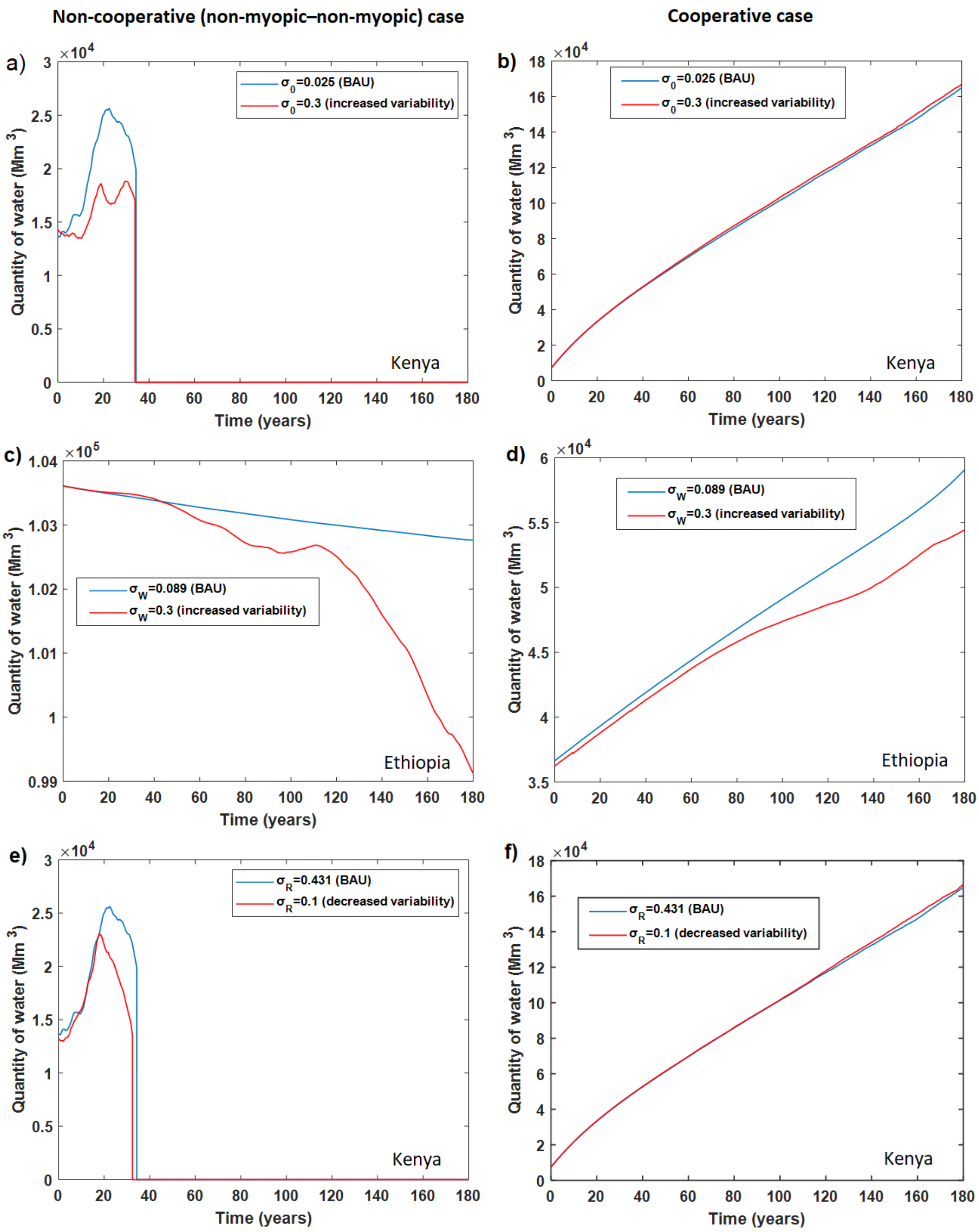

Figure 8: σ_O variability's effects on Kenya's water use (first row), σ_W variability's effects on Ethiopia's water use (second row), σ_R variability's effects on Kenya's water use (third row): Comparison of non-cooperative non-myopic–non-myopic (left column), and cooperative case (right column).

In the non-cooperative case (Fig.8a,b), Kenya tries to adjust its water consumption due to the increased outflow volatility (so to save water). In the cooperative case, no country changes its behaviour, as there is no risk of drought due to mutual assistance. Ethiopia's water consumption (Fig.8c,d) tends to zero, indicating the short-term planning. In cooperation, the behaviour is almost the same, allowing the trades, and NBs are also higher compared to the non-cooperative case. That difference would be enough to motivate both countries to keep on trading even under extremes. Although runoff decreased (Fig.8e,f), NBs and water consumption do not change significantly. Under cooperation, even with uncertain runoff, Ethiopia and Kenya can continue to use almost the same water quantity, unlike to the non-cooperative case, where Kenya slightly reduces its water use, to gain $2,002,000 more, but the lake depletes earlier than the BAU scenario.

In the studied basin, the life-dependance between water resources and survival (not just economy) is well described in the demand curves. This mandates a rational and sustainable water resources management that will lead to *globally-optimum* results. Under any conditions, cooperation seems to be a win-win sustainable strategy, for both countries, environment, and economy. The results are in agreement with previous studies, e.g. Dinar (2009) argues that under increased water supply variability, cooperation should be preferred to address the risks, and this is now proved. An international agreement would strengthen this strategy, because at the time any trade-offs depend on governmental decisions. Hydrological uncertainties put into risk most cooperative decisions: Dinar et al. (2010) found a bell-shaped relationship between water supply variations and cooperation agreements; Ansink and Ruijs (2008) also demonstrate that a decrease in average river flows reduces the stability of an agreement, while an increase in variance may have both positive and negative effects.

CONCLUSIONS

In this work, a framework for scarce transboundary water resources management was presented. Game theory, hydro-economics, and econometrics were combined to explore the optimal strategies in environmental and economic terms, while the whole system was tested under hydrological uncertainty.

The conceptual framework is quite simple, while the analytical solution is provided, to make possible its replication. It is based on the principles of water balance, marginal and total costs, net and social benefits, while a novel element was the stochastic consideration of its hydrological components. The stochastic Stackelberg differential game approach was successful and enabled the evaluation of numerous potential strategies. The econometric model's contribution is also deemed essential for planning, as it provided production functions for all sectors for both countries, which was expressed as their social benefits, and the derived water demand curves. A novel mathematical approach was demonstrated to address the endogeneity issues of the production functions' inputs, combining different tools, in order to provide a realistic representation of the problem. As said in the previous section, the findings that can be derived from the results of the technical inefficiency in water use, and the participation of all the five sectors of the economy, could not be obtained with any previous approach. The management insights that a policymaker can consider from the results are very important both in the short- and long-term planning. The conceptual hydro-economic model, with the game cases under uncertainty that we presented, completes the integrated character of the proposed framework. The novel character of this contribution is based on its detailed hydro-economic and sophisticated mathematical modelling, which identifies easily the most solid and "win-win" management strategies, supporting thus the sustainable decision-making and planning.

We chose not to present thoroughly the data collection and preparation (the techniques are well-known – for that reason, we provide Appendix C) because the aim of this work is to demonstrate the proposed framework, rather than a case-study application. However, some specific conclusions are worth-mentioned. The analysis proved the vital role of water resources to any continuation and development of the economic activities. It is well known that as the price of a good rises, buyers will choose to buy less of it, and as its price falls, they buy more: as water price increases over time due to water scarcity, the demand for all economic sectors reaches zero sequentially. The way this finding was proved (showing also the ordering of the sectors who will reach zero) is a novel element, and combined with the examined game strategies, it is proved that under any circumstances, cooperation is the overall optimal strategy. Under cooperation scenario, the upstream country realises the upcoming benefits coming from giving up a considerable amount of water to the downstream country, in exchange of their produced food supply, over time. The reaction of the downstream country is the increment of its water use to increase production. So over time, it turns out to be more profitable for both countries the case where the downstream one uses more water than the upstream, which currently seems utopic. A swift in selfish and opportunistic mindsets is required, so both countries can secure a future water availability, sustainable access to the input resource-driver of their economic growth, and exploit the mutual benefits of cooperation and collaboration.

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APPENDIX A.

NON-COOPERATIVE CASE

<u>*Upstream*</u>: The Hamiltonian for the j-exit stage is:

$$H_j^U\left(W_{jt}^U, w_{jt}^U, \dots, w_{5t}^U, \lambda_{jt}^U\right) = \sum_{i=j}^{5}\left[\frac{a_i^U}{b_i^U} w_{it}^U - \frac{1}{2b_i^U} \cdot (w_{it}^U)^2 - (k_2^U - k_1^U W_{jt}^U) w_{it}^U\right] + \lambda_{jt}^U\left[W_{jt}^U - \sum_{i=j}^{5} w_{it}^U\right], \quad j = 1,2,\dots,5 \qquad \text{(A1)}$$

where λ_{jt}^U is the j-exit stage adjoint variable that represents water scarcity rents for U country. The necessary conditions for the optimality are given as follows:

$$\frac{\partial H_j^U}{\partial w_{it}^U} = \frac{a_i^U}{b_i^U} - \frac{1}{b_i^U} w_{it}^U - \left(k_2^U - k_1^U W_{jt}^U\right) - \lambda_{jt}^U \quad i = j,\dots,5,\ j=1,2,\dots,5 \qquad \text{(A2)}$$

$$d\lambda_{jt}^U = \left[-\frac{\partial H_j^U}{\partial W_{it}^U} + r\lambda_{jt}^U\right]dt = \left[-k_1^U \sum_{i=j}^{5} w_{it}^U + r\lambda_{jt}^U\right]dt \quad \text{(A3)}$$

The first optimality condition gives:

$$w_{it}^U = a_i^U - b_i^U\left(k_2^U - k_1^U W_{jt}^U\right) - b_i^U \lambda_{jt}^U, \qquad i = j, \ldots, 5 \quad \text{(A4)}$$

Then substituting to the state equation (Eq.4), we have:

$$dW_{jt}^U = \left[-k_1^U\left(\sum_{i=j}^{5} b_i^U\right)W_{jt}^U + \left(\sum_{i=j}^{5} b_i^U\right)\lambda_{jt}^U + W_t - \sum_{i=j}^{5} a_i^U + k_2^U \sum_{i=j}^{5} b_i^U\right]dt \quad \text{(A5)}$$

while substituting to the adjoint Eq.(A3) we have

$$d\lambda_{jt}^U = \left\{-(k_1^U)^2\left(\sum_{i=j}^{5} b_i^U\right)W_{jt}^U + \left[k_1^U\left(\sum_{i=j}^{5} b_i^U\right) + r\right]\lambda_{jt}^U - k_1^U \sum_{i=j}^{5} a_i^U + k_1^U k_2^U \sum_{i=j}^{5} b_i^U\right\}dt \quad \text{(A6)}$$

Setting $A_j^U \triangleq \sum_{i=j}^{5} a_i^U$ and $B_j^U \triangleq \sum_{i=j}^{5} b_i^U$ we obtain the forward-backward stochastic differential equations system (FBSDEs):

$$\begin{aligned} dW_{jt}^U &= \left[-k_1^U B_j^U W_{jt}^U + B_j^U \lambda_{jt}^U + W_t - A_j^U + k_2^U B_j^U\right]dt, \\ d\lambda_{jt}^U &= \left[-(k_1^U)^2 B_j^U W_{jt}^U + \left(k_1^U B_j^U + r\right)\lambda_{jt}^U - k_1^U A_j^U + k_1^U k_2^U B_j^U\right]dt, \\ W_{00}^U &= w_0,\ \lim_{t\to\infty} \lambda_{jt}^U = 0,\ j = 1,2,\ldots,5 \end{aligned} \quad \text{(A7)}$$

To solve the above system of FBSDEs we impose a solution of the form:

$$\lambda_{jt}^U = N_{jt}^U W_{jt}^U + M_{jt}^U,\ i = j, \ldots, 5 \quad \text{(A8)}$$

where N_{jt}^U and M_{jt}^U are stochastic processes to be determined. Taking differentials, we have:

$$d\lambda_{jt}^U = W_{jt}^U dN_{jt}^U + N_{jt}^U dW_{jt}^U + dM_{jt}^U = W_{jt}^U dN_{jt}^U + dM_{jt}^U + \left\{\left[B_j^U\left(N_{jt}^U\right)^2 - k_1^U B_j^U N_{jt}^U\right]W_{jt}^U + B_j^U N_{jt}^U M_{jt}^U + \left[W_t - A_j^U + k_2^U B_j^U\right]N_{jt}^U\right\}dt \quad \text{(A9)}$$

while from the backward equation of the system (Eq.A7) we have:

$$d\lambda_{jt}^U = \left\{\left[\left(k_1^U B_j^U + r\right)N_{jt}^U - (k_1^U)^2 B_j^U\right]W_{jt}^U + \left(k_1^U B_j^U + r\right)M_{jt}^U - k_1^U A_j^U + k_1^U k_2^U B_j^U\right\}dt \quad \text{(A10)}$$

Sufficient conditions for the last two relationships to be equivalent are:

$$\begin{aligned} dN_{jt}^U &= \left[-B_j^U\left(N_j^U\right)^2 + \left(2k_1^U B_j^U + r\right)N_{jt}^U - (k_1^U)^2 B_j^U\right]dt, \\ \lim_{t\to\infty} N_{jt}^U &= 0 \end{aligned} \quad \text{(A11)}$$

which is a Backward Riccatti Equation (BRE) that can be solved numerically for N_{jt}^U,

And

$$\begin{aligned} dM_{jt}^U &= \left[\left(-B_j^U N_j^U + k_1^U B_j^U + r\right)M_{jt}^U - \left(W_t - A_j^U + k_2^U B_j^U\right)N_j^U - k_1^U A_j^U + k_1^U k_2^U B_j^U\right]dt \\ \lim_{t\to\infty} M_{jt}^U &= 0,\ j=1,2,\ldots,5 \end{aligned} \quad \text{(A12)}$$

Substituting the linear solution form of Eq.(A8) to the forward equation of the FBSDEs system:

$$\mathrm{dW^U_{jt}} = \left[\left(-\mathrm{k^U_1} + \mathrm{N^U_{jt}}\right)\mathrm{B^U_{jt}}\mathrm{W^U_{jt}} + \mathrm{B^U_j}\mathrm{M^U_j} + \mathrm{W_t} - \mathrm{A^U_j} + \mathrm{k^U_2}\mathrm{B^U_j}\right]\mathrm{dt}, \qquad \mathrm{W^U_{00}} = \mathrm{w_0} \tag{A13}$$

Which is a forward linear SDE that can be solved for $\mathrm{W^U_{jt}}$. Then the backward adjoint variable λ^U_{jt} follows from Eq.(A8) and the optimal water use $\mathrm{w^U_{jt}}$ follows from the optimality condition (Eq.A4).

Downstream: The maximization function, as described in the main text (water balance concept) and also shown in Fig.2, it subjects to water balance of U, the runoff in D, water stock of D (state equation), and the water use of D. For the (j,k)-th exit-stage, we have the Hamiltonian:

$$H^D_{jk}\left(S_{jk}, w^D_{jkt}, \ldots, w^D_{j5t}, \lambda^D_{jkt}\right) \triangleq \sum_{l=k}^{5}\left[\frac{a^D_l}{b^D_l} w^D_{jlt} - \frac{1}{2b^D_l}\cdot (w^D_{jlt})^2\right] - (k^D_2 - k^D_1 S_{jkt})\sum_{l=k}^{5} w^D_{jlt} + \lambda^D_{jkt}\left[W_t - \sum_{i=j}^{5} w^U_{it} - \sum_{l=k}^{5} w^D_{jlt} + R_t - O_t\right], \quad j,k = 1,2,\ldots,5 \tag{A14}$$

where λ^D_{jkt} is the (j,k)-th exit stage adjoint variable that represents water scarcity rents for D. The necessary conditions for optimality are given as follows:

$$\frac{\partial \mathrm{H^D_{jk}}}{\partial \mathrm{w^D_{jlt}}} = \frac{\mathrm{a^D_l}}{\mathrm{b^D_l}} - \frac{1}{\mathrm{b^D_l}}\cdot \mathrm{w^D_{jlt}} - \left(\mathrm{k^D_2} - \mathrm{k^D_1}\mathrm{S_{jkt}}\right) - \lambda^D_{\mathrm{jkt}} = 0, \;\; \mathrm{l} = \mathrm{k},\ldots,5, \;\; j,k = 1,2,..,5 \tag{A15}$$

$$\mathrm{d}\lambda^D_{\mathrm{jkt}} = \left[-\frac{\partial \mathrm{H^D_{jk}}}{\partial \mathrm{S_{jkt}}} + \mathrm{r}\lambda^D_{\mathrm{jkt}}\right]\mathrm{dt} \;\Leftrightarrow\; \mathrm{d}\lambda^D_{\mathrm{jkt}} = \left[-\mathrm{k^D_1}\sum_{\mathrm{l=k}}^{5}\mathrm{w^D_{jlt}} + \mathrm{r}\lambda^D_{\mathrm{jkt}}\right]\mathrm{dt} \tag{A16}$$

From the first condition we have that:

$$w^D_{jkt} = a^D_l - b^D_l\left(k^D_2 - k^D_1 S_{jkt}\right) - b^D_l\lambda^D_{jkt}, \;\; l = k,\ldots,5, \;\text{and}\; \mathrm{j,k} = 1,2,\ldots 5 \tag{A17}$$

Setting again $A^U_j \triangleq \sum_{i=j}^{5} a^U_i$ and $B^U_j \triangleq \sum_{i=j}^{5} b^U_i$, the water storage state Eq.(5), and the adjoint equation Eq(A16) are reformulated as:

$$\begin{aligned} dS_{jkt} &= \left[-k^D_1 B^D_k S_{jkt} + B^D_k\lambda^D_{jkt} + W_t - \sum_{i=j}^{5} w^U_{it} + R_t - O_t - A^D_k + k^D_2 B^D_k\right]dt \\ d\lambda^D_{jkt} &= \left\{-(k^D_1)^2 B^D_k S_{jkt} + (k^D_1 B^D_k + r)\lambda^D_{jkt} - k^D_1 A^D_k + k^D_1 k^D_2 B^D_k\right\}dt \\ S_{000} &= s_0, \;\; \lim_{t\to\infty}\lambda^D_{jkt} = 0, \quad j,k = 1,2,\ldots,5 \end{aligned} \tag{A18}$$

To solve the above system of FBDEs we impose a solution of the form:

$$\lambda^D_{jkt} = N^D_{jkt}S_{jkt} + M^D_{jkt} \tag{A19}$$

where N^D_{jkt} and M^D_{jkt} are stochastic processes to be determined. Taking differentials in Eq.(A19), we have:

$$d\lambda^D_{jkt} = S_{jkt}dN^D_{jkt} + N^D_{jkt}dS_{jkt} + dM^D_{jkt} \quad = S_{jkt}dN^D_{jkt} + dM^D_{jkt} + \left\{[B^D_k\left(N^D_{jkt}\right)^2 - k^D_1 B^D_k N^D_{jkt}]S_{jkt} + B^D_k N^D_{jkt}M^D_{jkt} + \left[W_t - \sum_{i=j}^{5} w^U_{it} + R_t - O_t - A^D_k + k^D_2 B^D_k\right]N^D_{jkt}\right\}dt \tag{A20}$$

while from the backward equation of the system (Eq.A18) we have:

$$d\lambda_{jkt}^{D} = \{[(k_1^D B_k^D + r)N_{jkt}^D - (k_1^D)^2 B_k^D]S_{jkt} + [k_1^D B_k^D + r]M_{jkt}^D - k_1^D A_k^D + k_1^D k_2^D B_k^D\}dt \quad \text{(A21)}$$

A sufficient condition for the latter to be equal is given by

$$dN_{jkt}^D = \left[-B_k^D \left(N_{jkt}^D\right)^2 + \left(2k_1^D B_k^D + r\right)N_{jkt}^D - (k_1^D)^2 B_k^D\right]dt, \quad \lim_{t\to\infty} N_{jkt}^D = 0 \quad \text{(A22)}$$

which is a BRE that can be solved numerically for N_{jkt}^D. Also:

$$dM_{jkt}^D = \left[\begin{matrix}\left(-B_k^D N_{jkt}^D + k_1^D B_k^D + r\right)M_{jkt}^D - \left(W_t - \sum_{i=j}^5 w_{it}^U + R_t - O_t - A_k^D + k_2^D B_k^D\right)N_{jkt}^D \\ - k_1^D A_k^D + k_1^D k_2^D B_k^D\end{matrix}\right]dt \quad \text{(A23)}$$

$$\lim_{t\to\infty} M_{jkt}^D = 0$$

which given the above solution is a backward linear first-order SDE that can be solved for M_{jkt}^D. Substituting the linear solution form of Eq.(A19) to the forward equation of the FBSDEs system (Eq.A18), we get:

$$\mathrm{dS_{jkt}} = \left[\left(-\mathrm{k_1^D} + \mathrm{N_{jkt}^D}\right)\mathrm{S_{jkt}} + \mathrm{B_k^D M_{jkt}^D} + \mathrm{W_t} - \sum_{\mathrm{i=j}}^5 \mathrm{w_{it}^U} + \mathrm{R_t} - \mathrm{O_t} - \mathrm{A_k^D} + \mathrm{k_2^D B_k^D}\right]\mathrm{dt} \quad \text{(A24)}$$

$$\mathrm{S_{000}} = \mathrm{s_0}\,, \quad j,k=1,2,\ldots,5$$

which is a forward linear SDE that can be solved numerically for $\mathrm{S_{jkt}}$. Thus, the backward adjoint variable λ_{jkt}^D follows from the linear transformation of Eq.(A19) and the optimal water use w_{jlt}^D follows from the optimality condition Eq.(A17).

COOPERATIVE CASE

Upstream: For the (j,k)-th exit stage we have the augmented Hamiltonian:

$$H_{jk}^U\left(W_{jt}^U, S_{jkt}, \lambda_{jkt}^D, w_{jkt}^U, \ldots, w_{5kt}^U, \mu_{j\kappa t}, \nu_{j\kappa t}, \xi_{j\kappa t}\right) \triangleq \sum_{i=j}^5 \left[\frac{a_i^U}{b_i^U} w_{it}^U - \frac{1}{2b_i^U}\cdot (w_{it}^U)^2 - (k_2^U - k_1^U W_{jt}^U) w_{it}^U\right] + \eta_1 S_{jkt} + \eta_2 + \mu_{jkt}\left[W_t - \sum_{i=j}^5 w_{it}^U\right] + \nu_{jkt}\left[W_t - \sum_{i=j}^5 w_{ikt}^U + R_t - O_t - A_k^D + B_k^D (k_2^D - k_1^D S_{jkt}) + B_k^D \lambda_{jkt}^D\right] + \xi_{jkt}\{-k_1^D [A_k^D - B_k^D (k_2^D - k_1^D S_{jkt})] + (r + k_1^D B_k^D)\lambda_{jkt}^D\} \quad \text{(A25)}$$

where $\left(\mu_{j\kappa}, \nu_{j\kappa}, \xi_{j\kappa}\right)$ is the vector of the associated adjoint variables.

The necessary conditions for optimality for the maximization problem of U are given below:

$$\frac{\partial H_{jk}^U}{\partial w_{ikt}^U} = \frac{a_i^U}{b_i^U} - \frac{1}{b_i^U}\cdot w_{ikt}^U - \left(k_2^U - k_1^U W_{jt}^U\right) - \mu_{jkt} - \nu_{jkt} = 0, \quad i = j, \ldots, 5,\ \mathrm{j,k} = 1,2, \ldots 5 \quad \text{(A26)}$$

$$d\mu_{jkt} = \left[-\frac{\partial H_{j\kappa}^U}{\partial W_{jt}^U} + r\mu_{jkt}\right]dt \quad \Leftrightarrow \quad d\mu_{jkt} = \left[-k_1^U \sum_{i=j}^5 w_{it}^U + r\mu_{jkt}\right]dt \quad \text{(A27)}$$

$$d\nu_{jkt} = \left[-\frac{\partial H_{j\kappa}^U}{\partial S_{jkt}} + r\nu_{jkt}\right]dt \quad \Leftrightarrow \quad \text{(A28)}$$

$$d\nu_{jkt} = \{-\eta_1 + k_1^D B_k^D \nu_{jkt} + (k_1^D)^2 B_k^D \xi_{jkt} + r\nu_{jkt}\}dt$$

$$d\xi_{jkt} = \left[-\frac{\partial H_{jk}^U}{\partial \lambda_{jt}^D} + r\xi_{jkt}\right]dt \quad \Leftrightarrow d\xi_{jkt} = [-B_k^D \nu_{jkt} - k_1^D B_k^D \xi_{jkt}]dt, \tag{A29}$$

$$\xi_{000} = 0$$

From the first optimality condition (Eq.A26) we have:

$$w_{ikt}^U = a_i^U - b_i^U(k_2^U - k_1^U W_{jt}^U) - b_i^U \mu_{jkt} - b_i^U \nu_{jkt},\ i = j, \ldots, 5, \mathrm{j,k} = 1,2,\ldots,5 \tag{A30}$$

It can be easily seen that the adjoint variables of both Eq.(A28) and Eq.(A29) satisfy the system of FBSDEs:

$$d\xi_{jkt} = -[k_1^D B_k^D \xi_{jkt} + B_k^D \nu_{jkt}]dt,$$

$$\mathrm{d}\nu_{\mathrm{jkt}} = \left\{(\mathrm{k}_1^{\mathrm{D}})^2 B_k^D \xi_{\mathrm{jkt}} + (\mathrm{r} + \mathrm{k}_1^{\mathrm{D}} B_k^D)\nu_{\mathrm{jkt}} - \eta_1\right\}\mathrm{dt}, \tag{A31}$$

$$\xi_{000} = 0\ , \quad \lim_{t\to\infty} \nu_{jkt} = 0, \quad \text{j,k=1,2,\ldots,5}$$

In order to solve this FBSDEs system we are looking for solutions (ξ_{jkt}, ν_{jkt}) that satisfy the linear transformation:

$$\nu_{ikt} = N_{jkt}\xi_{jkt} + M_{jkt}, \ \ \mathrm{j,k} = 1,2,\ldots,5 \tag{A32}$$

Where N_{jkt} and M_{jkt} are stochastic processes to be determined. Taking differentials in Eq.(A32) we get:

$$d\nu_{ikt} = N_{jkt}\xi_{jkt} + \xi_{jkt}dN_{jkt} + dM_{jkt} = \xi_{jkt}dN_{jkt} + dM_{jkt} - [(B_k^D N_{jkt}^2 + k_1^D B_k^D N_{jkt})\xi_{jkt} + B_k^D N_{jkt} M_{jkt}]dt \tag{A33}$$

While the backward equation of Eq.(A31) may be written as

$$d\nu_{ikt} = \{[(\mathrm{r} + \mathrm{k}_1^{\mathrm{D}} B_k^D)\mathrm{N}_{\mathrm{jkt}} + (k_1^D)^2 B_k^D]\xi_{jkt} + (\mathrm{r} + \mathrm{k}_1^{\mathrm{D}} B_k^D)\mathrm{M}_{\mathrm{jkt}} - \eta_1\}dt \tag{A34}$$

Sufficient conditions for the latter to be equivalent are provided by

$$dN_{jkt} = [B_k^D\ N_{jk}^2 + (2k_1^D B_k^D + r)N_{jkt} + (k_1^D)^2 B_k^D]dt,$$

$$\lim_{t\to\infty} N_{jkt} = 0, \quad \text{j,k=1,2,\ldots,5} \tag{A35}$$

which is a BRE that can be solved numerically for N_{jkt}^D and by

$$dM_{jkt} = [(B_k^D N_{jt}^U + r + k_1^D B_k^D)M_{jkt} - \eta_1]dt,$$

$$\lim_{t\to\infty} M_{jkt} = 0, \quad \text{j,k=1,2,\ldots,5} \tag{A36}$$

which given the above solution is a backward linear first-order SDE that can be easily solved for M_{jkt}.

Substituting the linear solution form of Eq.(A32) to the forward equation of the FBSDEs system Eq.(A31)**Error! Reference source not found.**, we obtain:

$$d\xi_{jkt} = [-B_k^D(N_{jkt} + k_1^D)\xi_{jkt} - B_k^D M_{jkt}]dt,$$

$$\xi_{000} = 0, \quad j,k = 1,2,\ldots,5 \tag{A37}$$

which is a forward linear SDE that can be solved for ξ_{jkt}. Then the backward adjoint variable v_{jkt} follows from the linear transformation of Eq.(A32)**Error! Reference source not found.**.

Given the obtained solution (ξ_{jk}, v_{jk}), $j,k = 1,2,\ldots,5$, of the FBSDEs system**Error! Reference source not found.**, as described above, we may put in use Eq.(A30) to derive that U's water resources state Eq.(4) and adjoint variable of Eq.(A27) form the subsequent system of FBSDEs:

$$\begin{aligned}
&dW_{jt}^U = \left[-k_1^U B_j^U W_{jt}^U + B_j^U \mu_{jkt} + B_j^U v_{jkt} + W_t - A_j^U + k_2^U B_j^U\right]dt,\\
&d\mu_{jkt} = \left[-(k_1^U)^2 B_j^U W_{jt}^U + \left(k_1^U B_j^U + r\right)\mu_{jkt} + k_1^U B_j^U v_{jkt} - k_1^U A_j^U + k_1^U k_2^U B_j^U\right]dt,\\
&W_{00}^U = wr_0,\ \lim_{t\to\infty}\mu_{jkt} = 0,\ j,k = 1,2,\ldots,5
\end{aligned} \tag{A38}$$

To find a solution process pair (W_{jt}^U, μ_{jk}), $j,k = 1,2,\ldots,5$, for this system of FBSDEs, we impose the linear transformation:

$$\mu_{jkt} = \Lambda_{jkt} W_{jt}^U + \Xi_{jkt}, \quad j,k = 1,2,\ldots,5 \tag{A39}$$

where Λ_{jk} and Ξ_{jkt} are stochastic processes to be determined. Taking differentials in Eq.(A39) we have that:

$$\begin{aligned}
&d\mu_{jkt} = \Lambda_{jkt} dW_{jt}^U + W_{jt}^U d\Lambda_{jkt} + d\Xi_{jkt} = W_{jt}^U d\Lambda_{jkt} + d\Xi_{jkt} +\\
&\left\{\left[B_j^U \Lambda_{jkt}^2 - k_1^U B_j^U \Lambda_{jkt}\right]W_{jt}^U + B_j^U \Lambda_{jkt}\Xi_{jkt} + B_j^U v_{jkt}\Lambda_{jkt} + \left[W_t - A_j^U + k_2^U B_j^U\right]\Lambda_{jkt}\right\}dt
\end{aligned} \tag{A40}$$

while the backward Eq.(A38) may be reformulated as:

$$\begin{aligned}
&d\mu_{jkt} = \left\{\left[-(k_1^U)^2 B_j^U + \left(k_1^U B_j^U + r\right)\Lambda_{jkt}\right]W_{jt}^U + \left(k_1^U B_j^U + r\right)\Xi_{jkt} +\right.\\
&\left. k_1^U B_j^U v_{jkt} - k_1^U A_j^U + k_1^U k_2^U B_j^U\right\}dt
\end{aligned} \tag{A41}$$

Sufficient conditions for the latter to be equivalent are given as follows:

$$\begin{aligned}
&d\Lambda_{jkt} = \left[-B_j^U \Lambda_{jkt}^2 + \left(2k_1^U B_j^U + r\right)\Lambda_{jkt} - (k_1^U)^2 B_j^U\right]dt,\\
&\lim_{t\to\infty}\Lambda_{jkt} = 0, \quad j,k = 1,2,\ldots,5
\end{aligned} \tag{A42}$$

which is a BRE that can be solved numerically for Λ_{jkt}, $j,k = 1,2,\ldots,5$, and

$$\begin{aligned}
&d\Xi_{jkt} =\\
&\left\{\begin{matrix}\left[-B_j^U \Lambda_{jkt} + \left(k_1^U B_j^U + r\right)\right]\Xi_{jkt} - B_j^U v_{jkt}\Lambda_{jkt} - \left[W_t - A_j^U + k_2^U B_j^U\right]\Lambda_{jkt}\\ + k_1^U B_j^U v_{jkt} - k_1^U A_j^U + k_1^U k_2^U B_j^U\end{matrix}\right\}dt,\\
&\lim_{t\to\infty}\Xi_{jkt} = 0, \quad j,k = 1,2,\ldots,5
\end{aligned} \tag{A43}$$

which given the above solution is a backward linear first-order SDE that can be easily solved for Ξ_{jkt}, $j,k = 1,2,\ldots,5$.

Substituting the linear transformation of Eq.(A39) to the forward equation of the FBSDEs system Eq.(A38) we deduce that:

$$\begin{aligned}
&dW_{jt}^U = \left\{\left[-k_1^U B_j^U + B_j^U \Lambda_{jkt}\right]W_{jt}^U + B_j^U \Xi_{jkt} + B_j^U v_{jkt} + W_t - A_j^U +\right.\\
&\left. k_2^U B_j^U\right\}dt,
\end{aligned} \tag{A44}$$

$$W_{00}^U = w_0^U, \quad j,k = 1,2,\dots,5$$

which is a forward linear SDE that can be solved for W_{jt}^U, $j = 1,2,\dots,5$. Then the backward adjoint variable μ_{jkt}, $j,k = 1,2,\dots,5$ follows readily from the linear transformation of Eq.(A39).

Given now the solutions $(\xi_{jk}, \nu_{jk}), (W_{jt}^U, \mu_{jkt})$, $j,k = 1,2,\dots,5$, of the FBSDEs systems Eq.(A31) and Eq.(A38), respectively, together with Eq.(A30) to write equivalently the Hamiltonian FBSDEs state system of the downstream country as:

$$\begin{aligned} &dS_{jkt} = \\ &\begin{bmatrix} -k_1^U B_j^U W_{jt}^U - k_1^D B_k^D S_{jkt} + B_k^D \lambda_{jkt}^D + B_j^U \mu_{jkt} + B_j^U \nu_{jkt} + W_t - A_j^U + k_2^U B_j^U \\ +R_t - O_t - A_k^D + k_2^D B_k^D \end{bmatrix} dt, \\ &d\lambda_{jkt}^D = \left[-(k_1^D)^2 B_k^D S_{jkt} + \left(r + k_1^D B_k^D\right)\lambda_{jkt}^D - k_1^D A_k^D + k_1^D k_2^D B_k^D - \eta_1\right] dt, \\ &S_{000} = s_0, \ \lim_{t\to\infty} \lambda_{jkt}^D = 0, \quad j,k = 1,2,\dots,5 \end{aligned} \tag{A45}$$

Imposing once again a solution $(S_{jkt}, \lambda_{jkt}^D)$, $j,k = 1,2,\dots,5$, that satisfies the linear transformation:

$$\lambda_{jkt}^D = \Pi_{jkt} S_{jkt} + \Sigma_{jkt}, \quad j,k = 1,2,\dots,5 \tag{A46}$$

To determine the stochastic processes Π_{jkt} and Σ_{jkt}, we take differentials in Eq.(A46) we have that:

$$\begin{aligned} d\lambda_{jkt}^D &= \Pi_{jkt} dS_{jkt} + S_{jkt} d\Pi_{jkt} + d\Sigma_{jkt} \\ &= S_{jkt} d\Pi_{jkt} + d\Sigma_{jkt} \\ &+ \begin{Bmatrix} \left[B_k^D \Pi_{jkt}^2 - k_1^D B_k^D \Pi_{jkt}\right] S_{jkt} + B_k^D \Pi_{jkt} \Sigma_{jkt} - k_1^U B_j^U W_j^U \Pi_{jkt} \\ +\left[B_j^U \mu_{jkt} + B_j^U \nu_{jkt} + W_t - A_j^U + k_2^U B_j^U + R_t - O_t - A_k^D + k_2^D B_k^D\right] \Pi_{jkt} \end{Bmatrix} dt, \end{aligned} \tag{A47}$$

while the backward Eq.(A45) may be written equivalently as:

$$\begin{aligned} d\lambda_{jkt}^D = &\left\{\left[-(k_1^D)^2 B_k^D + (r + k_1^D B_k^D)\Pi_{jkt}\right] S_{jkt} + (r + k_1^D B_k^D)\Sigma_{jkt} - \right. \\ &\left. k_1^D A_k^D + k_1^D k_2^D B_k^D\right\} dt \end{aligned} \tag{A48}$$

Sufficient conditions for the latter to be equivalent are provided by:

$$\begin{aligned} &d\Pi_{jkt} = \left[-B_k^D \left(\Pi_{jk}^U\right)^2 + (2k_1^D B_k^D + r)\Pi_{jkt}^U - (k_1^D)^2 B_k^D\right] dt, \\ &\lim_{t\to\infty} \Pi_{jkt} = 0, \quad j,k = 1,2,\dots,5 \end{aligned} \tag{A49}$$

which is a BRE that can be solved numerically for $\Pi_{jkt}, j,k = 1,2,\dots,5$, and by:

$$\begin{aligned} &d\Sigma_{jkt} = \\ &\begin{Bmatrix} \left[-B_k^D \Pi_{jkt} + \left(k_1^D B_k^D + r\right)\right] \Sigma_{jkt} \\ -\left[-k_1^U B_j^U W_j^U + B_j^U \mu_{jkt} + B_j^U \nu_{jkt} + W_t - A_j^U + k_2^U B_j^U + R_t - O_t - A_k^D + k_2^D B_k^D\right] \Pi_{jkt} \\ -k_1^D A_k^D + k_1^D k_2^D B_k^D \end{Bmatrix} dt, \\ &\lim_{t\to\infty} \Sigma_{jkt} = 0, \quad j,k = 1,2,\dots,5 \end{aligned} \tag{A50}$$

which given the above solution is a backward linear first-order SDE that can be easily solved for $\Sigma_{jkt}, j,k = 1,2,\dots,5$.

Substituting the linear transformation Eq.(A46) to the forward equation of the FBSDEs system Eq.(A45)**Error! Reference source not found.**, we obtain:

$$dS_{jkt} = \begin{Bmatrix} B_k^D [\Pi_{jkt} - k_1^D] S_{jkt} + B_k^D \Sigma_{jkt} - k_1^U B_j^U W_{jt}^U + B_j^U \mu_{jkt} + B_j^U \nu_{jkt} \\ +W_t - A_j^U + k_2^U B_j^U + R_t - O_t - A_k^D + k_2^D B_k^D \end{Bmatrix} dt, \quad \text{(A51)}$$
$$S_{000} = s_0, \quad j,k = 1,2,\ldots,5$$

which is a forward linear SDE that can be solved for $S_{jkt},\ j,k = 1,2,\ldots,5$. Then the backward adjoint variable $\lambda_{jkt}^D,\ j,k = 1,2,\ldots,5,$ follows immediately from the linear transformation of Eq.(A46)**Error! Reference source not found.**. Clearly, the optimal water abstraction policies $w_{jkt}^U, w_{jkt}^D,\ j,k = 1,2,\ldots,5,$ of U and D follow from Eq.(A30) and Eq.(A17)**Error! Reference source not found.**, respectively.

APPENDIX B.

SEQUENTIAL MONTE CARLO

The particle filter methodology can be applied to state space models of the general form:

$$y_T \sim p(y_t|x_t),\ s_t \sim p(s_t|s_{t-1}) \quad \text{(B1)}$$

where s_t is a state variable. For general introductions see Gordon (1997), Gordon et al. (1993), Doucet et al., (2001) and Ristic et al. (2004). Given the data Y_t the posterior distribution $p(s_t|Y_t)$ can be approximated by a set of (auxiliary) particles $\left\{s_t^{(i)}, i = 1,\ldots,.N\right\}$ with probability weights $\left\{w_t^{(i)}, i = 1,\ldots,N\right\}$ where

$\sum_{i=1}^N\ w_t^{(i)} = 1$. The predictive density can be approximated by:

$$p(s_{t+1}|Y_t) = \int p(s_{t+1}|s_t)p(s_t|Y_t)ds_t \simeq \sum_{i=1}^N\ p(s_{t+1}|s_t^{(i)})w_t^{(i)} \quad \text{(B2)}$$

and the final approximation for the filtering density is:

$$p(s_{t+1}|Y_t) \propto p(y_{t+1}|s_{t+1})p(s_{t+1}|Y_t) \simeq p(y_{t+1}|s_{t+1}) \sum_{i=1}^N\ p(s_{t+1}|s_t^{(i)})w_t^{(i)} \quad \text{(B3)}$$

The basic mechanism of particle filtering rests on propagating $\left\{s_t^{(i)}, w_t^{(i)}, i = 1,\ldots,N\right\}$ to the next step, viz. $\left\{s_{t+1}^{(i)}, w_{t+1}^{(i)}, i = 1,\ldots,N\right\}$ but this often suffers from the weight degeneracy problem. If parameters $\theta \in \Theta \in \Re^k$ are available, as is often the case, we follow Liu and West (2001) parameter learning takes place via a mixture of multivariate normals:

$$p(\theta|Y_t) = \sum_{i=1}^N w_t^{(i)}\ N(\theta|\alpha\theta_t^{(i)} + (1-a)\bar{\theta}_t, b^2 V_t) \quad \text{(B4)}$$

where $\bar{\theta}_t = \sum_{i=1}^N w_t^{(i)}\theta_t^{(i)}$, and $V_t = \sum_{i=1}^N w_t^{(i)}(\theta_t^{(i)} - \bar{\theta}_t)(\theta_t^{(i)} - \bar{\theta}_t)'$. The constants a and b are related to shrinkage and are determined via a discount factor $\delta \in (0,1)$ as $a = (1-b^2)^{1/2}$ and $b^2 = 1 - [(3\delta - 1)/2\delta]^2$ (see also Casarin and Marin, 2007). Andrieu and Roberts (2009), Flury and Shephard (2011) and Pitt et al. (2012) provide the Particle Metropolis-Hastimgs (PMCMC) technique which uses an unbiased estimator of the likelihood function $\hat{p}_N(Y\mid\theta)$ as $p(Y|\theta)$ is often not available in closed form.

Given the current state of the parameter $\theta^{(j)}$ and the current estimate of the likelihood, say $L^j = \hat{p}_N(Y \mid \theta^{(j)})$, a candidate θ^c is drawn from $q(\theta^c|\theta^{(j)})$ yielding $L^c = \hat{p}_N(Y \mid \theta^c)$. Then, we set $\theta^{(j+1)} = \theta^c$ with the Metropolis - Hastings probability:

$$A = \min\left\{1, \frac{p(\theta^c)L^c}{p(\theta^{(j)})L^j}\frac{q(\theta^{(j)}|\theta^c}{q(\theta^c|\theta^{(j)})}\right\} \tag{B5}$$

otherwise we repeat the current draws: $\left\{\theta^{(j+1)}, L^{j+1}\right\} = \left\{\theta^{(j)}, L^j\right\}$.

Hall et al. (2014) propose an auxiliary particle filter which rests upon the idea that adaptive particle filtering (Pitt et al., 2012) used within PMCMC requires far fewer particles that the standard particle filtering algorithm to approximate $p(Y|\theta)$. From Pitt and Shephard (1999) we know that auxiliary particle filtering can be implemented easily once we can evaluate the state transition density $p(s_t|s_{t-1})$. When this is not possible, Hall et al. (2014) present a new approach when, for instance, $s_t = g(s_{t-1}, u_t)$ for a certain disturbance. In this case we have:

$$p(y_t|s_{t-1}) = \int p(y_t|s_t)p(s_t|s_{t-1})ds_t \tag{B6}$$

$$p(u_t|s_{t-1}; y_t) = p(y_t|s_{t-1}, u_t)p(u_t|s_{t-1})/p(y_t|s_{t-1}) \tag{B7}$$

If one can evaluate $p(y_t|s_{t-1})$ and simulate from $p(u_t|s_{t-1}; y_t)$ the filter would be fully adaptable (Pitt & Shephard, 1999). One can use a Gaussian approximation for the first-stage proposal $g(y_t|s_{t-1})$ by matching the first two moments of$p(y_t|s_{t-1})$. So in some way we find that the approximating density$p(y_t|s_{t-1}) = N(E(y_t|s_{t-1}), V(y_t|s_{t-1}))$. In the second stage, we know that$p(u_t|y_t, s_{t-1}) \propto p(y_t|s_{t-1}, u_t)p(u_t)$. For $p(u_t|y_t, s_{t-1})$ we know it is multimodal so suppose it has M modes are $\hat{u}_t^m$, for$m = 1, \dots, M$. For each mode we can use a Laplace approximation.

Let $l(u_t) = log[p(y_t|s_{t-1}, u_t)p(u_t)]$. From the Laplace approximation we obtain:

$$l(u_t) = l(\widehat{u_t^m}) + \frac{1}{2}(u_t - \widehat{u_t^m})' \nabla^2 l(\widehat{u_t^m})(u_t - \widehat{u_t^m}) \tag{B8}$$

Then we can construct a mixture approximation:

$$g(u_t \mid x_t, s_{t-1}) = \sum_{m=1}^{M} \lambda_m (2\pi)^{-d/2} \mid \Sigma_m \mid^{-1/2} \exp\left\{\tfrac{1}{2}(u_t - \hat{u}_t^m)'\Sigma_m^{-1}(u_t - \hat{u}_t^m\right\}, \tag{B9}$$

where $\Sigma_m = -\left[\nabla^2 l(\hat{u}_t^m)\right]^{-1}$ and $\lambda_m \propto exp\{l(u_t^m)\}$ with $\sum_{m=1}^{M} \; = 1$. This is done for each particle s_t^i. This is known as the Auxiliary Disturbance Particle Filter (ADPF). An alternative is the independent particle filter (IPF) of Lin et al. (2005). The IPF forms a proposal for s_t directly from the measurement density $p(y_t|s_t)$ although Hall et al. (2014) are quite right in pointing out that the state equation can be very informative. In the standard particle filter of Gordon et al. (1993) particles are simulated through the state density $p(s_t^i|s_{t-1}^i)$ and they are re-sampled with weights determined by the measurement density evaluated at the resulting particle, viz. $p(y_t|s_t^i)$. The ADPF is simple to construct and rests upon the following steps, for $t = 0, \dots, T-1$ given samples $s_t^k \sim p(s_t|Y_{1:t})$ with mass π_t^k for $k = 1, \dots, N$:

1. For $k = 1, \dots, N$ compute $\omega_{t|t+1}^k = g(y_{t+1}|s_t^k)\pi_t^k$, $\pi_{t|t+1}^k = \omega_{t|t+1}^k / \sum_{i=1}^N \omega_{t|t+1}^i$.
2. For $k = 1, \dots, N$ draw $\widetilde{s_t^k} = \sum_{i=1}^N \pi_{t|t+1}^k \delta_{St}^i(ds_t)$
3. For $k = 1, \dots, N$ draw δ_{St}^i= g($u_{t+1}| \widetilde{s_t^k}, y_{t+1}$ and set $s_{t+1}^k = h(s_t^k; u_{t+1}^k)$

4. For $k = 1, \ldots, N$ compute $\omega_{t+1}^{k} = \frac{p(y_{t+1} \mid s_{t+1}^{k}) p(u_{t+1}^{k})}{g(y_{t+1} \mid s_{t}^{k}) g(u_{t+1}^{k} \mid \tilde{s}_{t}^{k}, y_{t+1})}, \qquad \pi_{t+1}^{k} = \frac{\omega_{t+1}^{k}}{\sum_{i=1}^{N} \omega_{t+1}^{i}}.$

(B10)

It should be mentioned that the estimate of likelihood from ADPF is:

$$p(Y_{1:T}) = \prod_{t=1}^{T} \left(\sum_{i=1}^{N} \omega_{t-1|t}^{i}\right)\left(N^{-1} \sum_{i=1}^{N} \omega_{t}^{i}\right) \tag{B11}$$

PARTICLE METROPOLIS ADJUSTED LANGEVIN FILTERS

Nemeth et al. (2014) provide a particle version of a Metropolis Adjusted Langevin algorithm (MALA). In Sequential Monte Carlo we are interested in approximating $p(s_t|Y_{1:t}, \theta)$. Given that:

$$p(s_t|Y_{1:t}, \theta) \propto g(y_t|x_t, \theta) \int f(s_t|s_{t-1}, \theta) p(s_{t-1}|y_{1:t-1}, \theta) ds_{t-1} \tag{B12}$$

where $p(s_{t-1}|y_{1:t-1}, \theta)$ is the posterior as of time $t-1$. If at time $t-1$ we have a set of particles $\{s_{t-1}^{i}, i = 1, \ldots, N\}$ and weights $\{w_{t-1}^{i}, i = 1, \ldots. N\}$ which form a discrete approximation for $p(s_{t-1}|y_{1:t-1}, \theta)$ then we have the approximation:

$$\hat{p}(s_{t-1}|y_{1:t-1}, \theta) \propto \sum_{i=1}^{N} w_{t-1}^{i} f(s_t|s_{t-1}^{i}, \theta) \tag{B13}$$

See Cappé at al. (2005) and Andrieu et al. (2010) for reviews. From (B13) Fernhead (2007) makes the important observation that the joint probability of sampling particle s_{t-1}^{i} and state s_t is:

$$\omega_t = \frac{w_{t-1}^{i} g(y_t|s_t, \theta) f(s|s_{t-1}^{i}, \theta)}{\xi_t^{i} q(s_t|s_{t-1}^{i}, y_t, \theta)} \tag{B14}$$

where $q(s_t|s_{t-1}^{i}, y_t, \theta)$ is a density function amenable to simulation and:

$$\xi_t^{i} q(s_t|s_{t-1}^{i}, y_t, \theta) \simeq c g(y_t|s_t, \theta) f\left(s_t \middle| s_{t-1}^{i}, \theta\right) \tag{B15}$$

and c is the normalizing constant in (B12).

In the MALA algorithm of Roberts and Rosenthal (1998)[13] we form a proposal:

$$\theta^{c} = \theta^{(s)} + \lambda z + \frac{\lambda^2}{2} \nabla \log p(\theta^{(s)}|Y_{1:T}) \tag{B16}$$

where $z \sim N(0, I)$ which should result in larger jumps and better mixing properties, plus lower autocorrelations for a certain scale parameter λ. Acceptance probabilities are: $a(\theta^{c}|\theta^{(s)}) = min\left\{1, \frac{p(Y_{1:T}|\theta^{c}) q(\theta^{(s)}|\theta^{c})}{p(Y_{1:T}|\theta^{(s)}) q(\theta^{c}|\theta^{(s)})}\right\}$

Using particle filtering it is possible to create an approximation of the score vector using Fisher's identity:

$$\nabla \log p\,(Y_{1:T}|\theta) = E[\nabla \log p\,(s_{1:T}, Y_{1:T}|\theta)|Y_{1:T}, \theta] \tag{B17}$$

which corresponds to the expectation of:

$$\nabla \log p\,(s_{1:T}, Y_{1:T}|\theta) = \nabla \log p\,(|s_{1:T-1}, Y_{1:T-1}|\theta) + \nabla \log g\,(y_T|s_T, \theta) + \nabla \log f\,(s_T|s|_{T-1}, \theta)$$

[13]The benefit of MALA over Random-Walk-Metropolis arises when the number of parameters n is large. This happens because the scaling parameter λ is $O(n^{-1/2})$ for Random-Walk-Metropolis but it is $O(n^{-1/6})$ for MALA, see Roberts et al. (1997) and Roberts and Rosenthal (1998).

over the path $s_{1:T}$. The particle approximation to the score vector results from replacing $p(s_{1:T}|Y_{1:T},\theta)$ with a particle approximation $\hat{p}(s_{1:T}|Y_{1:T},\theta)$.

With particle *i-th* at time *t-1*, we can associate a value $\alpha_{t-1}^{i} = \nabla\, log\, p\,(s_{1:t-1}^{i}, Y_{1:t-1}|\theta)$ which can be updated recursively. As we sample κ_i in the APF (the index of particle at time $t-1$ that is propagated to produce the ith particle at time t) we have the update:

$$\alpha_t^i = a_{t-1}^{\kappa_i} + \nabla\, log\, g\,(y_t|s_t^i,\theta) + \nabla\, log\, f\,(s_t^i|s_{t-1}^i,\theta) \quad \text{(B18)}$$

To avoid problems with increasing variance of the score estimate $\nabla\, log\, p\,(Y_{1:t}|\theta)$ we can use the approximation:

$$\alpha_{t-1}^i \sim N(m_{t-1}^i, V_{t-1}) \quad \text{(B19)}$$

The mean is obtained by shrinking α_{t-1}^i towards the mean of α_{t-1} as follows:

$$m_{t-1}^i = \delta\alpha_{t-1}^i + (1-\delta)\sum_{i=1}^{N} w_{t-1}^i\alpha_{t-1}^i \quad \text{(B20)}$$

where $\delta \in (0{,}1)$ is a shrinkage parameter. Using Rao-Blackwellization one can avoid sampling α_t^i and instead use the following recursion for the means:

$$m_t^i = \delta m_{t-1}^{\kappa_i} + (1-\delta)\sum_{i=1}^{N} w_{t-1}^i m_{t-1}^i + \nabla\, log\, g\,(y_t|s_t^i,\theta) + \nabla\, log\, f\,\left(s_t^i|s_{t-1}^{\kappa_i},\theta\right) \quad \text{(B21)}$$

which yields the final score estimate:

$$\nabla\, \widehat{log\, p}\,(Y_{1:t}|\theta) = \sum_{i=1}^{N} w_t^i m_t^i \quad \text{(B22)}$$

As a rule of thumb Nemeth et al. (2014) suggest taking $\delta = 0.95$. Furthermore, they show the important result that the algorithm should be tuned to the asymptotically optimal acceptance rate of 15.47% and the number of particles must be selected so that the variance of the estimated log-posterior is about 3. Additionally, if measures are not taken to control the error in the variance of the score vector, there is no gain over a simple random walk proposal.

Of course, the marginal likelihood is:

$$p(Y_{1:T}|\theta) = p(y_1|\theta)\prod_{t=2}^{T} p(y_t|Y_{1:t-1},\theta) \quad \text{(B23)}$$

where

$$p(y_t|Y_{1:t-1},\theta) = \int g(y_t|s_t)\int f(s_t|s_{t-1},\theta)p(s_{t-1}|Y_{1:T-1},\theta)ds_{t-1}ds_t \quad \text{(B24)}$$

provides, in explicit form, the predictive likelihood.

APPENDIX C.

DATA MENTIONED IN RESULTS SECTION

The Environmental Indices derived from the factors presented in Section 7, as well as the parameters of the hydro-economic model (same section) were obtained from official databases.

The Eora global supply chain database consists of a Multi-Region Input-Output table (MRIO) model that provides a time series of high-resolution Input-Output (IO) tables with matching environmental and social satellite accounts for 190 countries (35 types of EI air pollution, energy use, greenhouse gas emissions, water use, land occupation, N and P emissions, etc.). 16 IO tables, each for the period 2000-2015 for Ethiopia and Kenya were used. Additional data, as well as pumping costs and hydrological timeseries were collected from scientific journals, official reports, governmental websites, and other forms of grey literature databases, including: African development bank, including African development bank, ILO (International Labor Organization) and the World Bank Group: Climate Change Knowledge Portal For Development Practitioners and Policy Makers, the United Nations Statistics Division, Food and Agriculture Organization of United Nations (FAOSTAT, AQUASTAT), Unesco World Heritage list, OpenDataSoft, Environment & Climate Change Data Portal, and offices of national statistics.

Table C.1.: Description of Indices of Ecosystem Services

ES Services	**Indicator**	**Description**	**Units**	**Source**
	GVA (Gross Value Added) per sector	Represents the contribution of labor and capital to the production process. Gross value added at basic prices is defined as output valued at basic prices less intermediate consumption valued at purchasers' prices. Although the outputs and inputs are valued using different sets of prices, for brevity the value added is described by the prices used to value the outputs. From the point of view of the producer, purchasers' prices for inputs and basic prices for outputs represent the prices actually paid and received. Their use leads to a measure of gross value added that is particularly relevant for the producer. Net value added is defined as the value of output less the values of both intermediate consumption and consumption of fixed capital.	$	Input-OutputTables http://www.worldmrio.com/country
	Gross Fixed Capital Formation per sector	Gross fixed capital formation is measured by the total value of a producer's acquisitions, less disposals, of fixed assets during the accounting period plus certain additions to the value of non-produced assets (such as subsoil assets or major improvements in the quantity, quality or productivity of land) realized by the productive activity of institutional units.	$	Input-Output Tables http://www.worldmrio.com/country
	Employment Per sector	Persons in employment are defined as all those of working age who, during a short reference period, were engaged in any activity to produce goods or provide services for pay or profit. They comprise employed persons "at work", i.e. who worked in a job for at least one hour; and employed persons "not at work" due to temporary absence from a job, or to working-time arrangements (such as shift work, flexitime and compensatory leave for overtime).	Abs. Value	ILO (International LABOR Organization)
Provis. services	**WFN: Total Water Footprint per sector**	Total water use which includes: – WFN: Total water footprint - Green – WFN: Total water footprint - Blue – WFN: Total water footprint - Grey	Mm^3/yr	Input-Output Tables http://www.worldmrio.com
	Energy Use (Total) per sector	Natural Gas, Coal, Petroleum, Nuclear Electricity, Hydroelectric Electricity, Geothermal Electricity, Wind Electricity, Solar, Tide and Wave Electricity, Biomass and Waste Electricity	TJ	Input-Output Tables http://www.worldmrio.com/country
Regul. Services	**Water Quality**	Nitrogen Emissions exportable to water bodies from agriculture and household waste water	Gg	Input-Output Tables http://www.worldmrio.com/country
	Fertilizers: Total Nitrogen and Phosphate (N and P2O5)	Proportions of consumption of fertilizers (by nutrient group) per unit of agricultural land area are calculated by UNSD using available consumption and land use data from FAOSTAT.	kg/ha	Food and Agriculture Organization of United Nations (FAOSTAT)
Provis. services	**Agricultural Area**	Agricultural area, this category is the sum of areas under "Arable land", "Permanent crops" and "Permanent pastures"	10^3ha	Food and Agriculture Organization of United Nations (FAOSTAT)
Provis. services	**Raw Materials per Sector**	For agriculture total biomass and for mining-quarries total construction material and total fossil fuel	t	Input-Output Tables http://www.worldmrio.com/country
Provis. services	**Permanent Crops**	Permanent crops is the land cultivated with long-term crops which do not have to be replanted for several years (such as cocoa and coffee); land under trees and shrubs producing flowers, such as roses and jasmine; and nurseries (except those for forest trees, which should be classified under "forest").	10^3ha	Food and Agriculture Organization of United Nations (FAOSTAT)

		Permanent meadows and pastures are excluded from land under permanent crops.		
Provis. services	**Arable Land**	Arable land is the land under temporary agricultural crops (multiple-cropped areas are counted only once), temporary meadows for mowing or pasture, land under market and kitchen gardens and land temporarily fallow (less than five years). The abandoned land resulting from shifting cultivation is not included in this category. Data for “Arable land” are not meant to indicate the amount of land that is potentially cultivable.	10^3ha	Food and Agriculture Organization of United Nations (FAOSTAT)
Provis. services	**Crop Production**	Crop statistics are recorded for 173 products, covering the following categories: Crops Primary, Fibre Crops Crop statistics are recorded for 173 products, covering the following categories: Crops Primary, Fibre Crops Primary, Cereals, Coarse Grain, Citrus Fruit, Fruit, Jute & Jute-like Fibres, Oilcakes Equivalent, Oil crops Primary, Pulses, Roots and Tubers, Treenuts and Vegetables and Melons. Data are expressed in terms of area harvested, production quantity, yield and seed quantity. The objective is to comprehensively cover production of all primary crops for all countries and regions in the world.	t	Input-Output Tables http://www.worldmrio.com/country
Regul. Services	**Forest**	Forest area is the land spanning more than 0.5 hectares with trees higher than 5 metres and a canopy cover of more than 10 percent, or trees able to reach these thresholds in situ. It does not include land that is predominantly under agricultural or urban land use. Forest is determined both by the presence of trees and the absence of other predominant land uses. The trees should be able to reach a minimum height of 5 metres (m) in situ. Areas under reforestation that have not yet reached but are expected to reach a canopy cover of 10 percent and a tree height of 5 m are included, as are temporarily unstocked areas, resulting from human intervention or natural causes, which are expected to regenerate. Includes: areas with bamboo and palms provided that height and canopy cover criteria are met; forest roads, firebreaks and other small open areas; forest in national parks, nature reserves and other protected areas such as those of specific scientific, historical, cultural or spiritual interest; windbreaks, shelterbelts and corridors of trees with an area of more than 0.5 ha and width of more than 20 m; plantations primarily used for forestry or protective purposes, such as: rubber-wood plantations and cork, oak stands. Excludes: tree stands in agricultural production systems, for example in fruit plantations and agroforestry systems. The term also excludes trees in urban parks and gardens.	10^3ha	Food and Agriculture Organization of United Nations (FAOSTAT)
Provis. services	**Total Area Equipped For Irrigation**	Area equipped with irrigation infrastructure to provide water to the crops. This includes areas equipped for full and partial control irrigation, spate irrigation areas, and equipped wetland or inland valley bottoms.	10^3ha	Food and Agriculture Organization of United Nations (FAOSTAT)
Provis. services	**Total Fisheries Production**	Total fisheries production measures the volume of aquatic species caught by a country for all commercial, industrial, recreational and subsistence purposes. The harvest from mariculture, aquaculture and other kinds of fish farming is also included.	t	Food and Agriculture Organization of United Nations (FAOSTAT)
	Temperature	The yearly mean historical rainfall and temperature data can be mapped to show the baseline climate and seasonality yearly, and for rainfall and temperature.	°C	The World Bank Group Climate Change Knowledge PortalFor Development Practitioners and Policy Makers
	Rainfall	Yearly Mean historical rainfall	mm	The World Bank Group Climate Change Knowledge PortalFor Development Practitioners and Policy Makers
Habitat services	**Biodiversity and Habitats**	A “proximity-to-target methodology” is used to assess how close each country is to an identified policy target. Country scores are determined by how close or far countries are to targets. Scores are standardized (i.e., on a scale of 0 to 100) for comparability, weighting, and aggregation. The Environmental Performance Index (EPI) is constructed through the calculation and aggregation of 20 indicators reflecting national-level environmental data. These indicators	%	Environment and Climate Change Data Portal

		are combined into nine issue categories, each of which fit under one of two overarching objectives. The two objectives that provide the overarching structure of the EPI are Environmental Health and Ecosystem Vitality. Biodiversity & Habitats belongs to the Ecosystem Vitality which measures ecosystem protection and resource management. These two objectives are further divided into nine issue categories that span high-priority environmental policy issues, including air quality, forests, fisheries, and climate and energy, among others. The issue categories are extensive but not comprehensive. Underlying the nine issue categories, 20 indicators are calculated from country-level data and statistics. In this case the Biodiversity and Habitat category includes four indicators: Critical Habitat Protection, Terrestrial Protected Areas (National Biome Weight), Terrestrial Protected Areas (Global Biome Weight), and Marine Protected Areas. The targets are: 100% for Critical Habitat Protection; 17% for Terrestrial Protected Areas (National Biome Weights); 17% for Terrestrial Protected Areas (Global Biome Weights); 10% for Marine Protected Areas. (c.f. http://archive.epi.yale.edu/our-methods/biodiversity-and-habitat)		
Regul. Services	**Terrestrial Protected Areas**	The definition of a "protected area", as adopted by the International Union for Conservation of Nature (IUCN), is "an area of land and/or sea especially dedicated to the protection and maintenance of biological diversity, and of natural and associated cultural resources, and managed through legal or other effective means". (IUCN 1994. Guidelines for Protected Areas Management Categories. IUCN; Gland; Switzerland and Cambridge; UK). Protected areas increase with time and are not deleted from subsequent years. Only protected areas that are "nationally designated" are included in this indicator. The status "designated" is attributed to a protected area when the authority that corresponds, according to national legislation or common practice (e.g. by means of an executive decree), officially endorses a document of designation. The designation must be for conservation of biodiversity, not single species and not fortuitous de facto protection arising because of some other activity (e.g. military). Hence, a number of United States Marine Managed Areas as well as permanent fisheries closures are excluded. Data are adjusted to account for transboundary protected areas (protected areas that transcend international boundaries) to ensure that the appropriate area/extent from the total area for that site is attributed to the country in which it is contained. Similar adjustments have been made where a protected area transcends both marine and terrestrial environments. The size of the protected area (its "extent") is the officially documented total area provided by the national authority or as listed by the World Database on Protected Areas and may be generated from spatial (GIS) boundary data (see source for details). Many protected areas can contain proportions of both the marine and terrestrial environment, and the size of the protected area extent that falls into each environment is not always available. The table also includes some protected areas for which the year (date of establishment/designation) is unavailable. If no update is received for a given year, the total number and size of the protected area is assumed to be equal to the previous year's values.	Km^2	World Database on Protected Areas (WDPA) website at: www.wdpa.org/.
	Access to Electricity	Access to electricity is the percentage of population with access to electricity. Electrification data are collected from industry, national surveys and international sources.	% of population	World Bank, Sustainable Energy for All (SE4ALL) database
	People Using Basic Drinking Water Services	The percentage of people using at least basic water services. This indicator encompasses both people using basic water services as well as those using safely managed water services. Basic drinking water services is defined as drinking water from an improved source, provided collection time is not more than 30 minutes for a round trip. Improved water sources include piped water, boreholes or tubewells, protected dug wells, protected springs, and packaged or delivered water.	% of population	World Bank, from WHO/UNICEF Joint Monitoring Programme (JMP) for Water Supply, Sanitation and Hygiene (washdata.org).
	International Tourism, Number of Arrivals	International inbound tourists (overnight visitors) are the number of tourists who travel to a country other than that in which they have their usual residence, but outside their usual environment, for a period not exceeding 12 months and whose	Abs. Value	World Bank, World Tourism Organization, Yearbook of

		main purpose in visiting is other than an activity remunerated from within the country visited. When data on number of tourists are not available, the number of visitors, which includes tourists, same-day visitors, cruise passengers, and crew members, is shown instead. Sources and collection methods for arrivals differ across countries. In some cases data are from border statistics (police, immigration, and the like) and supplemented by border surveys. In other cases data are from tourism accommodation establishments. For some countries number of arrivals is limited to arrivals by air and for others to arrivals staying in hotels. Some countries include arrivals of nationals residing abroad while others do not. Caution should thus be used in comparing arrivals across countries. The data on inbound tourists refer to the number of arrivals, not to the number of people traveling. Thus a person who makes several trips to a country during a given period is counted each time as a new arrival.		Tourism Statistics, Compendium of Tourism Statistics and data files
Regul. Services	**Renewable Electricity Production**	Electricity production refers to gross production, which is the sum of the electrical energy production by all the generating units/installations concerned (including pumped storage) measured at the output terminals of the main generators. Renewable electricity production (%) refers to the proportion of total electricity produced that comes from a renewable origin. Electricity production refers to gross electricity production, which is the sum of the electrical energy production by all the generating units/installations concerned (including pumped storage) measured at the output terminals of the main generators. This includes the consumption by station auxiliaries and any losses in the transformers that are considered integral parts of the station. Renewable electricity production was calculated as the sum of electricity produced from hydro, geothermal, solar, wind, tide, wave and ocean sources. All electricity production from combustible fuels is considered non-renewable; therefore electricity produced from burning biomass or renewable waste is not included as renewable electricity in this table. However, this has been observed to be a relatively negligible proportion of electricity production in most cases.	%	United Nations Statistics Division, Energy Statistics http://unstats.un.org/unsd/energy/yearbook/default.htm.
Regul. Services	**CO_2**	Public electricity and heat production Other Energy Industries Manufacturing Industries and Construction Domestic aviation Road transportation Rail transportation Inland navigation Other transportation Residential and other sectors Fugitive emissions from solid fuels Fugitive emissions from oil and gas International aviation International navigation Production of minerals Cement production Lime production Production of chemicals Production of metals Production of pulp/paper/food/drink Production of halocarbons and SF6 Refrigeration and Air Conditioning Foam Blowing Fire Extinguishers Aerosols F-gas as Solvent Semiconductor/Electronics Manufacture Electrical Equipment Other F-gas use Non-energy use of lubricants/waxes (CO_2) Solvent and other product use: paint Solvent and other product use: degrease Solvent and other product use: chemicals Solvent and other product use: other Enteric fermentation Manure management Rice cultivation Direct soil emissions Manure in pasture/range/paddock	Gg	Input-Output Tables http://www.worldmrio.com/country

		Indirect N_2O from agriculture Other direct soil emissions Savanna burning Agricultural waste burning Forest fires Grassland fires Decay of wetlands/peatlands Other vegetation fires Forest Fires-Post burn decay Solid waste disposal on land Wastewater handling Waste incineration Other waste handling Fossil fuel fires Indirect N_2O from non-agricultural NO_x Indirect N_2O from non-agricultural NH_3 Other sources		
Regul. Services	**NO_2**	Public electricity and heat production Other Energy Industries Manufacturing Industries and Construction Domestic aviation Road transportation Rail transportation Inland navigation Other transportation Residential and other sectors Fugitive emissions from solid fuels Fugitive emissions from oil and gas Memo: International aviation Memo: International navigation Production of minerals Cement production Lime production Production of chemicals Production of metals Production of pulp/paper/food/drink Production of halocarbons and SF6 Refrigeration and Air Conditioning Foam Blowing Fire Extinguishers Aerosols F-gas as Solvent Semiconductor/Electronics Manufacture Electrical Equipment Other F-gas use Non-energy use of lubricants/waxes (CO_2) Solvent and other product use: paint Solvent and other product use: degrease Solvent and other product use: chemicals Solvent and other product use: other Enteric fermentation Manure management Rice cultivation Direct soil emissions Manure in pasture/range/paddock Indirect N_2O from agriculture Other direct soil emissions Savanna burning Agricultural waste burning Forest fires Grassland fires Decay of wetlands/peatlands Other vegetation fires Forest Fires-Post burn decay Solid waste disposal on land Wastewater handling Waste incineration Other waste handling Fossil fuel fires Indirect NO_2 from non-agricultural NO_x Indirect NO_2 from non-agricultural NH_3 Other sources		
Regul. Services	**Total Annual Freshwater Withdrawals**	Annual freshwater withdrawals refer to total water withdrawals, not counting evaporation losses from storage basins. Withdrawals also include water from desalination plants in	$10^9 m^3$	Food and Agriculture

		countries where they are a significant source. Withdrawals can exceed 100 percent of total renewable resources where extraction from non-renewable aquifers or desalination plants is considerable or where there is significant water reuse. Withdrawals for agriculture and industry are total withdrawals for irrigation and livestock production and for direct industrial use (including withdrawals for cooling thermoelectric plants). Withdrawals for domestic uses include drinking water, municipal use or supply, and use for public services, commercial establishments, and homes. Data are for the most recent year available for 1987-2002.		Organization, AQUASTAT data.
Regul. Services	**Floods and Droughts**	Number of floods/droughts events.	[-]	**The Emergency Events Database - Université catholique de Louvain (UCL) - CRED, D. Guha-Sapir,** www.emdat.be**, Brussels, Belgium.** http://emdat.be/emdat_db/
Cultural and amenity services	**Cultural-Natural-Mixed Heritage Sites**	To be included on the World Heritage List, sites must be of outstanding universal value and meet at least one out of ten selection criteria. These criteria are explained in the Operational Guidelines for the Implementation of the World Heritage Convention which, besides the text of the Convention, is the main working tool on World Heritage. The criteria are regularly revised by the Committee to reflect the evolution of the World Heritage concept itself. Access to an improved water source refers to the percentage of the population with reasonable access to an adequate amount of water from an improved source, such as a household connection, public standpipe, borehole, protected well or spring, and rainwater collection. Unimproved sources include vendors, tanker trucks, and unprotected wells and springs. Reasonable access is defined as the availability of at least 20 liters a person a day from a source within one kilometer of the dwelling.	[-]	UNESCO World Heritage Centre – World Heritage List

Table C.2.: Parameters of the hydro-economic model.

Symbol	Description	Parameter Value
B^u	Vector of the absolute values of the slope of the water demand for *i* sector in the upstream area.	[833.33; 31,152.65; 769.23; 714.29; 1,000] Mm^3/$
A^u	Vector of the intercepts of the water demand for *i* sector in the upstream area.	[1,233.33; 46,105.92; 1,269.23; 1,235.71; 1,800] Mm^3/$
B^d	Vector of the absolute values of the slope of the water demand for *i* sector in the downstream area.	[909.09; 1,000; 31,347.96; 769.23; 666.67] Mm^3/$
A^d	Vector of the intercepts of the water demand for *i* sector in the downstream area.	[1,400; 1,560; 48,902.82; 1,307.69; 1,180] Mm^3/$
k_1^U	Cost of pumping in the upstream area 1Mm^3 of water per Mm^3 of volume of the river	0.066 $/$m^3$
k_2^U	The intercept of the pumping cost equation for the upstream area	0.33 $/$m^3$
k_1^D	Cost of pumping in the downstream area 1Mm^3 of water per Mm^3 of volume of the lake	0.61 $/$m^3$
k_2^D	The intercept of the pumping cost equation for the downstream area	2.03 $/$m^3$
s_0	Initial storage of the lake	292,500 Mm^3

R_O	Initial runoff rate	16,666.155 Mm^3
O_0	Initial outflow rate	22,788.644 Mm^3
w_0	Initial precipitation	$6.8308 \cdot 10^5$ Mm^3
wr_0	Initial renewable water resources	$4.43 \cdot 10^7 Mm^3$
σ_R	Volatility of runoff	0.431
σ_O	Volatility of outflow	0.025
σ_W	Volatility of precipitation	0.089

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