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Mitigation and Solar Radiation Management in Climate Change Policies

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Mitigation and Solar Radiation Management in Climate Change Policies*

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Abstract

We couple a spatially homogeneous energy balance climate model with an economic growth model which incorporates two potential policies against climate change: mitigation, which is the traditional policy, and geoengineering. We analyze the optimal policy mix of geoengineering and mitigation in both a cooperative and a noncooperative framework, in which we study open loop and feedback solutions. Our results suggests that greenhouse gas accumulation is relatively higher when geoengineering policies are undertaken, and that at noncooperative solutions incentives for geoengineering are relative stronger. A disruption of geoengineering efforts at a steady state will cause an upward jump in global temperature.

Keywords: Climate change, mitigation, geoengineering, cooperation, differential game, open loop - feedback Nash equilibrium

JEL Classification: Q53, Q54.

1 Introduction

The issue of climate change and the development of policies that will slow down or even reverse current trends has become an important issue both in terms of scientific research and applied policy. The current discussions in both theory and practice focus on mitigation of emissions as the main policy instrument against climate change.[1]

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[1]Adaptation is another policy option which is not discussed here since it does not aim at changing the current trends in climate but rather at coping with the consequences of climate change.

In the last few years, however, it it has become technically feasible to use engineering methods for solar radiation management as a means to offset the warming caused by the accumulation of carbon dioxide (CO_2) and other greenhouse gases (GHGs) ([1], [9]). Solar radiation management, which is also referred to as geoengineering ([8]), includes methods that either block incoming solar radiation or equivalently increase the planet's albedo, that is the capacity of the planet to reflect incoming solar radiation. In this paper we examine a specific geoengineering proposal that suggests pumping sulphur dioxide into the stratosphere to shade the earth from the sun by spreading very small reflective particles. What are regarded as the main advantages of geoengineering that make it attractive for managing climate change are that it is quick and cheap ([1], [14]). It is quick because it directly affects global temperature by mimicking the impact of a large volcano explosion that blocks incoming solar radiation as it emits large amounts of sulfur dioxide.[2] It is also cheap relative to the cost of large scale mitigation as current studies indicate (e.g. [1], [14]). On the other hand, the currently accepted option of limiting global warming is mitigation, that is reduction of emissions of GHG, which reduce global temperature by reducing the stock of GHGs, thus allowing larger amounts of outgoing solar radiation (see for example [9]) . However, more time is required until significant results can be obtained in reversing current trends in temperature through mitigation. Furthermore mitigation is an expensive solution relative to geoengineering and suffers from the well known free rider problem. In contrast geoengineering reduces temperature by blocking incoming radiation. It is relatively cheap and works fast; that is, it is a 'quick fix'.

There are, however many potential global disadvantages in geoengineering techniques, which include negative effects on plants due to reduced sunlight; ozone depletion; more acid depositions; less solar radiation available for solar power systems; and inability of engineering methods to adjust regional climate to desired levels. Furthermore, if geoengineering is used as a substitute for GHG emission reductions, the acidification of oceans will be intensified. The most serious drawback of geoengineering is, however, that we do not really know the consequences of solar radiation management on the earth's climate. Moreover once these methods are applied it might be difficult to reverse outcomes ([14]).

As indicated by Weitzman [16], geoengineering is also associated with an important externality which is different from the free rider externality associated with climate change. Weitzman calls this externality the 'free driver' externality. The source of this externality is the fact that, because geoengineering is relatively cheap, it can be undertaken unilaterally by one country which is not willing to incur the cost of mitigation. This action, however, is very likely to generate potentially very large costs to all other countries. Thus while the private costs of geoengineering are low, the potentially very high social costs of geoengineering are spread among all countries.

[2] The generally accepted mechanism by which large eruptions affect climate is generally injection of sulfur into the stratosphere and conversion to sulfate aerosol, which in turn reduces the solar energy reaching the earth's surface. The most recent example of such a process is the explosion of Mount Pinatubo in 1991.

In this paper, we study the design of policy against climate change, by combining at the same time mitigation, the traditional approach to policy design, with the policy option of solar radiation management through geoengineering as a means of reducing global temperature. The model we develop consists of a traditional economic module along with a climate module based on a simplified energy balance climate model (EBCM). EBCMs are based on the idea of global radiative heat balance. In radiative equilibrium the rate at which solar radiation is absorbed matches the rate at which infrared radiation is emitted. The purpose of geoengineering as a policy instrument is to reduce global average temperature by controlling the incoming solar radiation.

Our main purpose is to study optimal policy design in terms of mitigation and geoengineering efforts. We seek to characterize and contrast cooperative and noncooperative emission strategies. On the modeling side we consider a world consisting of a number of identical countries with production activities that generate GHG emissions. The stock of GHGs blocks outgoing radiation and causes temperature to increase. Mitigation reduces emissions and the stock of GHGs, which allows a larger flow of outgoing radiation and eases the pressure on temperature to rise. Geoengineering, on the other hand, blocks incoming radiation which is expected to cause a drop in temperature. This drop does not, at least in the way that our model is developed, depend on the accumulated GHGs.

We analyze the problem, as it is usual in this type of problems, in the context of cooperative and noncooperative solutions. In the cooperative case there is international coordination for the implementation of geoengineering and mitigation in order to maximize the joint, or global, welfare. In the noncooperative case, each government chooses geoengineering and mitigation policies noncooperatively. In this case we analyze open-loop and feedback Nash equilibrium (FBNE) strategies. We are interested in analyzing and comparing the cooperative and the noncooperative solutions, regarding the steady state stock of GHGs and temperature, in examining the sensitivity of the steady state temperature to each instrument alone, as well as the substitutability between mitigation and geoengineering along optimal cooperative and noncooperative paths.

Our results suggest that when geoengineering is a policy option, the steady state accumulation of GHGs is higher relative to the case where geoengineering is not an option. This result holds at the cooperative and noncooperative solutions, with relatively stronger incentives for geoengineering at the noncooperative solutions. Higher GHGs could be compatible with lower global temperature, at least in the short run, since geoengineering increases global albedo which tends to reduce temperature. Thus geoengineering could lead to a solution of relatively higher GHGs and temperature, or relatively higher GHGs but lower temperature relative to the case where geoengineering is not an option. The outcome is largely an empirical issue with many deep structural uncertainties. Another result stemming from our analysis is that even if geoengineering leads to a lower temperature, maintaining this temperature requires a constant flow of geoengineering. Thus, if this flow cannot be kept constant at some point in time, then there will be a jump in the temperature which will be intensified since the stock of GHGs will already be high.

In section 2 we present our model consisting of an economic and a climate module. In sections 3 and 4 we determine cooperative and noncooperative solutions and derive optimality conditions. In section 5 we compare cooperative and noncooperative solutions, in section 6 we study and compare the two polar cases of 'mitigation only' or 'geoengineering only', while in section 7 we examine mitigation-geoengineering substitutability. Section 8 concludes.

2 The Basic Model

2.1 The economic module

In our model, $i = 1, ..., \mathcal{N}$ economies, or countries, produce output according to a standard neoclassical production function:

$$Y_i(t) = F(K_i(t), \mathcal{A}N_i(t)) \quad , \quad i = 1, ..., \mathcal{N} \tag{1}$$

where $K_i(t)$ is capital, $\mathcal{A}N_i(t)$ is effective labour and $t \in [0, \infty)$ is the time index.

Output at each point in time is allocated to net capital formation $\frac{dK(t)}{dt} = \dot{K}(t)$, depreciation $\delta K(t)$, consumption $C(t)$, and the cost of mitigation and geoengineering. It is assumed that mitigation effort of magnitude $X(t)$ will cost $p_\chi(X(t)), (p_\chi(X(t)))' > 0, (p_\chi(X(t)))'' \geqslant 0$ in terms of output, while geoengineering effort of magnitude $Z(t)$ will cost $p_\zeta(Z(t)), p_\zeta(Z(t))' > 0, p_\zeta(Z(t))'' \geqslant 0$ in terms of output.

To simplify we specify the cost of mitigation and geoengineering by the linear functions $p_\chi X_i(t)$, $p_\zeta Z_i(t)$. Thus the resource constraint for economy $i = 1, ..., \mathcal{N}$, omitting t to simplify notation, will be[3]

$$\dot{K}_i = F(K_i, \mathcal{A}N_i) - C_i - p_\chi X_i - p_\zeta Z_i - \delta K_i. \tag{2}$$

We normalize all the relevant variables in terms of labor efficiency units. Population grows at the exogenous rate $n > 0$ and labor efficiency $\mathcal{A}$ grows at the given rate of labor-augmenting technical progress $\pi > 0$. Effective labour is $\mathcal{A}N = \mathcal{A}_0 N_0 e^{(n+\pi)t}$, so we have that output per effective worker is:

$$\begin{aligned} Y &= F(K, \mathcal{A}N) \rightarrow \frac{F(K, \mathcal{A}N)}{\mathcal{A}N} = \frac{Y}{\mathcal{A}N} = y \text{ and} \\ f(k) &= F\left(\frac{K}{\mathcal{A}N}, \frac{\mathcal{A}N}{\mathcal{A}N}\right) = F(k, 1). \end{aligned}$$

Assuming that the production function is a Cobb-Douglas, we have that:

$$y = f(k) = k^a \quad , \quad 0 < a < 1. \tag{3}$$

[3] We are basically assuming closed economies, interacting only through the global externality of climate change. This simplifies the model without affecting the qualitative nature of our results about climate change.

Thus the capital stock accumulation equation can be written in efficiency units as:

$$\dot{k} = k_i^a - p_\chi \chi_i - p_\zeta \zeta_i - c_i - (n + \pi + \delta) k_i \tag{4}$$

where $f(k_i) = \frac{K_i}{\mathcal{A}N_i}$: output per effective worker, $y = f(k_i) = k_i^a$: production function, $\chi_i = \frac{X_i}{\mathcal{A}N_i}$: mitigation effort per effective worker, $\zeta_i = \frac{Z_i}{\mathcal{A}N_i}$: geoengineering effort per effective worker, $c_i = \frac{C_i}{\mathcal{A}N_i}$: consumption.

We assume for each country a linear utility function:

$$U(C_i) = C_i, U(C_i) = \mathcal{A}_{0i} N_{0i} e^{(n+\pi)t} c_i. \tag{5}$$

Due to the linear utility function, the problem of choosing the optimal consumption path can be considerably simplified because the capital accumulation problem can be written as a Most Rapid Approach Path (MRAP) problem and k can be eliminated as a state variable from the optimization, suggesting that capital stock in each country relaxes fast to its steady state value relative to the evolution of climate. This simplification helps to better reveal the main results about climate change policies given the complexity of the model.

We assume two types of damage functions related to climate change which affect utility. The first one reflects damages from the increase in the average global surface temperature because of GHGs emissions. This damage function is represented as usual by a convex, quadratic in our case, function,

$$\Omega_T(T) = \frac{1}{2} c_T T^2, \Omega_T(0) = 0, (\Omega_T(T))' > 0, (\Omega_T(T))'' > 0 \tag{6}$$

where $c_T T$ is the marginal damage cost from a temperature increase.

The second is the damage function associated with geoengineering effects, such as for example ocean acidification or increased acid depositions.[4] Assume that a country undertakes geoengineering ζ_i, which will generate total social damages $\frac{1}{2} c_\zeta \zeta_i^2$ in terms of ocean acidification and acid depositions. Assume that these damages will be spread uniformly among all countries. So each country will suffer $\frac{1}{2\mathcal{N}} c_\zeta \zeta_i^2$ from geoengineering undertaken by country i. If all countries undertake geoengineering, the aggregate damages that country i will suffer from geoengineering undertaken by all countries, including own induced damages, will be

$$\frac{1}{2\mathcal{N}} c_\zeta \zeta_1^2 + ... + \frac{1}{2\mathcal{N}} c_\zeta \zeta_i^2 + ... \frac{1}{2\mathcal{N}} c_\zeta \zeta_\mathcal{N}^2 = \frac{c_\zeta}{2\mathcal{N}} \sum_{i=1}^{\mathcal{N}} \zeta_i^2.$$

[4] As mentioned in the introduction, the use of geoengineering methods could intensify ocean acidification. Although the natural absorption of CO_2 by the world's oceans helps mitigate the climatic effects of anthropogenic emissions of CO_2, it is believed that since geoengineering will cause an increase in GHG emissions, the resulting decrease in pH will have negative consequences, primarily for oceanic calcifying organisms, and there will be an impact on marine environments. For a discussion of damage functions related to climate change see Weitzman[15].

Therefore global damages from geoengineering will be

$$\begin{aligned} \Omega_{\zeta}(\boldsymbol{\zeta}) &= \sum_{i=1}^{\mathcal{N}} \left(\frac{c_{\zeta}}{2\mathcal{N}} \sum_{i=1}^{\mathcal{N}} \zeta_i^2 \right) \\ \text{thus, } \Omega_{\zeta}(\mathbf{0}) &= 0, (\Omega_{\zeta}(\boldsymbol{\zeta}))' > 0, (\Omega_{\zeta}(\boldsymbol{\zeta}))'' > 0 \end{aligned} \tag{7}$$

where $c_{\zeta}\zeta_i$ is the social marginal damage cost from the geoengineering.[5]

The global welfare function to be maximized is the unweighted discounted life time utility of a representative household in each country minus the social damages related to the increase in global temperature and to geoengineering. Thus a cooperative case is equivalent to having a social planner solving:

$$W = \max_{c_i, \zeta_i, \chi_i} \int_0^{\infty} e^{-vt} \left\{ \sum_{i=1}^{\mathcal{N}} \mathcal{A}_{0i} N_{0i} \left[c_i - \frac{1}{2} c_T T^2 - \frac{1}{2} \frac{c_{\zeta}}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} \zeta_i^2 \right] \right\} dt, \ v = \rho - n - \pi$$

subject to resource and climate constraints. (8)

2.2 The climate module

We use North [11] – [13] as basis for our exposition in order to describe climate by a simplified "homogeneous-earth" EBCM[6] that describes the relation between outgoing infrared radiation $I(t)$ at time t, and the average global surface temperature $T(t)$ (measured in degrees Celsius) at time t. The infrared radiation flux to space $I(t)$ can be represented as a linear function of the surface temperature $T(t)$ by the empirical formula:

$$I(t) = A + BT(t), A = 201.4 W/m^2, \ B = 1.45 W/m^2 \tag{9}$$

where A, B are constants used to relate outgoing infrared radiation with the corresponding surface temperature.

In our model the change in the average global surface temperature $T(t)$ is determined by the sum of the absorbed solar heating (T_0), the reduction of incoming radiation due to the aggregate geoengineering effort (T_1) and the increase in the surface temperature due to the emissions of GHGs (T_2) which block outgoing radiation,

[5] There is a subtle difference between damages from the overall increase in temperature due to global warming and the damages from acidification due to geoengineering. Damages from an overall increase in temperature have public bad characteristics since all countries are affected by the global change. On the other hand ocean acidification or acid depositions due to geoengineering will have local effects depending on countries' reliance on oceans, soil compositions etc. Thus damages induced from a given geoengineering activity undertaken by a single country will spread among countries according to countries' specific characteristics. In an extreme case, geoengineering activities undertaken by one country might have negligible effects on this same country. In our approach we assume that geoengineering damages are evenly spread across countries. This a strong simplifying assumption and analysis of asymmetric damages is an interesting area for further research.

[6] A homogeneous-earth model is a "zero-dimensional" model since it does not contain spatial dimensions but only the temporal dimension.

$$\dot{T} = T_0 + T_1 + T_2 \tag{10}$$

$$T_0 = \frac{-(A+BT) + S(1-\alpha)}{B},\ T_1 = -\frac{\phi}{B}\sum_{i=1}^{\mathcal{N}} \zeta_i,\ T_2 = \frac{\psi}{B}\ln\left(1+\frac{G}{G_0}\right). \tag{11}$$

The term $(A+BT)$ reflects outgoing radiation, S is the mean annual distribution of radiation, α is the average albedo of the planet, the function $\varphi(\zeta) = \frac{\phi}{B}\sum_{i=1}^{\mathcal{N}} \zeta_i$ is the reduction in solar radiation due to aggregate geoengineering $\sum_{i=1}^{\mathcal{N}} \zeta_i$, $\phi > 0$ is the sensitivity of incoming radiation to geoengineering in reducing the average global temperature,[7] ψ is a measure of climate's sensitivity and G, G_0 are variables associated with the GHGs, where G is the current accumulation of GHGs and G_0 is the preindustrial GHGs accumulation.

We substitute T_0, T_1, T_2 into (10) to obtain:[8]

$$\dot{T} = \frac{-(A+BT) + S(1-\alpha)}{B} - \frac{\phi}{B}\sum_{i=1}^{\mathcal{N}} \zeta_i + \frac{\psi}{B}\ln\left(1+\frac{G}{G_0}\right). \tag{12}$$

From equation (12) we have that: a) the average global temperature increases when current accumulation of GHGs is above the preindustrial GHGs accumulation because GHGs block outgoing radiation and b) the average global temperature decreases when the application of geoengineering manages to reduce incoming radiation.

We assume that capital k and average global temperature T converge fast to their corresponding steady states relative to the accumulation of GHGs (G) (e.g. [3]) Then the steady state value of T can be used to express T as a function of G, or

$$\dot{T} = 0 \Longrightarrow \frac{-(A+BT) + S(1-\alpha)}{B} - \frac{\phi}{B}\sum_{i=1}^{\mathcal{N}} \zeta_i + \eta(G - G_0) = 0$$

$$T = \frac{-A + S(1-\alpha) - \phi\sum_{i=1}^{\mathcal{N}} \zeta_i + \eta(G-G_0)}{B} = \varphi(\zeta_i, G). \tag{13}$$

Note that in order to simplify the exposition we replace the term $\frac{\psi}{B}\ln\left(1+\frac{G}{G_0}\right)$ in (12) with its linear approximation around G_0. In this case $\eta = \frac{\psi}{2BG_0}$.

We specify net emissions of GHGs in each country to be a function of the capital stock and mitigation effort, or

$$E(t) = \Omega(t)\,\kappa K^a X^{-\varepsilon}\ , 0 < \varepsilon < 1$$

[7] Geoengineering can be regarded as increasing the global albedo, since it blocks incoming radiation. We use a sensitivity function which is linear in aggregate geoengineering instead of a nonlinear function in order to simplify the exposition.

[8] We do not consider at this stage the transportation of heat across the globe, which is a standard assumption of the EBCM developed by North [11] – [13]. Thus we study a homogeneous-earth, zero-dimensional model. This allows us to obtain tractable results regarding the mitigation/ geoengineering trade-off. The analysis of the mitigation/geoengineering trade-off in the context of a one-dimensional spatial model is an area for further research.

where $\Omega(t)$ is an efficiency parameter reflecting emission reducing technical progress and ε is a technical coefficient transforming, for fixed mitigation effort, the contribution of capital stock K^a into emissions of GHGs. Expressing K and Z in per effective worker terms we have

$$E(t) = \Omega(t)\,\kappa\mathcal{A}_0 N_0 \mathrm{e}^{(n+\pi)(a-\varepsilon)} k^a \chi^{-\varepsilon}.$$

To study steady states and avoid explosive dynamics in emissions per capita we impose the condition $\dot{\Omega}(t)/\Omega(t) = \hat{\omega} : \hat{\omega} + (n+\pi)(a-\varepsilon) = 0$. Assuming $a - \varepsilon > 0$, we set $\Omega(t) = \omega e^{-(n+\pi)(a-\varepsilon)t}$ (see [5], [4] for a similar assumption). Thus for country i, $E_i(t) = \omega\kappa k_i^a \chi_i^{-\varepsilon}$, and the evolution of the accumulation of GHGs can be written as:

$$\dot{G} = \beta \sum_{i=1}^{\mathcal{N}} \omega\kappa k_i^a \chi_i^{-\varepsilon} - mG \tag{14}$$

where β is the proportion of GHGs emissions remaining in the atmosphere, and m is the natural decay rate of GHGs.

Defining $\beta\omega\kappa = \gamma$ we can rewrite (14) as:

$$\dot{G} = \sum_{i=1}^{\mathcal{N}} \gamma k_i^a \chi_i^{-\varepsilon} - mG. \tag{15}$$

The problem of the social planner is to maximize global unweighted discounted life time utility by choosing paths for mitigation $\chi_i(t)$ (control of emissions of CO_2 and other GHGs), geoengineering $\zeta_i(t)$ and consumption $c_i(t)$ subject to the resource constraint, the constraint of the average global temperature and the constraint of the accumulation of GHGs. The planner's goal is to obtain an optimal policy mix that takes into account both mitigation and geoengineering (increase of albedo) and to find an optimal path which will provide a stable solution for managing global warming in an optimal way.

3 Cooperation among countries

We assume that $i = 1, ..., \mathcal{N}$ countries cooperatively decide emission and geoengineering paths. This is equivalent to having a social planner choose consumption, emissions and geoengineering paths to maximize global welfare, or

$$\begin{aligned} W &= \max_{c_i, \zeta_i, \chi_i} \int_0^\infty e^{-vt} \sum_{i=1}^{\mathcal{N}} \mathcal{A}_{0i} N_{0i} \left[c_i - \frac{1}{2} c_T T^2 - \frac{1}{2} \left(\frac{c_\zeta}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} \zeta_i^2 \right) \right] dt \\ v &= \rho - n - \pi \end{aligned} \tag{16}$$

subject to (13), the resource constraint, and

$$\dot{G} = \sum_{i=1}^{\mathcal{N}} \gamma k_i^a \chi_i^{-\varepsilon} - mG. \tag{17}$$

Linear utility allows to express c as a function of $\left(\chi, \zeta, k, \dot{k}\right)$, or

$$c_i = k_i^a - p_\chi \chi_i - p_\zeta \zeta_i - \dot{k}_i - (n + \pi + \delta) k_i. \tag{18}$$

Then, using the MRAP transformation, the optimization problem can be written as:[9]

$$\begin{aligned} W &= \max_{k_i, \zeta_i, \chi_i} \int_0^\infty e^{-\nu t} \sum_{i=1}^{\mathcal{N}} \mathcal{A}_{0i} N_{0i} \left\{ \left[\left(k_i^a - p_\chi \chi_i - p_\zeta \zeta_i\right) - (\rho + \delta) k_i \right] \right. \\ & \left. -\frac{1}{2} c_T T^2 - \frac{1}{2} \frac{c_\zeta}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} \zeta_i^2 \right] \Bigg\} dt. \end{aligned} \tag{19}$$

The current value Hamiltonian function is:

$$\begin{aligned} H &= \sum_{i=1}^{\mathcal{N}} \mathcal{A}_{0i} N_{0i} \left[\xi (\chi_i, \zeta_i, k_i) - \frac{1}{2} c_T (\varphi(\zeta_i, G))^2 - \frac{1}{2} \frac{c_\zeta}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} \zeta_i^2 \right] + \\ & + \lambda_i^C(t) \left[\sum_{i=1}^{\mathcal{N}} \gamma k_i^a \chi_i^{-\varepsilon} - mG \right] \end{aligned} \tag{20}$$

where $$\left[\left(k_i^a - p_\chi \chi_i - p_\zeta \zeta_i\right) - (\rho + \delta) k_i \right] = \xi (\chi_i, \zeta_i, k_i). \tag{21}$$

Imposing symmetry so that $k_i = k$, $\chi_i = \chi$, $\zeta_i = \zeta$ for all i, and setting $\mathcal{A}_{0i} N_{0i} = 1$ to simplify, we obtain the optimal controls as functions of $\left(G, \lambda^C\right)$. For interior solutions we obtain[10]

$$k^* = \left[\frac{a\left(\lambda^C \gamma \chi^{-\varepsilon} + 1\right)}{\rho + \delta} \right]^{\frac{1}{1-a}} = h_1\left(\lambda^C\right) \tag{22}$$

$$\chi^* = \left(-\frac{p_\chi}{\varepsilon \lambda^C \gamma k_i^a} \right)^{\frac{-1}{1+\varepsilon}} = h_2\left(\lambda^C\right) \tag{23}$$

$$\begin{aligned} \zeta^* &= \frac{\mathcal{N} \phi c_T \left[-A + S(1 - \alpha) + \eta (G - G_0) \right] - B^2 p_\zeta}{c_\zeta B^2 + \mathcal{N}^2 \phi^2 c_T} \\ &= h_3(G) \end{aligned} \tag{24}$$

with $(\beta \omega \kappa = \gamma) : \lambda^C \gamma\, \chi^{-\varepsilon} < 1$. We notice from equation (22) that the optimal level of capital stock at time t is determined by the equality of the extra benefits in terms of consumption from having an additional unit of capital with the extra

[9] Problem (8) is an approximation of the MRAP problem for very large S and $-S \leq \frac{dk_i}{dt} \leq S$. Thus in problem (8), k can be eliminated on a state variable.

[10] For the derivation of(22), (23), (24), see Appendix.

cost of the global damages due to the GHGs generated by this unit. From equation (23) the optimal level of mitigation is determined at the point where the extra global benefits from an additional unit of mitigation, in terms of reduced GHGs, equals the cost of mitigation in terms of output used to support mitigation efforts. In a similar way, the optimal level of geoengineering is determined by the equality between the private and the social costs of geoengineering with the marginal benefits from the reduction in global temperature due to geoengineering. It is also worth noting that mitigation depends on the shadow cost of the GHGs through (23), but geoengineering depends on the social and private costs of geoengineering c_ζ and p_ζ respectively. Thus a high social cost of geoengineering will reduce geoengineering efforts. On the other hand, low private costs will tend to increase geongineering.

The modified dynamic Hamiltonian system (MHDS) characterizing, under symmetry, the cooperative solution is:[11]

$$\dot{G} = \mathcal{N}\gamma \left[h_1\left(\lambda^C\right)\right]^a \left[h_2\left(\lambda^C\right)\right]^{-\varepsilon} - mG \tag{25}$$

$$\dot{\lambda}^C = (m+\nu)\lambda^C + \frac{\mathcal{N}\eta c_T T}{B} \tag{26}$$

where $T = \dfrac{-A + S(1-\alpha) - \phi\mathcal{N}\zeta^*(G) + \eta(G - G_0)}{B}$.

Intertemporal transversality conditions require:

$$\lim_{t\longrightarrow\infty} e^{-\nu t}\lambda^C G(t)\,dt = 0\ , \quad \lim_{t\longrightarrow\infty} e^{-\nu t} G(t)\,dt = 0\ . \tag{27}$$

The modified Hamiltonian dynamic system (MHDS) $(25), (26)$ with an initial condition for G and the transversality conditions (27) determine the evolution of the state (G) and costate $\left(\lambda^C\right)$ variables along the optimal path which characterizes optimal mitigation and optimal geoengineering.

A steady state GHGs accumulation and its corresponding shadow cost can be defined as $\overline{G} : \mathcal{N}\gamma\left[h_1\left(\overline{\lambda}^C(G)\right)\right]^a\left[h_2\left(\overline{\lambda}^C(G)\right)\right]^{-\varepsilon} - mG = 0$ and $\overline{\lambda}^C(G) = -\frac{\mathcal{N}\eta c_T T(G)}{B(m+\nu)}$. Assume that such a steady state exists in a closed interval $[G_1, G_2]$ when conditions described in Appendix A.2 are satisfied, then the stability properties of the long-run equilibrium for the GHGs accumulation that correspond to the social optimum are summarized in the following proposition.

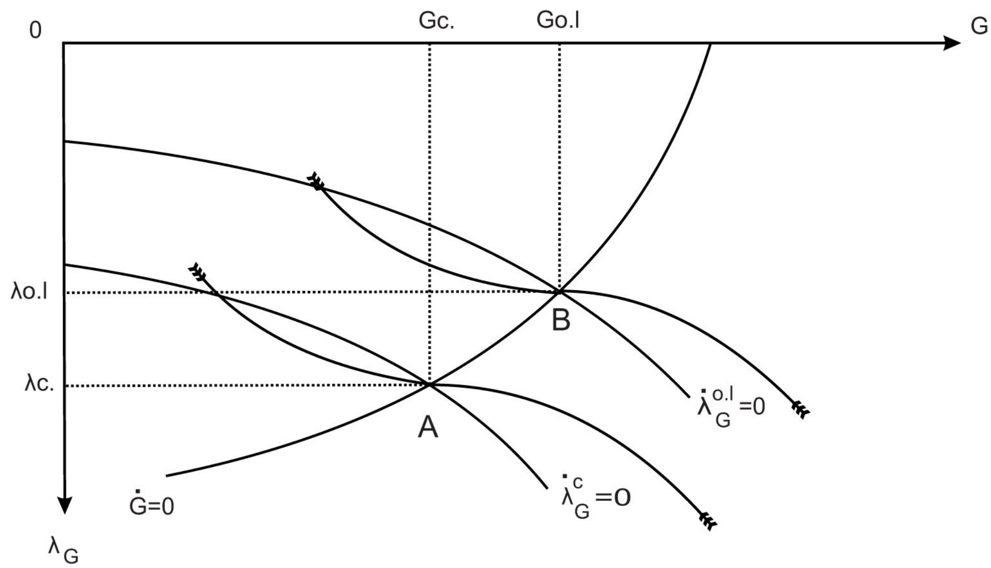
Proposition 1 *Consider a steady state of the MHDS* $(25), (26)$, *in a closed interval* $[G_1, G_2]$. *If* $Z = \eta - \phi\mathcal{N}\frac{\partial\zeta^*}{\partial G} \geq 0$, *then this steady state is a local saddle point. If* $Z = \eta - \phi\mathcal{N}\frac{\partial\zeta^*}{\partial G} < 0$, *then this steady state can be locally unstable.*

For proof, see Appendix A.3.

[11] For the derivation of $(25), (26)$, see Appendix.

This proposition suggests that the presence of geoengineering may introduce locally unstable steady states. This could happen if the total impact of geoengineering with respect to changes of current GHGs, $\phi\mathcal{N}\frac{\partial\zeta^*}{\partial G}$, is large relative to η.

10



$_1.pdf$

Figure 1

From (26) for $\dot{\lambda}^C = 0$ we have that $\lambda^C(G) = \frac{-\mathcal{N}\eta c_T T(G)}{(m+\nu)B}$ at the steady state. This quantity is expected to be negative, since the average global temperature $T(G)$ is expected to be positive for realistic values of G, and increasing in G. Then if G becomes very large, $\lambda^C(G)$ is expected to be negative and large in absolute value. When $G = G_0$ then $\zeta(G)$ will be small and $\lambda^C(G)$ will be finite and small in absolute value. Thus $\lambda^C(G)$ is negative, starts around zero and goes to minus infinity.

However $\lambda^C(G)$ might have increasing parts, because of the following argument. The derivative of $\lambda^C(G)$ with respect to G is $\frac{d\lambda^C}{dG} = \frac{-\mathcal{N}\eta c_T\left(\eta-\phi\mathcal{N}\frac{\partial\zeta^*}{\partial G}\right)}{(m+\nu)B^2}$. We expect $\zeta'(G) > 0$ therefore $\lambda^C(G)$ may have increasing and decreasing parts as it starts around zero and goes to minus infinity.

For $\dot{G} = 0$ the slope of (25) at the steady state under symmetry is determined by the total derivative of (25) which is given by:

$$\mathcal{N}\gamma\left[h_2\left(\lambda^C\right)\right]^{-\varepsilon-1}\left[h_1\left(\lambda^C\right)\right]^{a-1}\left[-\varepsilon h_1\left(\lambda^C\right)\frac{dh_2\left(\lambda^C\right)}{d\lambda^C} + a h_2\left(\lambda^C\right)\frac{dh_1\left(\lambda^C\right)}{d\lambda^C}\right]d\lambda^C = mdG \tag{28}$$

where

$$\left[-\varepsilon h_1\left(\lambda^C\right)\frac{dh_2\left(\lambda^C\right)}{d\lambda^C}+ah_2\left(\lambda^C\right)\frac{dh_1\left(\lambda^C\right)}{d\lambda^C}\right]>0,\text{ since }\frac{dh_2\left(\lambda^C\right)}{d\lambda^C}<0,\frac{dh_1\left(\lambda^C\right)}{d\lambda^C}>0. \tag{29}$$

Thus the curve described by (25) will be increasing and multiple steady states may exist, given the behavior of $\lambda^C(G)$.

The above argument suggests that in the closed interval $[G_1, G_2]$ the isocline of $\dot{G}=0$ is increasing and the isocline of $\dot{\lambda}^C=0$ may have increasing and decreasing parts. Assume that in the closed interval $[G_1, G_2]$ the isocline of $\dot{\lambda}^C=0$ has a positive slope $\left(\eta-\phi\mathcal{N}\frac{\partial\zeta^*}{\partial G}<0\right)$, then if it intersects the isocline of $\dot{G}=0$ we have local instability. If in the closed interval $[G_1, G_2]$ the isocline of $\dot{\lambda}^C=0$ has negative slope $\left(\eta-\phi\mathcal{N}\frac{\partial\zeta^*}{\partial G}>0\right)$, then we can define the isoclines for $\dot{G}=\dot{\lambda}^C=0$ as monotonic functions that intersect once at a local saddle point steady state. In this case there exists a one-dimensional stable manifold that contains the steady state. The solution is presented in Figure 1, where A is the steady state obtained at the intersection of the $\dot{G}=0$ locus with the $\dot{\lambda}^C=0$ locus.

4 Non-cooperative Solutions

We now examine noncooperative solutions where each country maximizes its own welfare subject to the resource constraint and the climate dynamics. We examine the two equilibrium concepts very often used for noncooperative equilibrium: the open loop and the feedback Nash equilibrium.

4.1 Open Loop Nash Equilibrium

In a noncooperative setup a country i decides unilaterally about the implementation of geoengineering and mitigation, by taking the actions of all other countries $j \neq i$ as given. In an open loop Nash equilibrium (OLNE), each country chooses open loop policies which depend only on initial values and time ([2]).

The noncooperative solution for country i is obtained as solution to the problem

$$\begin{aligned} W_i &= \max_{c_i,\zeta_i,\chi_i}\int_0^\infty e^{-\nu t}\left[\mathcal{A}_{0i}N_{0i}c_i-\frac{1}{2}c_T T^2-\frac{c_\zeta}{2\mathcal{N}}\left(\zeta_i^2+\sum_{j\neq i}^{\mathcal{N}}\bar{\zeta}_j^2\right)\right]dt \quad (30)\\ \nu &= \rho-n-\pi \end{aligned}$$

subject to the resource constraint and

$$\dot{G} = \gamma k_i^a \chi_i^{-\varepsilon} + \sum_{j \neq i}^{\mathcal{N}} \gamma \left(\bar{k}_j\right)^a \left(\bar{\chi}_j\right)^{-\varepsilon} - mG \tag{31}$$

$$T = \frac{-A + S(1-\alpha) - \phi\left(\zeta_i + \sum_{i \neq j}^{\mathcal{N}} \bar{\zeta}_j\right) + \eta(G - G_0)}{B}$$

where $\bar{\chi}_j, \bar{\zeta}_j$ means that country i takes the action of of all other countries $j \neq i$ as given in an OLNE.

Using the MRAP transformation, the current value Hamiltonian function underlying open loop strategies is:

$$H_i^{OL} = \mathcal{A}_{0i} N_{0i} \xi(\chi_i, \zeta_i, k_i) - \frac{1}{2} c_T T^2 - \frac{c_\zeta}{2\mathcal{N}} \left(\zeta_i^2 + \sum_{j \neq i}^{\mathcal{N}} \bar{\zeta}_j^2\right) + $$
$$+\lambda_i^{OL}(t) \left[\gamma k_i^a \chi_i^{-\varepsilon} + \sum_{j \neq i}^{\mathcal{N}} \gamma \left(\bar{k}_j\right)^a \left(\bar{\chi}_j\right)^{-\varepsilon} - mG\right]. \tag{32}$$

Imposing symmetry so that $\zeta_i = \zeta$ and $k_i = k$ and $\chi_i = \chi$, $\mathcal{A}_{0i} = \mathcal{A}_0, N_{0i} = N_0$ for all i, and setting $\mathcal{A}_{0i} N_{0i} = 1$ to simplify, we obtain the optimal controls as functions of $\left(G, \lambda^{OL}\right)$. For interior solutions we obtain:[12]

$$k^* = \left(\frac{a\left(\lambda^{OL} \gamma \chi^{-\varepsilon} + 1\right)}{\rho + \delta}\right)^{\frac{1}{1-a}} = h_1\left(\lambda^{OL}\right) \tag{33}$$

$$\chi^* = \left(-\frac{p_\chi}{\varepsilon \lambda^{OL} \gamma k_i^a}\right)^{\frac{-1}{1+\varepsilon}} = h_2\left(\lambda^{OL}\right) \tag{34}$$

$$\zeta^* = \frac{\mathcal{N}\phi c_T\left[-A + S(1-\alpha) + \eta(G - G_0)\right] - \mathcal{N} B^2 p_\zeta}{c_\zeta B^2 + \mathcal{N}^2 \phi^2 c_T} \tag{35}$$
$$= h_3(G).$$

The interpretation of the optimality conditions is the same as in the cooperative equilibrium. But the benefits or the costs in terms of changes in the GHGs accumulation are not global; they refer to a single country. It should be noted that the derivative $\partial\zeta^*/\partial p_\zeta$ is higher in absolute value in the noncooperative solution than the cooperative solution. This means that a decrease in the private geoengineering costs will increase geoengineering more when countries do not cooperate.

[12] For the derivation of $(33), (34), (35)$, see Appendix.

The MHDS characterizing the OLNE under symmetry implies[13]

$$\dot{G} = \mathcal{N}\gamma \left[h_2\left(\lambda^{OL}\right)\right]^{-\varepsilon}\left[h_1\left(\lambda^{OL}\right)\right]^{a} - mG \tag{36}$$

$$\dot{\lambda}^{OL} = (m+\nu)\,\lambda^{OL} + \frac{\eta c_T T}{B} \tag{37}$$

where $T = \dfrac{-A + S(1-\alpha) + \eta(G - G_0) - \mathcal{N}\phi\zeta^*(G)}{B}$.

Intertemporal transversality conditions require:

$$\lim_{t\longrightarrow\infty} e^{-\nu t}\lambda^{OL}G(t)\,dt = 0\ ,\quad \lim_{t\longrightarrow\infty} e^{-\nu t}G(t)\,dt = 0\ . \tag{38}$$

The MHDS (36),(37) with initial condition for G and the transversality conditions (38) determines the evolution of the state (G) and costate variables $\left(\lambda^{OL}\right)$ along the optimal path which characterizes optimal mitigation expenditures and optimal geoengineering.

The properties of the steady state of the GHGs accumulation for the OLNE regarding existence and stability are similar to the social optimum presented above. This is because the structure of the MHDS is the same in both cases.

The solution is presented in Figure 1, where B is the steady state obtained at the intersection of the $\dot{G} = 0$ locus with the $\dot{\lambda}^{OL} = 0$ locus. Analysis of the OLNE implies, as we show in section 5, that countries have an incentive to choose a higher level of GHGs emissions relative to cooperation leading to a higher accumulation of GHGs at the steady state relative to the cooperative solution.

4.2 Feedback Nash equilibrium

As is well known, an OLNE is not a strong time consistent solution ([2]). In this section we study feedback solutions which have the desired property of time consistency. Country i takes as given the feedback (or closed loop) time stationary strategies $\zeta_j(G(t))$ and $\chi_j(G(t))$ $j \neq i$, of other countries. The feedback strategies condition mitigation and geoengineering policies on the observed concentration of CO_2.[14] The result will not change if we condition geoengineering on the global temperature since by (13) temperature is monotonically increasing in G.

[13] For the derivation of $(36), (37)$, see Appendix.

[14] It should be noted that at this stage the feedback strategies are not known, since they emerge as part of the solution of the problem. Full determination of the feedback strategies requires the use of the dynamic programming approach and numerical methods, since our problem is not linear quadratic ([7]). In analyzing feedback solutions in this paper, we follow the Hamiltonian approach with unspecified feedback strategies, since this approach allows qualitative comparison of cooperative and non-cooperative solutions in a meaningful way, by comparing the shadow cost of GHGs which emerge as the costate variable of the Hamiltonian formulation.

We will assume the following properties for the feedback strategies:

$$\chi'(G) > 0 \ , \ \zeta'(G) > 0$$

The assumption that $\chi'(G)$ and $\zeta'(G)$ are positive implies that country i expects other countries to increase their mitigation and geoengineering efforts respectively when the level of GHGs increases, and other countries $(j \neq i)$ expect the same from country i. This can be regarded as a plausible assumption.[15]

The noncooperative feedback solution for country i is determined by the solution of the problem

$$\begin{aligned} W_i &= \max_{c_i,\zeta_i,\chi_i} \int_0^\infty e^{-\nu t} \left[\mathcal{A}_{0i} N_{0i} c_i - \frac{1}{2} c_T T^2 - \frac{c_\zeta}{2\mathcal{N}} \left(\zeta_i^2 + \sum_{j\neq i}^{\mathcal{N}} \left(\zeta_j(G)\right)^2 \right) \right] dt \\ \nu &= \rho - n - \pi > 0 \end{aligned} \tag{39}$$

subject to the resource constraint and

$$\dot{G} = \gamma k_i^a \chi_i^{-\varepsilon} + \sum_{j\neq i}^{\mathcal{N}} \gamma \left(\bar{k}_j\right)^a \left[\chi_j(G)\right]^{-\varepsilon} - mG \tag{40}$$

$$T = \frac{-A + S(1-\alpha) - \phi\left(\zeta_i + \sum_{j\neq i}^{\mathcal{N}} \left(\zeta_j(G)\right)\right) + \eta(G - G_0)}{B}.$$

Using the MRAP, the current value Hamiltonian function for this problem can be written as:

$$\begin{aligned} H_i^{FB} &= \mathcal{A}_{0i} N_{0i} \xi(\chi_i, \zeta_i, k_i) - \frac{1}{2} c_T T^2 - \frac{c_\zeta}{2\mathcal{N}} \left(\zeta_i^2 + \sum_{j\neq i}^{\mathcal{N}} \left(\zeta_j(G)\right)^2 \right) + \\ &\quad + \lambda_i^{FB}(t) \left[\gamma k_i^a \chi_i^{-\varepsilon} + \sum_{j\neq i}^{\mathcal{N}} \gamma \left(\bar{k}_j\right)^a \left[\chi_j(G)\right]^{-\varepsilon} - mG \right]. \end{aligned} \tag{41}$$

This implies that the optimal controls will be functions of $\left(G, \lambda^{FB}\right)$ or, under

[15] Regarding the stock of capital, we do not introduce the assumption of closed-loop strategies since it does not seem realistic, at least in the current world, to assume that aggregate investment in a country is conditioned on CO_2 accumulation. One might argue that investment in green technologies could be conditioned on CO_2 accumulation. This requires however a disaggregated model of technology choice which is beyond the scope of the present paper.

symmetry and by setting $\mathcal{A}_{0i}N_{0i}=1$ to simplify[16]

$$k^{*} = \left(\frac{a\left(\lambda^{FB}\gamma\chi^{-\varepsilon}+1\right)}{\rho+\delta}\right)^{\frac{1}{1-a}} = h_{1}\left(\lambda^{FB}\right) \tag{42}$$

$$\chi^{*} = \left(-\frac{p_{\chi}}{\varepsilon\lambda^{FB}\gamma k_{i}^{a}}\right)^{\frac{-1}{1+\varepsilon}} = h_{2}\left(\lambda^{FB}\right) \tag{43}$$

$$\zeta^{*} = \frac{\mathcal{N}\phi c_{T}\left[-A+S\left(1-\alpha\right)-\phi\sum_{j\neq i}^{\mathcal{N}}\zeta_{j}\left(G\right)+\eta\left(G-G_{0}\right)\right]-\mathcal{N}B^{2}p_{\zeta}}{c_{\zeta}B^{2}+\mathcal{N}\phi^{2}c_{T}} = h_{3}\left(G\right). \tag{44}$$

The interpretation of the optimally conditions is the same as in cooperative equilibrium, but in this case the benefits or the costs refer to a single country i, as i takes as given the feedback time stationary strategies $\zeta_{j}\left(G\left(t\right)\right)$ and $\chi_{j}\left(G\left(t\right)\right)$ $j\neq i$, of other countries.

The MHDS characterizing the FBNE under symmetry implies[17]

$$\dot{G}=\mathcal{N}\gamma\left[h_{2}\left(\lambda^{FB}\right)\right]^{-\varepsilon}\left[h_{1}\left(\lambda^{FB}\right)\right]^{a}-mG \tag{45}$$

$$\begin{aligned}\dot{\lambda}^{FB} = {} & \left(m+\nu\right)\lambda^{FB}+\frac{c_{T}T}{B}\left[\left(-\right)\phi\left(\mathcal{N}-1\right)\zeta'\left(G\right)+\eta\right]+c_{\zeta}\frac{\left(\mathcal{N}-1\right)}{\mathcal{N}}\zeta\left(G\right)\zeta'\left(G\right)\\ & +\varepsilon\lambda^{FB}\left(\mathcal{N}-1\right)\gamma\left(\chi_{j}\left(G\right)\right)^{-\varepsilon-1}\bar{k}_{j}^{a}\left(\chi_{j}\left(G\right)\right)'\end{aligned} \tag{46}$$

$$\text{where } T=\frac{-A+S\left(1-\alpha\right)-\phi\left(\zeta_{i}+\sum_{j\neq i}^{\mathcal{N}}\left(\zeta_{j}\left(G\right)\right)\right)+\eta\left(G-G_{0}\right)}{B}. \tag{47}$$

Intertemporal transversality conditions require

$$\lim_{t\longrightarrow\infty}e^{-\nu t}\lambda^{FB}G\left(t\right)dt=0\ ,\quad \lim_{t\longrightarrow\infty}e^{-\nu t}G\left(t\right)dt=0. \tag{48}$$

The MHDS of $(45),(46)$ with initial condition for G and the transversality condition (48) determine the evolution of the state (G) and costate variables $\left(\lambda^{FB}\right)$ along the optimal path which characterizes optimal mitigation expenditures and optimal geoengineering.

If, under the conditions described in Appendix A4, a steady state exists, then the stability properties of the symmetric long-run FBNE equilibrium for the GHGs accumulation are summarized in the following proposition.

[16] For the derivation of $(42),(43),(44)$, see Appendix.
[17] For derivation of $(45),(46)$, see Appendix.

Proposition 2 *Assume that functions $\zeta(G)$ and $\chi(G)$, with $\zeta'(G) > 0$ and $\chi'(G) > 0$, exist such that a steady state symmetric FBNE exists in a closed interval $[G_1, G_2]$. Then if $\frac{\partial H^{FB}}{\partial G} \geq 0$, the steady state is a local saddle point and if $\frac{\partial H^{FB}}{\partial G} < 0$, the steady state can be locally unstable.*

For proof, see Appendix A.5.

5 Comparisons

In this section we compare steady state GHGs accumulation under cooperative and noncooperative solutions.

5.1 Cooperation vs open loop strategies

The analysis of cooperative and open loop equilibrium implies that, in absolute values, the steady state shadow cost of GHGs accumulation in the case of cooperation among countries is higher than the steady state shadow cost in OLNE. We have:

Cooperative solution

$$\lambda^C = \frac{-c_T \mathcal{N} T \frac{\eta}{B}}{(m+\nu)} \tag{49}$$

OLNE

$$\lambda^{OL} = \frac{-c_T T \frac{\eta}{B}}{(m+\nu)} \tag{50}$$

By comparing $\dot{\lambda}^{OL} = 0$ and $\dot{\lambda}^{C} = 0$ for any given G, we can see that:

$$\left|\lambda^C\right| = \mathcal{N} \cdot \left|\lambda^{OL}\right| \Longrightarrow \left|\lambda^{OL}\right| < \left|\lambda^C\right|. \tag{51}$$

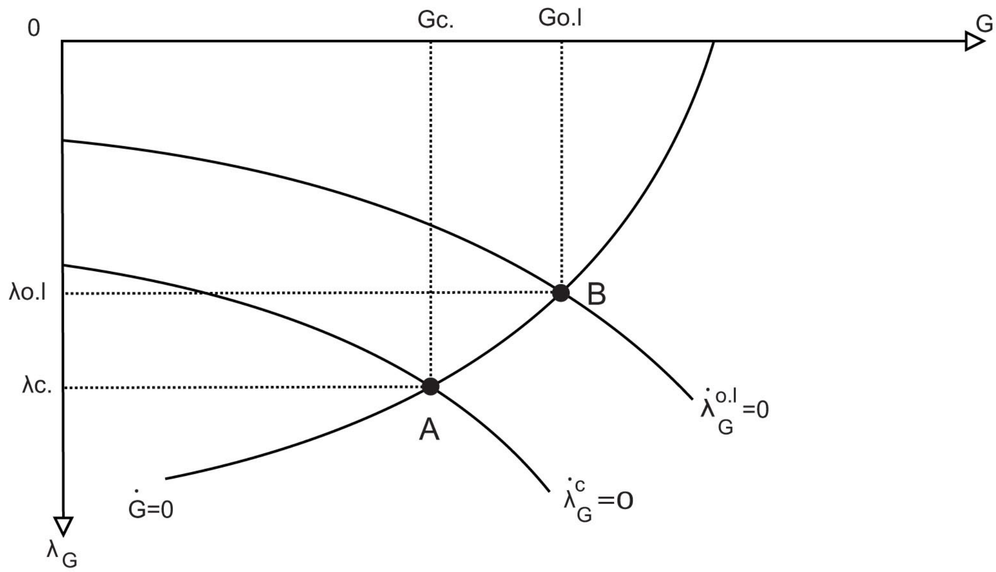


Figure 2

In an OLNE, countries commit to a particular emission path at the outset of the game; they do not respond to observed variations of the GHGs concentration and they do not take into account damages to other countries. From Figure 2 we can see that we have higher steady state stock of GHGs in OLNE than in the case of cooperation among countries.

It is interesting to note, however, that although at the steady state $G^C < G^{OL}$, this does not necessarily imply that at the steady state $T^C < T^{OL}$, as in models with mitigation only, since (24) and (35) imply that at the steady state $\zeta^C < \zeta^{OL}$, because $G^C < G^{OL}$. So the steady state temperature at the cooperative solution is pushed down because the stock of GHGs is less relative to the open loop case, but on the other hand this effect may be counterbalanced by the fact that more radiation is blocked in the open loop case because geoengineering is relatively more. Thus the results suggest that cooperation will reduce the stock of GHGs in the presence of both mitigation and geoengineering, relative to an open loop noncooperative solution but the effect on temperature is ambiguous.

Another issue is the path of global temperature at the steady state, if the steady state level of geoengineering cannot be sustained for a certain period. Since the steady state temperature depends on the steady state flow of geoengineering, a drop in this flow for a certain period will increase temperature. This increase will not be compensated by any reduction in the stock of GHGs, G, which has been already stabilized at high levels, and which will not be affected by any change in geoengineering.

5.2 Open Loop and Cooperation vs Feedback Nash strategies

Let λ^{FB} and λ^{OL} be the shadow cost of GHGs accumulation in the case of FBNE and OLNE respectively. The steady state shadow costs at the FBNE are:

$$\lambda^{FB} = \frac{-\frac{c_T T}{B}\left[(-)\,\phi\,(\mathcal{N}{-}1)\,\zeta'(G)+\eta\right]-\frac{c_\zeta(\mathcal{N}-1)}{\mathcal{N}}\zeta(G)\,\zeta'(G)}{\left[m+\nu+\varepsilon\,(\mathcal{N}{-}1)\,\gamma\left(\chi_j(G)\right)^{-\varepsilon-1}\bar{k}_j^a\left(\chi_j(G)\right)'\right]}. \tag{52}$$

It can easily be shown that the open loop is a special case of the feedback solution for $\zeta'(G)=0$ and $\chi'(G)=0$. Comparing open loop and feedback shadow costs we have:

Open loop

$$\lambda^{OL} = \frac{-c_T T\frac{\eta}{B}}{(m+\nu)} \tag{53}$$

Feedback

$$\lambda^{FB} = \frac{-c_T T\frac{\eta}{B}+(\mathcal{N}{-}1)\,\zeta'(G)\left[c_T T\frac{\phi}{B}-\frac{c_\zeta}{\mathcal{N}}\zeta(G)\right]}{\left(m+\nu+\varepsilon\,(\mathcal{N}{-}1)\,\gamma\left(\chi_j(G)\right)^{-\varepsilon-1}\bar{k}_j^a\left(\chi_j(G)\right)'\right)}$$

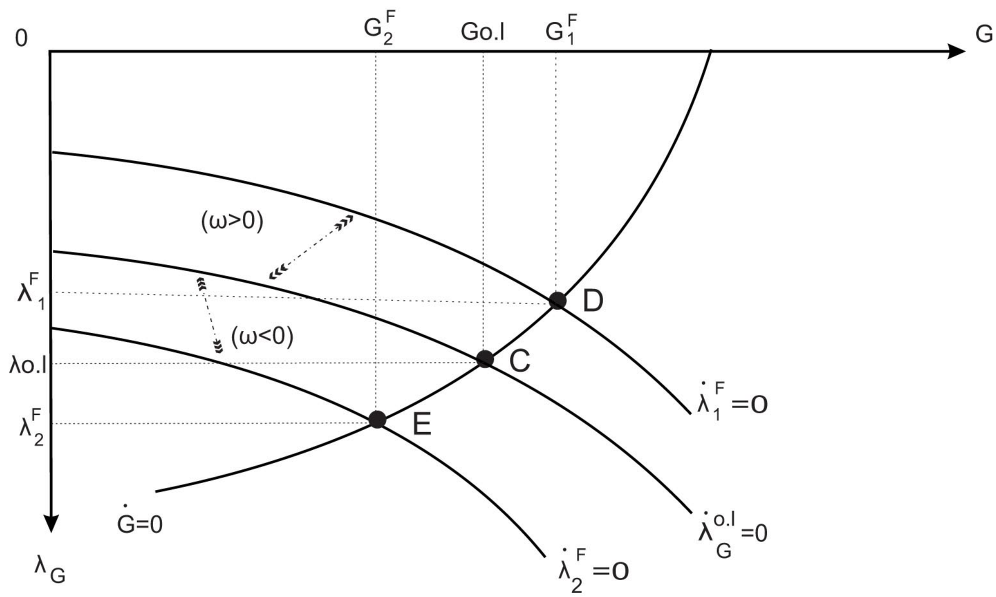


Figure 3

In the traditional analysis of international pollution control problems (e.g. [6], [17], [18]), the steady state GHGs accumulation or the steady state pollution stock under feedback strategies are greater than the corresponding GHGs accumulation under open loop strategies. This result may be revised, however, when both mitigation and geoengineering coexist. To see this, compare the open loop and the feedback steady state shadow costs given by λ^{OL} and λ^{FB}.

Assume that $\zeta'(G) \approx 0$, then since $\chi'(G) > 0$, the denominator of λ^{FB} is larger than the denominator of λ^{OL} and therefore $\left|\lambda^{FB}\right| < \left|\lambda^{OL}\right|$. In this case the steady state GHGs accumulation under feedback strategies exceeds the corresponding steady state GHGs accumulation under open loop strategies, which is the traditional result (Figure 3).[18] From Figure 3 it is clear that under feedback strategies the steady state equilibrium point is obtained at D and under open loop strategies the corresponding equilibrium point is obtained at C. Thus if country i expects little or no change in the geoengineering efforts from other countries when the level of GHGs increases, and other countries ($j \neq i$) expect also no reaction from country i, then the traditional result is verified and the steady state level of GHGs under feedback strategies is greater than the one under open loop strategies.

Assume that $\zeta'(G) > 0$ and not negligible. In this case the comparison between $\left|\lambda^{FB}\right|, \left|\lambda^{OL}\right|$ depends on the term $\omega = \left[c_T T \frac{\phi}{B} - \frac{c_\zeta}{N}\zeta(G)\right]$. In this term, $\frac{\phi}{B}$ from (12) is the temperature response to a unit of geoengineering. Thus $c_T T \frac{\phi}{B}$ is the steady state marginal savings in damages from a temperature increase due to a unit increase in geoengineering. On the other hand the term $\frac{c_\zeta}{N}\zeta(G)$ reflects the additional social damages due to this geoengineering unit. Thus if $\omega > 0$, damage savings from a unit of geoengineering exceeds the corresponding damages due to this unit, and the opposite is true when $\omega < 0$.

Suppose $\omega > 0$, then the nominator of λ^{FB} is further reduced relative to λ^{OL}, and the inequality $\left|\lambda^{FB}\right| < \left|\lambda^{OL}\right|$ remains. The presence of geoengineering causes the $\dot{\lambda}^{FB} = 0$ isocline to move farther to the right relative to the case where geoengineering is not available, i.e. when $\zeta(G) = 0$. This means that when geoengineering is present, the steady state GHGs accumulation at the FBNE increases further as compared to the steady state GHGs accumulation at the OLNE, or the cooperative solution .

The intuition behind this result can be described in the following way. If the stock of GHGs, G, increases, then country i will expect other countries to increase mitigation. This results in country i reducing mitigation and the same applies to all countries in symmetry. This is the free riding effect. So the stock of GHGs and the global temperature increase. An increase in G will trigger geoengineering. If $\omega > 0$, the benefits from geoengineering will exceed the social cost of geoengineering which is spread among all countries which means that countries will engage in geoengineering. Since mitigation has been reduced, and geoengineering does not affect emissions, the stock of GHGs increases further.

At the new steady state the stock of GHGs has increased, but it is not clear what happens to the steady state temperature T, since the increased G will

[18] Note that we always examine a steady state which is a local saddle point.

tend to increase T (through (47)) for a steady state FBNE, but the increased geoengineering activity will tend to reduce it.

Assume now that $\omega < 0$. This means that damage savings from a unit of geoengineering is less than the corresponding social damages due to this unit. If ω is sufficiently large in absolute value, this might reverse the $\left|\lambda^{FB}\right|, \left|\lambda^{OL}\right|$ relationship. If ω is such that $\left|\lambda^{FB}\right| > \left|\lambda^{OL}\right|$, then the $\dot{\lambda}^{FB} = 0$ isocline moves to the left below the $\dot{\lambda}^{OL} = 0$ isocline. This means that when geoengineering is present, the steady state GHGs accumulation at the FBNE is smaller as compared to the steady state GHGs accumulation at the OLNE . This is because countries turn to mitigation since the geoengineering option implies higher net damages.

A similar comparison can be carried out between the cooperative solution and the FBNE. If $\omega > 0$, then $\left|\lambda^{FB}\right| < \left|\lambda^{OL}\right| < \left|\lambda_G^C\right|$, which is the traditional result. If $\omega < 0$, then we might have a reversal in the ranking of the shadow costs.

-

FB_0

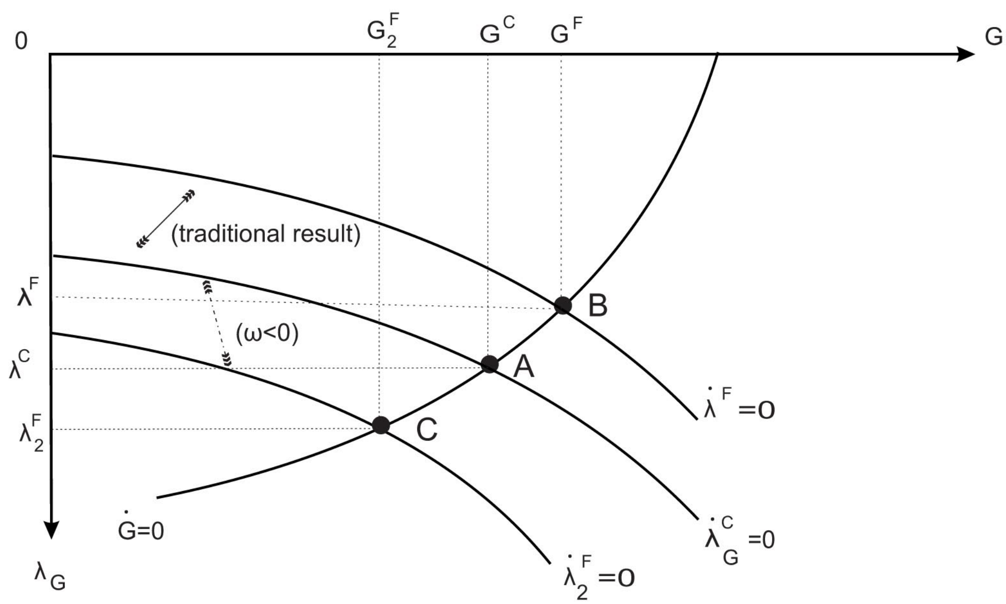


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Figure 4

The likelihood of this reversal depends on the size of marginal damages savings from temperature increase due to geoengineering, relative to social marginal damages due to geoengineering itself. Since damages from geoengineering undertaken by one country are spread among all countries, while geoengineering affects the evolution of temperature directly, through (13), and not indirectly, through changes in the stock of GHGs, it is more likely that $\omega > 0$. In this case geoengineering will result in larger GHGs concentration relative to the case where geoengineering is not available. This increase in the stock of GHGs relative to cooperation will be intensified by the strategic behavior of countries.

As in the case of comparing the cooperative case with the OLNE, a higher stock of GHGs at the FBNE does not necessarily imply a higher temperature, since the higher stock will induce more geoengineering that will tend to reduce steady state temperature. Maintaining the steady state temperature requires a steady geoengineering flow. If there is a sudden drop in geoengineering, global temperature will jump since the stock of GHGs is already high.

6 Two Polar Cases: Mitigation or Geoengineering

In this section we examine the implications of our model under two extreme cases. In the first case we analyze our model under the assumption that countries will engage in geoengineering only, and in the second case we assume that we will have only mitigation. The purpose is to examine conditions and characterize the steady-state temperature responses either to mitigation or to geoengineering.

Let T^g, G^g, λ^g denote the steady state values of temperature, GHGs accumulation, and shadow cost of GHGs accumulation respectively in the case of absence of mitigation (that is, 'geoengineering only'). Let T^m, G^m, λ^m denote the corresponding steady state values in the case of absence of geoengineering (that is, 'mitigation only'). The values are derived in Appendix A.7.

The impact of geoengineering will be determined by the steady state derivative $\frac{dT}{d\zeta}$, while the impact of mitigation by the derivative $\frac{dT}{d\chi}$, where all derivatives are evaluated at the appropriate $\left(T^{jg}, G^{jg}, \lambda^{jg}\right)$ or $\left(T^{jm}, G^{jm}, \lambda^{jm}\right)$, $j = C, OL, FB$.

Thus if we apply geoengineering methods only, then we observe a reduction in the steady state temperature by $\frac{\phi\mathcal{N}}{B}$, or

$$\frac{dT}{d\zeta} = -\frac{\phi\mathcal{N}}{B} \tag{54}$$

which is the same for the cooperative and noncooperative solution, where ϕ is the sensitivity of the average global temperature to geoengineering.[19] This result indicates that the sensitivity of global temperature under mitigation is the same under cooperative or noncooperative behavior. The actual change, however, can be approximated by $\Delta T = -\frac{\phi\mathcal{N}}{B}\Delta\zeta$. If $\Delta\zeta$ is higher at the noncooperative solution, then the temperature change will also be larger relative to the cooperative solution.

Mitigation will affect global temperature through the change in the stock of GHGs. Therefore if we apply mitigation only, then we observe a reduction in the steady state level of GHGs accumulation by $\frac{\mathcal{N}\varepsilon\gamma\left(\chi^*\left(\lambda^{jm\cdot}\right)\right)^{-\varepsilon-1}\left[k^*\left(\lambda^{jm\cdot}\right)\right]^a}{m}$, since

[19] All derivatives are obtained by using the steady state values from Appendix A.7.

$$\frac{dG}{d\chi} = -\frac{\mathcal{N}\varepsilon\gamma\left(\chi^*\left(\lambda^{jm\cdot}\right)\right)^{-\varepsilon-1}\left[k^*\left(\lambda^{jm\cdot}\right)\right]^a}{m} \tag{55}$$

where $j =$ cooperative, open loop, feedback.

Due to the reduction in the steady state level of GHGs accumulation, the steady state global temperature will be reduced by $\frac{\eta}{B}$, or

$$\frac{dT}{dG} = -\frac{\eta}{B} = -\frac{\frac{\psi}{2BG_0}}{B} = -\frac{\psi}{2B^2G_0} \tag{56}$$

where η is a measure of the steady state temperature sensitivity to a change in GHGs. Thus

$$\frac{dT}{d\chi} = \frac{dT}{dG}\cdot\frac{dG}{d\chi}.$$

We observe that geoengineering can affect the steady state level of temperature directly, in contrast to mitigation which affects the steady state temperature indirectly through the changes in the steady state level of the stock of greenhouse gasses.

To compare the impact of the two approaches on the steady state temperature we use (54), (55), and (56) to obtain:

$$\begin{array}{rcl} \frac{dT}{d\zeta} & \gtreqqless & \frac{dT}{dG}\cdot\frac{dG}{d\chi} \\ if\ \ |\phi| & \gtreqqless & \left|\frac{\eta}{m}\cdot\varepsilon\gamma\left(\chi^*\left(\lambda_{G,j}^{m\cdot}\right)\right)^{-\varepsilon-1}\left[k^*\left(\lambda_{G,j}^{m\cdot}\right)\right]^a\right|. \end{array}$$

Thus the effectiveness of the methods is an empirical issue but some observations are possible. The higher ϕ is, the more effective geoengineering is, while the higher η is, the more effective mitigation is. Furthermore mitigation will be more effective the smaller m is, which means that the GHGs stay for a long time in the atmosphere and are not absorbed by the oceans.

Also mitigation is more effective the higher β is, or the larger the proportion is of GHGs emissions remaining in the atmosphere, and the higher the base line emissions intensity is. It should also be noted that since steady state mitigation $\chi^*\left(\lambda_{G,j}^{m\cdot}\right)$ and capital stock $k^*\left(\lambda_{G,j}^{m\cdot}\right)$ will be different at the cooperative and the noncooperative solutions, the relative effectiveness of the two methods depends on whether countries cooperate or not.

7 Mitigation - Geoengineering Substitution

We can examine the substitution possibilities between mitigation and geoengineering by analyzing the marginal rate of substitution between mitigation and geoengineering, which is the rate at which the policy maker is willing to exchange mitigation for geoengineering, while maintaining his/her overall utility constant. The marginal rate of substitution is:

$$MRS_{\chi,\zeta} = \frac{\partial H^*/\partial \chi}{\partial H^*/\partial \zeta} = \frac{d\zeta}{d\chi}$$

where H^* is the Hamiltonian along the optimal trajectory. Since the Hamiltonian is constant along the optimal trajectory, $MRS_{\chi,\zeta}$ can be interpreted as the rate at which the decision maker is willing to exchange mitigation and geoengineering along the optimal trajectory.

We consider comparisons of the MRS at a steady state. Our previous results suggest that the steady state values for GHGs accumulation, shadow cost of GHGs accumulation, capital stock, geoengineering and mitigation for cases of cooperation (C), OLNE (OL), and FBNE (FB) are:[20]

$$\begin{aligned} G^C &< G^{OL} < G^{FB} \\ \lambda^C &< \lambda^{OL} < \lambda^{FB} \\ k^C &< k^{OL} < k^{FB} \\ \zeta^C &< \zeta^{OL} < \zeta^{FB} \\ \chi^C &> \chi^{OL} > \chi^{FB} \end{aligned}$$

Since the derivatives of the optimal capital stock, geoengineering and mitigation are the following

$$\begin{aligned} \frac{\partial k^j}{\partial \lambda} &= -\frac{\gamma \left(\frac{a\left(\lambda\gamma\chi^{-\varepsilon}+1\right)}{\rho+\delta} \right)^{\frac{1}{1-\alpha}}}{(a-1)\left(\lambda\gamma + \chi^{\varepsilon}\right)} = \frac{\gamma k^j}{(1-a)\left(\lambda^j\gamma + \chi^{\varepsilon}\right)} > 0 \\ \frac{\partial \zeta^l}{\partial G} &= \frac{\mathcal{N}\phi c_j \eta}{c_\zeta B^2 + \mathcal{N}^2\phi^2 c_j} > 0 \\ \frac{\partial \zeta^{FB}}{\partial G} &= \frac{\mathcal{N}\phi c_j \left(\eta - \phi\left(\mathcal{N}-1\right)\zeta'\left(G\right)\right)}{c_\zeta B^2 + \mathcal{N}\phi^2 c_j} \lesseqgtr 0 \\ \frac{\partial \chi^j}{\partial \lambda} &= \frac{\left(-\frac{p_\chi}{\varepsilon\lambda^j\gamma k_i^a}\right)^{\frac{-1}{1+\varepsilon}}}{\lambda^j\left(\varepsilon+1\right)} = \frac{\chi^j}{\lambda^j\left(\varepsilon+1\right)} < 0 \end{aligned}$$

where $j = C, OL, FB$ and $l = C, OL$

Under Cooperation the MRS is:

$$MRS^C = \frac{-p_\chi - \lambda^C\gamma\varepsilon\left(\chi^C\right)^{-\varepsilon-1}\left(k^C\right)^\alpha}{-p_\zeta - c_\zeta\zeta^C + \frac{\mathcal{N}\phi c_T\left[-A+S(1-a)-\phi\mathcal{N}\zeta^C+\eta(G^C-G_0)\right]}{B^2}}. \tag{57}$$

Under OLNE the MRS is:

$$MRS^{OL} = \frac{-p_\chi - \lambda^{OL}\gamma\varepsilon\left(\chi^{OL}\right)^{-\varepsilon-1}\left(k^{OL}\right)^\alpha}{-p_\zeta - \frac{c_\zeta}{\mathcal{N}}\zeta^{OL} + \frac{\phi c_T\left[-A+S(1-a)-\phi\mathcal{N}\zeta^{OL}+\eta(G^{OL}-G_0)\right]}{B^2}}. \tag{58}$$

[20] For the FBNE we consider that 'traditional' result where $G^C < G^{OL} < G^{FB}$.

Under FBNE the MRS is:

$$MRS^{FB} = \frac{-p_\chi - \lambda^{FB}\gamma\varepsilon\left(\chi^{FB}\right)^{-\varepsilon-1}\left(k^{FB}\right)^{\alpha}}{-p_\zeta - \frac{c_\zeta}{\mathcal{N}}\zeta^{FB} + \frac{\phi c_T\left[-A+S(1-a)-\phi\mathcal{N}\zeta^{FB}++\eta(G^{FB}-G_0)\right]}{B^2}}. \tag{59}$$

Although the MRSs are well defined unambiguous comparisons are not possible. If the third term in the denominator is sufficiently large due to $\mathcal{N}$ then one may expect that the MRS under cooperation is smaller in absolute value relative to OLNE and FBNE. More exact results however require calibration of the model which is beyond the scope of the current paper.

8 Concluding Remarks

The efforts to deal with the problem of global warming have focused on limiting the emissions of GHGs in the atmosphere. The attempts of international cooperation to limit the emissions of GHGs seem, however, to encounter coordination and implementation problems, so other approaches are discussed. In this paper we study both the traditional approach to policy design, that of mitigation, and recent ideas about solar radiation management though geoengineering, an approach that has been discussed as a potential future alternative. After introducing the two approaches in a coupled model of the economy and the environment, we make a first attempt to compare the two policy instruments.

Our main findings indicate that when geoengineering is present, the expected steady state accumulation of GHGs is higher relative to the case where geoengineering is not an option. This result holds under cooperative and noncooperative behavior among countries. Furthermore, the presence of geoengineering as an alternative policy instrument seems to induce higher geoengineering effort and GHGs emissions at the noncooperative solutions when compared to cooperation, relative to the case where geoengineering is not a policy option. Thus in the context of our model, cooperation implies more mitigation and less geoengineering, while noncooperative behavior implies less mitigation and more geoengineering. This result suggests that if international cooperation to reduce GHG emissions cannot be reached and countries move to unilateral actions, geoengineering rather than mitigation is the policy to be expected. Stronger incentives for geoengineering at the noncooperative solutions can be attributed to the interplay between the free rider and the free driver externality. Although our model is symmetric and does not allow for unilateral actions, even at a symmetric equilibrium, free rider incentives tend to reduce mitigation, while low private costs and spreading of social costs - the free driver incentives - tend to increase geoengineering and GHGs emissions.

Higher GHGs emissions do not, however, necessarily imply higher temperature, at least in the short run, because geoengineering efforts increase global albedo which tends to reduce temperature. These results suggest therefore that geoengineering could lead to a solution of relatively higher GHGs and temperature, or relatively higher GHGs but lower temperature relative to the case where geoengineering is not an option. The outcome depends on many factors, the most important of which are the sensitivity of temperature to the increase of the global albedo of the planet, the average time that GHGs remain in the atmosphere, and the social cost of geoengineering. Low sensitivity might lead to high GHGs and temperature. On the other hand, high social cost of geoengineering will lead to low geoengineering efforts, while underestimation of the potential social geoengineering costs could lead to excess geoengineering, and excess GHG emissions with high social costs.

Another important issue relates to the fact that even if geoengineering leads to a lower temperature, maintaining this temperature requires a constant flow of geoengineering. If for some reason this flow cannot be kept at its steady state level, then there will be a jump in the temperature. This jump will be intensified since the stock of GHGs will already be high. A drop in the mitigation flow at the steady state is not expected to have a similar result because the effect of mitigation on temperature is indirect and realized through the change in the stock of GHGs. In any case, the final outcome is an empirical issue, the qualitative response of temperature to the two policy instruments is different and this may have important implications for applied policy issues. We think that our analysis is suggestive of the factors affecting the final outcomes regarding climate change under mitigation and/or geoengineering and the mechanisms through which these outcomes are realized.

Although the issue of geoengineering and its impacts embody deep uncertainties, our analysis is deterministic. This is because we wanted to study the basic mechanisms involved, without the complications induced by stochastic factors. Introduction of uncertainty - especially as deep structural uncertainty - including characteristics such as model uncertainty, ambiguity aversion, robust control methods, or regime shifts, is a very important area of further research. The deterministic structure presented here can be used as a basis for introducing these elements.

Finally, our results were based on a number of simplifying assumptions such as symmetric countries, a simplified zero-dimensional climate model, no spatial impacts, no impact of geoengineering on precipitation. All these extensions are areas for further research which will substantially increase our insight into the relative impact of geoengineering as an alternative policy option against climate change, especially regarding our main result that geoengineering will increase the stock of GHGs with ambiguous results on the temperature.

Appendix

A.1 Optimality Conditions

In deriving the optimality conditions, we impose at the appropriate derivation stage symmetry: $\zeta_i = \zeta$ and $k_i = k$ and $\chi_i = \chi$, for all i and set $\mathcal{A}_{0i} N_{0i} = 1$ to simplify.

Cooperation among countries

The current value Hamiltonian function:

$$\begin{aligned} H &= \sum_{i=1}^{\mathcal{N}} \left[\left(k_i^a - p_\chi \chi_i - p_\zeta \zeta_i \right) - (\rho + \delta) k_i \right] - \\ & -\frac{1}{2} \sum_{i=1}^{\mathcal{N}} c_T \left(\frac{-A + S(1-\alpha) - \phi \sum_{i=1}^{\mathcal{N}} \zeta_i + \eta (G - G_0)}{B} \right)^2 - \\ & - \sum_{i=1}^{\mathcal{N}} \left(\frac{c_\zeta}{2\mathcal{N}} \sum_{i=1}^{\mathcal{N}} \zeta_i^2 \right) + \lambda_i^C(t) \left[\sum_{i=1}^{\mathcal{N}} \gamma \chi_i^{-\varepsilon} k_i^a - mG \right]. \end{aligned}$$

First order necessary conditions for k_i, χ_i and ζ_i

$$\frac{\partial H}{\partial k_i} = 0$$

$$\sum_{i=1}^{\mathcal{N}} \left[a k_i^{a-1} - (\rho + \delta) \right] + a \lambda^C \sum_{i=1}^{\mathcal{N}} \gamma \chi_i^{-\varepsilon} k_i^{a-1} = 0$$

$$\Longrightarrow k^* = \left(\frac{a \left(\lambda^C \gamma \chi_i^{-\varepsilon} + 1 \right)}{(\rho + \delta)} \right)^{\frac{1}{1-a}} \tag{60}$$

$$\frac{\partial H}{\partial \chi_i} \leq 0, \chi_i \geq 0$$

$$-\sum_{i=1}^{\mathcal{N}} p_\chi - \varepsilon \lambda^C \sum_{i=1}^{\mathcal{N}} \gamma \chi_i^{-\varepsilon-1} k_i^a \leq 0$$

$$\Longrightarrow \varepsilon \lambda^C \gamma \chi_i^{-\varepsilon-1} k_i^a = -p_\chi \ \text{(interior solution)}$$

$$\Longrightarrow \chi^* = \left(-\frac{p_\chi}{\varepsilon \lambda^C \gamma k_i^a} \right)^{-\frac{1}{\varepsilon+1}} \tag{61}$$

$$\frac{\partial H}{\partial \zeta_i} \leq 0, \zeta_i \geq 0$$

$$-p_\zeta + \mathcal{N} \frac{\phi}{B} c_T \left(\frac{-A + S(1-\alpha) - \phi \sum_{i=1}^{\mathcal{N}} \zeta_i + \eta (G - G_0)}{B} \right) - \sum_{i=1}^{\mathcal{N}} \frac{c_\zeta}{\mathcal{N}} \zeta_i \leq 0$$

imposing symmetry and considering interior solution:

$$\mathcal{N} \frac{\phi}{B} c_T \left(\frac{-A + S(1-\alpha) - \phi \mathcal{N} \zeta_i + \eta (G - G_0)}{B} \right) - c_\zeta \zeta_i = p_\zeta \ \text{(interior solution)}$$

$$\Longrightarrow \zeta^{*}=\frac{\mathcal{N}\phi c_{T}\left[-A+S\left(\alpha-1\right)+\eta\left(G-G_{0}\right)\right]-B^{2}p_{\zeta}}{c_{\zeta}B^{2}+\mathcal{N}^{2}\phi^{2}c_{T}}. \tag{62}$$

The MHDS is:

$$\dot{G}=\frac{\partial H}{\partial\lambda^{C}}=\mathcal{N}\gamma\left(\chi^{*}\right)^{-\varepsilon}\left(k^{*}\right)^{a}-mG \tag{63}$$

$$\dot{\lambda}^{C}=\nu\lambda^{C}-\frac{\partial H}{\partial G}$$

$$\Longrightarrow \dot{\lambda}^{C}=\left(m+\nu\right)\lambda^{C}+\frac{\mathcal{N}\eta c_{T}\left[-A+S\left(1-\alpha\right)-\mathcal{N}\phi\zeta^{*}+\eta\left(G-G_{0}\right)\right]}{B^{2}}. \tag{64}$$

OLNE

The current value noncooperative Hamiltonian function is:

$$\begin{aligned} H_{i}^{OL} &= \left[\left(k_{i}^{a}-p_{\chi}\chi-p_{\zeta}\zeta_{i}\right)-\left(\rho+\delta\right)k_{i}\right] \\ &-\frac{1}{2}c_{T}\left(\frac{-A+S\left(1-\alpha\right)-\phi\left(\zeta_{i}+\sum_{j\neq i}^{\mathcal{N}}\bar{\zeta}_{j}\right)+\eta\left(G-G_{0}\right)}{B}\right)^{2} \\ &-\frac{c_{\zeta}}{2\mathcal{N}}\left(\zeta_{i}^{2}+\sum_{j\neq i}^{\mathcal{N}}\bar{\zeta}_{j}^{2}\right)+\lambda_{i}^{OL}\left(t\right)\left[\gamma\chi_{i}^{-\varepsilon}k_{i}^{a}+\sum_{j\neq i}^{\mathcal{N}}\gamma\left(\bar{\chi}_{j}\right)^{-\varepsilon}\bar{k}_{j}^{a}-mG\right]. \end{aligned}$$

First order necessary conditions under symmetry

$$\frac{\partial H}{\partial k_{i}}=0$$

$$\left[ak_{i}^{a-1}-\left(\rho+\delta\right)\right]+a\lambda^{OL}\gamma\chi_{i}^{-\varepsilon}k_{i}^{a-1}=0$$

$$\Longrightarrow k^{*}=\left(\frac{a\left(\lambda^{OL}\gamma\chi_{i}^{-\varepsilon}+1\right)}{\left(\rho+\delta\right)}\right)^{\frac{1}{1-a}} \tag{65}$$

$$\frac{\partial H}{\partial\chi_{i}}\leq 0,\chi_{i}\geq 0$$

$$-p_{\chi}-\varepsilon\lambda^{OL}\gamma\chi_{i}^{-\varepsilon-1}k_{i}^{a}\leq 0$$

$$\Longrightarrow \varepsilon\lambda^{OL}\gamma\chi_{i}^{-\varepsilon-1}k_{i}^{a}=-p_{\chi}\ \text{(interior solution)}$$

$$\Longrightarrow \chi^{*}=\left(-\frac{p_{\chi}}{\varepsilon\lambda^{OL}\gamma k_{i}^{a}}\right)^{-\frac{1}{\varepsilon+1}} \tag{66}$$

$$\frac{\partial H}{\partial\zeta_{i}}\leq 0,\zeta_{i}\geq 0$$

$$-p_\zeta + \frac{\phi}{B}c_T\left(\frac{-A+S\left(1-\alpha\right)-\phi\left(\zeta_i+\sum_{j\neq i}^{\mathcal{N}}\bar{\zeta}_j\right)+\eta\left(G-G_0\right)}{B}\right)-\frac{c_\zeta}{\mathcal{N}}\zeta_i \leq 0$$

$$\frac{\phi}{B}c_T\left(\frac{-A+S\left(1-\alpha\right)-\phi\mathcal{N}\zeta+\eta\left(G-G_0\right)}{B}\right)-\frac{c_\zeta}{\mathcal{N}}\zeta = p_\zeta \text{ (interior solution)}$$

$$\Longrightarrow \zeta^* = \frac{\mathcal{N}\phi c_T\left[-A+S\left(1-\alpha\right)+\eta\left(G-G_0\right)\right]-\mathcal{N}B^2p_\zeta}{c_\zeta B^2+\mathcal{N}^2\phi^2c_T}. \tag{67}$$

The MHDS is:

$$\dot{G} = \frac{\partial H}{\partial\lambda^{OL}} = \mathcal{N}\gamma\left(\chi^*\right)^{-\varepsilon}\left(k^*\right)^a - mG \tag{68}$$

$$\dot{\lambda}^{OL} = \nu\lambda^{OL} - \frac{\partial H}{\partial G}$$

$$\Longrightarrow \dot{\lambda}^{OL} = \left(m+\nu\right)\lambda^{OL} + \frac{\eta c_T\left[-A+S\left(1-\alpha\right)-\sum_{i=1}^{\mathcal{N}}\phi\zeta^*+\eta\left(G-G_0\right)\right]}{B^2}. \tag{69}$$

FBNE

The current value noncooperative Hamiltonian function:

$$\begin{aligned} H_i^{FB} &= \left[\left(k_i^a - p_\chi\chi - p_\zeta\zeta_i\right)-\left(\rho+\delta\right)k_i\right] \\ &\quad -\frac{1}{2}c_T\left(\frac{-A+S\left(1-\alpha\right)-\phi\left(\zeta_i+\sum_{j\neq i}^{\mathcal{N}}\left(\zeta_j\left(G\right)\right)\right)+\eta\left(G-G_0\right)}{B}\right)^2 \\ &\quad -\frac{c_\zeta}{2\mathcal{N}}\left(\zeta_i^2+\sum_{j\neq i}^{\mathcal{N}}\left(\zeta_j\left(G\right)\right)^2\right)+\lambda_i^{FB}\left(t\right)\left[\gamma\chi_i^{-\varepsilon}k_i^a+\sum_{j\neq i}^{\mathcal{N}}\gamma\left(\chi_j\left(G\right)\right)^{-\varepsilon}\bar{k}_j^a - mG\right]. \end{aligned}$$

First order necessary conditions for k_i, χ_i and ζ_i

$$\frac{\partial H}{\partial k_i} = 0$$

$$\left[ak_i^{a-1}-\left(\rho+\delta\right)\right]+a\lambda^{FB}\gamma\chi_i^{-\varepsilon}k_i^{a-1} = 0$$

$$\Longrightarrow k^* = \left(\frac{a\left(\lambda^{FB}\gamma\chi_i^{-\varepsilon}+1\right)}{\left(\rho+\delta\right)}\right)^{\frac{1}{1-a}} \tag{70}$$

$$\frac{\partial H}{\partial\chi_i} \leq 0, \chi_i \geq 0$$

$$-p_\chi - \varepsilon\lambda^{FB}\gamma\chi_i^{-\varepsilon-1}k_i^a \leq 0$$

$$\Longrightarrow \varepsilon\lambda^{FB}\gamma\chi_i^{-\varepsilon-1}k_i^a = -p_\chi \text{ (interior solution)}$$

$$\Longrightarrow \chi^* = \left(-\frac{p_\chi}{\varepsilon \lambda^{FB} \gamma k_i^a}\right)^{-\frac{1}{\varepsilon+1}} \tag{71}$$

$$\frac{\partial H}{\partial \zeta_i} \leq 0, \zeta_i \geq 0$$

$$-p_\zeta + \frac{\phi}{B} c_T \left(\frac{-A + S(1-\alpha) - \phi\left(\zeta_i + \sum_{j\neq i}^{\mathcal{N}} \left(\zeta_j(G)\right)\right) + \eta(G - G_0)}{B}\right) - \frac{c_\zeta}{\mathcal{N}} \zeta_i \leq 0$$

$$\mathcal{N} \frac{\phi}{B} c_T \left(\frac{-A + S(1-\alpha) - \phi\left(\zeta_i + \sum_{j\neq i}^{\mathcal{N}} \left(\zeta_j(G)\right)\right) + \eta(G - G_0)}{B}\right) - c_\zeta \zeta_i = \mathcal{N} p_\zeta$$

(interior solution)

$$\Longrightarrow \zeta^* = \frac{\mathcal{N} \phi c_T \left[-A + S(1-\alpha) - \phi \sum_{j\neq i}^{\mathcal{N}} \zeta_j(G) + \eta(G - G_0)\right] - \mathcal{N} B^2 p_\zeta}{c_\zeta B^2 + \mathcal{N} \phi^2 c_T}. \tag{72}$$

The MHDS characterizing the FBNE under symmetry implies

$$\dot{G} = \frac{\partial H}{\partial \lambda^{FB}} = \mathcal{N} \gamma (\chi^*)^{-\varepsilon} (k^*)^a - mG \tag{73}$$

$$\dot{\lambda}^{FB} = \nu \lambda^{FB} - \frac{\partial H}{\partial G}$$

$$\begin{aligned} \dot{\lambda}^{FB} = {} & (m+\nu) \lambda^{FB} + c_T T \left[\frac{(-)\phi(\mathcal{N}-1)}{B} \zeta'(G) + \frac{\eta}{B}\right] + c_\zeta \frac{(\mathcal{N}-1)}{\mathcal{N}} \zeta(G) \zeta'(G) \\ & + \varepsilon \lambda^{FB} (\mathcal{N}-1) \gamma \left(\chi_j(G)\right)^{-\varepsilon-1} \bar{k}_j^a \left(\chi_j(G)\right)'. \end{aligned} \tag{74}$$

A.2 Existence conditions, cooperative solution

The MHDS is:

$$\dot{\lambda}^C = (m+\nu) \lambda^C + \frac{\mathcal{N} \eta c_T T}{B} = 0 \tag{75}$$

$$\Longrightarrow \lambda^C = -\frac{\mathcal{N} \eta c_T T}{B(m+\nu)}$$

$$\dot{G} = \sum_{i=1}^{\mathcal{N}} \gamma \left(h_2\left(\lambda^C\right)\right)^{-\varepsilon} \left(h_1\left(\lambda^C\right)\right)^a - mG. \tag{76}$$

From $(75), (76)$ we have at a steady state:

$$\sum_{i=1}^{\mathcal{N}}\gamma\left(-\frac{p_\chi}{\varepsilon\left(-\frac{\mathcal{N}\eta c_T T}{B(m+\nu)}\right)\gamma k_i^a}\right)^{-\frac{\varepsilon}{\varepsilon+1}}\cdot\left(\frac{a\left(\left(-\frac{\mathcal{N}\eta c_T T}{B(m+\nu)}\right)\gamma\chi_i^{-\varepsilon}+1\right)}{(\rho+\delta)}\right)^{\frac{a}{1-a}} = mG.$$

We can define the following functions of G, as:

$$\overline{\lambda}^C(G) = -\frac{\mathcal{N}\eta c_T T(G)}{B(m+\nu)}.$$

Note that since $\overline{\lambda}^C(G)$ is the social cost of GHGs and $T(G)$ is increasing in G, an increase in G will increase the social cost of GHGs.

$$g_1\left(G,\overline{\lambda}^C(G)\right) = \left(-\frac{p_\chi}{\varepsilon\left(-\frac{\mathcal{N}\eta c_T T}{B(m+\nu)}\right)\gamma k_i^a}\right)^{-\frac{\varepsilon}{\varepsilon+1}} = \left(-\frac{p_\chi}{\varepsilon\overline{\lambda}^C(G)\gamma\left(h_1\left(\lambda^C\right)\right)^a}\right)^{-\frac{\varepsilon}{\varepsilon+1}}$$

$$g_2\left(G,\overline{\lambda}^C(G)\right) = \left(\frac{a\left(\left(-\frac{\mathcal{N}\eta c_T T}{B(m+\nu)}\right)\gamma\chi_i^{-\varepsilon}+1\right)}{(\rho+\delta)}\right)^{\frac{a}{1-a}} = \left(\frac{a\left(\overline{\lambda}^C(G)\gamma\left(h_2\left(\lambda^C\right)\right)^{-\varepsilon}+1\right)}{(\rho+\delta)}\right)^{\frac{a}{1-a}}$$

then we have that:

$$\dot{G} = \sum_{i=1}^{\mathcal{N}}\gamma g_1\left(G,\overline{\lambda}^C(G)\right)\cdot g_2\left(G,\overline{\lambda}^C(G)\right) - mG = R(G). \tag{77}$$

The roots of equation $R(G) = 0$ determine the steady states. A steady state will exist in an interval $[G_1, G_2]$ where $R(G)$ is continuous, if $R(G_1)\cdot R(G_2) < 0$. For low $G = G_1$ we expect $R(G_1) > 0$ since the social cost of GHGs is low and this allows for more emissions and and therefore growth in the accumulation of GHGs. For high G we expect that a sufficiently high level G_2 exists, that the high social cost of GHGs will induce reduced emissions, and that the already high G will make the mG term dominate the first term in (77) so that $R(G_2) < 0$. If a steady state exists, it may not be unique.

□

A.3 Proof of Proposition 1

For the Jacobian matrix of the current value Hamiltonian at the steady state defined as

$$J = \begin{bmatrix} \frac{\partial H_{\lambda^C}}{\partial G} & \frac{\partial H_{\lambda^C}}{\partial \lambda^C} \\ \frac{\partial H_G}{\partial G} & \frac{\partial H_G}{\partial \lambda^C} \end{bmatrix}$$

we have:

$$\frac{\partial H_{\lambda^C}}{\partial G} = -m \Longrightarrow \frac{\partial H_{\lambda^C}}{\partial G} < 0$$

$$\frac{\partial H_{\lambda^C}}{\partial \lambda^C} = -\varepsilon \mathcal{N}\gamma \left(\chi^*\left(\lambda^C\right)\right)^{-\varepsilon-1}\left[k^*\left(\lambda^C\right)\right]^a \frac{\partial \chi^*}{\partial \lambda^C} + $$
$$+a\mathcal{N}\gamma\left(\chi^*\left(\lambda^C\right)\right)^{-\varepsilon}\left[k^*\left(\lambda^C\right)\right]^{a-1}\frac{\partial k^*}{\partial \lambda^C} \Longrightarrow \frac{\partial H_{\lambda^C}}{\partial \lambda^C} > 0$$

$$\frac{\partial H_G}{\partial G} = \frac{\mathcal{N}\eta c_T}{B^2}\cdot\left(\eta - \phi\mathcal{N}\frac{\partial \zeta^*}{\partial G}\right)$$

$$\frac{\partial H_G}{\partial \lambda^C} = m+\nu \Longrightarrow \frac{\partial H_G}{\partial \lambda^C} > 0$$

$$\det(j) = \frac{\partial H_{\lambda^C}}{\partial G}\cdot\frac{\partial H_G}{\partial \lambda^C} - \frac{\partial H_{\lambda^C}}{\partial \lambda^C}\cdot\frac{\partial H_G}{\partial G}$$

where G and λ^C are evaluated at the steady state.

We can see that $tr(j) = \nu > 0$, then from the Kurz theorem we know that we will have either a saddle point or a locally unstable steady state.

$$If\ \ \phi\mathcal{N}\frac{\partial \zeta^*}{\partial G} < \eta \rightarrow \frac{\partial H_G}{\partial G} > 0 \tag{78}$$

then $\det(j) < 0$ and the steady state is a saddle point. (79)

$$If\ \ \phi\mathcal{N}\frac{\partial \zeta^*}{\partial G} > \eta \rightarrow \frac{\partial H_G}{\partial G} < 0 \tag{80}$$

and the natural decay rate of GHGs is low and near zero $\longrightarrow \frac{\partial H_{\lambda^C}}{\partial G}\cdot\frac{\partial H_G}{\partial \lambda^C} < \frac{\partial H_{\lambda^C}}{\partial \lambda^C}\cdot\frac{\partial H_G}{\partial G}$

then $\det(j) > 0$ and the steady state is locally unstable.

□

A.4 Existence Conditions, FBNE

The MHDS is:

$$\dot{G} = \mathcal{N}\gamma\left(h_2\left(\lambda^{FB}\right)\right)^{-\varepsilon}\left[h_1\left(G,\lambda^{FB}\right)\right]^a - mG$$

$$\dot{\lambda}^{FB} = (m+\nu)\lambda^{FB} + \frac{c_T T}{B}\left[(-)\phi(\mathcal{N}-1)\zeta'(G)+\eta\right] + c_\zeta\frac{(\mathcal{N}-1)}{\mathcal{N}}\zeta(G)\zeta'(G)$$
$$+\varepsilon\lambda^{FB}(\mathcal{N}-1)\gamma\left(\chi_j(G)\right)^{-\varepsilon-1}\bar{k}_j^a\left(\chi_j(G)\right)'.$$

From the MHDS we obtain

$$\sum_{i=1}^{\mathcal{N}}\gamma\left(-\frac{p_\chi}{\varepsilon\left(-\frac{\frac{c_T T}{B}[(-)\phi(\mathcal{N}-1)\zeta'(G)+\eta]+c_\zeta\frac{(\mathcal{N}-1)}{\mathcal{N}}\zeta(G)\zeta'(G)}{(m+\nu)+\varepsilon(\mathcal{N}-1)\gamma\left(\chi_j(G)\right)^{-\varepsilon-1}\bar{k}_j^a\left(\chi_j(G)\right)'}\right)\gamma k_i^a}\right)^{-\frac{\varepsilon}{\varepsilon+1}}.$$

$$\cdot\left(\frac{a\left(\left(-\frac{\frac{c_T T}{B}\left[(-)\phi(\mathcal{N}-1)\zeta'(G)+\eta\right]+c_\zeta\frac{(\mathcal{N}-1)}{\mathcal{N}}\zeta(G)\zeta'(G)}{(m+\nu)+\varepsilon(\mathcal{N}-1)\gamma\left(\chi_j(G)\right)^{-\varepsilon-1}\bar{k}_j^a\left(\chi_j(G)\right)'}\right)\gamma\chi_i^{-\varepsilon}+1\right)}{(\rho+\delta)}\right)^{\frac{a}{1-a}}=mG.$$

We can define three functions of G as:

$$\overline{\lambda}^{FB}(G)=\left(-\frac{\left[-\frac{c_T T}{B}\phi+\frac{c_\zeta}{\mathcal{N}}\zeta(G)\right](\mathcal{N}-1)\zeta'(G)+\frac{c_T T}{B}\eta}{(m+\nu)+\varepsilon(\mathcal{N}-1)\gamma\left(\chi_j(G)\right)^{-\varepsilon-1}\bar{k}_j^a\left(\chi_j(G)\right)'}\right),\ \overline{\lambda}^{FB}(G)<0$$

$$\begin{aligned}
g_1\left(G,\overline{\lambda}^{FB}(G)\right) &= \left(-\frac{p_\chi}{\varepsilon\left(-\frac{\frac{c_T T}{B}\left[(-)\phi(\mathcal{N}-1)\zeta'(G)+\eta\right]+c_\zeta\frac{(\mathcal{N}-1)}{\mathcal{N}}\zeta(G)\zeta'(G)}{(m+\nu)+\varepsilon(\mathcal{N}-1)\gamma\left(\chi_j(G)\right)^{-\varepsilon-1}\bar{k}_j^a\left(\chi_j(G)\right)'}\right)\gamma k_i^a}\right)^{-\frac{\varepsilon}{\varepsilon+1}} \\
&= \left(-\frac{p_\chi}{\varepsilon\overline{\lambda}^{FB}(G)\gamma k_i^a}\right)^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned}$$

$$\begin{aligned}
g_2\left(G,\overline{\lambda}^{FB}(G)\right) &= \left(\frac{a\left(\left(-\frac{\frac{c_T T}{B}\left[(-)\phi(\mathcal{N}-1)\zeta'(G)+\eta\right]+c_\zeta\frac{(\mathcal{N}-1)}{\mathcal{N}}\zeta(G)\zeta'(G)}{(m+\nu)+\varepsilon(\mathcal{N}-1)\gamma\left(\chi_j(G)\right)^{-\varepsilon-1}\bar{k}_j^a\left(\chi_j(G)\right)'}\right)\gamma\chi_i^{-\varepsilon}+1\right)}{(\rho+\delta)}\right)^{\frac{a}{1-a}} \\
&= \left(\frac{a\left(\overline{\lambda}^{FB}(G)\gamma\chi_i^{-\varepsilon}+1\right)}{(\rho+\delta)}\right)^{\frac{a}{1-a}}
\end{aligned}$$

then we have that:

$$\dot{G}=\sum_{i=1}^{\mathcal{N}}\gamma g_1\left(G,\overline{\lambda}^{FB}(G)\right)\cdot g_2\left(G,\overline{\lambda}^{FB}(G)\right)-mG=R(G).$$

The roots of equation $R(G)=0$ determine the steady states. A steady state will exist in an interval $[G_1,G_2]$ where $R(G)$ is continuous, if $R(G_1)\cdot R(G_2)<0$. Existence of a steady state requires that the equilibrium feedback functions $\chi(G)$ and $\zeta(G)$ be such that functions for low G_1 and high G_2, $R(G_1)\cdot R(G_2)<0$. If a steady state exists, it may not be unique.

□

A.5 Proof of Proposition 2

The Jacobian matrix of the MHDS evaluated at the steady state is defined as:

$$J=\begin{bmatrix}\frac{\partial H_{\lambda^{FB}}}{\partial G} & \frac{\partial H_{\lambda^{FB}}}{\partial\lambda^{FB}}\\ \frac{\partial H_G}{\partial G} & \frac{\partial H_G}{\partial\lambda^{FB}}\end{bmatrix}$$

where

$$\frac{\partial H_{\lambda^{FB}}}{\partial G} = -m \Longrightarrow \frac{\partial H_{\lambda^{FB}}}{\partial G} < 0$$

$$\begin{aligned}\frac{\partial H_{\lambda^{FB}}}{\partial \lambda^{FB}} &= -\varepsilon \mathcal{N} \gamma \left(\chi^* \left(\lambda^{FB}\right)\right)^{-\varepsilon-1} \left[k^* \left(\lambda^{FB}\right)\right]^a \frac{\partial \chi^*}{\partial \lambda^{FB}} + \\ &\quad + a \mathcal{N} \gamma \left(\chi^* \left(\lambda^{FB}\right)\right)^{-\varepsilon} \left[k^* \left(\lambda^{FB}\right)\right]^{a-1} \frac{\partial k^*}{\partial \lambda^{FB}} \Longrightarrow \frac{\partial H_{\lambda^{FB}}}{\partial \lambda^{FB}} > 0\end{aligned}$$

$$\frac{\partial H_G}{\partial G} = \Phi\left(\chi^*, \zeta^*, \chi_j(G), \zeta_j(G)\right)$$

$$\frac{\partial H_G}{\partial \lambda^{FB}} = m + \nu + \varepsilon (\mathcal{N}-1) \gamma \left(\chi_j(G)\right)^{-\varepsilon-1} \bar{k}_j^a \left(\chi_j(G)\right)' \Longrightarrow \frac{\partial H_G}{\partial \lambda^{FB}} > 0$$

$$\det(j) = \frac{\partial H_{\lambda^{FB}}}{\partial G} \cdot \frac{\partial H_G}{\partial \lambda^{FB}} - \frac{\partial H_{\lambda^{FB}}}{\partial \lambda^{FB}} \cdot \frac{\partial H_G}{\partial G}$$

where G and λ^{FB} are evaluated at the steady state.

We can see that $tr(j) = \nu > 0$, which means that the steady state is a saddle point or locally unstable.

$$If \ \ \frac{\partial H_G}{\partial G} > 0 \text{ then } \det(j) < 0 \tag{81}$$

and the steady state is a saddle point.

$$\begin{aligned} If \ \ \frac{\partial H_G}{\partial G} &< 0 \text{ and the natural decay rate of GHGs is low} \\ \text{and near zero} \ \ \longrightarrow & \ \ \frac{\partial H_{\lambda^{FB}}}{\partial G} \cdot \frac{\partial H_G}{\partial \lambda^{FB}} < \frac{\partial H_{\lambda^{FB}}}{\partial \lambda^{FB}} \cdot \frac{\partial H_G}{\partial G} \end{aligned} \tag{82}$$

then $\det(j) > 0$ and the steady state is locally unstable.

□

A.6 Two Polar Cases

A.6.1 No mitigation

Policy makers can apply only geoengineering. We assume that $i = 1, ..., \mathcal{N}$ countries are involved in emissions and geoengineering. Cooperative and non-cooperative solutions are obtained as solutions of the following optimization problems

- **Cooperation**

$$W=\max_{k_i,\zeta}\int_0^{\infty}e^{-\nu t}\left[\sum_{i=1}^{\mathcal{N}}c_i-\frac{1}{2}\sum_{i=1}^{\mathcal{N}}c_TT^2-\sum_{i=1}^{\mathcal{N}}\frac{c_\zeta}{2\mathcal{N}}\sum_{i=1}^{\mathcal{N}}\zeta_i^2\right]dt$$

subject to

$$\begin{aligned} i)\ \dot{G} &= \sum_{i=1}^{\mathcal{N}}\gamma k_i^a-mG \\ ii)\ T &= \frac{-A+S\left(1-\alpha\right)-\phi\sum_{i=1}^{\mathcal{N}}\zeta+\eta\left(G-G_0\right)}{B} \end{aligned}$$

- **Open Loop**

$$W=\max_{k_i,\zeta}\int_0^{\infty}e^{-\nu t}\left[c_i-\frac{1}{2}c_TT^2-\frac{c_\zeta}{2\mathcal{N}}\left(\zeta_i^2+\sum_{j\neq i}^{\mathcal{N}}\bar{\zeta}_j^2\right)\right]dt$$

subject to

$$i)\ \dot{G}=\gamma k_i^a+\sum_{j\neq i}^{\mathcal{N}}\gamma\bar{k}_j^a-mG$$

$$ii)\ T=\frac{-A+S\left(1-\alpha\right)-\phi\left(\zeta+\sum_{i\neq j}^{\mathcal{N}}\bar{\zeta}_j\right)+\eta\left(G-G_0\right)}{B}$$

- **Feedback**

$$W=\max_{k_i,\zeta}\int_0^{\infty}e^{-\nu t}\left[c_i-\frac{1}{2}c_TT^2-\frac{c_\zeta}{2\mathcal{N}}\left(\zeta_i^2+\sum_{j\neq i}^{\mathcal{N}}\left(\zeta_j\left(G\right)\right)^2\right)\right]dt$$

subject to

$$i)\ \dot{G}=\gamma k_i^a+\sum_{j\neq i}^{\mathcal{N}}\gamma\bar{k}_j^a-mG$$

$$ii)\ T=\frac{-A+S\left(1-\alpha\right)-\phi\left(\zeta+\sum_{j\neq i}^{\mathcal{N}}\left(\zeta_j\left(G\right)\right)\right)+\eta\left(G-G_0\right)}{B}.$$

Then we will have the following MHDS for each case:

- **Cooperation**

$$\begin{aligned} \dot{G} &= \sum_{i=1}^{\mathcal{N}}\gamma\left(k^*\left(\lambda^C\right)\right)^a-mG \\ \dot{\lambda}^{C,g} &= \left(m+\nu\right)\lambda^C+\frac{\mathcal{N}\eta c_T}{B}\cdot T \end{aligned}$$

- **Open Loop**

$$\begin{aligned}\dot{G} &= \sum_{i=1}^{\mathcal{N}} \gamma \left(k^{*}\left(\lambda^{OL}\right)\right)^{a} - mG \\ \dot{\lambda}^{OL,g} &= (m+\nu)\,\lambda^{OL} + \frac{\eta c_T}{B}\cdot T\end{aligned}$$

- **Feedback**

$$\begin{aligned}\dot{G} &= \sum_{i=1}^{\mathcal{N}} \gamma \left(k^{*}\left(\lambda^{FB}\right)\right)^{a} - mG \\ \dot{\lambda}^{FB,g.} &= (m+\nu)\,\lambda^{FB} + \frac{c_T T}{B}\cdot\left[-\phi\,(\mathcal{N}-1)\,\zeta'(G)+\eta\right] + c_\zeta \frac{(\mathcal{N}-1)}{\mathcal{N}}\zeta(G)\,\zeta'(G)\,.\end{aligned}$$

A.6.2 No geoengineering

Policy makers can apply only mitigation. We assume that $i = 1,...,\mathcal{N}$ countries are involved in emissions and mitigation. Thus

- **Cooperation**

$$W = \max_{k,\chi_i} \int_0^\infty e^{-\nu t}\left[\sum_{i=1}^{\mathcal{N}} c_i - \frac{1}{2}\sum_{i=1}^{\mathcal{N}} c_T T^2\right] dt$$

subject to

$$\begin{aligned} i)\ \ \dot{G} &= \sum_{i=1}^{\mathcal{N}} \gamma \chi_i^{-\varepsilon} k_i^{a} - mG \\ ii)\ T &= \frac{-A + S(1-\alpha) + \eta\,(G-G_0)}{B}\end{aligned}$$

- **Open Loop**

$$W = \max_{k,\chi_i} \int_0^\infty e^{-\nu t}\left[c_i - \frac{1}{2} c_T T^2\right] dt$$

subject to

$$i)\ \dot{G} = \gamma \chi_i^{-\varepsilon} k_i^{a} + \sum_{j\neq i}^{\mathcal{N}} \gamma\left(\bar{\chi}_j\right)^{-\varepsilon} \bar{k}_j^{a} - mG$$

$$ii)\ T = \frac{-A + S(1-\alpha) + \eta\,(G-G_0)}{B}$$

- **Feedback**

$$W = \max_{k,\chi_i} \int_0^\infty e^{-\nu t} \left[c_i - \frac{1}{2} c_T T^2 \right] dt$$

subject to

$$i)\ \dot{G} = \gamma \chi_i^{-\varepsilon} k_i^a + \sum_{j \neq i}^{\mathcal{N}} \gamma \left(\bar{\chi}_j\right)^{-\varepsilon} \bar{k}_j^a - mG$$

$$ii)\ T = \frac{-A + S(1-\alpha) + \eta (G - G_0)}{B}$$

Then we will have the following MHDSs for each case:

- **Cooperation**

$$\begin{aligned} \dot{G} &= \sum_{i=1}^{\mathcal{N}} \gamma \left(\chi^* \left(\lambda^C \right) \right)^{-\varepsilon} \left(k^* \left(\lambda^C \right) \right)^a - mG \\ \dot{\lambda}^{C,m.} &= (m+\nu) \lambda^C + \frac{\mathcal{N} \eta c_T}{B} \cdot T \end{aligned} \tag{83}$$

- **Open Loop**

$$\begin{aligned} \dot{G} &= \mathcal{N} \gamma \left(\chi^* \left(\lambda^{OL} \right) \right)^{-\varepsilon} \left[k^* \left(\lambda^{OL} \right) \right]^a - mG \\ \dot{\lambda}^{O.L,m.} &= (m+\nu) \lambda^{OL} + \frac{\eta c_T}{B} \cdot T \end{aligned} \tag{84}$$

- **Feedback**

$$\begin{aligned} \dot{G} &= \mathcal{N} \gamma \left(\chi^* \left(\lambda^{FB} \right) \right)^{-\varepsilon} \left[k^* \left(\lambda^{FB} \right) \right]^a - mG \\ \dot{\lambda}^{FB,m.} &= \left(m + \nu + \varepsilon (\mathcal{N} - 1) \gamma \left(\chi^* \left(\lambda^{FB} \right) \right)^{-\varepsilon - 1} \bar{k}_j^a \left(\chi_j (G) \right)' \right) \lambda^{FB} + \frac{\eta c_T}{B} \cdot T. \end{aligned}$$

A.7 Steady State Values

Cooperation:

$$G^{g\cdot} = \frac{\mathcal{N} \gamma \left[k^* \left(\lambda^{g\cdot} \right) \right]^a}{m} \quad , \quad G^{m\cdot} = \frac{\mathcal{N} \gamma \left(\chi^* \left(\lambda^{m\cdot} \right) \right)^{-\varepsilon} \left[k^* \left(\lambda^{m\cdot} \right) \right]^a}{m}$$

$$T^{g\cdot} = \frac{-A + S(1-\alpha) - \phi \mathcal{N} \zeta^* \left(G^{g\cdot} \right) + \eta \left(G^{g\cdot} - G_0 \right)}{B} \quad , \quad T^{m\cdot} = \frac{-A + S(1-\alpha) + \eta \left(G^{m\cdot} - G_0 \right)}{B}$$

$$\lambda^{g\cdot} = -\frac{\mathcal{N} \eta c_T T^{g\cdot}}{B(m+\nu)} \quad , \quad \lambda^{m\cdot} = -\frac{\mathcal{N} \eta c_T T^{m\cdot}}{B(m+\nu)}$$

Open loop:

$$G^{g\cdot} = \frac{\mathcal{N}\gamma\left[k^*\left(\lambda^{g\cdot}\right)\right]^a}{m} \quad , \quad G^{m\cdot} = \frac{\mathcal{N}\gamma\left(\chi^*\left(\lambda^{m\cdot}\right)\right)^{-\varepsilon}\left[k^*\left(\lambda^{m\cdot}\right)\right]^a}{m}$$

$$T^{g\cdot} = \frac{-A + S\left(1-\alpha\right) - \phi\sum_{i=1}^{\mathcal{N}}\zeta^*\left(G^{g\cdot}\right) + \eta\left(G^{g\cdot} - G_0\right)}{B} \quad , \quad T^{m\cdot} = \frac{-A + S\left(1-\alpha\right) + \eta\left(G^{m\cdot} - G_0\right)}{B}$$

$$\lambda^{g\cdot} = -\frac{\eta c_T T^{g\cdot}}{B\left(m+\nu\right)} \quad , \quad \lambda^{m\cdot} = -\frac{\eta c_T T^{m\cdot}}{B\left(m+\nu\right)}$$

Feedback:

$$G^{g\cdot} = \frac{\mathcal{N}\gamma\left[k^*\left(\lambda^{g\cdot}\right)\right]^a}{m} \quad , \quad G^{m\cdot} = \frac{\mathcal{N}\gamma\left(\chi^*\left(\lambda^{m\cdot}\right)\right)^{-\varepsilon}\left[k^*\left(\lambda^{m\cdot}\right)\right]^a}{m}$$

$$T^{g\cdot} = \frac{-A + S\left(1-\alpha\right) - \phi\sum_{i=1}^{\mathcal{N}}\zeta^*\left(G^{g\cdot}\right) + \eta\left(G^{g\cdot} - G_0\right)}{B} \quad , \quad T^{m\cdot} = \frac{-A + S\left(1-\alpha\right) + \eta\left(G^{m\cdot} - G_0\right)}{B}$$

$$\begin{aligned} \lambda^{g\cdot} &= -\frac{\frac{c_T T^{g\cdot}}{B}\left[\left(-\right)\phi\left(\mathcal{N}-1\right)\zeta'\left(G^{g\cdot}\right) + \eta\right] + c_\zeta\frac{\left(\mathcal{N}-1\right)}{\mathcal{N}}\zeta^*\left(G^{g\cdot}\right)\zeta'\left(G^{g\cdot}\right)}{\left(m+\nu\right)} \\ \lambda^{m\cdot} &= -\frac{\frac{c_T T^{m\cdot}}{B}\eta}{\left(m+\nu\right) + \varepsilon\left(\mathcal{N}-1\right)\gamma\left(\chi_j\left(G^{m\cdot}\right)\right)^{-\varepsilon-1}\bar{k}_j^a\left(\chi_j\left(G^{m\cdot}\right)\right)'}. \end{aligned}$$

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