DEPARTMENT OF INTERNATIONAL AND EUROPEAN ECONOMIC STUDIES

ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

Optimal Scheduling of Greenhouse Gas Emissions under Carbon Budgeting and Policy Design

Elettra Agliardi

Anastasios Xepapadeas

Working Paper Series

18-08

March 2018

Optimal Scheduling of Greenhouse Gas Emissions under Carbon Budgeting and Policy Design

Elettra Agliardi
University of Bologna

Anastasios Xepapadeas
Athens University of Economics and Business
and University of Bologna

January 18, 2018

Abstract

We solve a problem of optimal scheduling of GHG emissions for a climate change policy that is consistent with the COP21 targets and has to be monitored at a fixed time horizon. Our model is dynamic and stochastic where production, and therefore well-being, increase in carbon emissions, but, at the same time, anthropogenic cumulative emissions determine a super-linear impact on the observed stochastic damage. We compare the optimal unconstrained path of emissions and the constrained path of emissions and evaluate when the carbon budget is exhausted. A sensitivity analysis is also developed to examine the effects of resilience, impact of emissions on damage and uncertainty. Our results have direct implications in terms of policy and show that uncertainty and the way we introduce it in the model are likely to influence the efficacy of the climate change policies for the foreseeable future.

1 Introduction

Global warming is one of the biggest challenges of the planet and (anthropogenic) emissions of a broad range of greenhouse gases (GHGs) of varying lifetimes and radiative forcing are taken as major contributors (Matthews et al, 2009; Meinshausen et al, 2006). The atmospheric concentration of carbon dioxide (CO_2) has increased from a pre-industrial level of about 280 ppm to over 400 ppm in recent years. At the same time, average land and surface temperatures have increased by approximately 0.9 degrees C since pre-industrial times. The concentration of all GHGs (CO_2 equivalent) is the highest in the last 800000 years (see IPCC, 2014) and, in the absence of mitigation policies, is expected to increase in this century above 1000 ppm. Consequently, an unbroken average temperature increase of about 4 degrees C might result by the end of this century, which is well above global mean temperature targets for dangerous anthropogenic interference (DAI) (see, Zickfeld et al., 2009; Solomon et al, 2010). Therefore, the ultimate objective of climate policies is to reduce emissions in order to achieve “stabilization of GHGs concentrations in the atmosphere at a level that would prevent DAI with the climate system” (United Nations Framework Convention on Climate Change, article 2).

The limit temperature level recognized by international climate policies, and adopted in the COP21 agreement, is 2 degrees C (or preferably 1.5 degrees C). This temperature stabilization target has since been reaffirmed on a number of occasions and has led to develop a range of allowable cumulative emissions targets that are compatible with the specified temperature target (Wigley, 2004; Kriegler and Bruckner, 2004; Knutti et al, 2005; Meinshausen, 2009; Matthews, et al 2009; Zickfeld et al, 2009). In particular, it has been estimated that in order to stabilize global mean temperature increase at 2 degrees C the carbon budget of cumulative emissions should be approximately 0.8 Tt C – see Matthews et al, 2009 -, even though large uncertainties in equilibrium climate sensitivity prevent highly confident estimates of the emissions stabilization levels (see Pindyck, 2013; Heal and Millner, 2014, Freeman, Wagner and Zeckhauser, 2015). In view of the persistence in the atmosphere of the effects of cumulative emissions, it would imply almost net zero new emissions by 2050 (UNFCCC, 2016).

Although there is no specific mechanism to force a country to set a target by a specific date, nor enforcement measures if the set target is not met, a monitoring system has been agreed, such that the implementation has to be evaluated every 5 years, with the first evaluation in 2023. Actually, the absence of enforcement measures and specific penalties for countries that do not fulfil the agreed targets has been questioned a lot, because it may put at risk the implementation of the COP21 agreement itself, and therefore appropriate mechanism designs to enforce the agreement are called for.

In this paper we study a setting to evaluate whether such implementation is feasible. More specifically, we consider a dynamic and stochastic model where production, and therefore well-being, increase in carbon emissions, but, at the same time, anthropogenic cumulative emissions determine DAI, with a superlinear impact of emissions on the observed damage. Moreover, we add a stochastic component into the damage function. In particular, total damage (per unit of emission) is represented by a stochastic process which consists of two components. One component of damage is supposed to be affected by the sequence of past GHG emissions, so that damage is increased by an amount which is proportional to emissions at any time. The other component is exogenous and allows us to describe fluctuations in actual economic damages, since we assume that its impact may fluctuate around its deterministic part in a random way. Finally, in order to capture the complexities of climate inertia and the carbon-climate response feedbacks, we introduce a parameter governing the absorbing capacity of the environment, which represents its resilience property. We suppose that carbon emissions are gradually dissipated and thus their negative economic impact is partly absorbed by the environment. Let $\rho \geq 0$ denote the parameter governing this absorbing capacity of the environment, which represents resiliency. In other words, when $\rho = 0$ the negative effect of carbon emissions persists forever; at the other extreme, $\rho = \infty$ describes immediate absorption of the carbon emissions. This specification allows us to have a pretty general characterization of damage, which is consistent with recent literature (see e.g. Weitzman, 2010, for a discussion of alternative damage functions).

Firstly, we study the optimal scheduling of GHG emissions for a climate change policy design that has to be monitored at a fixed time horizon and is consistent with the above-mentioned objectives (Section 2). In particular, we study how the optimal scheduling of emissions is affected by targets in the cumulative levels of emissions that have to be achieved at specified dates, as stated in the monitoring procedures of the international climate change agreements. A sensitivity analysis is developed to examine the effects of resilience, impact of emissions on damage and uncertainty (Section 3).

Then we examine a case where a penalty is set by an institutional regulator in order to provide incentives to reduce the impact of GHG emissions within a fixed time horizon (Section 4). For example, one can think of emissions that need not to surpass a prescribed level in order to mitigate ecological and social damage in future times. If an upper threshold is surpassed at the specified monitoring time, then a penalty - in proportion to cumulative emission damages - should be paid on the exceeding amount. We compare the path of emissions we find in this case with the path of emissions in the absence of penalties to evaluate under which circumstances the proposed penalty may act as an incentive device to enforce the agreement.

This paper contributes to the literature in at least three ways. First, in contrast to the majority of the literature, which uses an arbitrary specification for the damage function, it develops an approach for deriving a convex damage function which incorporates the effects of cumulative past carbon emissions. Second, it allows for damage uncertainty by explicitly introducing into the objective function of the optimization problem the variance of climate change damages weighted by the regulator's concerns regarding uncontrolled effects. Third, it introduces the possibility of a penalty which is potentially paid by the representative firm if the damages from cumulative emissions exceed the voluntary agreed carbon budget at some point during the planning horizon.

The organization of this paper is as follows. Section 2 describes the model. Section 3 presents the main result in Proposition 1, providing the solution to the optimization problem, develops a numerical simulation with calibrated data, which allows us to evaluate whether the target of COP21 can be implemented, and presents a sensitivity analysis of the optimal scheduling of emissions to changes in the relevant parameters. Section 4 introduces a penalty as a mechanism design (Proposition 2) and discusses its effects. Section 5 contains the conclusions. Finally, all proofs are in the Appendix.

2 The model

Let us consider the problem of a social planner who has to maximize global well-being with a fixed time horizon by choosing paths for GHG emissions subject to technology and the climate constraints.

We adopt a discrete time schedule, where $t_1 < t_2 < ....$ denote the times at which the decisions on the production process are made by observing the information available at the current time. The information set is modelled throughout a filtered probability space where $\{\Im_k\}_{k=0,\ldots,N}$ denotes an increasing family of σ-algebras on a given probability space. In the sequel, the conditional expectations $E(. \mid \Im_k)$ will be denoted as E_k.

Let x_1, ... x_N denote the amount of emissions at time $t_1, ...t_N$ as a consequence of the production process. We assume that the process $\{x_k\}$ is adapted to the given filtration, that is, the decision maker at each decision time is aware of the negative environmental effects of his policy.

The total social damage (per unit of emission) due to GHG emissions is represented by a stochastic process D_t. There is substantial discussion and, perhaps more importantly, uncertainty, about the appropriate damage function, due to imperfect understanding of feedback effects, among other things. Therefore, it is relevant to allow for uncertainty, as we do in this paper. We suppose that total damage consists of two components. One component, D_t^0, which is related to the global warming potential of GHGs, allows us to describe fluctuations in actual economic damages, since we assume its impact may fluctuate around its deterministic component in a random way. A simple model for these random fluctuations is an arithmetic Brownian motion, i.e. $dD_t^0 = \sigma dW_t$, where W_t is a Wiener process with respect to the fixed filtration that represents the information available to the decision maker. More generally, this assumption can be replaced by any martingale with respect to the reference filtration. Here we adopt an arithmetic Brownian motion, because it is the easiest way to model randomness and allows us to obtain an explicit analytic solution[1].

The other component of damage is supposed to be affected by the sequence of emissions, x_k, that is, D_{t_k} is increased by an amount εx_k, $\varepsilon > 0$. Notice that our damage function has emissions as its argument (as also in Nordhaus, 2007, and van der Ploeg, 2014), whereas other models express damages as a function of a climate indicator, such as global temperature (see, e.g., Weitzman, 2010). Here we follow the argument in Golosov et al (2014), where the D_{t_k} mapping can be thought of in two steps. The first is the mapping from emissions to the global mean temperature, as outlined since Matthews et al. (2009). The second is the mapping from temperature to damage. Thus, our taking emissions as an input should be viewed as a composition of the typical damage function, with temperature as an argument, and another function mapping carbon emissions into temperature[2].

[1] The occurrence of negative damage due to the assumption of an arithmetic Brownian motion is avoided by taking small values of the volatility σ.

[2] As recalled by Golosov et al (2014), typical approximations used in climate science make the former mapping convex and the latter mapping concave, so it is not clear whether the

Our assumption results in a superlinear impact of the emissions on the observed damage, which is consistent with the literature showing strong carbon/climate feedbacks and more persistent warmings due to GHGs (see, e.g., Solomon et al, 2010). We suppose that GHG emissions are gradually dissipated and thus their negative economic impact is partly absorbed by the environment. Let $\rho \geq 0$ denote the parameter governing this absorbing capacity of the environment, which represents its resilience property. In other words, when $\rho = 0$ the negative effect of emissions persists forever; at the other extreme, $\rho = \infty$ describes immediate absorption of emissions.To summarize, the dynamics for D are as follows:

$$D_k = D_k^0 + \varepsilon \sum_{j=1}^{k-1} e^{-\rho(t_k - t_j)} x_j \tag{1}$$

where D^0 follows an arithmetic Brownian motion with zero drift and variance parameter σ^2, and the notation D_k stands for D_{t_k}, for notational brevity. As an emission $\Delta x = \frac{1}{\varepsilon}\Delta D$ is responsible for an incremental damage ΔD, the incremental damage resulting soon after an emission x_k can be computed as follows:

$$\int_{D_k}^{D_k+\varepsilon x_k} \tfrac{1}{\varepsilon}\delta d\delta = \tfrac{1}{2\varepsilon}[(D_k + \varepsilon x_k)^2 - (D_k)^2] = D_k x_k + \tfrac{\varepsilon x_k^2}{2}$$

In general, the total cumulated damage, soon after time t_k, can be computed recursively as follows:

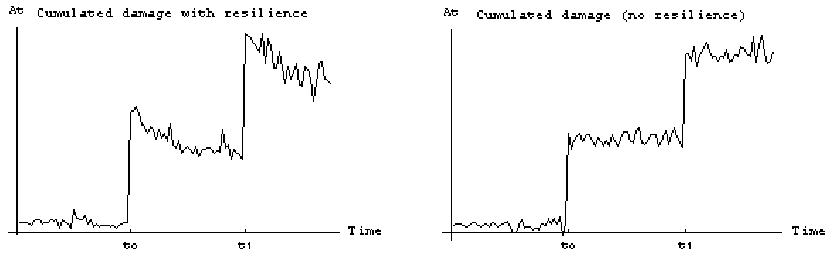
$$\sum_{j=1}^{k} x_j D_j^0 + \varepsilon \sum_{j=1}^{k} \left(\frac{x_j^2}{2} + \sum_{i<j} e^{-\rho(t_j - t_i)} x_i x_j\right) \tag{2}$$

Therefore, the total cumulated damage is characterized by a superlinear effect of emissions. Unless $\rho = \infty$, in which case damage is affected by the sum of x_j and x_j^2 only, if there is persistence of climate changes due to emissions (that is, for the other values $\rho \geq 0$ different from ∞), then total damage is strengthened by previous emissions (the term $\sum_{i<j} e^{-\rho(t_j - t_1)} x_i x_j$). This non linear feature captures the exceptional persistence displayed by CO_2, that renders its warming nearly irreversible for more than 1000 years, and also by other GHGs, which, although not irreversible, persist notably longer than the anthropogenic changes in the GHGs concentrations themselves (see Solomon et al, 2010).

Figure 1 plots a path for the total cumulated damage after two subsequent emissions. In panel (b), there is a full permanent impact of the emissions, while in panel (a) a resilience effect mitigates the impact over time.

overall function mapping into damages should be convex or concave.

Figure 1 in Golosov (2014) shows the composition of the emissions-to-temperature and temperature-to-net-of-damages mappings, as calibrated by Nordhaus. The composition implied by Nordhaus's formulation is first concave, then convex, while Golosov et al's function is exponential, which makes our approximation rather reasonable as well.



(a) Case $\rho = 0.1$ (b) Case $\rho = 0$

Figure 1. Cumulated damage with two emissions

The objective of the decision maker is to minimize the expected total damage over a time horizon $T = t_N$, while maximizing the total utility. We will study the case where a regulatory target in terms of cumulative emissions is set by international agreements within the time horizon.

In this paper a quadratic utility will be adopted, that is, $U(x) = ux - \frac{1}{2}wx^2$, which is common in the literature (e.g.,Dockner and Van Long, 1993). This function could be seen as a reduced form of a problem where the utility function is a function of consumption, which itself depends on economic output, which is a function of emissions[3]. The parameter u measures the effect on marginal benefits from emissions, while w the strength of their diminishing returns. For simplicity, no discount rates are considered here (moreover, real-world interest rates are close to zero, and see also the argument in Stern (2008) in favour of intergenerational equity). Thus, the objective function to be maximized is of the form:

$$\sup_{x_1,\ldots,x_N} \; E_0[\sum_{k=1}^{N}(ux_k - \tfrac{1}{2}wx_k^2 - x_k D_k^0 - \varepsilon(\tfrac{x_k^2}{2} + \sum_{i<k} e^{-\rho(t_k - t_i)} x_i x_k))]$$

If we compare it with the so-called Chichilnisky's criterion (1997), which proposes a weighted average between the discounted sum of instantaneous costs and the long run cost associated with pollution, here more emphasis is placed on future generations, because the cumulated effect of emissions, that is, the final environmental damage, is given weight one. Such a specification is consistent

[3] One can think of a simple growth model with the utility function $U(c) = ac - \frac{1}{2}bc^2$, where c denotes consumption. Let us define the budget constraint for the economy with an Ak production function such as $c_t = Ak_t - k_{t+1} + (1-d)k_t$, where k is the capital stock and d is the depreciation rate. If we define emissions as proportional to output, that is $x_t = sAk_t$,where s is an exogenous parameter of emission intensities, then we get the following expression for the the utility function: $U(x_t) = a(\frac{x_t}{s} - \frac{x_{t+1}}{sA} + (1-d)\frac{x_t}{sA}) - \frac{1}{2}b(\frac{x_t}{s} - \frac{x_{t+1}}{sA} + (1-d)\frac{x_t}{sA})^2$
and therefore the sum of the utility functions over t is: $\sum_t U(x_t) = a\sum_t(\frac{x_t}{s} - \frac{x_{t+1}}{sA} + (1-d)\frac{x_t}{sA}) - \frac{1}{2}b\sum_t(\frac{x_t}{s} - \frac{x_{t+1}}{sA} + (1-d)\frac{x_t}{sA})^2$. If $b = 0$, then $\sum_t U(x_t) = \frac{a}{s}(1 - \frac{d}{A})\sum_t x_t = u\sum_t x_t$. Therefore, if utility is also linear in consumption (see Weitzman, 1998), then it becomes a function of emissions too. This argument can be applied to the general case of a quadratic utility function whenever w is small.

with the notion of sustainability (see also the discussion in Asheim and Mitra, 2010).

Note that the expectation of the cumulated effect of emissions can be rewritten as follows:

$$E_0[\sum_{k=1}^N x_k D_k^0 + \varepsilon \sum_{k=1}^N (\frac{x_k^2}{2} + \sum_{i<k} e^{-\rho(t_k - t_i)} x_i x_k)] =$$
$$= D_0 \sum_{k=1}^N x_k + \varepsilon \sum_{k=1}^N [x_k V_{k-1} + \frac{x_k^2}{2}]$$

where $V_k = \sum_{j=1}^k e^{-\rho(t_{k+1}-t_j)} x_j$ for $k \geq 1$ (and $V_0 = 0$) is the volume of cumulated emissions (up to time t_{k+1}) if the dissipating effect is taken into account.

If the decision maker is also concerned about the risk of random fluctuations in damage, then an additional term in the form of a variance should be added, so the optimization problem takes the form:

$$\sup_{x_1,\dots,x_N} \; E_0[(u - D_0) \sum_{k=1}^N x_k - \sum_{k=1}^N (\frac{(\varepsilon+w)x_k^2}{2} + \varepsilon x_k V_{k-1})] -$$
$$-\frac{\gamma}{2} var_0[\sum_{k=1}^N x_k D_k^0 + \varepsilon \sum_{k=1}^N (\frac{x_k^2}{2} + \sum_{i<k} e^{-\rho(t_k - t_i)} x_i x_k)]$$

where γ denotes the risk aversion parameter. This assumption can be justified in the framework of the standard mean-variance approach, which has been employed in financial economics extensively. One can approximate the objective function according to Taylor expansion. Then, in view of the assumption of an arithmetic Brownian motion, which implies that we are dealing with Gaussian random variables and the solution $\{x_k\}_{k=1,\dots N}$ is deterministic, the expression above is justified. The variance term can be written as follows:

$$E_0[\sigma^2 (\sum_{k=1}^N x_k W_{t_k})^2] = \sigma^2 \sum_{k=1}^N [x_k^2 t_k + 2 x_k \sum_{i<k} x_i t_i].$$

Thus, the optimization problem is as follows:

$$\sup_{x_1,\dots,x_N} (u - D_0) \sum_{k=1}^N x_k - \sum_{k=1}^N [(\varepsilon + w + \gamma\sigma^2 t_k) \frac{x_k^2}{2} + \varepsilon x_k V_{k-1} + \gamma\sigma^2 x_k \sum_{i<k} x_i t_i] \quad (3)$$

with $V_{k+1} = e^{-\rho(t_{k+1}-t_k)}(V_k + x_k)$ for $k \geq 0$, $V_0 = 0$.

Here the parameter γ embodies the concern of the decision maker regarding the uncontrolled effect of the outstanding volume of emissions, measured by the variance. Notice that the introduction of a variance term has not been explored much in the literature, while it is relevant to study the effects of the variability of system behaviour changes (see Brock and Carpenter, 2006, where they stress that increased variance may provide a leading indicator of regime shifts that can be used in ecosystem management).

In the following we confine ourselves to the case of equally spaced time intervals, $t_k - t_{k-1} = \Delta t$, to simplify the exposition. Furthermore, $e^{-\rho \Delta t}$ will be denoted by β and $\gamma\sigma^2 \Delta t$ will be denoted by Γ.

3 Adding the carbon budget constraint

Our objective is to study how the optimal scheduling of emissions is affected by targets in the cumulative levels of emissions that have to be achieved at specified dates, as stated in the monitoring procedures of the international climate change agreements. For example, in case of CO_2 emissions, climate policy sets an upper threshold to mitigate global warming. This is consistent with the use of the cumulated carbon budget which should not be exceeded for a given threshold temperature, as formulated by Matthews et al (2009) and Matthews et al (2012)[4]. One can safely think of other pollutant emissions that need not to surpass a prescribed level (set by a regulator) in order to mitigate ecological and social damage in future times. As a first analysis, we assume that an upper threshold, Z, is set by the regulator on the total amount of emissions up to the monitoring time T. In our setting, it amounts to adding a constraint of the form:

$$\textstyle\sum_{k=1}^{N} x_k \leq Z \tag{4}$$

Then the constrained optimization problem can be solved through Kuhn-Tucker method. Let us define the Lagrangian as follows:

$$\begin{aligned}\mathfrak{L}(x_1, ..., x_N, \lambda) \quad &= \quad (u - D_0) \textstyle\sum_{k=1}^{N} x_k - \displaystyle\sum_{k=1}^{N} [(\varepsilon + w + k\Gamma)\frac{x_k^2}{2} + \varepsilon x_k \sum_{j=1}^{k-1} \beta^{k-j} x_j + \\ &\quad\quad \Gamma x_k \sum_{i<k} i x_i] - \lambda[\textstyle\sum_{k=1}^{N} x_k - Z]\end{aligned}$$

where $\lambda \geq 0$ is a Lagrange multiplier.

Then the first-order condition for positive x_k is $\frac{\partial \mathfrak{L}}{\partial x_k} = 0$, $k = 1, ..., N$. It amounts to solving a linear system of the form:

$$M \begin{pmatrix} x_1 \\ ... \\ x_N \end{pmatrix} = \begin{pmatrix} u - D_0 - \lambda \\ ... \\ u - D_0 - \lambda \end{pmatrix}$$

where the matrix M is as follows:

$$\begin{pmatrix} \varepsilon + w + \Gamma & \varepsilon\beta + \Gamma & \varepsilon\beta^2 + \Gamma & ... & \varepsilon\beta^{N-1} + \Gamma \\ \varepsilon\beta + \Gamma & \varepsilon + w + 2\Gamma & \varepsilon\beta + 2\Gamma & ... & \varepsilon\beta^{N-2} + 2\Gamma \\ ... & & & & ... \\ \varepsilon\beta^{N-1} + \Gamma & \varepsilon\beta^{N-2} + 2\Gamma & ... & ... & \varepsilon + w + N\Gamma \end{pmatrix} \tag{5}$$

[4] Matthews et al (2009) found that the increase in mean global yearly temperature is approximately proportional to cumulated carbon emissions in all their simulations. They suggest that such relationship could be used for policy purposes, so that the coefficient of proportionality allows us to set a cumulated carbon budget that should not be exceeded if a given temperature has to be maintained.

The following proposition provides the solution to our optimization problem. A proof is in the Appendix.

Proposition 1 *Assume that $u > D_0$ and that an upper threshold, Z, is set by the regulator on the total amount of emissions. Then the optimal policy is obtained as follows:*

$$\begin{pmatrix} x_1^* \\ ... \\ x_N^* \end{pmatrix} = M^{-1}(u - D_0 - \lambda)I,$$

where $I = \begin{pmatrix} 1 \\ ... \\ 1 \end{pmatrix}$, the matrix M is defined in (4), and $\lambda = u - D_0 - \frac{Z}{\sum_{i,j} \widetilde{m}_{i,j}}$, with $M^{-1} = (\widetilde{m}_{i,j})$.

In principle, this can be easily accomplished, but for high dimensions one has to resort to numerical computation, which will be performed in Section 3.1.

Observe that λ in Proposition 1 is the shadow price of the carbon budget, which can be interpreted as a measure of the social cost of carbon. If we take, as an example, $N = 1$, then $\lambda = u - D_0 - (\varepsilon + w + \Gamma)Z$. Since $\lambda \geq 0$, then $Z \leq \frac{u-D_0}{\epsilon+w+\Gamma}$, which implies that Z should be decreased, if the impact of each emission on damage increases (ε), or aversion to uncertainty and /or variance increase (Γ), or the strength of diminishing returns on marginal benefits from emissions increases (w).

Finally, we compute the optimal carbon tax. In this context, it amounts to solving the tax rate at which the representative firm's profit maximizing emissions equal the optimal emissions chosen by the regulator, as from Proposition 1. The representative firm faces an exogenous tax τ on emissions and solves a static problem

$$\max_{x_k} ux_k - \frac{1}{2}wx_k^2 - \tau_k x_k \ , \ k = 1, ...N$$

which means that emissions are

$$x_k^0 = \frac{u - \tau_k}{w}$$

We want to choose the tax so that the firm's profit maximizing emissions are equal to the optimal emissions chosen by the regulator. This means that

$$\frac{u - \tau_k}{w} = M^{-1}(u - D_0 - \lambda)I$$

Solving for τ_k we obtain

$$\tau_k = u - wM^{-1}(u - D_0 - \lambda)I$$

or

$$\tau_k = u - wx_k^* \ , k = 1, ...N \tag{6}$$

where x_k^* are optimal emissions chosen by the regulator. Thus, taxes are increasing over time.

3.1 A numerical simulation

In this section we perform a numerical simulation with calibrated data which will allow us to evaluate whether the target of COP21 can be implemented.

We take a monitoring period of 40 years, $N = 40$, in most of our analysis, and start in year 2015, so that T=2055. For our calculations we need to calibrate the following parameters: ε, β, u and w.

In order to calibrate ε, we follow the argument in Golosov et al (2014), which is based on the calibration of damages in Nordhaus (2008). When calibrating the damage function, using a bottom-up approach collecting a large number of studies on various effects of global warming, a 0.48% loss of global GDP can be estimated at 2.5 degrees C heating, corresponding to about 1,035 GtC, while the pre-industrial atmospheric CO_2 concentration is 581 GtC. Thus, equalling the calibrated damage function in Golosov et al. (2014) to our expression (1), we get $\varepsilon = 1,057 \times 10^{-5}$. In order to calibrate β, we also follow the argument in Golosov et al (2014) for their calibration of the parameter describing the total fraction of a unit emitted at time 0 that is left in the atmosphere at time s. For simplicity, we consider a constant resilience over time, which corresponds to s=1. Thus, we get $\beta = 0.507$.

The parameter w of the utility function is taken from Karp and Zhang (2006), $w = 1.9212$, who estimate a quadratic benefit-of-emissions function equivalent to the quadratic abatement function in Nordhaus. Then the corresponding parameter u in the utility function is obtained considering that emissions in 2015 (the starting year in our calibration) were 36.3 GtC (as from from Carbon Dioxide Information Analysis Center (CDIAC) of the U.S. Oak Ridge National Laboratory, Global Carbon Budget 2016). Thus, $u = 69.74$. Moreover, we take the risk aversion parameter $\gamma = 2$, consistently with the literature that usually takes values between 1 and 3 (see Pindyck, 2013). Finally, we need to calibrate σ. It is well known that it is hard to estimate how climate change will affect the economy, a problem on which we have very little data to base empirical work (see Pindyck, 2013). In order to get an estimate of the variance of damage, we first consider the estimate of the variance of the gamma distribution employed by Pindyck (2012) to describe the temperature change distribution, and then the estimates of the economic impact of temperature changes. If we take the estimates of the economic impact of temperature changes as in Golosov et al. (2014) at 2.5 degrees C heating, then we get $\sigma = 0.01$ approximately. If we follow Nordhaus (2008) estimates at 2 degrees C heating, then $\sigma = 0,02$, which is higher but of the same order of magnitude.

The target Z in terms of cumulative emissions that has not to be surpassed in order to be compatible with the 2 degrees C temperature target is (approximately) 800 GtC, as stated also in the Global Carbon Project (2016).

In Figure 2 the solution of the unconstrained problem is depicted. In Figure 3 it is shown that the carbon budget is exhausted in year 2037, if we employ the variance of damage, as deducted from Golosov et al (2014). A slightly different result would be obtained if we had based our estimate of the variance on Nordhaus (2008), that is, the carbon budget is exhausted in year 2040.

Thus, emissions have to be reduced drastically over next years if the constraint has to be satisfied. Figure 4 compares the unconstrained emission and the constrained emission schedules over the next 40 years.

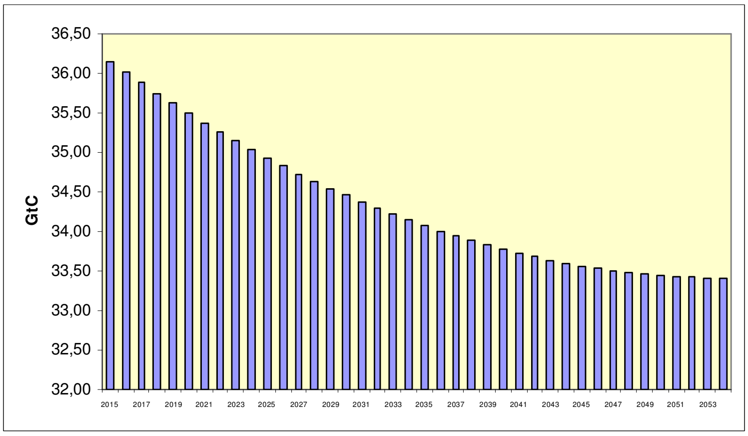


Figure 2

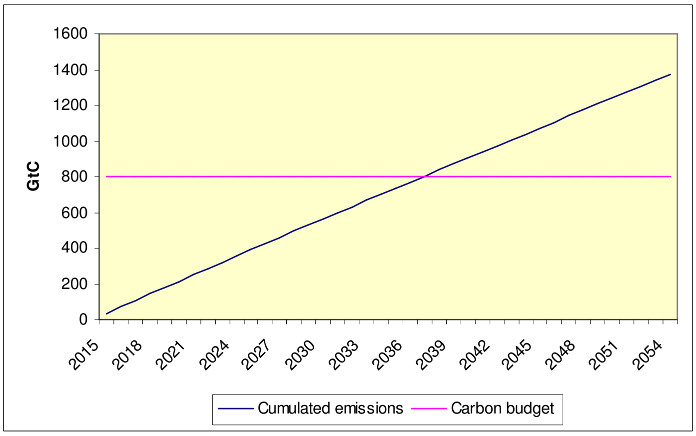


Figure 3

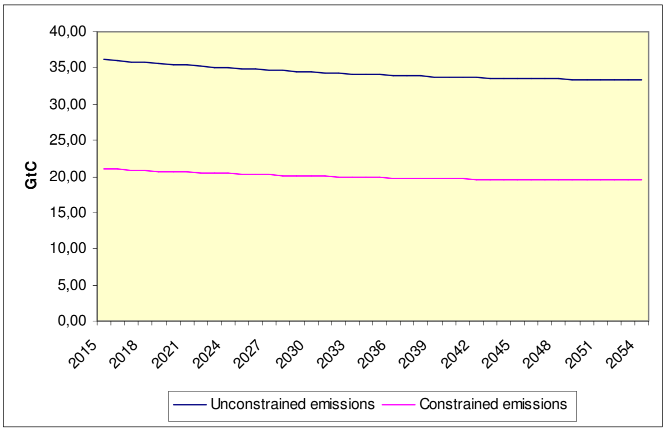


Figure 4

With the parameter values above, we can compute λ, the social cost of carbon, which becomes $\lambda = 29.12$ approximately (if we employ the variance of damage as deducted from Golosov et al. (2014), while it is $\lambda = 22.82$, if we base our estimate of the variance on Nordhaus (2008)).

Finally, we can compute the optimal carbon taxes, as from expression (6). Figure 5 represents the path of the optimal carbon taxes consistent with constrained emissions, which is increasing. Similarly, one can compute the path of carbon taxes in case emissions are unconstrained, and, of course, carbon taxes would be lower.

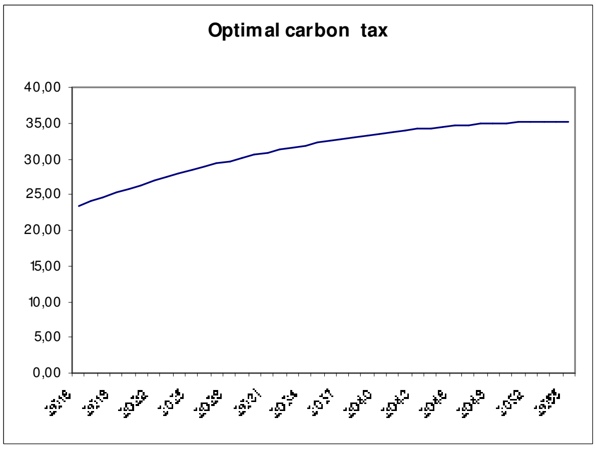
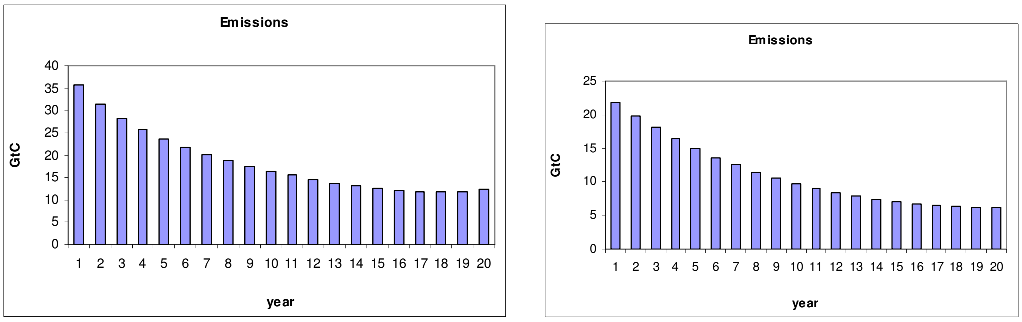


Figure 5

3.2 Sensitivity analysis

In what follows a sensitivity analysis is presented where the base case is the unconstrained case with the parameter values above. Figure 6 shows the optimal scheduling of emissions for N=20 points in time, as the resilience parameter is decreased.

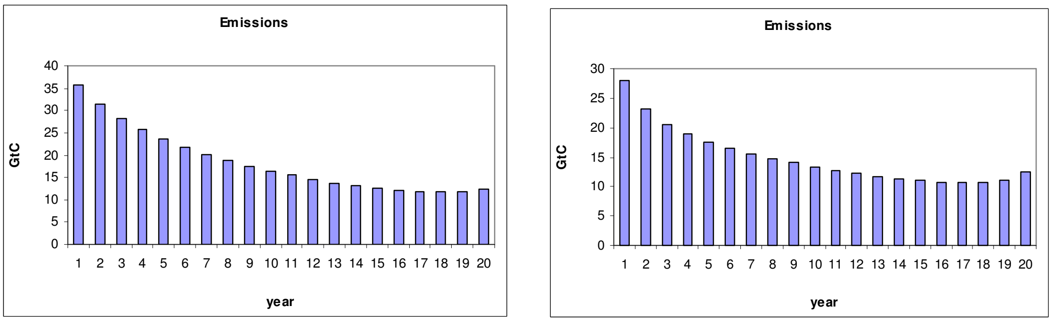


Base case $\qquad\qquad\qquad$ $\rho \approx 0$

Figure 6. Dependence of the emission schedule on the resilience parameter.

Figure 6 shows that the optimal amount of each emission is larger if the resilience parameter is higher, that is, if the environment has a better absorption capacity, then more emissions are allowed

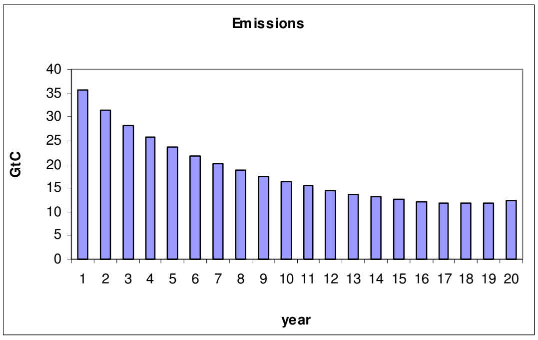
Figure 7 depicts the effect of an increased impact of each emission on damage, which is captured by the parameter ε. The result is as expected: an increase in ε forces a reduction in the optimal scheduling.

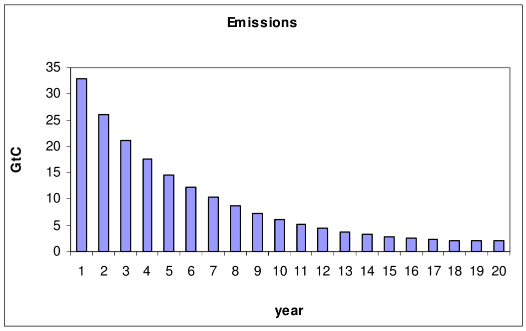


Base case $\qquad\qquad\qquad$ ε increased

Figure 7. Dependence of the emission schedule on ε.

In Figure 8 the effects of aversion to uncertainty and /or increased variance are explored by increasing the parameter Γ. The parameter Γ embodies the concern regarding the actual negative effect of pollution and the effectiveness of technology to harness future damage. In Figure 7 the parameter σ is two times the base case parameter.





Base case σ doubled

Figure 8. Dependence of the emission schedule on risk aversion and volatility.

The concern about uncertainty has a dramatic effect on the optimal scheduling of emissions: the larger risk aversion and/or volatility, the smaller the amount of emissions. This result is in keeping with the "precautionary principle" (see, e.g., Taleb et al., 2014; Athanassoglou and Xepapadeas, 2012). Thus, if the decision maker has high concern about uncertainty, then a more restrained policy should be adopted, reducing emissions.

4 A penalty as an alternative mechanism design

In this section a different perspective is taken regarding the form of constraint on the emissions that has to be imposed in order to achieve an optimal policy. We suppose that if an upper threshold, $\widetilde{Z}$, on the damages related to cumulative emissions is surpassed, then a penalty - in the proportion η - should be paid on the exceeding amount.

Assume that $T = t_N$ and thus a penalty is paid on $max[\Delta_N - \widetilde{Z}, 0]$. In our setting, Δ is related to prior emissions and, at the same time, incorporates the environmental resilience. It pays the role of a 'climate vector' defined in the literature (see Iverson and Karp (2017), for example).Then, the objective function needs to be modified as follows:

$$\sup_{x_1,\dots,x_N} (u - D_0) \textstyle\sum_{k=1}^N x_k - \displaystyle\sum_{k=1}^{N} [(\varepsilon + w + k\Gamma)\frac{x_k^2}{2} + \varepsilon x_k V_{k-1} + \Gamma x_k \sum_{i<k} i x_i]$$
$$-\eta E_0[max(\Delta_N - \widetilde{Z}, 0)] \tag{7}$$

Note that $E_0[max(\Delta_N - \widetilde{Z}, 0)] = E_0[max(\sigma W_T + D_0 + \varepsilon V_N - \widetilde{Z}, 0)]$, where W_t is a Wiener process and V is assumed to be a deterministic quantity. Proposition 2 gives the expression for the penalty. A proof is given in the Appendix.

Proposition 2 *The expectation at time* 0 *of the penalty takes the form:* $-\eta p(V_N)$, *where* $p(V)$ *is computed as follows:* $p(V) = (D_0 + \varepsilon V - \widetilde{Z})\Phi(\frac{D_0+\varepsilon V-\widetilde{Z}}{\sigma\sqrt{T}}) + \sigma\sqrt{T}\varphi(\frac{D_0+\varepsilon V-\widetilde{Z}}{\sigma\sqrt{T}})$, *with* Φ *and* φ *denoting the cumulative distribution function and the density function, respectively, of the standard Gaussian distribution.*

Note that

$\partial_V p(V) = \varepsilon\Phi(\frac{D_0+\varepsilon V-\widetilde{Z}}{\sigma\sqrt{T}})$.

Thus, the optimal solution to (7) can be obtained along the lines of Proposition 1. Unfortunately, x_k^* cannot be written in an explicit form, but needs to be computed numerically. To be more explicit, consider the equation yielding x_k^*. The first-order condition is:

$$M \begin{pmatrix} x_1 \\ ... \\ x_N \end{pmatrix} = \begin{pmatrix} u - D_0 - \eta\varepsilon\beta\Phi(\frac{D_0+\varepsilon V-\widetilde{Z}}{\sigma\sqrt{T}}) \\ ... \\ u - D_0 - \eta\varepsilon\beta\Phi(\frac{D_0+\varepsilon V-\widetilde{Z}}{\sigma\sqrt{T}}) \end{pmatrix} \tag{8}$$

that can be solved for x_k only numerically.

One can think of the penalty as another type of tax. In this case from the problem of the representative firm we can set $\widehat{x}_k = \frac{u-\widehat{\tau}_k}{w}$, where $\widehat{x}_k$ is obtained from the first-order condition (8), and then solve for $\widehat{\tau}_k$. The tax will be paid only if $\Delta_N > \widetilde{Z}$.

We can compare the constrained optimization problem, as from Proposition 1, with the problem with the penalty as from equation (7). If $\varepsilon = 1$ and $\rho = 0$, for an easy comparison, then we find that the boundary 800 GtC is satisfied, if $\eta = \lambda$. When $\sigma \to 0$, the problem with the penalty collapses into the constrained problem with $\eta = \lambda$ (see equation (8)). If we compute $p(.)$ for $\sum_{k=1}^N x_k = Z$, $\varepsilon = 1$ and $\rho = 0$, we get: $p = \frac{\sigma\sqrt{T}}{\sqrt{2\pi}}$. Thus, $p(.)$ increases with σ, while $\eta(= \lambda)$ decreases with σ.

Overall, we can compute the penalty $\eta p(.)$, as from Proposition 2, as a function of σ. Figure 9 shows that as σ increases, the penalty has to be increased because of the effect of uncertainty in order to keep emissions under control. This result has relevant policy implications, since it implies that in the presence of uncertainty the penalty should be increased, in order to achieve the target, a result which is in keeping with a precautionary argument.

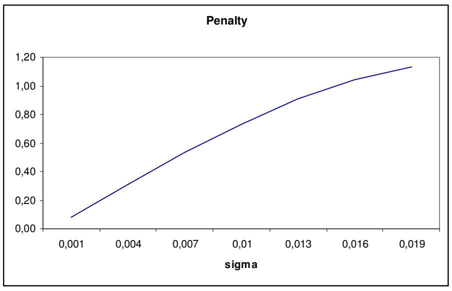


Figure 9

The argument above does not consider the beneficial effect of a positive resilience parameter, ρ, and small values of ε (< 1). In this case, the expression of the penalty obtained in Proposition 2 shows that one can afford a lower η, if the regulator accounts for the mitigating effects of resilience.

5 Conclusion

This paper examines a problem of optimal scheduling of GHG emissions for a climate change policy that is consistent with the COP21 targets and has to be monitored at a fixed time horizon. In particular, we consider a dynamic and stochastic model where production, and therefore well-being, increase in carbon emissions, but, at the same time, anthropogenic cumulative emissions determine a super-linear impact on the observed damage. Moreover, we add a stochastic component into the damage function. We compare the optimal unconstrained path of emissions and the constrained path of emissions and evaluate when the carbon budget is exhausted. A sensitivity analysis is also developed to examine the effects of resilience, impact of emissions on damage and uncertainty.

Then we examine an alternative mechanism to achieve the target, that is, through a penalty set by an institutional regulator in order to provide incentives to reduce the impact of GHG emissions within a fixed time horizon. We compute the value of the penalty and show that the introduction of a penalty can be an effective instrument to keep the cumulated emissions below the agreed threshold. Our results have direct implications in terms of policy and show that uncertainty and the way we introduce it in the model are likely to influence the efficacy of the climate change policies for the foreseeable future.

Some extensions of the basic model can be explored. First, one could develop our model with two countries - or two regions of the world, say, the north and the south - and explore the effects on the optimal path of emissions, in case the north is producing more carbon emissions than the south, but the south is affected more negatively by the world cumulated total damage. Both cases of two countries acting cooperatively or non-cooperatively could be examined and the effects of the various parameters (resilience, impact of emissions on damage, correlation of the impact of emissions, uncertainty) could be studied. Moreover, alternative enforcement measures and specific penalties could be studied for countries that do not fulfill the agreed targets in terms of total emissions.

Another extension refers to the stochastic process describing the damage function. Here an arithmetic Brownian motion was assumed, because it is the easiest way to model randomness and allowed us to obtain an explicit analytic solution. Economic models hardly ever account in a meaningful way for the uncertainty over climate sensitivity as well as damage uncertainty, so a framework with alternative stochastic processes, able to account for this drawback, is called for. Of course, the analytic of the model would become more involved and an explicit analytic solution cannot be obtained.

6 Appendix

Proof of Proposition 1.

The result follows from the first order conditions on $\mathfrak{L}(x_1, ..., x_N, \lambda)$.

From $M \begin{pmatrix} x_1 \\ ... \\ x_N \end{pmatrix} = \begin{pmatrix} u - D_0 - \lambda \\ ... \\ u - D_0 - \lambda \end{pmatrix}$ we get

$\begin{pmatrix} x_1 \\ ... \\ x_N \end{pmatrix} = M^{-1}(u - D_0 - \lambda)I$, where $I = \begin{pmatrix} 1 \\ ... \\ 1 \end{pmatrix}$

Moreover, since $Z = \sum_{k=1}^{N} x_k = X^T I = (u - D_0 - \lambda)I^T M^{-1} I$, we get $\lambda = u - D_0 - \frac{Z}{\sum_{i,j} \widetilde{m}_{i,j}}$, with $M^{-1} = (\widetilde{m}_{i,j})$.

For example, if $N = 1$, then $\lambda = u - D_0 - (\varepsilon + w + \Gamma)Z$, and $\lambda \geq 0$.

Proof of Proposition 2.

Let us compute $E_0[max(\sigma W_T - S, 0)]$, with $S = \widetilde{Z} - D_0 - \varepsilon V_N$. Rewrite:

$$max(\sigma W_T - S, 0) = [\sigma W_T - S]1_{\{\sigma W_T - S > 0\}}$$

where 1_A denotes the indicator function of the set A, that is $1_A = 1$ on A and $1_A = 0$ elsewhere. As $\frac{W_T}{\sqrt{T}}$ follows a standard normal distribution,

$$E_0[S1_{\{\sigma W_T - S > 0\}}] = S.Prob[\frac{W_T}{\sqrt{T}} > \frac{S}{\sigma\sqrt{T}}] = S\Phi(-\frac{S}{\sigma\sqrt{T}}),$$

where Φ denotes the cumulative distribution function of the standard Gaussian distribution. On the other hand,

$$E_0[W_T 1_{\{\sigma W_T - S > 0\}}] = \sqrt{T} E_0[\frac{W_T}{\sqrt{T}} 1_{\left\{\frac{W_T}{\sqrt{T}} > \frac{S}{\sigma\sqrt{T}}\right\}}] = \sqrt{T} \int_{\frac{S}{\sigma\sqrt{T}}}^{+\infty} x.\varphi(x)dx$$

where φ denotes the density function of the standard Gaussian distribution. An antiderivative of $x\varphi(x)$ is $\frac{-1}{\sqrt{2\pi}}\exp[-\frac{x^2}{2}] = -\varphi(x) = -\varphi(-x)$ and thus the integral on the right-hand side is computed as:

$\int_{\frac{S}{\sigma\sqrt{T}}}^{+\infty} x.\varphi(x)dx = \varphi(\frac{-S}{\sigma\sqrt{T}})$.

Therefore

$$E_0[max(\sigma W_T - S, 0)] = \sigma\sqrt{T}\varphi(\frac{-S}{\sigma\sqrt{T}}) - S\Phi(-\frac{S}{\sigma\sqrt{T}}).$$

7 References

Asheim, G.B. and T. Mitra, 2010. Sustainability and discounted utilitariansm in models of economic growth. Mathematical Social sciences, 59, 148-169.

Athanassoglou, S and A. Xepapadeas, 2012. Pollution Control with Uncertain Stock Dynamics: When and How to be Precautios, Journal of Environmental Economics and Management, 62, 304-320

Brock, W.A. and S.R. Carpenter, 2006. Rising variance: a leading indicator of ecological transition, Ecology Letters, 9, 311-18

Chichilniski G,1997. What is sustainable development? Land Economics, 73, 476-491

Dockner, E.J and N. Van Long, 1993, International Pollution Control: Cooperative versus Noncooperative Strategies, Journal of Environmental Economics and Management, 25, 13-29.

Freeman, M.C, Wagner, G and R.J. Zeckhauser, 2015. Climate sensitivity uncertainty: when is good news bad? NBER WP 20900

Golosov, M., Hassler, J, Krusell, P and A. Tsyvinski, 2014, Optimal taxes on fossil fuel in general equilibrium, Econometrica,82, 41-88

Hansen,J., R. Ruedy, M. Sato and K. Lo, 2010. Global surface temperature change, Review of Geophysics, 1-29

Hansen, J., Sato, M., Ruedy, R., Nazarenko, L., Lacis, A., Schmidt, G.A., Russell, G., Aleinov, I., Bauer, M., Bauer, S., Bell, N., Cairns, B., Canuto, V., Chandler, M., Cheng, Y., Del Genio, A., Faluvegi, G., Fleming, E., Friend, A., Hall, T., Jackman, C., Kelley, M., Kiang, N., Koch, D., Lean, J., Lerner, J., Lo, K., Menon, S., Miller, R., Minnis, P., Novakov, T., Oinas, V., Perlwitz, Ja., Perlwitz, Ju., Rind, D., Romanou, A., Shindell, D., Stone, P., Sun, S., Tausnev, N., Thresher, D., Wielicki, B.,Wong, T., Yao, M., Zhang, S., 2005. Efficacy of climate forcings. Journal of Geophysical Research 110, D18104.

Hansen J. et Lebedeff S.,1987.. Global trends of measured surface air temperature. Journal of Geophysical Research, 92, 11, 13345–13372

Heal G. and A. Millner, 2014. Uncertainty and Decision-Making in Climate Change Economics, Review of Environmental Economics and Politics

IPCC, 2014. Climate Change (2014), Synthesis Report, Summary for Policymakers. Available online at:

https://www.ipcc.ch/pdf/assessment-report/ar5/syr/AR5_SYR_FINAL_SPM.pdf

Iverson T. and L. S. Karp, 2017. Carbon taxes and commitment with non constant time discounting, mimeo, https://escholarship.org(uc/item/3hw6s14v

Karp, L. S. and J. Zhang, 2006. Regulation with anticipated learning about environmental damages, Journal of Environmental Economics and Management, 51 (3), 259–279.

Knutti et al (2005), Probabilistic climate change projections for CO2 stabilization profiles, Geophys Research Letters, 32, L 22707

Kriegler, E. and T. Bruckner, 2004. Sensitivity analysis of emissions corridors for the 21 century, Climate Change 66, 345-387

Laepple, T., Huybers, P.,2014.. Global and regional variability in marine surface temperatures. Geophys. Res. Lett. 41, 2528–2534, http://dx.doi.org/10.1002/2014GL059345.

Latif, M., 2010. Uncertainty in climate change projections. Journal of Geochemical Exploration, 110, 1–7.

Le Quere et al (2016) Global Carbon Project. Global carbon budget 2016, Futurearth

Manoussi, V. and A. Xepapadeas, 2017. Cooperation and competition in climate change policies: Mitigation and climate engineering when countries are asymmetric. Environmental and Resource Economics, 66, 605-627.

Matthews, D., N.G. Gillett, P.A. Stott and K. Zickfeld, 2009. The proportionality of global warming to cumulative carbon emissions, Nature, 829-833

Matthews, D., Solomon, S. and R. Pierrehumbert, 2012. Cumulative carbon as a policy framework for achieving climate stabilization, Philosophical Transactions of the Royal Society of London, 370, 4365-4379.

Meinshausen, M., B. Hare, T.M. Wigley, D Van Vuuren, M.G. J Den Elzen and R. Swart, 2006.. Multi-gas emissions pathways to meet climate targets, Climate Change, 75, 151-194

National Centers For Environmental Information (NOAA), 2016. Global Surface Temperature Anomalies. Available online at:

https://www.ncdc.noaa.gov/oa/climate/research/1998/anomalies/anomalies.html

Nazarenko, L., et al. , 2015. Future climate change under RCP emission scenarios with GISS ModelE2, J. Adv. Model. Earth Syst., 7, 244–267, doi:10.1002/2014MS000403

Nordhaus, W. , 2007. To Tax or Not to Tax: The Case for a Carbon Tax,Review of Environmental Economics and Policy, 1 (1), 26–44.

Nordhaus, W , 2008. A Question of Balance: Weighing the Options on Global Warming Policies, Yale University Press

Pierrehumbert, R. , 2014. Short-lived climate pollution, Annual Review of Earth and Planetary Sciences, 42, 341-379

Pindyck, R. 2012. Uncertain outcomes and climate change policy, Journal of Environmental Economics and Management, 63, 289-303

Pindyck, R. , 2013. Climate Change Policy: what do the models tell us?, Journal of Economic Literature, 51 (3), 860-872

Stern, N. , 2008. The Economics of Climate Change. The Stern Review, Cambridge University Press.

Solomon S., Daniel J. S, Todd, J, S, Murphy, D.M., Plattner, G-K, Kuntti, R. and Friedlinstein P. , 2010. Persstence of climate changes due to a range of GHGs, PNAS, 107, 43, 18354-9

Taleb, N.N., Read, R., Douady, R., Norman, J., Bar-Yam Y., 2014 The Precautionary Principle (with applications to the genetic modification of organisms). Extreme Risk Initiative. NYU School of Engineering Working Paper Series, 1-24

Tol, R. , 2005. Adaptation and Mitigation: trade-offs in substance and methods, Environmental Science and Policy, 572-578

Tollefson, J., 2014. Climate change: the case of the missing heat. Nature 505, 276–278, http://dx.doi.org/10.1038/505276a.

Van der Ploeg, F. 2014, Abrupt positive feedback and the social cost of carbon, European Economic Review, 67, 28-41

Weitzman, M. , 1998. Why the far-distant future should be discounted at its lowest possible rate, Journal of Environmental Economics and Management, 36, 201-208

Weitzman, M. , 2009. On modeling and interpreting the economics of catastrophic climate change, Review of Economics and Statistics, 91, 1-19.

Weitzman, M , 2010. What is the damage function for global warming and what difference might it make, Climate Change Economics, 1, 57-69

Wigley, TML 2004. Choosing a stabilization target for CO2,Climate Change, 67, 1-11

Witt D.A., J de Boor, N. Hedlund and P. Osseweijer, 2016. A new tool to map the major worldviews in the Netherlands and USA, and explore how they relate to climate change. Environmental Science & Policy, 63, 101-112.

Zickfeld, K., M. Eby, H., Matthews D.and A. J. Weaver , 2009. Setting cumulative emission targets to reduce the risk of dangerous climate change, PNAS, 16129-16134