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On the optimal management of environmental stock externalities

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Following a brief review of the management of environmental externalities under strategic interactions in the traditional temporal domain, results are extended to the spatiotemporal domain. Conditions for spatial open-loop and feedback Nash equilibria, along with conditions for the benchmark cooperative solution, are presented and compared. A simplified numerical example illustrates the spatial patterns emerging at a steady state under Fickian diffusion and dispersal kernels, and the inefficiency of spatially-flat emission taxes. This conceptual framework could provide new research areas.

Keywords: spatial diffusion, dispersion kernel, open-loop Nash equilibrium, feedback Nash equilibrium, cooperative solution, steady state, optimal emission tax

A very important class of pollutants are those for which the stock is built into the ambient environment as emissions accumulate at a rate exceeding that which natural processes can absorb. For a stock pollutant, the damages are not caused by the flow of emissions per unit time but rather by the stock of the accumulated pollutants. Stock pollutants are associated with a number of very important environmental problems.

Climate change is a very good example – most likely the ultimate example – of a global stock externality, since it is the accumulated stock of greenhouse gases (GHGs) in the atmosphere and not the flow of GHGs emissions that causes the climate damages. The stock of GHGs has increased from approximately 600 GtC in the preindustrial period to a value of approximately 830 GtC in the present time with a corresponding increase in the average global temperature of approximately 1°C (1). Other examples of stock externalities include: the accumulation of nutrients, especially phosphorus in ground and surface water, which contributes to nitrate pollution; the accumulation of heavy metals such as lead in the soil; acid depositions in soil; or the uncontrollable accumulation of non-degradable waste in landfills. In all these cases it is the accumulation of the pollutant that creates the environmental damages. Typically the stock externality which accumulates in the ambient environment is the outcome of decisions taken by many decision makers (DMs), such as countries in the case of GHGs accumulation, or farmers in the case of phosphorus loadings.

Negative stock externalities are also associated with harvesting open access or common pool resources when property rights among the DMs (harvesters, appropriators) are lacking (2). In this case DMs do not take into account the full depletion cost of the resources in their harvesting decisions. In both types of problems the action of one DM affects the utility of the others through the damages generated by the ambient stock of the externality (e.g., GHG, phosphorous) or by reducing the stock of the resource beyond the socially-optimal level (e.g., fishery collapse).

The presence of many DMs implies that each of them takes actions, such as emissions or harvesting, in a forward-looking optimization context by making assumptions about the actions of the rest of the

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agents, since it is this collective action that determines the evolution of the stock and the strength of the externality. The stock externality generates damages to all agents but each individual agent acts strategically by maximizing non-cooperatively own net benefits less own externality damages subject to the constraints imposed by the evolution of the stock externality.

The evolution of the externality takes place in a temporal domain, which is clear given the accumulation process, but also in a spatial domain since pollutants and resources in general diffuse in space. There is transport of environmental state variables across geographical space due to natural processes. Airborne contaminants in the atmosphere are transported from the source of emissions due to turbulent eddy motion and wind. Resources move in space due to biological characteristics or in search of food or because of other exogenous forces such as climate change. In patchy groundwater aquifers, water flows across patches according to hydrological rules (e.g., Darcy's law). There is heat and moisture transport across the globe – from the Equator to the Poles – in non-zero dimensional energy balance climate models.

When the forward-looking optimizing agents face the spatiotemporal constraint, new issues emerge which are not captured by the traditional approach that considers only the temporal domain and implicitly assumes that the spatial diffusion of the externality is infinite and that the externality is therefore uniform in space. This uniform mixing characteristic is true for the accumulation of GHGs in the atmosphere,[1] but can only be regarded as an approximation in other cases.

In this context the purpose of this paper is twofold: first to briefly review results related to the management of environmental and resource externalities under strategic interactions in the traditional temporal domain, and second to provide extensions when the strategic interactions take place in a spatiotemporal domain. In this respect conditions for spatial open-loop and feedback Nash equilibria along with conditions for the benchmark cooperative solution are presented and compared. The objective is not to provide closed form or numerical solutions to these problems but rather to provide a conceptual framework in which to study forward-looking optimization problems under strategic spatiotemporal interactions, to discuss and partially characterize solution concepts and, most importantly, to indicate paths for further research in this area.

1 Modeling Stock Externalities in the Spatiotemporal Domain

Environmental stock externalities are characterized by strategic interdependence since the actions of a forward-looking DM (or economic agent/player such as an individual, firm, community, region, nation) affect own welfare (benefit, payoff, utility) but also, through the DM's impact on the stock externality, affect the other DMs' welfare as it evolves in a spatiotemporal domain.

Differential games are suitable analytical tools to represent time, space, strategic behavior, and interdependencies in models of environmental and resource economics (4,5,6). A differential game is a state-space game that contains a set of state variables which include stock externalities, or resource stocks that describe the main features of a dynamic system at any instant of time during the game. The state variables adequately summarize all relevant consequences of the past history of the game and their evolution is affected by the actions of the DMs. The dynamics of a differential game are described by ordinary or partial differential equations (ODEs or PDEs respectively), depending on whether the spatial dimension of the problem is taken into account.

The study of competing DMs in a dynamic deterministic setting over a long time period can be analyzed in the framework of an infinite horizon differential game. This game is defined in terms of states $x(t, z)$ which describe the state of the system (e.g., the stock of pollutant, stock of biomass or the surface temperature) at time $t \in [0, \infty)$ and location $z \in \mathcal{D}$, where $\mathcal{D}$ is a spatial domain of appropriate

[1] Although the accumulation of GHGs is uniform, the temperature anomaly has a clear spatial structure due to albedo feedback and heat and moisture transport from the Equator to the Poles which induces Polar amplification (3).

dimensions, and controls $u_i(t,z)$ which describe the actions of the DMs (e.g., emissions or harvesting) at time t and location z.

When the spatial dimension is not taken into account, the externality dynamics are described by systems of ODEs,

$$\dot{x}_j(t) = f_j\left(x_1(t), ..., x_J(t), u_1(t), ..., u_n(t)\right), x_j(0) = x_{j0}, j = 1, ..., J, \quad (1)$$

for J states and n controls. An example that uses nonlinear dynamics as described by equation (1) can be found in the modeling of lake pollution. Due to heavy use of fertilizers on surrounding land and an increased inflow of wastewater from human settlements and industries, lakes at some point tend to flip from a clear oligotrophic state to a turbid eutrophic state with a greenish look caused by a dominance of phytoplankton (7,8). The essential dynamics of the eutrophication process can be modeled, after an appropriate transformation, by the differential equation:

$$\dot{x}(t) = \sum_{i=1}^{n} a_i(t) - bx(t) + f(x(t)), \; x(0) = x_0, \quad (2)$$

where $a_i(t),\; x(t)$ represent phosphorous loadings by the ith DM (e.g., farmer) and the phosphorous stock in the lake respectively; $f(x(t)) = \frac{x(t)^2}{1+x(t)^2}$ represents nonlinear positive feedback; and b is an exponential natural pollution decay rate. If the nonlinear term $f(x(t))$ is absent, then linear dynamics emerge which have been used, for example, to study a dynamic non-point source pollution problem (see (9)) or to study international pollution control and climate policy issues with appropriate interpretations of the state and control variables (see (10,11,12)). In equation (1) or (2), the action of each DM reflects its strategy.[2]

To introduce space, consider a spatial domain $z \in \mathcal{Z} = [0, Z]$ along with the time domain $t \in [0, \infty)$. Let $x(t,z), u(t,z)$ be the state and the control, respectively, at time t and spatial point z. The state could be the concentration of an environmental stock (e.g., phosphorus in a lake, GHGs, fish biomass), while the control could be emissions, harvesting or abatement.

Assume that the stock located at point z moves to nearby locations and that the direction of the movement is such that stock from locations where it is abundant, i.e., locations of high concentration, moves toward locations of low concentration. This is the assumption of Fickian diffusion, or Fick's first law, and is equivalent to stating that the flux of stock $x(t,z)$ is proportional to the gradient of the biomass concentration, i.e., the spatial derivative of concentration or $J(t,z) = -D\frac{\partial x(t,z)}{\partial z}$, where D is the diffusion coefficient, or diffusivity, measuring how fast the stock moves from locations of high concentration to locations of low concentration.

Consider an abstract pollution management problem in which a continuum of DMs that are potential polluters is distributed evenly at each location in the space $[0, Z]$. In this context, z denotes the DM that is located at z and generates emissions $a(t,z)$. Let $\alpha(t,z) = \int_0^Z \gamma_z(z')w(z-z')a(t;z')dz'$ represent the aggregate emissions of a pollutant (e.g., aggregate air pollutants or phosphorous loadings) at location z and time t. Aggregate emissions at z is a weighted average of emissions generated at z and emissions generated at other locations z' which travel to location z due to weather conditions and the natural characteristics of the landscape. This emission dispersal could be modeled by a dispersal kernel $w(z-z')$ indicating that the impact of emissions emitted at z' on aggregate emissions at z is reduced as the distance between z and z' increases. A common representation for such a kernel is $w(z-z') = \exp\left[-\delta(z-z')^2\right], \delta > 0$ and finite (14).[3] The pollution stock accumulated at a location could diffuse

[2] Nonlinear dynamics are typical in renewable resource management (see, e.g., (13)).

[3] $\gamma_z(z') = \left\{ \begin{array}{ll} 1, & z' = z \\ 0, & z' \neq z \end{array} \right\}$ when emissions do not spread once emitted and only the stock diffuses, $\gamma_z(z') = 1$ for all z when emissions spread soon after generation or mobile sources emit away from their location and the accumulated stock in locations such as "hot spots" further diffuse (e.g., traffic congestions). Furthermore, $\int_0^Z \gamma_z(z')e^{-\delta(z-z')^2}dz' \leq 1$, to indicate

in the spatial domain under Fick's first law to neighboring sites with relatively less concentration of accumulated pollutants (15).

The spatiotemporal evolution of pollution dynamics can be written as:

$$\frac{\partial x(t,z)}{\partial z} = \alpha(t,z) - bx(t,z) + f(x(t,z)) + D\frac{\partial x^2(t,z)}{\partial z^2}, x(0,z) = x_0(z) \text{ given}, \tag{3}$$

where $f(\cdot)$ represents nonlinear positive feedbacks as in equation (2). In equation (3), spatial boundary conditions could take different forms for all t: (a) a circle or $x(t,0) = x(t,Z)$; (b) hostile boundaries or $x(t,0) = x(t,Z) = 0$; or (c) zero flux at the boundaries or $\frac{\partial x(t,0)}{\partial z} = \frac{\partial x(t,Z)}{\partial z} = 0$.

The spatiotemporal dynamics of (3) are quite general and can be used to model renewable resource harvesting or temperature evolution in a spatial domain. In harvesting models, the term $\alpha(t,z)$ should be interpreted as harvesting by all potential appropriators at z with the dispersal kernel indicating that the efficiency of harvesting effort for an appropriator that is located at z but harvests at z' is reduced as the distance between z and z' increases, because for example of transportation costs. Property rights structure can be modeled by $\gamma_z(z')$, so $\gamma_z(z') = 0$ means that the appropriator at z' has no rights to harvest at z, while $\gamma_z(z') = 1$ for all z implies an open access resource. The feedback term $f(x(t,z))$ should be interpreted as biological resource growth at z, while the diffusion term indicates that resource biomass moves from high to low concentration locations. Equation (3) can model temperature evolution at a specific geographical location in the context of one-dimensional energy balance climate models. The first three terms should be interpreted, with appropriate modifications, as incoming solar radiation, outgoing infrared radiation and forcing due to global GHG emissions, respectively. Heat transport from the Equator to the Poles is modeled by the nonlinear diffusion term $D\frac{\partial}{\partial z}\left[\left(1-z^2\right)\frac{\partial x(t,z)}{\partial z}\right]$ where z is the sine of the latitude[4] and D is the heat transport coefficient (16,17).

2 Forward-Looking Optimizing Behavior

Assume that emissions $a(t,z)$ generated per unit time at z produce benefits to the polluter represented by a strictly concave benefit function $B(a(t,z)), B' \geq 0, B'' < 0, \lim_{a \to 0} B'(a) = \infty$. The stock of pollution accumulated at each point of the spatial domain generates cost to each DM which is given by a strictly increasing convex cost function $C(x(t,z))$. Then, net benefits for a DM located at point z can be defined as $B(a(t,z)) - C(x(t,z))$. The definition of net benefits for each DM exhibits "spatial myopia" since the DM takes into account pollution costs associated with the stock of pollution accumulated only in its location and ignores the impact of its actions on the rest of the spatial domain. A social planner that seeks to internalize the pollution externality would take into account the aggregate pollution costs associated with the stock of pollution accumulated in each location. Furthermore, pollution cost could exhibit spatial dependence emerging from the fact that damages at a location z do not depend on stock accumulation at z only but on accumulation in nearby locations as well, with the impact declining with distance. Thus the social damage function at z can be defined as $SD(t,z) = \int_{\mathcal{Z}} \varphi(z-z')C(x(t,z'))\,dz'$. The kernel $\varphi(z-z')$, defined by $\exp\left[-\delta_{SD}(z-z')^2\right], \delta_{SD} > 0$, for example, indicates the impact of pollution accumulated at z' on damages at z, while pollution dynamics are given by equation (3).

A social planner or a regulator seeks spatiotemporal emissions paths for emissions generated at each point in time that will maximize aggregate net benefits over the entire spatial domain. Then the planner's problem is to maximize aggregate payoff J (17,18,19),

$$\max_{\{a(t,z)\}} J = \max_{\{a(t,z)\}} \int_0^\infty e^{-\rho t} \int_{\mathcal{Z}} \left[B(a(t,z)) - \int_{\mathcal{Z}} \varphi(z-z')C(x(t,z'))\,dz'\right] dz dt, \tag{4}$$

that some emissions generated in $[0,Z]$ could travel outside the spatial domain.

[4] $z=0$ denotes the Equator and $z=\pm 1$ denotes the North and the South Pole respectively.

subject to (3) and appropriate spatial boundary conditions, with ρ the utility discount rate.

In a non-cooperative solution each DM acts myopically and considers damages from pollution stock at its site. The DM will in general follow an emission strategy which is a function of the state of the system in the set of strategies for this DM. This strategy indicates the emissions choice of a DM located at z whose emissions could spread to the spatial domain and whose damages could be affected by pollution accumulation in nearby locations. Non-cooperative solutions can be analyzed, therefore, in the context of a differential game evolving in a spatiotemporal domain.

A differential game is said to have an open-loop informational structure if the DMs follow open-loop strategies: $a(t,z) = \theta(x(0,z), t, z)$. The differential game is said to have a feedback information structure if the DMs follow time and space stationary feedback strategies: $a(t,z) = \theta(x(t,\mathcal{Z}))$ (20).

In the feedback information structure it is assumed that an individual DM's emissions follow a feedback rule which depends on the pollutant stock for the entire spatial domain. When feedback rules are considered for similar problems without spatial interactions, the underlying assumption is that $\theta'(x(t)) < 0$ because each DM believes that when aggregate stock increases, the other DMs will reduce emissions to counterbalance increased damages from aggregate pollution. This is an incentive for a given DM to increase emissions since it expects that others will reduce theirs. Since all DMs behave in the same way, aggregate emissions increase. In the spatial domain, a similar argument – based this time on diffusion – can be used to justify a negative derivative. The DM believes that an increased stock of pollutant at its site will be reduced because Fickian diffusion will move to locations with lower pollutant concentrations. Since all DMs behave in the same way, aggregate emissions will eventually increase.

A Nash equilibrium in the class of open-loop strategies is called an open-loop Nash equilibrium (OLNE), while an equilibrium in the class of feedback strategies is called a feedback or closed-loop Nash (FBNE) equilibrium.[5] OLNE and FBNE solutions without spatial interactions have been analyzed in the literature for a number of cases such as international pollution control, the lake problem and resource harvesting (10,23,24,25,26,27). The main results indicate that steady-state pollutant accumulation under FBNE is higher relative to OLNE, confirming the insight associated with the negative derivative of the feedback rule (i.e., $\theta'(x(t)) < 0$) and both are higher relative to the cooperative solution, while in terms of the welfare indicator used, FBNE is inferior relative to OLNE and both are inferior relative to the cooperative solution.

The comparison of solutions allows the characterization of the optimal policy in terms of externality taxes that can attain the socially-optimal solution when DMs follow open-loop or feedback strategies. Results vary from closed form solutions for linear-quadratic problems to numerical solutions for problems with nonlinearities, such as the lake problem. This conceptual framework is extended to the spatiotemporal domain. Cooperative OLNE and FBNE solution concepts are defined and characterized below.

In OLNE each DM follows a spatially-myopic strategy, takes the actions of all other DMs located at $z' \in Z \setminus z$ as exogenous, and commits to an emission path that optimizes its own objective, that is:

$$\max_{\{a(t,z)\}} J^O = \max_{\{a(t,z)\}} \int_0^\infty e^{-\rho t} \left[B\left(a(t,z)\right) - \int_{\mathcal{Z}} \varphi(z-z') C\left(x(t,z')\,dz'\right] dt, \text{ subject to} \tag{5}$$

$$\frac{\partial x(t,z)}{\partial z} = a(x(t,z)) + \int_{\mathcal{Z}\setminus z} \gamma_z(z') w(z-z') \bar{a}(t,z') dz' - bx(t,z) + f(x(t,z)) + D\frac{\partial x^2(t,z)}{\partial z^2}. \tag{6}$$

The integral term in (6) reflects long-range effects of emissions generated at z' by DMs other than the DM located at z, in addition to the local diffusion effects of the stock on location z. Interpreting the dynamics of (3) within a resource-harvesting context, the integral term represents harvesting in z of

[5] Since the DMs are located in the interval $\mathcal{Z}$ that represents a continuum, the concept of non-cooperative Nash equilibrium is defined in the context of a continuum (21,22).

appropriators located in z', if this is possible by the property rights structure (e.g., open access).[6]

In FBNE each DM does not recall the previous history of the system,[7] and assumes that the other DMs' emissions are conditioned on the pollution accumulation in the spatial domain. The optimization problem in this case can be written as:

$$\max_{\{a(t,z)\}} J_i^F = \max_{\{a(t,z)\}} \int_0^\infty e^{-\rho t} \left[B\left(a(t,z)\right) - \int_{\mathcal{Z}} \varphi(z-z') C\left(x(t,z')\, dz'\right] dt \text{ , subject to} \tag{7}$$

$$\frac{\partial x(t,z)}{\partial z} = a(t,z) + \left[\theta_0 + \theta_1 \left(x(t,z) \int_{\mathcal{Z} \setminus z} \gamma_z(z') w(z-z') dz' \right) \right] - bx(t,z) + f(x(t,z)) + D \frac{\partial x^2(t,z)}{\partial z^2}, \tag{8}$$

where the term in brackets on the right side of (8) represents a simplified linear feedback rule which implies that, under symmetry, each emitter considers emissions as a linear function of a stock of pollutant equal to the stock at its site and the relevant dispersion parameters which affect its site. The value for the parameters (θ_0, θ_1) should be derived along with the solution. A linear equilibrium feedback rule is consistent with a linear-quadratic problem in the spirit of temporal-only linear-quadratic differential games.

3 Characterizing Cooperative and Non-cooperative Outcomes

The optimality conditions characterizing cooperative and unregulated non-cooperative outcomes can be obtained by an extension of Pontryagin's maximum principle to the case where the transition equation includes linear diffusion in space and spatial kernels (19,29). We focus on characterizing the steady-state solutions because they reveal the difference in the shadow cost of pollution among the solution concepts, but most importantly because these differences can be used to gain insights into the structure of long-run spatially-structured policies.[8]

The social planner The current value Hamiltonian for the problem is

$$\mathcal{H} = \int_{\mathcal{Z}} \left[B\left(a(t,z)\right) - SD(t,z) \right] dz + \lambda\left(t,z\right) \frac{\partial x(t,z)}{\partial t}. \tag{9}$$

The costate variable $\lambda(t,z)$ should be interpreted as the social shadow cost of the accumulated pollutant stock at time t and location z, and will be the basis for the formulation of an efficient emission tax. Optimality conditions indicate that socially-optimal emissions at time t and location z should be chosen such that marginal emission benefits equal the corresponding spatial shadow cost of the pollution stock. Using the optimality conditions and ignoring the feedback term to simplify the exposition, the steady states for the planner's problem are defined as:

$$\lambda^\infty(z) = \frac{1}{(\rho+b)} \left[- \int_{\mathcal{Z}} \varphi(z-z') C'\left(x(t,z)\, dz' + D \frac{\partial \lambda^2(t,z)}{\partial z^2} \right] \tag{10}$$

$$x^\infty(z) = \frac{1}{b} \left[\alpha^*(t,z) + D \frac{\partial x^2(t,z)}{\partial z^2} \right]. \tag{11}$$

Open-loop Nash equilibrium Assuming symmetry among the DMs, the steady state is defined as:

[6] A similar approach without the long-range effects has been developed by de Frutos and Martín-Herrán (28) and solved through discretization of space.

[7] In the economics literature, feedback strategies are also called Markov perfect strategies (lack of memory).

[8] Derivations for the social planner and DMs under OLNE and FBNE are presented in the supplementary material.

$$\lambda_O^{\infty}(z) = \frac{1}{(\rho+b)}\left[-C'\left(x(t,z) + D\frac{\partial\lambda^2(t,z)}{\partial z^2}\right]\right. \tag{12}$$

$$x_O^{\infty}(z) = \frac{1}{b}\left[\alpha_O(t,z) + D\frac{\partial x^2(t,z)}{\partial z^2}\right], \tag{13}$$

with $\lambda_O^{\infty}(z)$ the open-loop shadow cost for pollution for the DM located at z.

Feedback Nash equilibrium Assuming again symmetry among the DMs and disregarding feedbacks in dynamics, the steady state under the simple feedback rule used in equation (8) is defined as

$$\lambda_F^{\infty}(z) = \frac{1}{(\rho+b)}\left[-C'\left(x(t,z) + \theta_1\int_{\mathcal{Z}\backslash z}\gamma_z(z')w(z-z')dz' + D\frac{\partial\lambda_F^2(t,z)}{\partial z^2}\right]\right. \tag{14}$$

$$x_F^{\infty}(z) = \frac{1}{b}\left[\alpha_F(t,z) + D\frac{\partial x^2(t,z)}{\partial z^2}\right], \tag{15}$$

with $\lambda_F^{\infty}(z)$ the feedback shadow cost for pollution for the DM located at z. Feedback solutions can be derived by applying dynamic programing techniques involving the solution of the Hamilton-Jacobi-Bellman (HJB) equation in the spirit of the approach used for temporal only differential games. Quadratic or isoelastic value functions could be appropriate solutions of the HJB equation for quadratic or isoelastic objective functions. The study of feedback Nash equilibria with integrodifferental dynamics stemming from the various interpretations of (3) is an area for further research.[9]

4 Spatial Steady States and Optimal Spatial Policies

Spatially-differentiated policy instruments have been studied in the literature in various models without explicit strategic interactions. Goetz and Zilberman (30) derive spatially-dependent tax policies to regulate phosphorous loadings when the run-off is location dependent. Xabadia *et al.* (31) introduce spatially-differentiated land quality in a problem of maximizing discounted net margin in agricultural production with pollution stock externality and technology choice; they derive spatially-differentiated dynamic input taxes and compare them to non-differentiated policies. Brock and Xepapadeas (18) derive spatially-differentiated taxes for the optimal regulation of semi-arid systems when plant biomass and soil water exhibit reaction-diffusion characteristics. Brock *et al.* (19) provide examples of spatially-differentiated policies in pollution control and resource harvesting under linear spatial diffusion, and – through an extension of the classic Turing mechanism for pattern formation in optimizing systems – show how, in an optimizing context, spatial diffusion could generate optimal patterns and associated optimal spatially-dependent policies. Spatially-dependent optimal emission taxes in urban models with interacting industrial and residential clusters are derived in Kyriakopoulou and Xepapadeas (32), while spatial policies are also relevant for congestion and air pollution regulation in cities. Spatially-dependent optimal policy schemes under strategic interactions in spatiotemporal models are less frequent. In de Frutos and Martín-Herrán (28), spatially-dependent policies in the context of a discretized model of a differential game with spatial diffusion are derived. Brock and Xepapadeas (33) and Cai *et al.* (34) derive spatial climate policies with strategic interactions among regions in the context of discrete spatial

[9]Solutions can also be obtained by discretization of the spatial component which will turn dynamics into a system of ODEs. The linear diffusion term can be discretized by a discrete version of the second derivative or using the approach described in de Frutos and Martín-Herrán (28). Discretization or nonlinear diffusion in climate models can be obtained by using even-numbered Legendre polynomials (17).

climate models with heat transport towards the Poles, while van der Ploeg and de Zeeuw (12) also analyze geographically-differentiated responses to potential climate tipping points in a North-South model.

Some characteristics of spatially-dependent emission taxes emerging from the models described here can be explored by noting that, at the socially-optimal emission path, marginal emission benefits are equal to the social shadow cost of the pollutant stock, or $B'(a(z,t)) = -\lambda(t,z)$. At the unregulated non-cooperative solutions, marginal emission benefits should be equal to the corresponding non-cooperative shadow cost of the pollutant, or $B'(a(z,t)) = -\lambda_j(t,z)$, $j = OLNE, FBNE$. Fig. 1 provides a simplified numerical simulation of the accumulated pollutant stock and the corresponding shadow cost steady states defined by (10)-(15), along with the optimal emission tax and the kernel function. A quadratic benefit function, a linear damage function without dispersion for emissions (i.e., $\alpha(t,z) = a(t,z)$) but with a spatial externality for damages that is declining with distance, low diffusion and a linear feedback rule were used.[10]

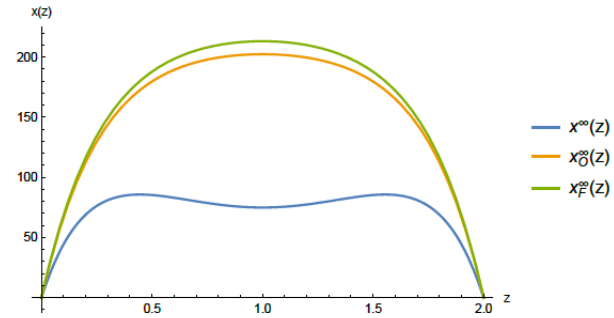

Fig. 1a: Stock of pollution

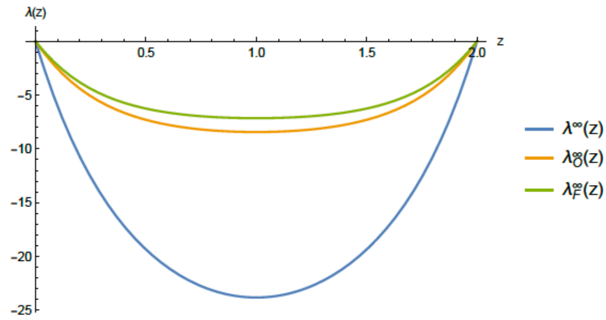

Fig. 1b: Shadow cost of pollution stock

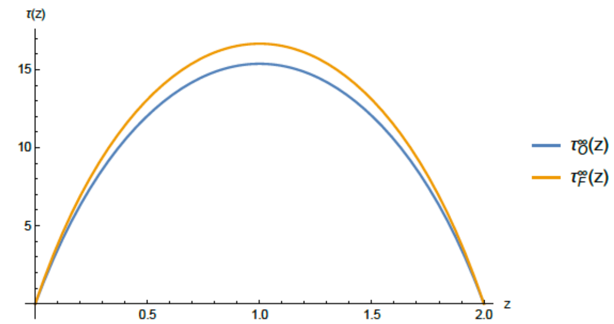

Fig. 1c: Optimal emission tax

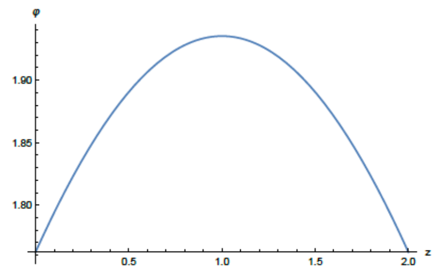

Fig. 1d: The kernel function

The social cost of the pollutant stock is higher relative to the non-cooperative cases and has a well-defined spatial structure as shown in Fig. 1b, with the kernel function in Fig. 1d, and $\lambda_O^\infty(z) \leq \lambda_F^\infty(z)$, $z \in [0,2]$ as expected under a negative-sloping equilibrium feedback rule. This is because at the social optimum aggregate damages are taken into account as shown in equation (10). The shadow cost at the social optimum is high in the middle of the spatial domain which induces a pollution accumulation spatial pattern with two modes (Fig. 1a). This is because when the full cost of emissions is internalized across space, then clustering around the central site generates high social shadow cost which makes a single cluster undesirable from the social point of view. This contrasts with the unregulated non-cooperative outcome in which the lower shadow cost generates a nearly flat high accumulation pattern in the middle

[10]The choices are arbitrary for a linear space $z \in [0,2]$ with boundary conditions $x(0) = x(2) = 0$. Similar boundary conditions for the shadow cost are used since it is reasonable to attach zero shadow cost to zero pollution stock at the boundary. Details are provided in the supplementary material. The full solution of these models, including the nonlinear feedbacks in dynamics, with potential nonlinearities in diffusion and long-range effects, are very interesting areas for further research. A starting point could be the solution of the HJB equation in infinite dimensional Hilbert space following the approach of Boucekkine *et al.* (35) for a problem without strategic interactions, in which the solution for an isoelastic utility function is obtained by looking for solutions of the HJB with an isoelastic value function. A feedback solution to a spatial differential game with linear diffusion and isoelastic objective is provided by de Frutos *et al.* (36) by considering affine functions as a solution for the the infinite dimensional HJB equation.

and promotes single clustering.

Since the objective of an optimal emission tax is to induce DMs to follow an emission path that coincides with the social optimum, and emission paths are chosen such that marginal emission benefits are equal to the shadow cost of the pollutant stock, the optimal emission tax should be equal to the difference between the shadow pollutant cost at the socially-optimal and non-cooperative solutions, or $\tau_l^\infty(z) = \lambda_l^\infty(z) - \lambda^\infty(z) > 0\,, l = O, F$. In this case the regulated steady-state optimal emission paths will coincide with the socially-optimal path. The spatial pattern of optimal taxes is depicted in Fig. 1c. The high tax in the middle of the spatial domain will push the non-cooperative accumulation patterns of Fig. 1a towards the two-mode socially-optimal pattern. It is clear that a flat (non-spatially differentiated) tax will not attain the social optimum. For example, with the numerical parametrization used for Fig. 1, $\lambda^\infty(z) = -20$ and $\lambda_O^\infty(z) = -4$, implying a flat emission tax $\tau_O^\infty(z) = 16$ which is clearly suboptimal and the same applies to the FBNE emission tax.

Focussing on steady states allows us to compare equilibrium spatial patterns but not to study issues related to the evolution of spatial distributions from some initial state or the emergence of optimal spatial patterns by extending the Turing mechanism (17,19) in a spatial differential game context. Given the complexity of the spatial forward-backward systems characterizing the evolution of the states and the costates in the models discussed, this is an area for further research.

5 Conclusions

In the management of stock externalities, strategic interactions among DMs play an important role due to public bad characteristics of the externality. In a real world, DMs could be committed to actions paths. In the implementation of the Paris Accord, for example, countries commit voluntarily to carbon emission paths. Since it is reasonable to assume that these paths are decided on the basis of own welfare given expected damages from climate change, this commitment implies strategic interactions with possible feedback characteristics if revision of paths depends on the evolution of temperature. This paper seeks to provide a preliminary extension of the existing conceptual framework for managing stock externalities in a temporal domain to a spatiotemporal domain – since realistically stock externalities evolve in time but also diffuse in space – by introducing spatial local diffusion along with long-range spatial effects.

Results supported by the current literature suggest that optimal management in the spatiotemporal domain implies space-dependent policy instruments which internalize the spatial externality which would otherwise have been ignored. The structure of policy instruments depends on the type of local diffusion (linear/nonlinear), long-range effects and the degree of commitment of the DMs. Areas for further research stemming from this framework could be directed toward: i) technical aspects of optimal control constrained by – potentially stochastic – PDEs with nonlinear diffusion terms or integrodifferential equations; (ii) policy design issues which could answer questions about the efficiency and the desirability of spatially-differentiated policy instruments, such as issues related to geographical differentiation of carbon taxes, or tradable permits; and (iii) the interaction of property rights and spatially-dependent policies under local diffusion and long-range effects.

Further extensions could involve spatially-differentiated ambiguity aversion and concerns for model misspecification in the spirit of the recent robust control approach (37). In such an extension, a differential game among the DMs could be combined with a "max-min" problem where the DMs choose the best course under the worst-case weighted family of models. Such a model could extend results obtained for the "taxes vs quotas" for a stock pollutant problem analyzed by Hoel and Karp (38) to the the case of "spatial taxes vs spatial quotas" for a diffusing stock pollutant.

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Supplementary Material

The social planner The current value Hamiltonian for the problem is

$$\mathcal{H} = \int_{\mathcal{Z}} \left[B\left(a(t,z)\right) - SD(t,z)\right] dz + \lambda\left(t,z\right) \frac{\partial x(t,z)}{\partial t}.$$

The costate variable $\lambda(t,z)$ should be interpreted as the social shadow cost of the accumulated pollutant stock at time t and location z, and will be the basis for the formulation of an efficient emission tax. The optimality conditions are:

$$B'\left(a(z,t)\right) = -\lambda\left(t,z\right) \Rightarrow a^*(t,z) = h\left(\lambda(t,z)\right) \tag{16}$$

$$\frac{\partial \lambda(t,z)}{\partial t} = \left(\rho + b - f'(x(t,z))\right)\lambda(t,z) + \int_{\mathcal{Z}} \varphi(z-z')C'\left(x(t,z\right)dz' - D\frac{\partial \lambda^2(t,z)}{\partial z^2} \tag{17}$$

$$\frac{\partial x(t,z)}{\partial t} = \alpha^*(t,z) - bx(t,z) + f(x(t,z)) + D\frac{\partial x^2(t,z)}{\partial z^2}, \tag{18}$$

with temporal transversality conditions at infinity

$$\lim_{t\to\infty} e^{-\rho t} x(t,z)\lambda(t,z) = 0 \; for \; all \; z,$$

and appropriate spatial boundary conditions.

Conditions (16) indicate that socially-optimal emissions at time t and location z should be chosen such that marginal emission benefits equal the corresponding spatial shadow cost of the pollution stock. The steady state for the pollution stock and its shadow cost are defined as $(\lambda^\infty(z), x^\infty(z)) : \frac{\partial \lambda(t,z)}{\partial t} = 0, \frac{\partial \lambda(t,z)}{\partial t} \frac{\partial x(t,z)}{\partial t} = 0$. Ignoring the feedback term to simplify the exposition, a spatial steady state – if it exists – is given as solution of the system:

$$\lambda^\infty(z) = \frac{1}{(\rho+b)}\left[\int_{\mathcal{Z}} C'\left(x(t,z\right)dz - D\frac{\partial \lambda^2(t,z)}{\partial z^2}\right]$$

$$x^\infty(z) = \frac{1}{b}\left[\alpha^*(t,z) + D\frac{\partial x^2(t,z)}{\partial z^2}\right].$$

Open-Loop Nash Equilibrium The current value Hamiltonian for the problem is

$$\begin{aligned}\mathcal{H}_{\mathrm{O}} = &\int_{\mathcal{Z}} \left[B\left(a(t,z)\right) - C(t,z)\right] dz \\ &+ \lambda_O\left(t,z\right)\left[a(x(t,z) + \int_{\mathcal{Z}\setminus z} \gamma_z(z')w(z-z')\bar{a}(t,z')dz' - bx(t,z) + f(x(t,z)) + D\frac{\partial x^2(t,z)}{\partial z^2}\right].\end{aligned}$$

The optimality conditions, assuming symmetry among the DMs and disregarding feedbacks, are:

$$\begin{aligned} B'\left(a(z,t)\right) &= -\lambda_o\left(t,z\right) \Rightarrow a_o(t,z) = h\left(\lambda_o(t,z)\right) \\ \frac{\partial \lambda_o(t,z)}{\partial t} &= \left(\rho+b\right)\lambda_o(t,z) + C'\left(x(t,z\right) - D\frac{\partial {\lambda_o}^2(t,z)}{\partial z^2} \\ \frac{\partial x(t,z)}{\partial t} &\alpha_o(t,z) - bx(t,z) + D\frac{\partial x^2(t,z)}{\partial z^2}, \end{aligned}$$

with appropriate transversality and boundary conditions and steady state:

$$\lambda_o^{\infty}(z) = \frac{1}{(\rho+b)}\left[-C'\left(x(t,z) + D\frac{\partial \lambda^2(t,z)}{\partial z^2}\right]\right.$$
$$x^{\infty}(z) = \frac{1}{b}\left[\alpha_o(t,z) + D\frac{\partial x^2(t,z)}{\partial z^2}\right].$$

Feedback Nash Equilibrium The current value Hamiltonian for the problem under symmetry is

$$\mathcal{H}_{\mathrm{O}} = \int_{\mathcal{Z}} \left[B\left(a(t,z)\right) - C(t,z)\right] dz$$
$$+\lambda_F\left(t,z\right)\left[a(t,z) + \left[\theta_0 + \theta_1\left(x(t,z)\int_{\mathcal{Z}\backslash z}\gamma_z(z')w(z-z')dz'\right)\right] - bx(t,z) + f(x(t,z)) + D\frac{\partial x^2(t,z)}{\partial z^2}\right].$$

Disregarding feedbacks, the optimality conditions are:

$$B'\left(a(z,t)\right) = -\lambda_F\left(t,z\right) \Rightarrow a_F(t,z) = h\left(\lambda_F(t,z)\right)$$
$$\frac{\partial \lambda_F(t,z)}{\partial t} = \left(\rho+b\right)\lambda_F(t,z) + C'\left(x(t,z) - \theta_1\int_{\mathcal{Z}\backslash z} w(z-z')dz' - D\frac{\partial \lambda_o^2(t,z)}{\partial z^2}\right.$$
$$\frac{\partial x(t,z)}{\partial t}\alpha_F(t,z) - bx(t,z) + D\frac{\partial x^2(t,z)}{\partial z^2},$$

with appropriate transversality and boundary conditions and steady state:

$$\lambda_F^{\infty}(z) = \frac{1}{(\rho+b)}\left[-C'\left(x(t,z) + \theta_1\int_{\mathcal{Z}\backslash z}\gamma_z(z')w(z-z')dz' - D\frac{\partial \lambda_F^2(t,z)}{\partial z^2}\right]\right.$$
$$x^{\infty}(z) = \frac{1}{b}\left[\alpha_o(t,z) + D\frac{\partial x^2(t,z)}{\partial z^2}\right].$$

Numerical example: The choice of the functional forms are compatible with the theoretical model. The parameters' values were arbitrarily chosen for presentation purposes.

$B(a(t,z) = a(t,z) - \frac{1}{2}a(t,z)^2$, $C(x(t,z)) = x(t,z)$, $D = 0.01$,$b = 0.6$, $\rho = 0.01$, $\theta_1 = -0.1, \delta = 0$.

$\int_{\mathcal{Z}} \varphi(z-z')dz' = 1.85\int_0^2 \exp\left[-0.75(z-z')^2\right]dz'$,

$\int_0^2 \exp\left[-0.75(z-z')^2\right]dz' = -1.023\mathrm{erf}\left(-0.866z\right) + 1.023\mathrm{erf}\left(1.732 - 0.866z\right)$, $\mathrm{erf}(y) = 2/\sqrt{\pi}\int_0^y e^{-u^2}du$.