The Cost of Ambiguity and Robustness in International Pollution Control

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Abstract

This paper examines robustness in international pollution control emerging from the regulator's concerns regarding possible misspecification of the natural system that is used to model pollution dynamics. Cooperative and noncooperative robust policy rules are determined along with the cost in terms of value loss of being robust relative to conventional policy rules.

Keywords: Ambiguity, Robustness, Precaution, Differential games, Open Loop and Feedback Nash equilibrium

1 Introduction

International pollution control is an issue that has acquired great significance during recent decades, both in terms of academic research and in terms of applied policy making. As Barrett (2005, p. 1459) points out, "It is a cliche to say that nature obeys no borders - but it is also true." Thus many major environmental problems entail situations in which activities in one country create negative externalities not only in the country itself but also in other countries. Such problems include the pollution of rivers and lakes that border more than one country - a transboundary pollution problem - and regional or global environmental problems, such as acid rains, ozone depletion and climate change.

From the point of view of resource allocation, problems associated with global pollution, such as climate change, belong to the theory of the voluntary provision of public goods, or more precisely 'public bads', since global pollution satisfies the basic characteristics of a public good, namely nonrivalry in consumption and nonexcludability.

The general methodological approach in dealing with these problems is to: (i) determine a noncooperative solution through which countries choose their emission levels by optimizing individual objectives without taking into account the external costs imposed on other countries, (ii) determine a cooperative solution through which countries determine their emissions by optimizing a global objective so that a Pareto efficient outcome is obtained, and (iii) compare the cooperative and noncooperative solutions. In this way one can explore the inefficiency of the noncooperative equilibrium and propose a course of action that can achieve the efficient outcome, which is the global pollution level that maximizes global welfare.

International pollution problems are very often analyzed in a dynamic setup, since the pollutants associated with these problems are of a stock or fund type and environmental damages are associated with the stock of the accumulated pollutant in the ambient environment. When cooperative solutions are analyzed, standard optimal control techniques which are applied to environmental and resource economics are used (e.g. Xepapadeas, 1997, chapter 2). When conflict and strategic interactions among countries make necessary the analysis of noncooperative solutions, then the differential games framework has been extensively used.[1]

Uncertainty is another issue that has been addressed extensively in these problems. Apart from a general kind of uncertainty associated with the future costs and benefits of an action, there is the specific uncertainty associated with the evolution of the natural system. This uncertainty could arise from sources such as major gaps in knowledge, limited modelling capacity and lack of theories to anticipate thresholds, and emergence of surprises and unexpected consequences. These uncertainties may impede adequate scientific understanding of the underlying natural system mechanisms and the impacts of policies applied to these systems. The discussion about the uncertainty surrounding climate sensitivity and the implications of fat tailed distributions in climate policies has highlighted the types of uncertainty surrounding the evolution of natural systems (e.g. Stern, 2007; Weitzman, 2009). For the purposes of our analysis we will refer to the overall uncertainty associated with these sources as scientific uncertainty.

One feature of the above structure of uncertainty is that it might be difficult or even impossible to associate probabilities with uncertain shocks affecting the natural system evolution. This is close to the concept of uncertainty as introduced by Frank Knight (1921) to represent a situation where there is ignorance, or not enough information to assign probabilities to events. Knight argued that uncertainty in this sense of unmeasurable uncertainty is more common in economic decision making. It seems that this type of uncertainty might also be relevant for modeling the evolution of natural systems.

Knightian uncertainty is contrasted to risk (measurable or probabilistic uncertainty) where probabilities can be assigned to events and are summarized by a subjective probability measure or a single Bayesian prior. The concept of Knightian uncertainty or ambiguity has been associated formally with a concept of multiple priors (Gilboa and Schmeidler, 1989), as well as with a concept of uncertainty or ambiguity aversion, which in general in-

[1] For a recent survey of the use of dynamic games in economics and pollution control, see Jorgensen et al. (2010).

creases with an ignorance parameter (Chen and Epstein, 2002).

In economics, decision making under risk is typically modelled as expected utility maximization. Gilboa and Schmeidler (1989), motivated by the Elsberg (1961) paradox, provided an axiomatic foundation of Wald's (1950) maxmin criterion, and showed that a maxmin expected utility theory based on the least favorable prior (LFP) can be used under conditions of Knightian uncertainty.[2]

Ambiguity aversion and decisions based on maxmin criteria and LFPs can be associated with the concept of the precautionary principle (PP), which is an approach wherein actions are taken to anticipate and avert serious or irreversible harm, such as for example the prevention of severe damages or an irreversible catastrophic event associated with climate change, in advance of or without a clear demonstration that such action is necessary. As Marchant (2003, p. 1799) states, "By formalizing and bringing precaution to the forefront, the precautionary principle has the potential to make environmental decision making more deliberative, transparent, and coherent."

The idea of an LFP, or a worst-case scenario, and serious or possibly irreversible changes can be intuitively put together, since the emergence of an LFP could lead to serious damages or an irreversible change. Therefore a direct link can be made between LFP ideas and the PP. Scientific uncertainty or model uncertainty underlying the natural systems can be manifested in multiple priors. The decision maker cannot choose among them but one or more of these priors, the LFP, could lead to severe damages or irreversible change. To prevent these damages, which are not clearly demonstrated since the decision maker does not know that the LFP will prevail, precaution might be desirable in designing specific policy rules, which implies that the decision rule could be based on the LFP. Thus, the maxmin expected utility could be used as a conceptual framework for designing management rules which adhere to a precautionary behavior.

The purpose of this paper is to explore the implications of introducing

[2] Given a set of prior probability distributions associated with the multiple priors framework, the LFP is the one that corresponds to the least favorable outcomes. It can be associated with the concept of the worst-case scenario. Under Knightian uncertainty the researcher cannot choose one prior to define expected utility as is done under risk.

ambiguity associated with uncertainty in the evolution of natural systems and precautionary concerns based on preferences for robustness under scientific uncertainty, in a stylized international pollution control problem. By comparing solutions under risk and under ambiguity, we provide a measure of the impact of adopting robust approaches in international control along with an approach to determine the extra cost of being robust and precautionary.

2 Modeling ambiguity

The type of uncertainty described above can be modeled by associating it with the case of a decision maker who is trying to make good choices when he regards his model not as the correct one, but as an approximation of the correct one or, to put it differently, when the decision maker has concerns about possible misspecifications of the correct model and wants to incorporate these concerns into the decision-making rules (e.g., Salmon, 2002; Hansen and Sargent, 2001a,b; 2008; Hansen et al., 2006; JET, 2006). The misspecification concerns emerge because the regulator cannot assign probabilities to events or, to put it in the Gilboa and Schmeidler context, the regulator is faced with multiple priors.

Having concerns about model misspecification, following Hansen et al. (2006) or Hansen and Sargent (2008), means that the regulator distrusts his model and wants good decisions over a cloud of models that surrounds the regulator's benchmark model. The models in the cloud are difficult to distinguish with finite data sets.

The cloud of models or the set of approximate models is obtained by disturbing a benchmark model by introducing a misspecification error, so that the admissible disturbances reflect the set of possible probability measures that the decision maker is willing to consider, or alternatively how ambiguous the decision maker is about the benchmark model. The more ambiguous the regulator is, the larger is the cloud of approximate models that he is willing to consider. In this setup the good or robust decisions are obtained by introducing a fictitious 'adversarial agent' which we will refer to as Nature. Nature promotes robust decision rules by forcing the regulator, who seeks to maximize an objective, to explore the fragility of decision rules to departures from the benchmark model. A robust decision rule to model misspecification means that lower bounds to the rule's performance are determined by Nature, the adversarial agent who acts as a minimizing agent when constructing these lower bounds. Hansen et al. (2006) show that robust control theory can be interpreted as a recursive version of maxmin expected utility theory (Gilboa and Schmeidler, 1989). In this context the decision maker cannot or does not formulate a single probability model and maximizes expected utility assuming the probability weights are chosen by Nature.

In this paper ambiguity is modeled in terms of a robust control problem. As will become clear later, the standard expected utility maximizing model could be derived as a special case of the robust control model when the regulator has no concerns about model misspecification and completely trusts the benchmark model. By comparing the decision rules between the two cases – mistrust versus complete trust in the benchmark model – it is possible to compare the impact of ambiguity and robustness on decision rules derived from international pollution control models as well as the cost implied by concerns about model misspecification and the desire to be precautionary when designing regulation.[3]

3 International pollution control under model misspecification

The cooperative and the noncooperative setup of international pollution control is modeled in the standard way (e.g. van der Ploeg and de Zeeuw (1992) or Dockner and van Long (1993)). To make the model simple, so that it is easier to trace the impact of ambiguity and precaution, the two-country linear quadratic specification of Dockner and van Long is adopted. Thus there are two countries indexed by $i = 1, 2$. Output in each country is a function of emissions $Q_i = F_i(E_i)$, where $F_i(\cdot)$ is strictly concave with $F_i(0) = 0$.

[3]For a similar approach to resource management, see for example Roseta-Palma and Xepapadeas (2004) and Vardas and Xepapadeas (2010).

Emissions contribute to the stock of a global pollutant $P(t)$. The evolution of the pollution stock is described by the usual linear differential equation,[4]

$$\dot{P} = E_1 + E_2 - mP \ , P(0) = P_0, \tag{1}$$

where $m > 0$ reflects the environment's self cleaning capacity and t is dropped to ease notation. Utility in each country, assuming constant population normalized to one, is $u_i(F_i(E_i)) - C(P)$ with $C(P)$ being the cost of the global pollutant where

$$\begin{aligned} u_i(F_i(E_i)) &= AE_i - \frac{1}{2}E_i^2, A > 0 && (2) \\ C(P) &= \frac{s}{2}P^2, s > 0. && (3) \end{aligned}$$

Thus each country's objective is to maximize individual welfare or

$$\max_{E_i \geq 0} \int_0^\infty \mathrm{e}^{-\rho t} \left(AE_i - \frac{1}{2}E_i^2 - \frac{s}{2}P^2 \right) dt, \tag{4}$$

subject to pollution dynamics, where $\mathrm{e}^{-\rho t}$ is the appropriate discount factor.. Uncertainty is introduced in the standard way, so that the stock of the pollutant accumulates according to the stochastic differential equation

$$dP = (E_1 + E_2 - mP)\, dt + \sigma d\hat{z}, \ \ P(0) = P_0 \text{ fixed} \tag{5}$$

where $\{\hat{z}(t) : t \geq 0\}$ is a Brownian motion on an underlying probability space $\{\Omega, \mathcal{F}, G\}$. In the terminology of the previous section, this is the benchmark model.[5]

Following Hansen and Sargent (2001a,b; 2008), concerns about model misspecification are introduced by a family of stochastic perturbations of the Brownian motion, so that the probabilities implied by (5) are distorted by replacing the measure G by a measure Q. The main idea is that stochastic

[4]To keep things simple nonlinearities and nonconvexities in the dynamics are not considered. For their implication, see for example Kossioris et al. (2008).

[5]It should be noted that for any fixed $(E_1, E_2) > (0, 0)$, (5) is an Ornstein-Uhlenbeck process with long-term mean $E_1 + E_2$ and long-term variance $\sigma^2/2$.

processes under Q will be difficult to distinguish from those under G using finite data. The perturbed model is obtained by replacing $\hat{z}(t)$ with:

$$\hat{z}(t) = z(t) + \int_0^t v(s)\, ds \tag{6}$$

where $\{z(t) : t \geq 0\}$ is a Brownian motion and $\{v(t) : t \geq 0\}$ is a measurable drift distortion which can be interpreted as a misspecification error of the pollution dynamics which is expressed in terms of deviations from the benchmark case. The benchmark case is defined for $v(t) := 0$. The discrepancy between the distributions G and Q is measured by the relative entropy,

$$R(Q) = \int_0^\infty e^{-\rho t} \mathcal{E}_Q \frac{|v(t)|^2}{2} dt. \tag{7}$$

By requiring that

$$R(Q) \leq \eta, \tag{8}$$

the decision maker can restrict the size of the relative entropy and establish the set of distributions that will be considered. By choosing η, the decision maker can determine the 'size' of the cloud of approximate models that will be considered given a benchmark model, which in a sense could determine how much misspecification is justified given the existing knowledge and history of the natural system. Considering a least favorable prior which would correspond to a large η does not need to imply a catastrophic event, but rather reflects the 'maximum' misspecification that the regulator wants to embody into the decision rule, given the existing data and history of the phenomenon under consideration. Pollution dynamics under model misspecification can be written, by replacing $d\hat{z}$ with dz, as:

$$dP = (E_1 + E_2 - mP + \sigma v)\, dt + \sigma dz, \; P(0) = P_0 \text{ fixed.} \tag{9}$$

When the regulator has no concerns about model misspecification, that is

$v = 0$, then we are in the case of decision making under risk where expected utility theory is appropriate. When the regulator has concerns about model misspecification, that is $v \neq 0$, then we are in the case of decision making under ambiguity or uncertainty where maxmin expected utility theory is appropriate. Two robust control problems have been associated with the problem of maximizing (4) subject to (9) (Hansen and Sargent (2001a,b; 2006; 2008): the constrained robust control problem

$$\begin{aligned} J(\eta) &= \max_{E_1,E_2} \min_{v} \mathcal{E}_0 \int_0^\infty e^{-\rho t} \left(AE_i - \frac{1}{2}E_i^2 - \frac{s}{2}P^2 \right) dt \\ &\quad \text{subject to (9) and (8)} \end{aligned} \tag{10}$$

and the multiplier robust control problem

$$\begin{aligned} J(\theta) &= \max_{E_1,E_2} \min_{v} \mathcal{E}_0 \int_0^\infty e^{-\rho t} \left(AE_i - \frac{1}{2}E_i^2 - \frac{s}{2}P^2 + \frac{1}{2}\theta v^2 \right) dt \\ &\quad \text{subject to (9).} \end{aligned} \tag{11}$$

In both problems, the minimizing agent, Nature, chooses v. In the multiplier problem, $\theta \in (\underline{\theta}, +\infty]$, $\underline{\theta} > 0$ is a penalty parameter restraining the minimizing choice of the $v(t)$ function. The lower bound $\underline{\theta}$ is a so-called breakdown point beyond which it is fruitless to seek more robustness because the minimizing agent is sufficiently unconstrained so that he can push the criterion function to $-\infty$ despite the best response of the maximizing agent. Thus when $\theta < \underline{\theta}$, robust control rules cannot be attained. On the other hand when $\theta \to \infty$, then there are no concerns about model misspecification. As shown by Hansen and Sargent (2006), under certain regularity assumptions the penalty parameter θ can be interpreted as the Lagrangian multiplier of the constrained robust control problem. Thus there is a direct link between θ and the size of entropy the the regulator is willing to incorporate into the policy rule. Although the constrained robust control problem is more intuitive, the approach that has been followed in general is the solution of the more tractable multiplier robust control problem.

It should be noted that a non-negativity (or irreversibility) constraint on emissions $E_i \geq 0$ is not imposed in the robust control problems (10) or (11), which implies that clean up is possible. There are of course many cases in pollution control where a more realistic assumption would be to impose the non-negativity constraint. If under emissions non-negativity there are models where a certain emission profile may cause irreversible damages, low emissions have a positive option value, since if the LFP that causes the irreversible damage is realized, the regulator is not irrevocably committed to a high stock of the pollutant implying unavoidable costs. Thus the irreversibility is a consequence of the policy maker's inability to reduce the stock of pollution which is captured by the non-negativity constraint, $E_i \geq 0$. Without the nonegativity constraint low emissions have no option value since clean up is possible.[6] In this paper emission irreversibility is not considered, which would have required the use of a Kuhn-Tucker multiplier, or as is more common in these problems, a real options approach, where the 'stopping' domain $E_i = 0$ and the interior domain $E_i > 0$ are joined by value matching and smooth pasting conditions.[7] The relation between emission irreversibility and ambiguity is beyond the purpose of the present paper and could be an interesting area for future research. Since, however, irreversibility of emissions and damages, a case which is closer to the concept of the PP, is not explicitly considered, the policies described in this paper will be referred to as robust policies rather than as policies associated with a PP.

In this robust control framework, the following sections analyze the cooperative and noncooperative solutions of the international pollution problem under risk and under ambiguity.

4 The cooperative solution under risk

The cooperative solution is obtained by maximizing expected joint welfare defined by (4). Given the linear quadratic structure of the problem, a quadratic

[6] I would like to thank an anonymous reviewer for pointing this out.

[7] See for example Dixit and Pindyck (1994) for the general approach of real options and Xepapadeas (1998), Wirl (2008) for application to environmental policy issues.

value function

$$W(P) = -\frac{1}{2}\alpha P^2 - \beta P - \gamma \tag{12}$$

with first and second derivatives

$$DW = -\alpha P - \beta \ , D^2W = -\alpha \tag{13}$$

is considered. The Hamilton-Jacobi-Bellman (HJB) equation for this problem, where DW and D^2W denote the first and second derivatives of the value function respectively, is:

$$\begin{aligned} \rho W &= \max_{E_1, E_2} \left\{ A(E_1 + E_2) - \frac{1}{2}\left(E_1^2 + E_2^2\right) - sP^2 + \right. \\ &\quad \left. DW(E_1 + E_2 - mP) + \frac{1}{2}\sigma^2 D^2 W \right\}. \end{aligned} \tag{14}$$

Optimality implies

$$E_1 = E_2 = A - \beta - \alpha P. \tag{15}$$

Then

$$\begin{aligned} &\rho\left(-\frac{1}{2}\alpha P^2 - \beta P - \gamma\right) = \\ &2A(A - \beta - \alpha P) - (A - \beta - \alpha P)^2 - sP^2 \\ &- (\alpha P + \beta)\left[2(A - \beta - \alpha P) - mP\right] - \frac{\alpha}{2}\sigma^2. \end{aligned} \tag{16}$$

The parameters of the value function are obtained as usual by equating coefficients of the same power. In this case the optimal cooperative emissions in a feedback form will be

$$E^* = (A - \beta) - aP. \tag{17}$$

Substituting optimal cooperative emissions from (17) into (5), the evolution of the pollutant stock under the cooperative solutions will be determined by the solution of the stochastic differential equation

$$dP = [2(A-\beta) - (2\alpha + m)P]\,dt + \sigma dz. \tag{18}$$

This an Ornstein-Uhlembeck process with solution

$$\begin{aligned} P(t) &= P_0 e^{-r_0 t} + \mu\left(1 - e^{-r_0 t}\right) + \int_0^t \sigma e^{r_0(u-t)} dz_u & (19)\\ r_0 &= 2\alpha + m, \mu = \frac{2(A-\beta)}{2\alpha + m}. & (20)\end{aligned}$$

The mean and variance corresponding to the cooperative solution are

$$\begin{aligned} \mathcal{E}P(t) &= P_0 e^{-r_0 t} + \mu\left(1 - e^{-r_0 t}\right) & (21)\\ \mathrm{var}P(t) &= \frac{\sigma^2}{2r_0}\left(1 - e^{-2r_0 t}\right), & (22)\end{aligned}$$

with long-run expected value $\mathcal{E}P^* = \mu = \frac{2(A-\beta)}{(2\alpha+m)}$, $\mathrm{var}P^* = \frac{\sigma^2}{2r_0} = \frac{\sigma^2}{2(2\alpha+m)}$. It is obvious that if $\alpha > 0$ this steady state will be stable.

To make the solution clear and to make possible comparisons with the noncooperative and the ambiguity cases, we use a numerical example where

$$\rho = 0.05, \sigma = 1, A = 100, m = 0.03, s = 1. \tag{23}$$

Then $\alpha = 0.972878$, $\beta = 96.0509$, $E^*(t) = 3.94914 - 0.972878P(t)$, and $\mathcal{E}P^* = 3.9976$.

Figure 1 below presents the time path for $\mathcal{E}P^*(t)$ (thick line) along with a belt of $\pm 3\sqrt{\mathrm{var}P^*(t)}$ (dashed lines) from an initial stock accumulation $P_0 = 2$.

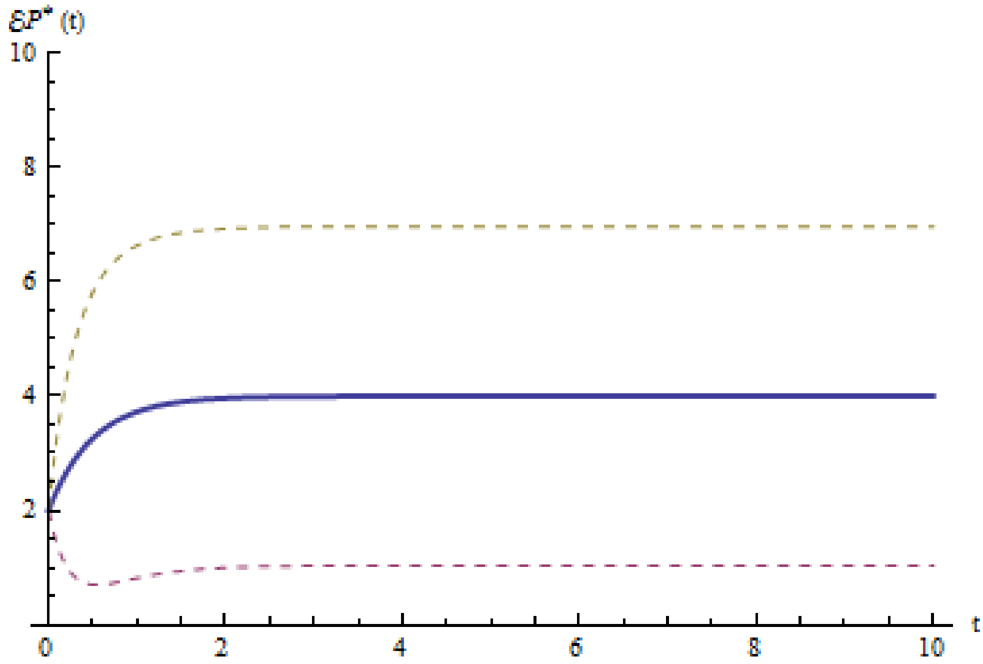

Figure 1: $\mathcal{E}P^*(t) \pm 3\sqrt{varP^*(t)}$

5 Robust control and the cooperative solution under ambiguity

When concerns about misspecification of the pollution dynamics exist, then the cooperative solution can be obtained as the solution of the following multiplier extremization problem, which has already been defined above, as:

$$\max_{E_1,E_2} \min_{v} \mathcal{E}_0 \int_0^\infty e^{-\rho t} \left[\sum_{i=1,2} \left(AE_i - \frac{1}{2}E_i^2 - \frac{s}{2}P^2 \right) + \frac{\theta v^2}{2} \right] dt \tag{24}$$

subject to (9). (25)

The benchmark optimal control problem is a special case of (24) for $v(t) \equiv 0$, and corresponds to the cooperative solution under risk. Using again a quadratic value function $W^0(P) = -\frac{1}{2}\alpha^0 P^2 - \beta^0 P - \gamma^0$, the Isaacs condition (Fleming and Souganidis, 1989) leads to the HJB equation:

$$
\begin{aligned}
\rho W^0 &= \max_{E_1,E_2} \min_{v} \left\{ A\left(E_1+E_2\right) - \frac{1}{2}\left(E_1^2+E_2^2\right) - sP^2 + \frac{1}{2}\theta v^2 + \right. \quad (26) \\
& \left. DW\left(E_1+E_2-mP+\sigma v\right) + \frac{1}{2}\sigma^2 D^2 W \right\}.
\end{aligned}
$$

Optimality implies

$$
v^0 = \frac{\left(\alpha^0 P + \beta^0\right)\sigma}{\theta} \quad (27)
$$

and

$$
E_1^0 = E_2^0 = A - \beta^0 - \alpha^0 P. \quad (28)
$$

It is clear that if $\theta \to \infty$, then $v \to 0$ and we are back to the benchmark model. The value function becomes:

$$
\begin{aligned}
&\rho\left(-\frac{1}{2}\alpha^0 P^2 - \beta^0 P - \gamma^0\right) = \quad (29) \\
&2A\left(A-\beta^0-\alpha^0 P\right) - \left(A-\beta^0-\alpha^0 P\right)^2 - sP^2 + \frac{1}{2}\theta\left[\frac{\left(\alpha^0 P+\beta^0\right)\sigma}{\theta}\right]^2 \\
&-\left(\alpha^0 P+\beta^0\right)\left[2\left(A-\beta^0-\alpha^0 P\right) - mP + \sigma\frac{\left(\alpha^0 P+\beta^0\right)}{\theta}\right] - \frac{\alpha}{2}\sigma^2,
\end{aligned}
$$

and the parameters of the value function are obtained as before by equating coefficients of the same power. Substituting the optimal choice of the adversarial agent (27) and optimal robust cooperative emissions (28) into (9), the evolution of the pollutant stock under the cooperative solution will be determined by the solution of the Ornstein-Uhlembeck process

$$
dP = \left[\left(2\left(A-\beta^0\right) - \frac{\sigma}{\theta}\beta^0\right) - \left(\left(2+\frac{\sigma}{\theta}\right)\alpha^0 + m\right)P\right]dt + \sigma dz \quad (30)
$$

with mean and variance

$$\mathcal{E}P(t;\theta) = P_0 e^{-r_0^0 t} + \mu^0 \left(1 - e^{-r_0^0 t}\right) \tag{31}$$

$$\text{var}P(t;\theta) = \frac{\sigma^2}{2r_0^0}\left(1 - e^{-2r_0^0 t}\right) \tag{32}$$

$$r_0^0 = \left(2 + \frac{\sigma}{\theta}\right)\alpha^0 + m, \ \mu^0 = \frac{2\left(A - \beta^0\right) - \frac{\sigma}{\theta}\beta^0}{\left(2 + \frac{\sigma}{\theta}\right)\alpha^0 + m}. \tag{33}$$

Thus our results are a function of the penalty parameter θ. Provided that α^0 and β^0 converge to finite values as $\theta \to \infty$, and this has been verified for all numerical simulations, then the limit of $\mathcal{E}P(t;\theta)$ and $\text{var}P(t;\theta)$ as $\theta \to \infty$ converge to their benchmark value, indicating that the model with no misspecification concerns can be regarded as a special case of the robust control model when $\theta \to \infty$. Thus when θ becomes large, the results regarding the parameters of the value function, the optimal emissions and the expected steady state pollution accumulation should converge to the results obtained for the benchmark model. Figure 2 presents α^0 as a function of θ.

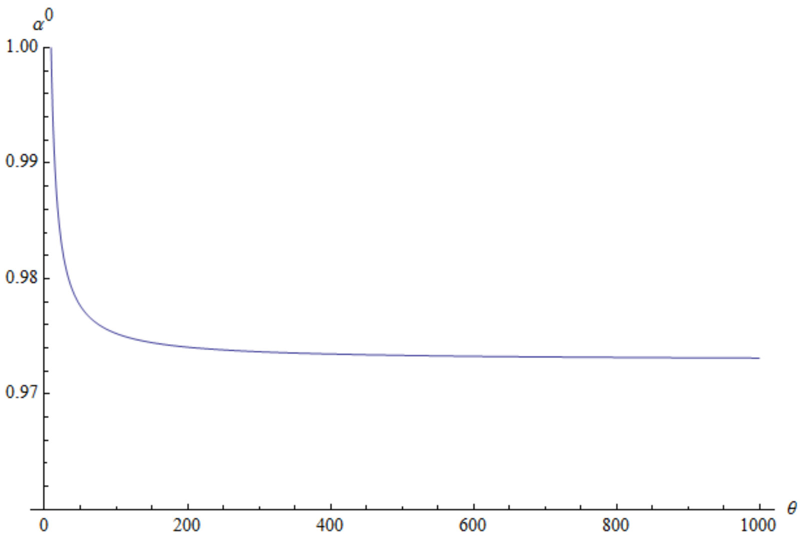

Figure 2: α^0 vs θ

The parameter remains positive so the stability requirement is satisfied and as θ increases it tends to the benchmark case value of $\alpha = 0.972878$.

Figure 3 presents the value function for three values of the penalty parameter $\theta = \{50, 100, 500\}$.

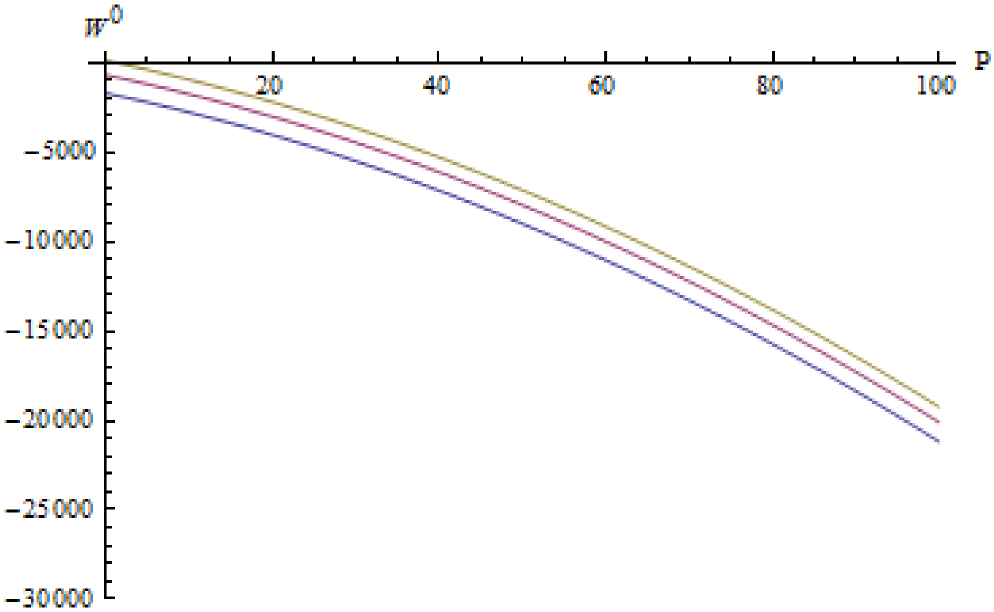

Figure 3: The value function

The value function shifts downward as θ decreases (the top line corresponds to $\theta = 500$ while the bottom to $\theta = 50$) indicating that as concerns about model misspecification increase, the robust control of the system becomes more costly as the system loses value. Table 1 shows the values of the value function at different levels of θ with P set at the initial value $P_0 = 2$. Therefore robust control under concerns about model misspecification becomes more costly. As misspecification concerns increase, the changes in the value function, as θ is reduced, can be interpreted as the cost of robustness or the cost of being more precautionary in order to avoid potentially severe damages associated with the emergence of an LFP from the cloud of models that satisfy the entropy constraint.

Table 1: The cost of robustness under cooperation

θ	$W^0(2)$
50	-1909.69
100	-896.69
500	-93.58
1000000	106

Figure 4 presents optimal robust emissions in feedback form for different values if $\theta = \{50, 100, 500\}$. As concerns about misspecification increase, optimal robust cooperative emissions are reduced and the emission function shifts downwards as shown in figure 4, where the top line corresponds to $\theta = 500$ and the bottom line to $\theta = 50$.

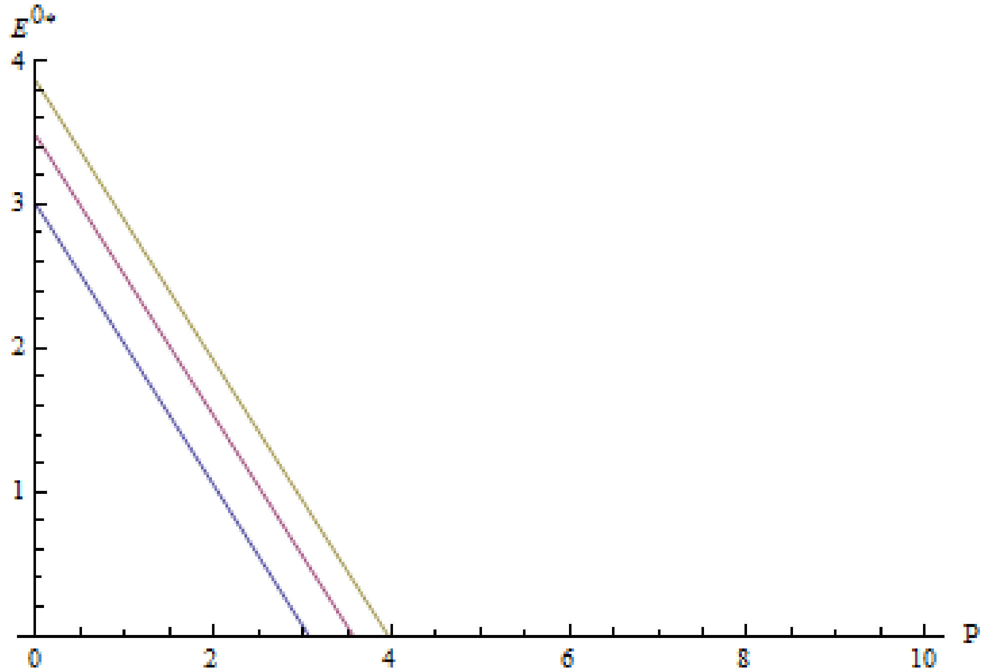

Figure 4: Optimal robust emissions

Finally figure 5 presents the time path of expected robust cooperative pollution accumulation for $\theta = \{50, 100, 500\}$. As misspecification concerns increase, that is θ is reduced, the steady state robust pollution accumulation is reduced. This is in line with the behavior of the emissions function. An increase in misspecification concerns (decrease of θ), because the regulator increases the size of entropy to be incorporated into the decision rule, will reduce the robust emission policy function and will lead to reduced expected pollution accumulation. This is indicated by a shift of the $\mathcal{E}P^0(t)$ path downwards in figure 5, where the top line corresponds to $\theta = 500$ and the bottom line to $\theta = 50$. As θ increases, ambiguity goes down and misspecification concerns decrease, robust emissions increase and as a result steady state stock increases too and eventually converges to the benchmark value.

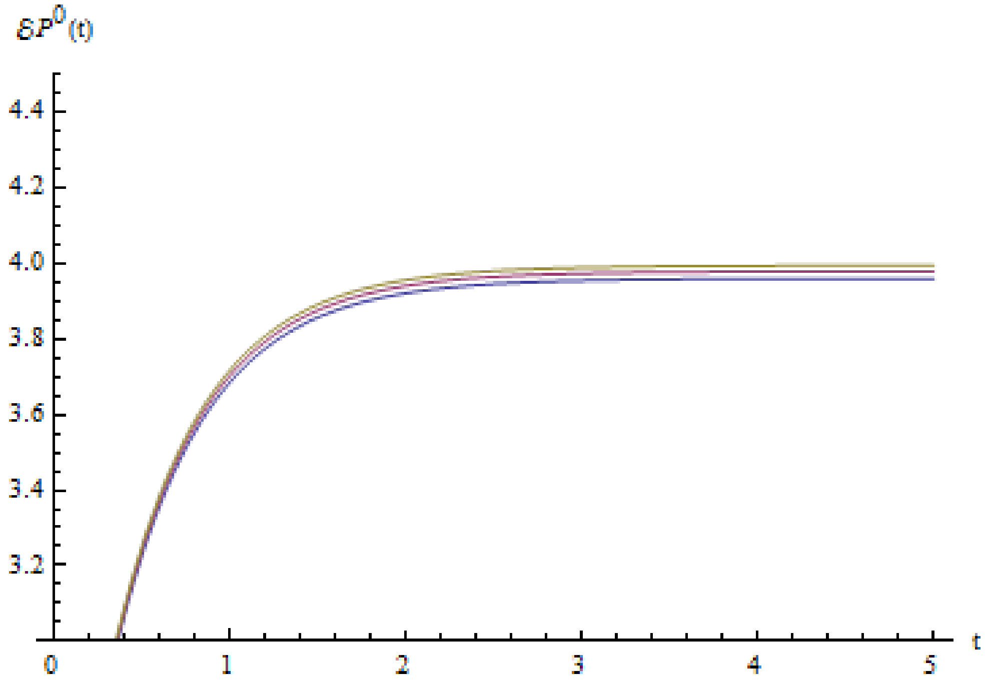

Figure 5: Robust cooperative pollution accumulation

Finally figure 6 below presents the time path for $\mathcal{E}P^0\,(t) \pm 3\sqrt{\text{var}P^*\,(t)}$ from the initial stock accumulation $P_0 = 2$ for $\theta = 50$.

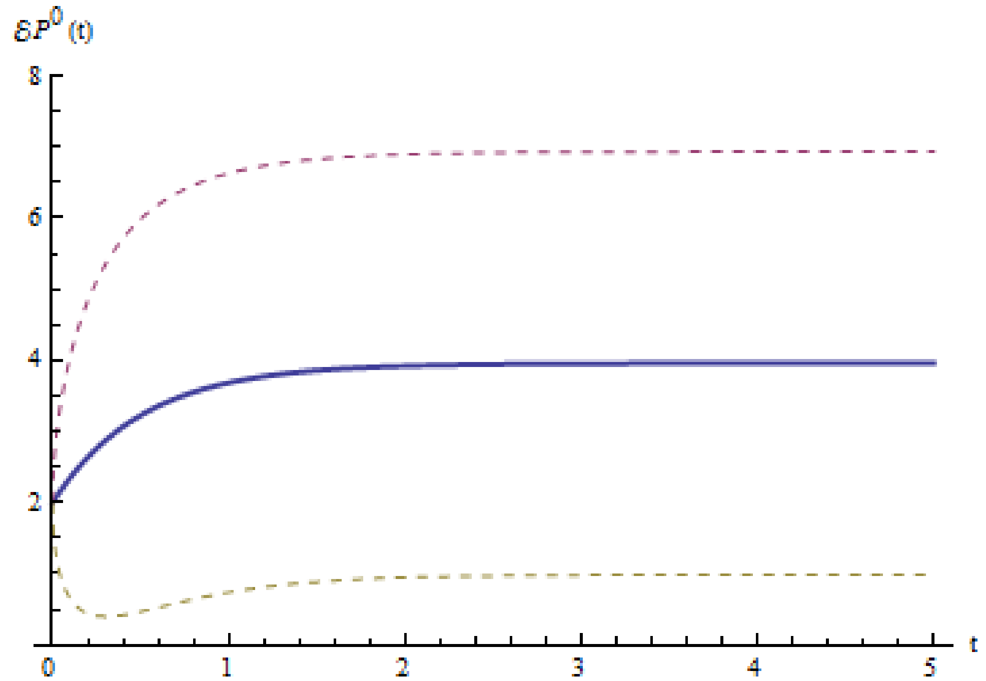

Figure 6: $\mathcal{E}P^0(t) \pm 3\sqrt{\mathrm{var}P^*(t)}$, $\theta = 50$

Note that the expected steady state pollution accumulation for $\theta = 50$ is 3.959, for $\theta = 1$ is 2.701, while the expected accumulation at the benchmark model ($\theta \to \infty$) is 3.9976.

6 The noncooperative solution under risk

To study the noncooperative solution where each country maximizes expected individual welfare subject to pollution dynamics, we assume that each country follows linear time stationary feedback strategies (or closed-loop) strategies (Basar and Olsder, 1982), which are decision rules which condition individual emissions on the current stock of the global pollutant in a linear fashion,[8] or $E_i(t) = \zeta_0 + \zeta_1 P(t)$. Feedback strategies are associated with the concept of feedback Nash equilibrium which is a strongly time-consistent noncooperative solution (Basar, 1989).

The feedback Nash equilibria (FBNE) for the linear quadratic international pollution game result from solving the dynamic programming or Hamilton-Jacobi-Bellman equations in the value functions W_i. The functions and parameters of our problem do not directly depend on time, so the problem is stationary. Therefore the equilibrium strategies can be represented in a time-stationary feedback form, $E_i(t) = \zeta_0 + \zeta_1 P(t)$, $i = 1, 2$, and the value functions W_i depend only on the state x. Furthermore, since the problem is symmetric, only symmetric equilibria are considered.

The value function for each country is

$$W_i(P) = -\frac{1}{2}\alpha_i P^2 - \beta_i P - \gamma_i \tag{34}$$

[8]For the analysis of nonlinear strategies, see Dockner and van Long (1993), Rubio and Casino (2002), and Kossioris et al. (2008).

and the corresponding HJB for each country is

$$\rho W_i = \max_{E_i}\left\{AE_i - \frac{1}{2}E_i^2 - \frac{s}{2}P^2 + \right. \tag{35}$$
$$\left. DW_i\left(E_1 + E_2 - mP\right) + \frac{1}{2}\sigma^2 D^2 W_i\right\}.$$

Optimality implies

$$E_i = A - \beta_i - \alpha_i P = E_1 = E_2, \tag{36}$$

so individual country strategies are in a feedback or closed loop form. Dropping the index i due to symmetry, the HJB satisfies

$$\rho\left(-\frac{1}{2}\alpha P^2 - \beta P - \gamma\right) = A\left(A - \beta - \alpha P\right) - \frac{1}{2}\left(A - \beta - \alpha P\right)^2 \tag{37}$$
$$-\frac{s}{2}P^2 + -\left(\alpha P + \beta\right)\left[2\left(A - \beta - \alpha P\right) - mP\right] - \frac{\alpha}{2}\sigma^2.$$

To compare cooperative and noncooperative solutions under risk, we continue our numerical example. The feedback equilibrium strategy is defined as $E_i\left(t\right) = 36.3672 - 0.55931 P\left(t\right), i = 1, 2,\ \alpha_1 = \alpha_2 = 0.55931$, while the expected FBNE pollution steady state is $\mathcal{E}P_{FBNE} = 63.3235$. The comparison of the cooperative equilibrium with the FBNE confirms the well known result that the FBNE results in higher emissions and higher steady-state pollution accumulation than the cooperative equilibrium. The FBNE is stochastically stable since $\alpha_i > 0$. Figure 7 presents the time path for $\mathcal{E}P_{FBNE}\left(t\right) \pm 3\sqrt{varP_{FBNE}\left(t\right)}$ from an initial stock accumulation $P_0 = 2$. The comparison of figure 7 with figure 1 clearly shows the differences between the cooperative and the feedback Nash equilibrium in terms of the equilibrium path of the state variable.

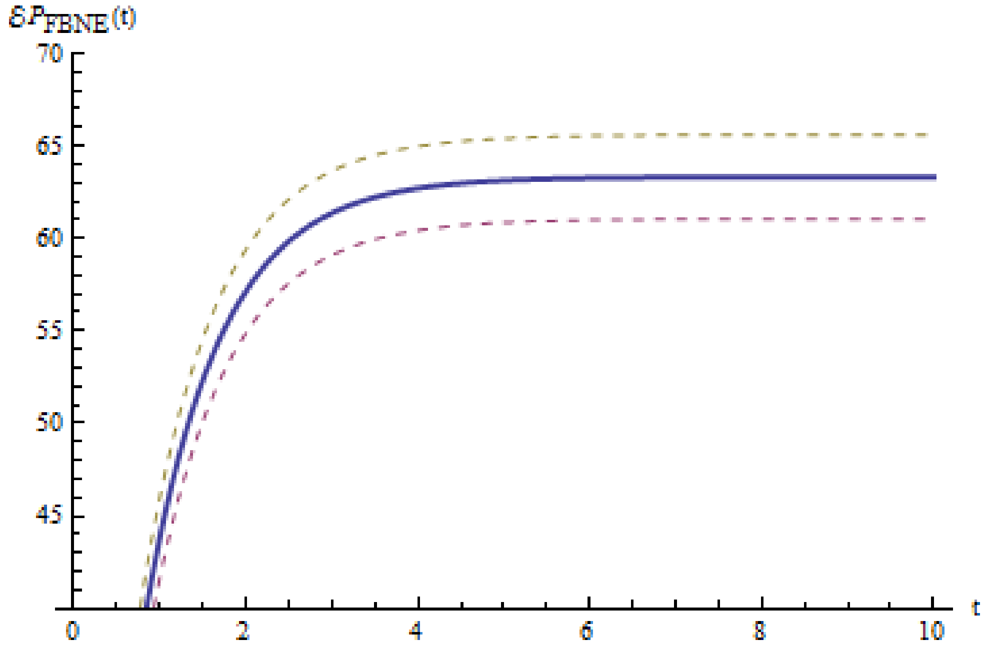

Figure 7: $\mathcal{E}P_{FBNE}(t) \pm 3\sqrt{varP_{FBNE}(t)}$

7 Robust control and the noncooperative solution under ambiguity

A robust FBNE can be obtained as the solution of the extremization multiplier problem

$$\max_{E_i} \min_{v_i} \int_0^\infty e^{-\rho t} \left[\left(AE_i - \frac{1}{2}E_i^2 - \frac{s}{2}P^2 \right) + \frac{\theta v_i^2}{2} \right] dt \quad (38)$$

$$\text{subject to} \quad (39)$$

$$dP = (E_1 + E_2 - mP + \sigma v_i)\, dt + \sigma P dz, \quad (40)$$

where v_i is the misspecification error for country i when countries follow time stationary linear feedback strategies. Assuming again a quadratic value function $W_i^0(P) = -\frac{1}{2}\alpha_i^0 P^2 - \beta_i^0 P - \gamma_i^0$, the Isaacs condition leads to the

HJB equation for each country:

$$\rho W_i^0 = \max_{E_i \geq 0} \min_{v_i} \left\{ AE_i - \frac{1}{2}E_i^2 - \frac{s}{2}P^2 + \frac{1}{2}\theta_i v_i^2 + \right. \tag{41}$$
$$\left. DW_i^0 \left(E_1 + E_2 - mP + \sigma v_i\right) + \frac{1}{2}\sigma^2 D^2 W_i^0 \right\}.$$

Optimality implies

$$v_i = \frac{\left(\alpha_i^0 P + \beta_i^0\right)\sigma_i}{\theta_i} \tag{42}$$
$$E_i^* = A - \beta_i^0 - \alpha_i^0 P, \tag{43}$$

and the HJB satisfies

$$\rho\left(-\frac{1}{2}\alpha_i^0 P^2 - \beta_i^0 P - \gamma_i^0\right) = \tag{44}$$
$$A\left(A - \beta_i^0 - \alpha_i^0 P\right) - \frac{1}{2}\left(A - \beta_i^0 - \alpha_i^0 P\right)^2 - \frac{s}{2}P^2 + \frac{1}{2}\theta_i\left[\frac{\left(\alpha_i^0 P + \beta_i^0\right)\sigma}{\theta_i}\right]^2 +$$
$$DW_i^0\left(2\left(A - \beta_i^0 - \alpha_i^0 P\right) - mP + \sigma\frac{\left(\alpha_i^0 P + \beta_i^0\right)}{\theta_i}\right) - \frac{\alpha_i^0}{2}\sigma^2.$$

The HJB equation (44) implies that the parameters of the value function and the optimal feedback strategy for each country depend on the penalty parameter θ. Thus (44) can be used to determine a robust FBNE which is the FBNE under conditions of ambiguity. As $\theta \to \infty$ the robust FBNE tends to the FBNE under conditions of risk. The numerical example is used again to obtain a clearer picture of the results.

Figure 8 presents α_i^0 as a function of θ.

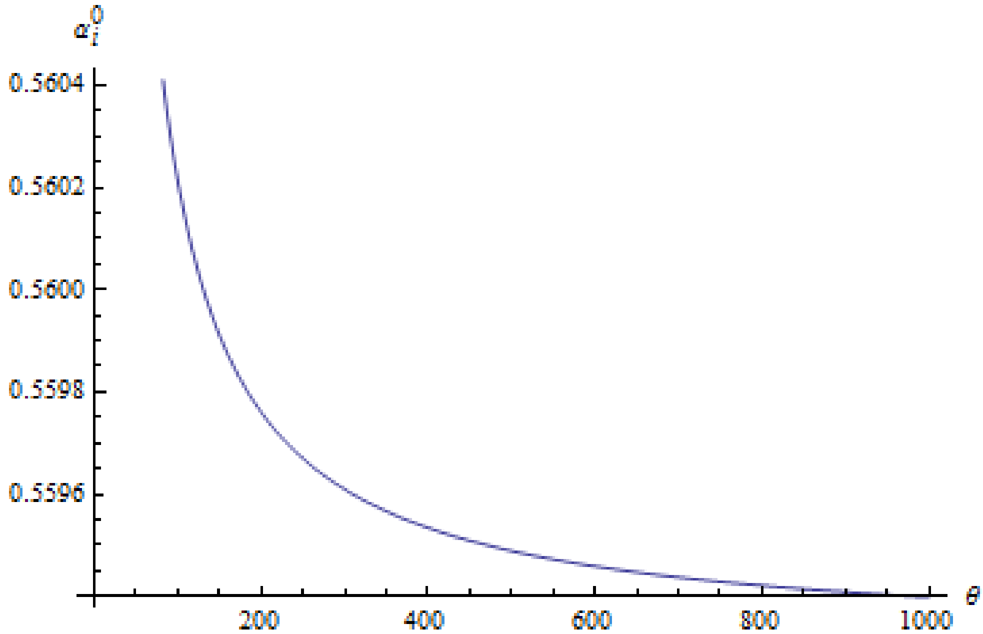

Figure 8: a_i^0 vs θ

The parameter remains positive so the stability requirement is satisfied, and as θ increases it tends to the benchmark case value of $\alpha_i = 0.55931$.

Figure 9 presents the value function for two values of the penalty parameter $\theta = \{10, 500\}$.

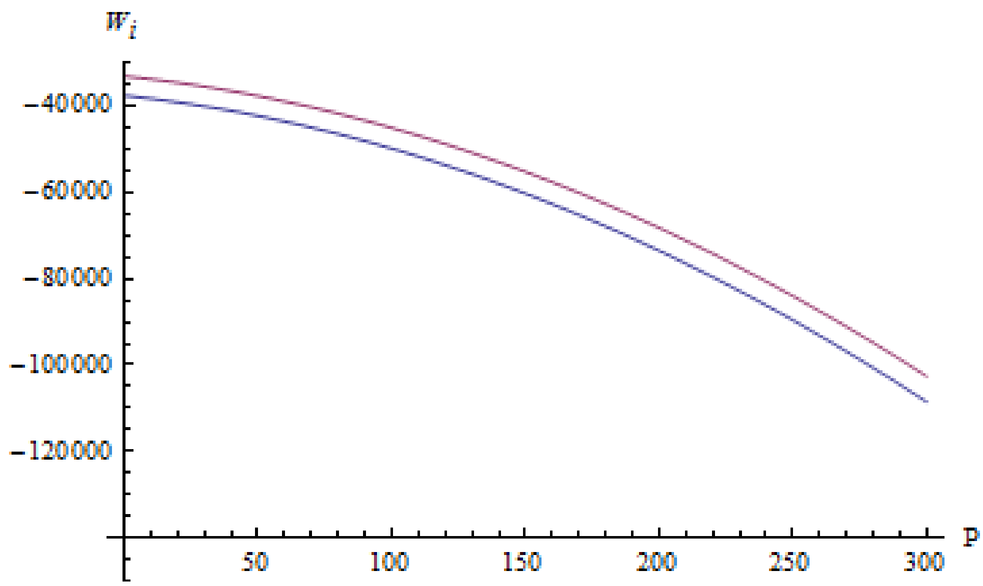

Figure 9: The value function for FBNE

The value function shifts downward as θ decreases (in figure 9 the top line corresponds to $\theta = 500$ and the bottom line to $\theta = 10$), indicating that as concerns about model misspecification increase the robust control of the system in each country becomes more costly as in the cooperative solution. Table 2 shows the values of the value function at different levels of θ with $P_0 = 2$. The changes in the value function as θ reduces can be interpreted as the cost of each country being robust when no cooperation is taking place.

Table 2: The cost of robustness at the FBNE

θ	$W_i^0(2)$
5	-34084.3
10	-33636.8
10^2	-33281
10^6	-33192.3

Figure 10 presents optimal noncooperative country emissions in feedback form for different values of $\theta = \{10, 500\}$. These are the feedback equilibrium strategies parametrized by the parameter θ. As in the cooperative case, increased misspecification concerns reduce the noncooperative equilibrium emissions.

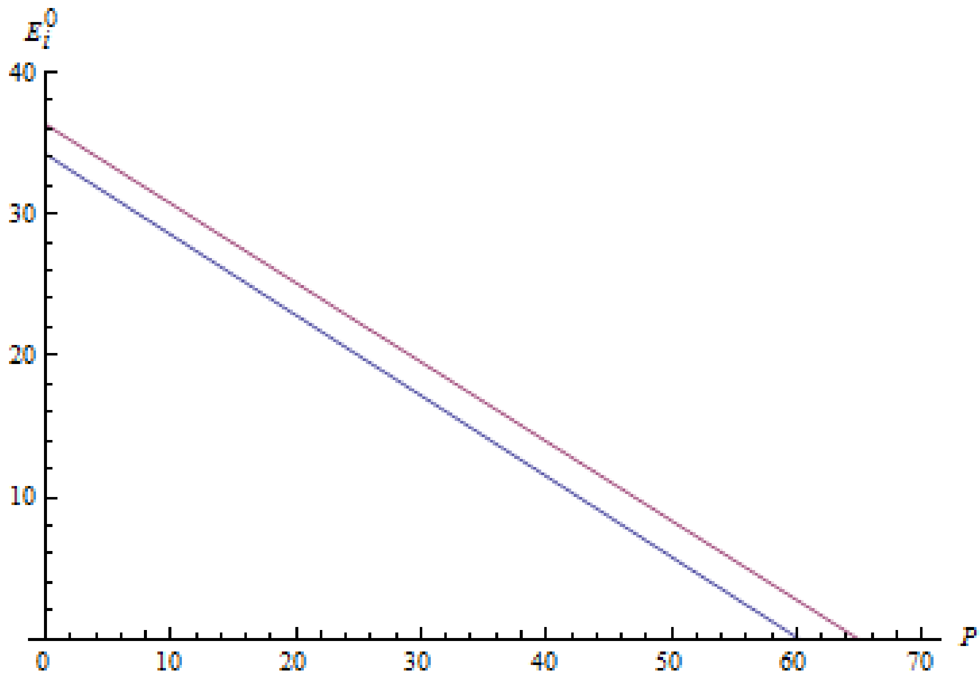

Figure 10: Optimal feedback emissions at the FBNE

Figure 11 presents the time path of the expected robust FBNE pollution accumulation for $\theta = \{50, 500\}$. As individual misspecification concerns increase, that is θ is reduced, the expected steady-state robust pollution accumulation is reduced. This is indicated by a shift of the $\mathcal{E}P^0_{FBNE}(t)$ path downwards in figure 11, where the top line corresponds to $\theta = 500$ and the bottom line to $\theta = 50$.

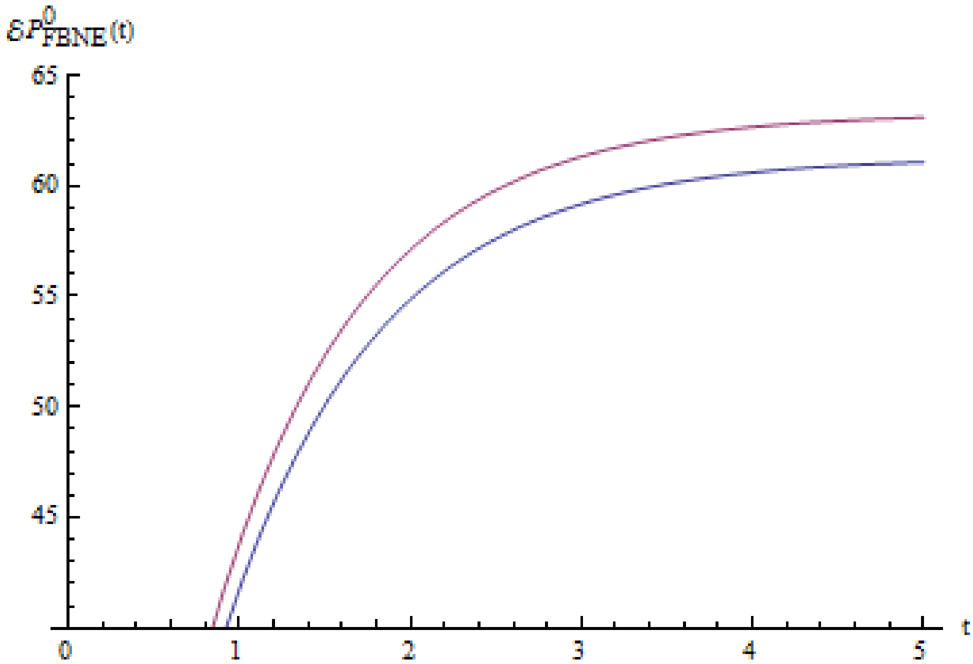

Figure 11: FBNE robust pollution accumulation

As θ increases, that is ambiguity is reduced, the steady-state stock increases and eventually converges to the noncooperative benchmark value. Finally figure 12 below presents the time path for $\mathcal{E}P^0_{FBNE}(t) \pm 3\sqrt{\mathrm{var}P^0_{FBNE}(t)}$ from the initial stock accumulation $P_0 = 2$ for $\theta = 50$.

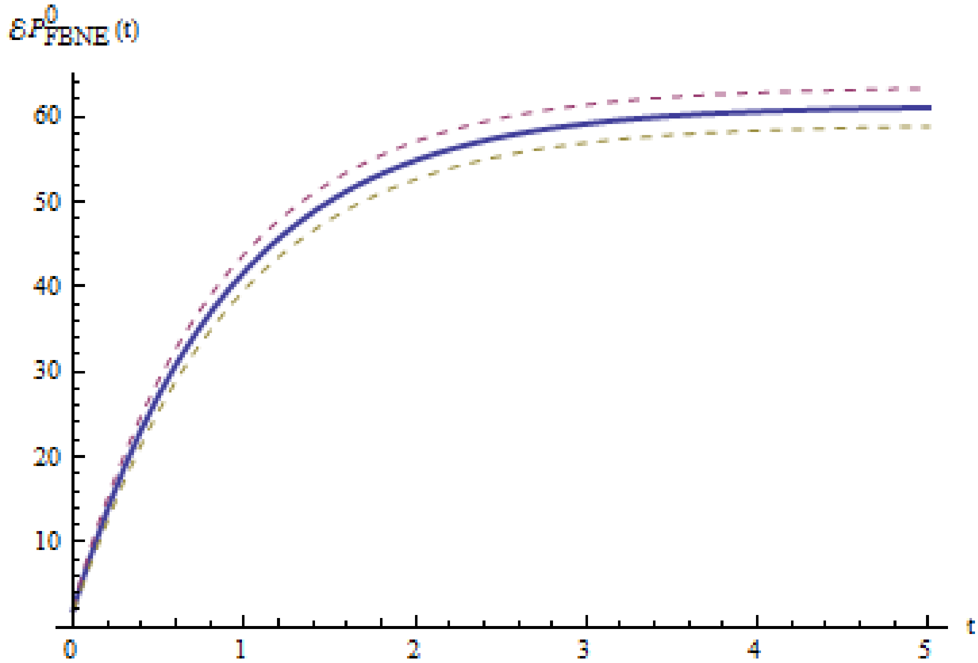

Figure 12: $\mathcal{E}P^0_{FBNE}(t) \pm 3\sqrt{\text{var}P^0_{FBNE}(t)}$, $\theta = 50$

8 Cooperation and the Cost of Robustness

Comparison of the results obtained from the numerical example, although not conclusive, suggest that concerns about model misspecification formulated in the context of robust control induce conservative behavior in the sense of reducing emissions both at the cooperative and the noncooperative solution relative to the pure risk case. Furthermore, reduced emissions under robust control lead to lower expected steady-state pollution accumulation. Comparisons of the value functions for different levels of the penalty parameter θ indicate that the more the regulator is concerned about model misspecification, the more costly is the design of robust control policies, that is, policies which perform well even when the emerging model is not the benchmark model. Increased concerns about model misspecification can be interpreted as 'increased ambiguity' regarding the laws governing the phenomenon, and the desire to design good rules as ambiguity increases.

In our approach the level of ambiguity is related to the choice of θ. Although this might be seen as arbitrary in the sense that θ is a free parameter, the relation between the multiplier and the constraint control problems might be used to discipline the choice of θ. In particular, the solutions of all the problems considered in this paper for a specific $\hat{\theta} < \infty$ will determine the relative entropy between the benchmark model and the distorted model that corresponds to this $\hat{\theta}$. If prior knowledge about the natural phenomenon can be used to specify η, that is to determine the misspecification or the size of entropy that the regulator is willing to consider, the relation between the chosen $\hat{\theta}$ and the specified η can be established. The problem can be solved for a set of θ until the relative entropy implied by a specific θ is sufficiently close to η. Alternatively, as suggested by Hansen and Sargent (2008) detection probabilities obtained from data of the past history of the phenomenon can be used in more general setups to discipline the choice of θ.

Allowing for misspecification concerns and preferences for robustness to shape the policy rules introduces a contex-specific precaution. Hansen and Sargent (2001b) identify in this context precautionary savings, or boosting of the price of risk. In our case this precaution can be identified with reduced emissions in order to prevent damages which might arise if the benchmark model is used for designing the policy, but due to misspecification, another model, from the cloud of models which are considered, emerges. However in this context more precaution is costly since the value of the system is reduced under robust policies and increased ambiguity which is captured by reductions in the ambiguity parameter θ, or the increase of η.

Some insight about the relative costs of robustness between the cooperative and the noncooperative solutions can be gained from figure 13 which depicts the loss in the value for the cooperative solution and the FBNE for both countries as θ is reduced from 500 to 200 at steps of 10. The results suggest that for the specific numerical example, robustness seems to be more costly under cooperation (solid line) relative to the FBNE. On the average the loss at the cooperative solution is 13% higher relative to the FBNE.

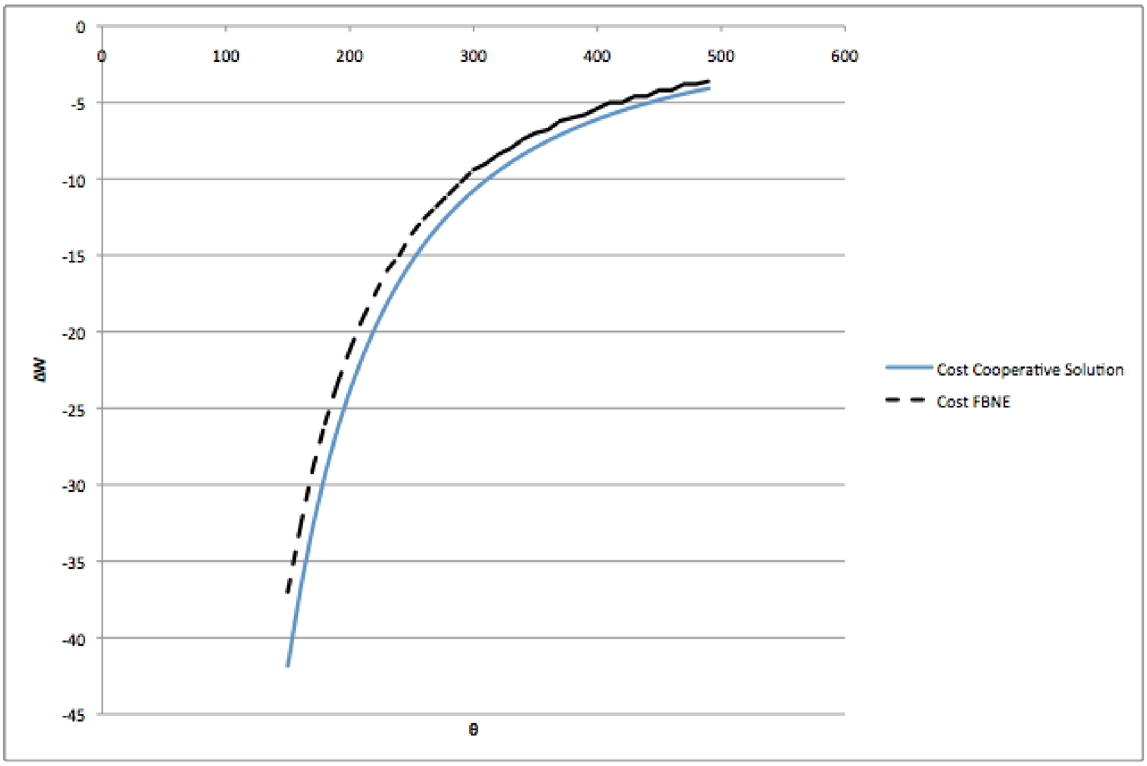

Figure 13: Cost of robusness, cooperative vs FBNE

9 Concluding remarks

This paper studies an approach for incorporating ambiguity and concerns about model misspecification in a problem of international pollution control. Since our knowledge regarding the dynamics underlying the accumulation of transboundary or global pollutants has gaps – sometimes significant gaps – it seems that this type of approach is relevant for decision making. It should be noted that in many pollution control problems, ambiguity could be more profound in the cost implied by any level of stock and not in the laws of motion for the stock itself, which implies that ambiguity could be associated with the damage function. In the model presented in this paper, the distinction of whether ambiguity arises in the processes governing the pollutant cost or the pollutant stock does affect the qualitative results which are derived from the presence of ambiguity in the relationship between future emissions and current welfare. Ambiguity that directly affects damages can be introduced by writing the damage function as $B(t)\, C(P)$ where $B(t)$ is a Brownian motion. Ambiguity in this context implies that $B(t)$ should be perturbed by measurable drift distortions. Although this approach increases the complexity of the model since it introduces a second state variable it might be an interesting area for further research.

The international pollution control problem was formulated in terms of robust control decision rules which are decision rules that might perform well for a set of models around a benchmark model which the decision maker thinks might be misspecified. Since this approach is directly related to decision making when a worst-case scenario might emerge, these robust decision rules can be associated with context-specific precaution.

Using this framework the paper derives robust decision rules for cooperative and noncooperative solutions regarding the emissions of two countries which contribute to a global pollutant. It is shown that decision rules derived when there is no concern about misspecification and the benchmark model is trusted, are a special case of the robust control model where concerns are parametrized by a penalty parameter. Using a simple linear quadratic model, robust decisions rules for the cooperative and the feedback Nash equilibrium with linear feedback strategies were derived.

A concern for precaution and the consequent adoption of a more precautionary approach does, however, incur higher costs. The dynamic programming approach followed here allowed the determination of the value function, and the determination of the costs involved in terms of value loss from increased concerns about misspecification and increased precaution. By comparing the robustness costs under cooperation and noncooperation, it might be possible to study the structure of the incentives for precaution in the two solutions. Better understanding of the structure of costs associated with precaution and the relevant trade-offs might be useful in the design of policies to deal with global pollutants, since the deep uncertainties associated with the evolution of these pollutants are what generate the precautionary needs.

An open issue in robust control is learning.[9] In the robust control ap-

[9] Gollier et al. (2000), Asano (2010) have shown how the anticipation of future learning

proach, learning is not explicitly incorporated into the modelling. One reason (Hansen and Sargent 2008) is that because the regulator discounts the future, the regulator cannot disregard current concerns about model misspecification and wait for enough data to gather so that these concerns can be eliminated. Hansen and Sargent suggest estimation and filtering approaches that could eliminate specific misspecifications concerns. Concerns that cannot be eliminated are incorporated into the lifetime entropy constraint (8). The more misspecifications concerns can be eliminated by learning at this stage, the smaller is η, the larger is θ and the smaller the cost of robustness and concerns for precaution. The question remains, however, of how to incorporate new knowledge into the model when enough data have been gathered to justify a revision of the entropy constraint constant η.

The analysis was kept at the linear quadratic level in order to make clear certain key issues. Possible extensions could be symmetry breaking so that the concerns about misspecification are different among countries, more extensive simulations in order to better trace relative precautionary costs, and allowing for nonconvexities in pollution dynamics so that flips and multiple basin of attractions are possible.

can affect the precautionary principle.

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