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Regional Climate Policy under Deep Uncertainty: Robust Control, Hot Spots and Learning

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Regional Climate Policy under Deep Uncertainty: Robust Control, Hot Spots and Learning

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Abstract

We study climate change policies using the novel pattern scaling approach of regional transient climate response, to develop a regional economy-climate model under conditions of deep uncertainty associated with: (i) temperature dynamics, (ii) regional climate change damages, and (iii) policy in the form of carbon taxes. We analyze cooperative and noncooperative outcomes. Under deep uncertainty, robust control policies are more conservative regarding emissions, the higher the aversion to ambiguity is, while damage uncertainty seems to produce more conservative behavior than climate dynamics uncertainty. As concerns about uncertainty increase, cooperative and noncooperative policies tend to move close together. Asymmetries in concerns about uncertainty tend to produce large deviations in regional emissions policy at the noncooperative solution. We calculate the cost of robustness in terms of welfare. If aversion to ambiguity is sufficiently high, optimal regulation might not be possible. The result is associated with the existence of regional hot spots and temperature spillovers across regions, a situation which emerges in the real world. In such cases, deep uncertainty about the impacts of climate change makes robust regulation infeasible. We show that, if resources are devoted to learning, which reduces uncertainty concerns, robust regulation is facilitated.

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1 Introduction

The need for regional analysis of the impacts of climate change, in contrast to the global approach taken by Integrated Assessment Models (IAMs) such as DICE (Nordhaus and Sztorc, 2013; Nordhaus, 2014), has been clearly recognized in the literature (see, for example, Easterling, 1997). In fact, major IAMs such as RICE (e.g., Nordhaus, 2011), FUND (e.g., Anthoff and Tol, 2013), or PAGE (e.g., Hope, 2006) explicitly include regional components. The regional aspects have been extended to both regional temperature effects and regional economic effects (e.g., FUND, PAGE), or to regional economic effects with predictions about mean global temperature (e.g., RICE).

Multi-region modeling in climate change economics has been developed since RICE. Desmet and Rossi-Hansberg (2015) developed a spatial model of climate change, Krusell and Smith (2017) introduced a 20,000 region spatial model, and Hassler and Krusell (2018) discuss approaches to multi-region climate modeling. Regional aspects of climate change and associated policies have been introduced in low-dimensional IAMs in which regional temperature dynamics are driven by endogenous mechanisms of heat and precipitation transport from the Equator to the Poles (see Brock et al., 2013, 2014a; Brock and Xepapadeas, 2017, 2018; Cai et al., 2018). The climate science part of these models is based on one- or two-dimensional dynamic energy balance models, defined either in continuous space (e.g., North et al., 1981) or in discrete South-North "two-box" models (e.g., Langen and Alexeev, 2007). Energy balance climate models generate spatial variability of temperature across regions through the endogenous mechanism of heat transfer. Another approach which climate science uses to generate spatial temperature variation across regions is pattern or statistical downscaling, or statistical emulation methods (see, for example, Castruccio et al., 2014; Hassler et al., 2016; Krusell and Smith, 2017).

Regional temperature differentiation also emerges from the use of the transient climate response to cumulative carbon emissions (TCRE) on a regional basis. The TCRE embodies both the physical effect of CO_2 on climate and the biochemical effect of CO_2 on the global carbon cycle (e.g., Matthews et al., 2009, 2012; Knutti, 2013; Knutti and Rogelj, 2015; MacDougal et al., 2017). The TCRE, denoted by λ, is defined as $\lambda = \frac{\Delta T(t)}{CE(t)}$, where $CE(t)$ denotes cumulative carbon emissions up to time t and $\Delta T(t)$ the change in temperature during the same period. The approximate constancy of λ suggests an approximately linear relationship between a change in global average temperature and cumulative emissions. This roughly linear relationship has also been recognized by the IPCC (2013).

In a recent paper, Leduc et al. (2016) identify an approximately linear relationship between cumulative CO_2 emissions and regional temperatures. This relationship is quantified by regional TCREs or RTCREs. The RTCRE parameters range from less than 1°C per TtC for some ocean regions, 5°C per Ttc in the Arctic, and Leduc et al. (2016) consider their approach to be a novel application of pattern scaling. The high RTCRE in the Arctic is indicative of Arctic amplification. It is well-known that Arctic amplification could cause serious detrimental environmental effects which could be diffused to other regions south of the Arctic (IPCC, 2013; Brock and Xepapadeas, 2017). Thus one implication of adopting a regional representation of climate is that changes in the temperature in one region could generate damages in another region. It should be noted that the existence of geographical spillover damage effects across regions is supported by recent studies[1] and that this issue could be important for policy purposes but, as far as we know, is not addressed by large-scale IAMs.

In models of climate and economy, the use of the RTCRE approach to model regional differences instead of the structural approach based on an endogenous heat transfer mechanism could provide a simplified but realistic reduced-form mechanism for modeling regional temperature dynamics. Once explicit regional modeling for temperature dynamics has been adopted, an issue that has to be addressed is the concept of the economic optimum or economic equilibrium which will emerge in a coupled climate-economy model which seeks to explore climate policy. In major IAMs which involve

[1] See, for example, Francis et al. (2018) who suggest that further Arctic warming may favor persistent weather patterns that can lead to weather extremes, or Wu and Francis (2019).

optimization at the global or regional level such as DICE or RICE, the objective is the maximization of a global welfare criterion (as with DICE) or the sum of welfare criteria across regions (as with RICE). In the case of RICE, the solution for the given objective corresponds to a cooperative solution in which a social planner chooses emissions paths to maximize aggregate regional welfare subject to economic and climate constraints. This assumption implies that regions or countries have agreed, through some kind of international agreement, to follow cooperative emission paths.

This approach is useful in identifying optimal cooperative emission paths and indicating policy instruments such as carbon taxes to attain these paths. However, when it comes to the real world, countries or regions might not be willing to follow a cooperative solution. Although they may recognize the impact of climate change on global welfare, a specific region or country might be willing to choose emission paths which will maximize own welfare, which will be in general gross benefits from using fossil fuels net of own climate damages. When, however, the climate change is studied for the global economy, with countries or regions seeking to maximize own net welfare, the solution concept for analyzing policy issues is not that of a cooperative equilibrium. The appropriate solution concept is the solution of a noncooperative dynamic game.

Thus the explicit introduction of regional temperature dynamics enlarges the set of possible solution concepts. If all regions agree to cooperate then the appropriate solution is the cooperative one, but if they decide to maximize own net welfare then the appropriate solution if that of a noncooperative game. In the noncooperative case it is important to distinguish two cases. In the first case, countries maximize their own welfare by committing to an emission path and taking the response paths of other countries as given. This corresponds to the open loop Nash equilibrium (OLNE). In the second case, countries choose their emissions conditional on the observed temperature levels. In this case countries follow Markov perfect strategies and the solution concept is that of a feedback Nash equilibrium (FBNE).

The comparison of cooperative and noncooperative solutions was studied by Brock and Xepapadeas (2018) in the context of a two-box regional model (South-North) based on the Langen and Alexeev (2007) model with heat transport Polar-wise. In the present paper we study cooperative and noncooperative solutions in the context of a multi-regional model in which temperature dynamics are based on the RTCRE approach of Leduc et al. (2016). The contribution of this approach is that it allows for the study of an explicit multi-regional model, not just a two-region model, by using the simplified but realistic framework of RTCRE.

It is well-known that the study of climate change, and more specifically the economics of climate change, is characterized by fundamental uncertainties (Heal and Millner, 2014). As Pindyck (2017) points out, we know very little or nothing about parameters or functions which are fundamental in climate change economics, such as climate sensitivity or the damage function. In the same context, Anthoff and Tol (2013) and Gillingham et al. (2015) characterize parameters of climate-economy modeling which embody considerable uncertainties, while Lemoine (2010), Nordhaus and Moffat (2017) and Hassler et al. (2018) discuss in detail the impacts of uncertainty on climate sensitivity. Brock and Hansen (2017) distinguish three forms of uncertainty: (i) risk, which is the traditional case studied in economics in which objective or subjective probabilities are assigned to stochastic events; (ii) ambiguity, which is the case where the decision maker has concerns and is uncertain about how to weight alternative models for explaining a phenomenon, in a case where a benchmark model is "surrounded" by these alternative models or probability measures; and (iii) misspecification, which is associated with the way in which we use models which are imperfect approximations of the true model. We will refer to cases (ii) and (iii) as deep uncertainty.

Thus the second contribution of our paper is to introduce deep uncertainty or ambiguity and aversion to ambiguity or concerns about model misspecification into a multi-regional model of climate and the economy by using the robust control approach of Hansen and Sargent (e.g., Hansen and Sargent, 2001, 2008; Hansen et al., 2006).[2] Robust control methods have been applied to the economics of climate change (e.g., Hennlock, 2009; Athanassoglou and Xepapadeas, 2012; Anderson et al., 2014). In this literature, ambiguity or deep uncertainty was mainly associated with uncertainty of temperature dynamics or, equivalently, carbon stock dynamics. This type of deep uncertainty indirectly affects damages since the damage function depends on temperature or, equivalently, on carbon stock in the atmosphere. In the present paper we allow for deep uncertainty and aversion to ambigu-

[2]Note that since the case of risk can be analyzed as a limiting case of ambiguity, this approach encompasses risk analysis.

ity, from the regulator's point of view, with regard to damage uncertainty and uncertainty in temperature dynamics.

However climate and damage uncertainty is not the only source of uncertainty that we study in this paper. Recently, policy makers have pointed out that climate change policy introduces transition risks, which are risks that firms will face as climate policy is introduced (e.g., Carney, 2015), as well as physical risks, which in principle are captured by the introduction of uncertainty in temperature dynamics and the damage function. These transition risks which are associated with changes in policy and technology are not faced by the regulator who designs climate policy but by firms which are the subject of climate policy. In this context we also allow for policy uncertainty and study the decisions of a robust firm which has concerns about the actual climate policy.

To summarize, the present paper contributes to climate change economics by studying climate change policies in a multi-regional model based on the novel pattern scaling approach of RTCREs under conditions of deep uncertainty associated with temperature dynamics, regional climate change damages, and policy in the form of carbon taxes. Since our model is regional, we analyze both cooperative solutions in which a social planner chooses carbon emission policies to maximize global regional welfare and noncooperative solutions in which each region decides carbon emission policy by maximizing own welfare.

Our results suggest that in general under deep uncertainty robust control policies are more conservative regarding emissions, the higher aversion to ambiguity is, while damage uncertainty seems to produce more conservative behavior than climate dynamics uncertainty. Cooperative policies tend to be more conservative than noncooperative policies for similar concerns about uncertainty, but as concerns about uncertainty increase, policies tend to move closer to each other. Asymmetries in concerns about uncertainty tend to produce large deviations in regional emissions policy at the noncooperative solution. Cooperative solutions produce a global carbon tax, provided that the regulator equally weights regional welfares; noncooperative solutions on the other hand produce different regional carbon taxes. This could be an obstacle for global policy cooperation and could provide incentives for carbon leakage. We also show that robust climate change policies are more costly in terms of welfare relative to deterministic policy. Thus regulation when there are concerns about model misspecification and ambiguity aversion is costly. This brings forward the issue of learning. Thus, in the final section we consider the possibility of diverting resources to learning, which will reduce concerns about model misspecification.

Finally, a result which is important for policy design, and which does not emerge from traditional IAMs, is that if concerns about deep uncertainty in a region are sufficiently high and an increase in temperature in one region – such as Polar amplification – might affect damages in another region, optimal regulation might not be possible. This means that optimal policy aiming to attain a cooperative steady state or a steady state that satisfies conditions for a Nash equilibrium might not be possible. The result is associated with the existence of regional hot spots in which deep uncertainty about the impacts of climate change makes robust regulation infeasible. Recent reports suggesting that the Greenland Ice Sheet (GIS) might be getting close to a tipping point,[3] or the Francis and Vavrus (2014) jet stream instability research, are good examples of the existence of hot spots and make clear why our robustness formulation which emphasizes emergent hot spots and spillovers across regions is important for policy analysis. It is important to note that our results suggest that learning, which reduces model misspecification concerns, could make optimal robust regulation feasible

2 Modeling multiple-source deep uncertainty in multi-regional climate change economics

It is clearly understood that the climate modules used in the coupled models of climate and economy, whether they are embedded in high- or low-dimensional IAMs, represent an approximation of more complex models. Therefore, in order to obtain tractability and better understanding of the basic mechanisms driving the results, we will adopt the modeling approach which is based on the approximate linear relation between cumulative emissions and regional temperatures and which is quantified by the RTCREs. Having chosen an approximate model, we concentrate on deep uncertainty and concerns about model misspecification.

[3]See https://gracefo.jpl.nasa.gov/resources/33/greenland-ice-loss-2002-2016/, https://news.osu.edu/greenland-ice-melting-rapidly-study-finds/, and Bevis et al. (2018).

In the context of robust control methodology, ambiguity is introduced by allowing for a family of stochastic perturbations to a Brownian motion characterizing stochastic dynamics. The perturbations are defined in terms of measurable drift distortions. The misspecification error which expresses the decisions maker's concerns regarding departures from a benchmark model is reflected in an entropic constraint (Hansen et al., 2006; Hansen and Sargent, 2008). Ambiguity and concerns about the possibility that "an adversarial agent" often referred to as "Nature" will choose not the benchmark model but another one within an entropy ball, which will harm the decision maker's objective, are reflected in a quadratic penalty term which is added to the regulator's objective. This type of ambiguity has also been referred to as model uncertainty and Hansen and Sargent call the decision maker's optimization problem with a quadratic penalty the multiplier robust control problem. A crucial parameter of the problem is the robustness parameter, which reflects the decision maker's concerns about model uncertainty or his/her aversion to ambiguity. It has been shown that as the robustness parameter, which is positive, tends to to the limiting value of zero or infinity,[4] the decision problem is reduced to the standard optimization problem under risk – that is, a problem with no ambiguity aversion. When the robustness parameter increases from zero, then concerns about model uncertainty increase.[5]

If the distortion of the dynamics benchmark model at time t is denoted by h_{it}, then the drift distortion of the stochastic dynamics is expressed by

$$\sqrt{\varepsilon}\sigma\left(h_t + dW_t\right), \tag{1}$$

where σ is the volatility of the stochastic dynamics, W_t is a Brownian motion, and ε is a small noise parameter. If the term $h_t = 0$, then the problem is reduced to the case of risk. In the multiplier problem, the penalty associated with the distortion is expressed by

$$\frac{1}{2\theta\left(\varepsilon\right)} h_t^2, \tag{2}$$

where $\theta\left(\varepsilon\right)$ is the robustness parameter. It has been shown by Anderson et al. (2012, 2014) that if $\theta\left(\varepsilon\right) = \theta_0\varepsilon$, then if $\varepsilon \to 0$ the stochastic robust control

[4]The limiting value depends on the way in which the problem is formulated.

[5]If ambiguity vanishes when the robustness parameter tends to infinity, then increased ambiguity is associated with reduction in the robustness parameter.

problem is reduced to a simpler "deterministic robust control problem". To simplify and increase tractability, we adopt the assumption leading to a deterministic robust control problem.

To develop the climate model we assume that the globe is divided into $i = 1, ..., N$ regions. Note that Leduc et al. (2016) divide the globe into 21 land regions. Following the RTCRE approach, regional temperature dynamics, $\dot{T}_{it}$, under model uncertainty can be written as

$$\dot{T}_{it} = \lambda_i \mathbb{E}_t - B_i T_{it} + \sigma_i h_{it}, i = 1, ..., N, \tag{3}$$

where $\mathbb{E}_t = \sum_{i=1}^{N} E_{it}$ is aggregate global carbon emissions from all regions. Taking into account that a fraction of the heat stored in the atmosphere escapes, we assume that this is captured by the term $B_i T_{it}$, where $B_i > 0$ is the heat dissipation parameter in region i (see Naevdal and Oppenheimer, 2007; Lemoine and Rudik, 2014; and Heutel et al., 2016). In (3), the parameter σ_i represents volatility of regional temperature dynamics, and h_{it} the corresponding drift distortion reflecting deep uncertainty and concerns about misspecification of temperature dynamics. We assume that concerns about regional temperature dynamics are global for the region and, therefore, embody concerns about the RTCRE, which is also an uncertain parameter.[6]

To construct the economic part of the model, we follow Brock and Xepapadeas (2017, 2018) and consider a simple welfare maximization problem with logarithmic utility, where global world welfare is expressed by the sum of welfare in each region and is given by:

$$\int_{t=0}^{\infty} \mathrm{e}^{-\rho t} \sum_{i=1}^{N} v_i L_{it} \ln(y_{it} E_{it}^{\alpha} \mathrm{e}^{-\psi_i(T)}) dt, T = (T_1, ..., T_N), \tag{4}$$

where $y_{it} E_{it}^{\alpha}$, $0 < \alpha < 1$, E_{it}, $T = (T_1, ..., T_N)$, and L_{it} are regional output per capita, fossil fuel input or carbon emissions, temperatures in each region i at date t, and fully employed population, respectively. We assume exponential damages (see also Golosov et al., 2014)[7] and a quadratic ψ, to allow for the possibility of increasing regional marginal damages. Thus,

[6]For a thorough discussion of uncertainties associated with climate change and approaches which do not rely on expected utility, see for example Heal and Millner (2014).

[7]A large body of research in climate change economics assumes that damages are not exponential (e.g., Weitzman, 2010; Nordhaus and Sztorc, 2013). We use exponential damages for the same tractability reasons as in Golosov et al. (2014).

$$\psi_i(T) = \sum_{j=1}^{N} \left(d_{ij} T_j + \frac{1}{2} v_{ij} T_{ij}^2 + k_{it} T_j \right), d_{ij}, v_{ij} \geq 0, i = 1, ..., N, \quad (5)$$

where k_i represents ambiguity about damages in region i. Thus the damage function in region i embodies geographical damage spillovers, or cross effects, which are damages caused by temperature increases in other regions. For example, Arctic amplification may generate damages in terms of sea level rise or greenhouse gasses emitted by permafrost melting in southern regions. It is assumed that y_{it} and L_{it} are exogenously given. That is, we are abstracting away from the problem of optimally accumulating capital inputs and other inputs in order to focus on optimal emissions paths and fossil fuel taxes. In this context, y_{it} could be interpreted as the component of a Cobb-Douglas production function that embodies all other inputs along with technical change that evolves exogenously. We assume autarky for the multi-region model and no world market for loans (see also Hassler and Krusell (2012) for this approximation). Finally, v_i represents welfare weights associated with region i. To increase tractability, we assume that regional populations are immobile and normalize them to one and write $\omega_i = v_i L_i$, $\sum_i \omega_i = 1$. Furthermore, to simplify the exposition even more, we assume that fossil fuels are abundant in both regions and provided at zero cost. The use of fossil fuels is, however, costly in terms of climate.

Under these assumptions, the part which is relevant for the optimization of the world's welfare that corresponds to the cooperative solution for designing climate policies can be written as

$$W^c = \int_{t=0}^{\infty} \mathrm{e}^{-\rho t} \sum_{i=1}^{N} \omega_i [\alpha \ln E_{it}. - \left(\sum_{j=1}^{N} d_{ij} T_{jt} + \frac{1}{2} v_{ij} T_{jt}^2 \right)] dt, \quad (6)$$

where $\omega_i > 0$, $\sum_i \omega_i = 1$ are welfare weights associated with each region. If we impose ambiguity concerns regarding damages and temperature dynamics in region i, the cooperative solution will be the outcome of the following deterministic multiplier robust control problem:

$$\max_{\{E_{it}\}} \min_{\{k_{it},h_i\}} \tag{7}$$
$$\int_{t=0}^{\infty} \mathrm{e}^{-\rho t} \sum_{i=1}^{N} \omega_i \left[\alpha \ln E_{it} - \left(\sum_{j=1}^{N} d_{ij} T_{jt} + \frac{1}{2} v_{ij} T_{jt}^2 + k_{it} T_{it} \right) + \frac{k_{it}^2}{2\eta_i} + \frac{h_{it}^2}{2\theta_i} \right] dt$$
subject to (3) .

The noncooperative solution will be the outcome of regional welfare maximization under temperature dynamics, or

$$\max_{\{E_{it}\}} \min_{\{k_{it},h_i\}} \tag{8}$$
$$\int_{t=0}^{\infty} \mathrm{e}^{-\rho t} [\alpha \ln E_{it} - \left(\sum_{j=1}^{N} d_{ij} T_{jt} + \frac{1}{2} v_{ij} T_{jt}^2 + k_{it} T_{it} \right) + \frac{k_{it}^2}{2\eta} + \frac{h_{it}^2}{2\theta_i}] dt$$
subject to (3) .

3 Cooperative regional climate change policies

The cooperative regional climate policy emerges from the solution of problem (7). For this problem, the current value Hamiltonian is:

$$\mathcal{H}^C = \{ \sum_{i=1}^{N} \omega_i \left[\alpha \ln E_{it} - \left(\sum_{j=1}^{N} d_{ij} T_{jt} + \frac{1}{2} v_{ij} T_{jt}^2 + k_{it} T_{it} \right) + \frac{k_{it}^2}{2\eta_i} + \frac{h_{it}^2}{2\theta_i} \right]$$
$$\sum_{i=1}^{N} \mu_i [.\lambda_i \mathbb{E}_t - B_i T_{it} + \sigma_i h_{it}] \}. \tag{9}$$

In this robust control problem, the social planner chooses emissions E_{it} to maximize the Hamiltonian but the adversarial agent chooses distortions (k_{it}, h_{it}) to minimize the Hamiltonian. The optimality conditions for the control choices are:

$$\frac{\alpha\omega_i}{E_{it}} + \sum_{i=1}^{N} \mu_{it}\lambda_i = 0 \Rightarrow E_{it}^* = \frac{-\alpha\omega_i}{\sum_i \mu_{it}\ \lambda_i} \tag{10}$$

$$h_{it} = -(\theta_i/\omega_i)\sigma_i\mu_{it}, k_{it} = \eta_i T_{it}, i = 1, ..., N \tag{11}$$

$$\dot{\mu}_{it} = \rho\mu_{it} - \frac{\partial \mathcal{H}^C}{\partial T_i} \Rightarrow \tag{12}$$

$$\dot{\mu}_{it} = (\rho + B_i)\,\mu_{it} + \sum_{j=1}^{N} \omega_j\,(d_{ji} + v_{ji}T_{it}) + \omega_i\eta_i T_{it} \tag{13}$$

$$\dot{T}_{it} = \lambda_i \mathbb{E}_t^* - B_i T_{it} - \sigma_i^2\,(\theta_i/\omega_i)\,\mu_{it}. \tag{14}$$

From (58), it follows that if the social planner weights all regions equally, or $\omega_i = \omega$ for all i, then both regions should have the same emission paths,

$$E_{it} = E_{jt} = \frac{-\alpha\omega}{X_t} \equiv E_t^*, X_t = \sum_i \mu_{it}\lambda_i, i, j = 1, ..., N. \tag{15}$$

System (61)-(62) with $\mathbb{E}_t$,h_{it}, k_{it} substituted by their optimal values from (58)-(60) is the dynamic Hamiltonian system for the social planner. Since the robustness parameters $\{\eta_i, \theta_i\}$ reflect the "intensity" of the social planner's ambiguity, the impact of deep uncertainty on optimal policy can be studied by performing comparative analysis with respect to the robustness parameters.

Another characteristic of the solution is that $X_t = \sum_i \mu_{it}\lambda_i$ is the cost of the climate externality which consists of the sum of regional shadow temperature costs weighted by RTCREs. Thus the solution of the regional problem provides information about the contribution of each region to the global cost of the climate externality. The issue of regional contributions, which has been examined recently at the empirical level by Ricke et al. (2018), could help characterize the heterogeneity of climate impacts across the globe and provide information which could help policy design.

We examine the steady state of the cooperative solution. From (61), we obtain at a steady state:

$$\mu = -\frac{1}{(\rho + B_i)} \left[\sum_{j=1}^{N} \omega_j\,(d_{ij} + v_{ij}T_i) + \eta_i T_i \right]. \tag{16}$$

Substituting into (62) we obtain that the steady-state regional temperatures are solutions of the system:

$$(\omega_i \alpha) / \left(\sum_i \left[\frac{\lambda_i}{(\rho + B_i)} (\Gamma_i + (\Delta_i + \eta_i) T_i) \right] \right) - B_i T_i$$
$$+ \frac{\theta_i}{\omega_i} \frac{\sigma_i^2}{(\rho_i + B_i)} [\Gamma_i + (\Delta_i + \eta_i) T_i] = 0, \tag{17}$$

where

$$\Gamma_i = \sum_{j=1}^{N} \omega_j d_{ij}, \bar{v}_i = T_i \sum_{j=1}^{N} \omega_j v_{ij}. \tag{18}$$

Proposition 1 *If $B_i \neq 0$ for all i, then in an open neighborhood of the point $\mathbf{o} = (v_{11}, ...v_{1N}, ..., v_{N1}, ..., v_{NN}, \eta_1, ...\eta_N) = 0$, a steady state for the regional temperature anomalies which is determined by the the system (61)-(62) for $\dot{\mu}_{it} = 0, \dot{T}_{it} = 0$ exists.*

For the proof see Appendix 1.

Thus it is expected that for small second-order parameters in the damage function and small robustness parameters for damages, a steady state will exist. Furthermore at point $\mathbf{o}$, the Jacobian determinant of the linearized Hamiltonian system (61)-(62) has N negative eigenvalues $\{-B_1, ..., -B_N\}$ and N positive eigenvalues $\{(\rho_1 + B_1), ..., (\rho_N + B_N\}$ and therefore the steady state at $\mathbf{o}$ has the saddle point property.

The saddle point property implies that the social planner can choose initial values and a path for regional emissions, determined by (58), so that the world economy will converge along a two-dimensional manifold to the socially optimal steady state. The paths of the costate variables μ_{it} will determine the optimal carbon tax. The steady-state distortions $(\bar{h}_i, \bar{k}_i)$ are obtained directly from (60) by substituting the corresponding steady states for regional temperatures and their shadow costs.

To get a clearer picture of robustness on the steady state associated with the regulator's global optimum, we consider a two-region problem and we derive the following result.

Proposition 2 *Sufficiently high aversion to ambiguity reflected in the robustness parameters $\eta_i, \theta_i, i = 1, 2$ or damage spillovers may result in the loss of saddle point stability of the cooperative steady state.*

For the proof see Appendix 1.

The meaning of this is that if the Hamiltonian system characterizing the steady state has less than two negative eigenvalues, then the regulator cannot choose appropriate initial values for regional robust emissions in order to determine the robust policy which will steer the system toward the cooperative steady state. As shown in the proof of proposition 2, this loss of saddle point stability could be caused by high robustness parameters in either region, or strong damage spillovers. In this case, the region in which concerns about model misspecification prevent robust policies toward a cooperative steady state can be regarded as a hot spot. Given that such hot spots have been identified in reality,[8] this result could be important for policy purposes.

The inability to design optimal robust policies under ambiguity could become more profound if the robustness parameters, η, θ, are parametrized as increasing functions of the temperature vector $(T_1(t), T_2(t))$. The rationale for such a parameterization comes from climate scientists who worry that as the stock of carbon in the atmosphere rises and regional temperatures rise, the climate system is being pushed into unknown realms, e.g., the ppm atmospheric carbon is beyond levels seen for the last 800,000 years. Thus damage concerns are presumably being pushed beyond past experiences documented in data.[9]

The trace of the the linearization matrix in this case will be

$$traceJ = [-B_1 - \left(\frac{\sigma_1^2 \mu_1}{\omega_1}\right) \frac{\partial \theta_1}{\partial T_1}] + [-B_2 - \left(\frac{\sigma_2^2 \mu_2}{\omega_2}\right) \frac{\partial \theta_2}{\partial T_2}] + (\rho + B_1) + (\rho + B_2). \tag{19}$$

Since $\mu_i > 0$, the terms on the principal diagonal of K could be positive, making the possibility of having a positive K even stronger. This strengthens the argument that high ambiguity could make robust control infeasible.

[8] See for example https://gracefo.jpl.nasa.gov/resources/33/greenland-ice-loss-2002-2016/, https://news.osu.edu/greenland-ice-melting-rapidly-study-finds/, or Francis and Vavrus (2015).

[9] Diffenbaugh et al. (2017) study the impact of global warming on unprecedented extreme climate events and provide results indicating that global warming has increased the severity and probability of the hottest monthly and daily events in more than 80% of the observed area and has increased the probability of the driest and wettest events in approximately half of the observed area.

The possibility that a large robustness parameter which indicates high ambiguity and concerns regarding the estimation of regional damages or temperature dynamics might prevent convergence to a steady state can be associated with the emergence of spatial hot spots in stochastic robust control problems in which high ambiguity may impede optimal regulation for the whole spatial domain (see Brock et al., 2014b; Xepapadeas and Yannacopoulos, 2017).[10]

Although it is clear from (17) and (16) that ambiguity affects steady states, emission policies and carbon taxes, the nonlinearities and the dimensionality of the problem do not allow the derivation of tractable comparative static results. To obtain some insights into the impacts of ambiguity, we resort to simulations.

3.1 Cooperation under ambiguity: simulation results

In designing our simulations, we chose to concentrate on a two-region model. More specifically, we used the two-box geographical structure of Langen and Alexeev (2007) and Alexeev and Jackson (2013), which consists of a single hemisphere, the Northern hemisphere, with two regions divided by the 30th latitude, which yields a similar surface area for the two regions.[11] Using Leduc et al. (2016), we associated approximate RTCREs with each region. The next step was to calibrate regional damage functions.

Using data from Berkeley Earth Surface Temperatures (BEST),[12] we set approximate average annual mean land temperature for 1951-1980 at

average temperature: $0°$- $33°$N $\approx 26°$C, $33°$N-$90°$N $\approx 12°$C.

Then, adding temperature anomalies from NASA data,[13] we calculated 2017 average temperatures. We used these temperatures as inputs in the

[10] In this case, the breakdown of robust control was associated with the nonexistence of a solution for the corresponding Hamilton-Jacobi-Bellman equation of the robust control multiplier problem.

[11] This specific two-region model provides results which can be seen in terms of economic development between North and South. It also facilitates numerical calculations. We focus on the Northern Hemisphere because the geography is very different from that of the Southern Hemisphere, and most of the world's economic activity takes place north of the Equator. Evidence indicates that 88% of the global population lives in the Northern Hemisphere (http://www.radicalcartography.net/index.html?histpop).

[12] See https://climatedataguide.ucar.edu/climate-data/global-surface-temperatures-best-berkeley-earth-surface-temperatures).

[13] See https://data.giss.nasa.gov/gistemp/tabledata_v3/ZonAnn.Ts+dSST.csv) with a base period 1951-1980.

regression reported by Tol (2018) for estimating climate damages as a proportion of GDP. For the 0°- 33°N we used the World Bank's GDP per capita for "low and middle income countries", which for 2017 is 4,494.8 in 2010 $US, while for the 33°N- 90°N we used the World Bank's GDP per capita for "high income countries", which for 2017 is 41,538.6 in 2010 $US. The losses from climate change as a proportion of GDP were calculated as:

loss as % of GDP (0°- 33°N) = 11.52%,

loss as % of GDP (33°N- 90°N) = 1.58%.

Then, the parameters of the value functions were calibrated using the relations:

$$(1-\gamma_1) = \exp\left[d_{11}\Delta T_1 + d_{12}\Delta T_2 + \frac{1}{2}v_{11}\left(\Delta T_1\right)^2 + \frac{1}{2}v_{12}\left(\Delta T_2\right)^2\right] \tag{20}$$

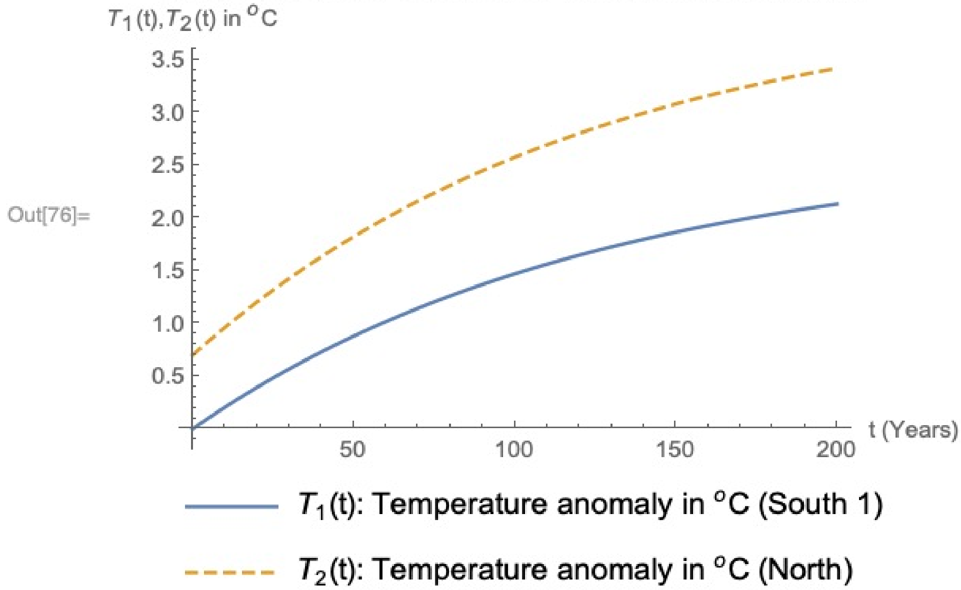
$$(1-\gamma_2) = \exp\left[d_{21}\Delta T_1 + d_{22}\Delta T_2 + \frac{1}{2}v_{21}\left(\Delta T_1\right)^2 + \frac{1}{2}v_{22}\left(\Delta T_2\right)^2\right], \tag{21}$$

where $\gamma_i, i = 1, 2$ are the damages as a proportion of GDP, and ΔT_i the temperature anomalies in each region. We consider two scenarios. In the first – which we call "No cross effects (NCE)" – it is assumed that the temperature anomaly in one region does not affect damages in the other region, or $d_{ij} = v_{ij} = 0, i, j = 1, 2, i \neq j$.

In the second, "Cross effects (CE)", we assume that the temperature anomaly in the North increases damages in the South by 1% of GDP. This implies that $(d_{12}, v_{12}) \neq (0, 0$, but $(d_{21}, v_{21}) = (0, 0)$. This assumption reflects Polar amplification effects which could cause additional damages to the South.[14] The parameters of the damage function, along with the rest of the parameters used in the simulation, are shown in table A1 of Appendix 2. In the simulations, we first obtain numerical solutions for the steady state of the nonlinear system (61)-(62). This corresponds to a steady state for the temperature anomalies and the corresponding shadow cost for the anomaly – that is, the costate variable – in each region. Then the Hamiltonian system (61)-(62) is linearized at the steady state and its Jacobian matrix is

[14] In Liu et al. (2017), exposure to climate change refers to damages from climate change and it is pointed out that for the high IPCC emissions scenario 8.5, the average exposure for Africa is over 118 times greater than it has been historically, while the exposure for Europe increases by only a factor of four.

Figure 1: Optimal Cooperative Temperature Anomalies



calculated. It is verified that this matrix has two negative and two positive eigenvalues; therefore the steady state is a saddle point, and transversality conditions at infinity are satisfied. The system of the four linear ordinary differential equations (ODEs) resulting from the linearization of (61)-(62) is solved with initial values for the temperature anomalies and terminal values for the steady state vector, by setting the constants corresponding to positive eigenvalues equal to zero. This allows us to obtain the optimal transition paths toward the steady state in the neighborhood of the steady state. The results regarding the steady state and the transition paths are shown below.

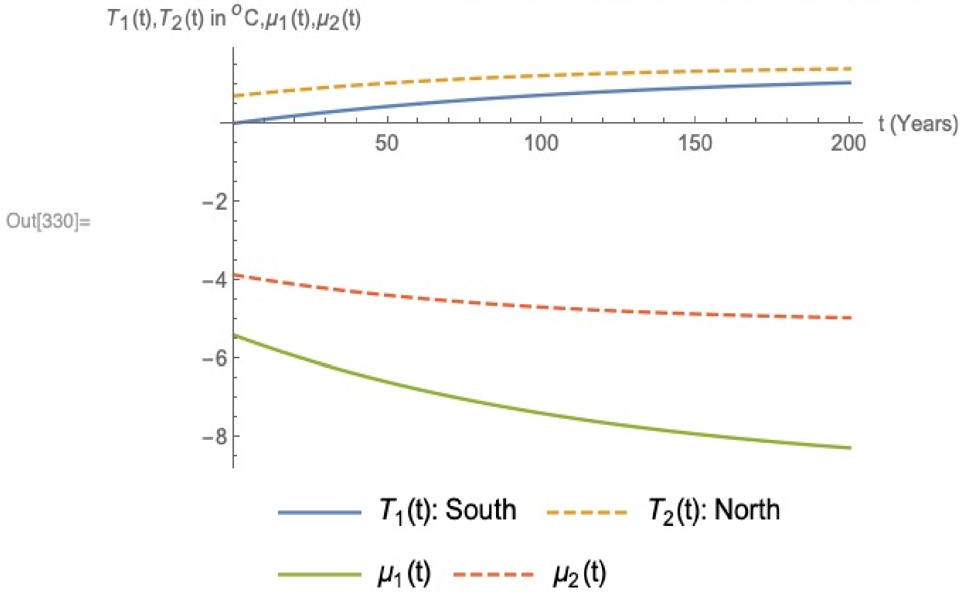
Table 1: Cooperative steady state – no ambiguity and misspecification concerns, $\eta_i = \theta_i = 0$, $i = 1, 2$

- NCE: $\bar{T}_1 = 2.67$ $\bar{T}_2 = 4.10$ $\bar{\mu}_1 = -4.37$ $\bar{\mu}_2 = -0.20$
- CE: $\bar{T}_1 = 1.01$ $\bar{T}_2 = 1.55$ $\bar{\mu}_1 = -4.37$ $\bar{\mu}_2 = -5.24$

The results suggest that cross effects increase global damages by increasing damages in the South and therefore reduce the optimal steady-state anomalies. In Figure 1 the paths are shown without cross effects.

In Figure 2 we present the cooperative temperature anomaly paths for a robust problem with robustness parameters $(\eta_1, \eta_2, \theta_1, \theta_2) = (0.1, 0, 0.05, 0.05)$

Figure 2: Optimal Robust Temperature Anomalies and Shadow Costs



and cross effects. This parametrization corresponds to a situation in which there is ambiguity about damages in the South, but in the North these ambiguities vanish. A possible justification is that since damages in the South are much higher as a proportion of regional GDP than in the North, and per capita GDP in the South is relatively smaller, the planner should be more concerned about misspecification in the damage function of the South when optimal policy is calculated. In this simulation it is assumed that the planner's concerns about regional temperature dynamics are the same for both regions.

In the linear approximation of Figure 2, the optimal robust emission policy means that for given initial values of the temperature anomaly, (0, 0.5) degrees Celsius, there are initial values for the costate variables paths, shown in Figure 2, such that if emissions are determined according to (58), the whole system will converge to the steady state along the stable manifold. Given the paths for the costate variables, the optimal robust emission path is a hyperbola which converges to the steady-state emissions. The cost of externality $X_t = -\sum_i \mu_{it}\lambda_i$, which determines the optimal robust carbon tax, is concave increasing and converges to the steady state.

3.2 The impact of ambiguity and preferences for robustness

To understand the impact of ambiguity and robust control we need to observe that the "choices" of distortions by the fictitious adversarial agent determined by (60) will provide guidelines for the regulator in order to design robust policies. These policies will be optimal if the "optimal distortions" which correspond to the worst scenario are realized. Since these distortions may not emerge, the steady states for the temperature anomalies implied by the solution of (17) and (16) can be interpreted as the steady states which will be attained under optimal robust control if the worst scenario emerges. On the other hand, since the regulator is trying to design regulation appropriate for the worst scenario, the emission policy and the tax policy realized will depend on ambiguity and preferences for robustness, i.e., parameters η, θ.

3.2.1 Impact on steady-state temperature anomalies

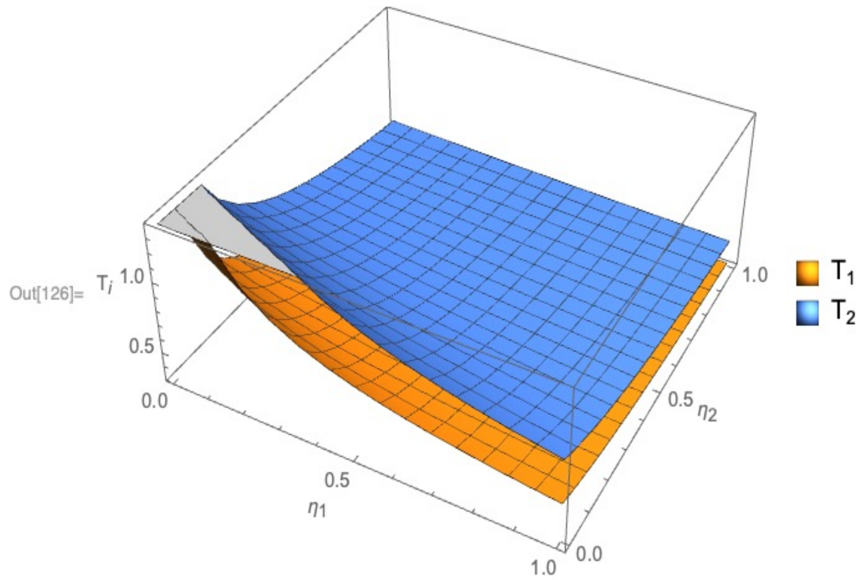
To provide a better picture of the mechanisms governing steady-state temperatures when the robustness parameters change, we consider two polar cases of the model. In the first, the regulator is concerned about misspecification in temperature dynamics but not in damages (i.e., $\eta_i = 0, \theta_i > 0$, $i = 1, 2$), while in the second, the regulator is concerned only about damage-related ambiguity (i.e., $\eta_i > 0, \theta_i = 0$). To make things simple, we assume a linear damage function and thus constant marginal damages, and no cross effects.

Substituting $\eta_i = 0, \theta_i > 0$ into (16), (17), solving for the steady state and taking the derivative with respect to θ, we obtain

$$\frac{\partial T_1}{\partial \theta_1} = \frac{\sigma_1 d_{11}}{B_1 (B_1 + \rho)} > 0, \frac{\partial T_2}{\partial \theta_2} = \frac{\sigma_2 d_{22}}{B_2 (B_2 + \rho)}. \tag{22}$$

We do the same, by substituting $\eta_i > 0, \theta_i = 0$ into (16), (17). However, due to nonlinearities, the result is not tractable, so we provide a graph of the functions $T_i (\eta_1, \eta_2)$, $i = 1, 2$, in Figure 3.

Figure 3: The impact of $\eta -$ ambiguity



The results indicate that, under robust control and provided that the worst scenario emerges, increasing the $\theta -$ ambiguity with no $\eta -$ ambiguity leads to higher steady-state emissions, because the choice of the adversarial agent is equivalent to increasing the impact of emissions on the change in temperature. In this case, the regulator's policy is to reduce emissions and increase the carbon tax, but the distortion which increases the temperature rate of growth eventually leads to a relatively higher steady-state anomaly relative to the no ambiguity case. On the other hand, increasing $\eta -$ ambiguity with no $\theta -$ ambiguity leads to lower steady-state temperatures under robust control. When both types of ambiguity exist, there are two opposite impacts on steady-state anomalies and the final outcome will depend on the relative strength of the effects.

To examine the impact of simultaneous changes in both robustness parameters, we calculate optimal cooperative steady states and paths for the values of $(\eta, \theta) \in \{0, 0.05, 0.1, 0.125\}$. In Figures 4 and 5 we depict the cooperative steady states when η takes the values $\{0, 0.05, 0.1, 0.125\}$ and θ is kept constant at 0 in panel (a), at 0.05 in panel (b), at 0.1 in panel (c), and at 0.125 in panel (d).[15] In Figure 4, "symmetric" refers to the case in which the robustness parameters are the same in both regions. In Figure 5,

[15]The way in which the robust control problem is set up means that an increase in the robustness parameters indicates an increase in concern about model misspecification and ambiguity.

“asymmetric” refers to the case in which the damage robustness parameter η_2 associated with the North is zero, while $\eta_1 = \{0, 0.05, 0.1, 0.125\}$.

Figure 4: Robust cooperative steady states, symmetric, no cross effects

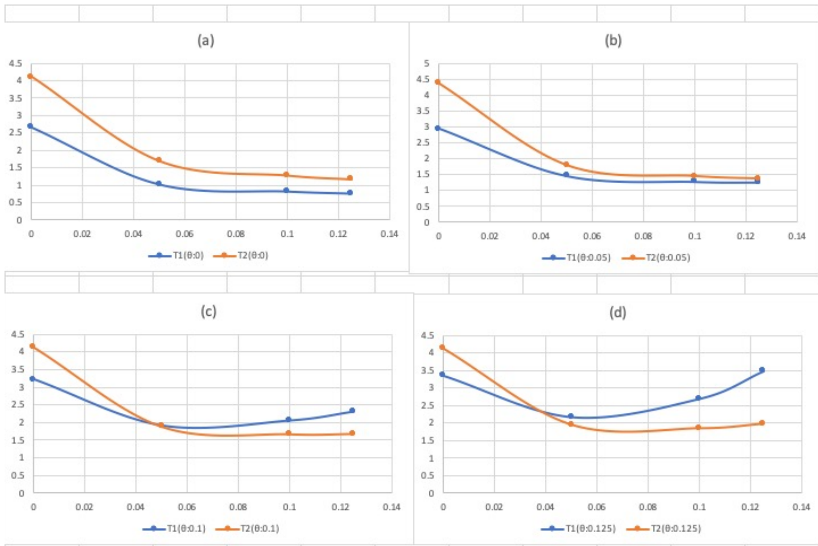
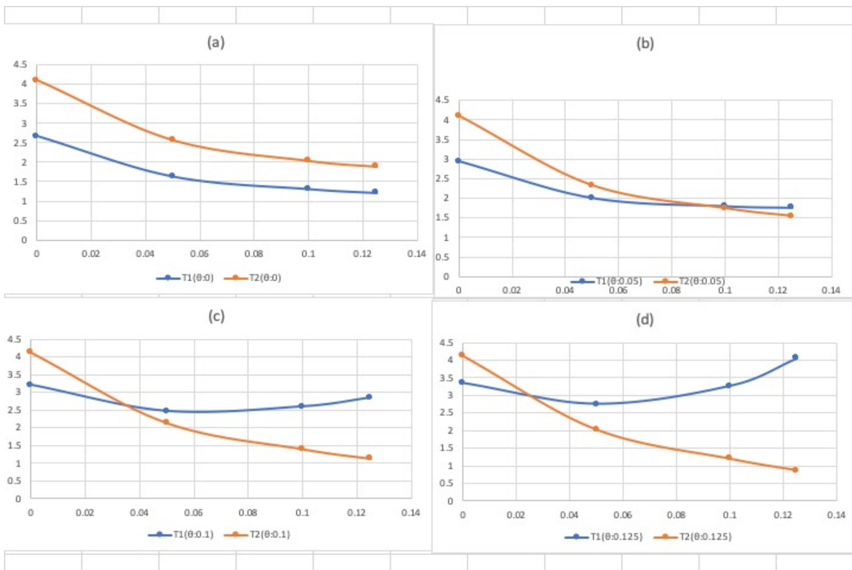


Figure 5: Robust cooperative steady states, asymmetric, no cross effects



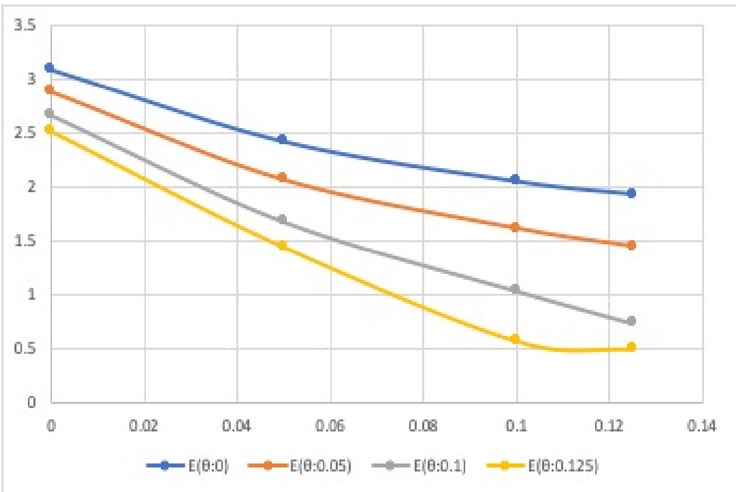
The graphs support the previous results. The steady-state anomalies are declining with η when θ is low but become increasing in η for larger values of θ.

3.2.2 Impact on policy

The impact of ambiguity and preference for robustness on the steady state will emerge if the worst scenario emerges. The emergence of this scenario is by no means certain since this case serves the purpose of helping the regulator design an emission policy which will be optimal *even if* the worst scenario is realized. Thus the best way to study the impact of changes in the strength of ambiguity is through its impact on emission policy.

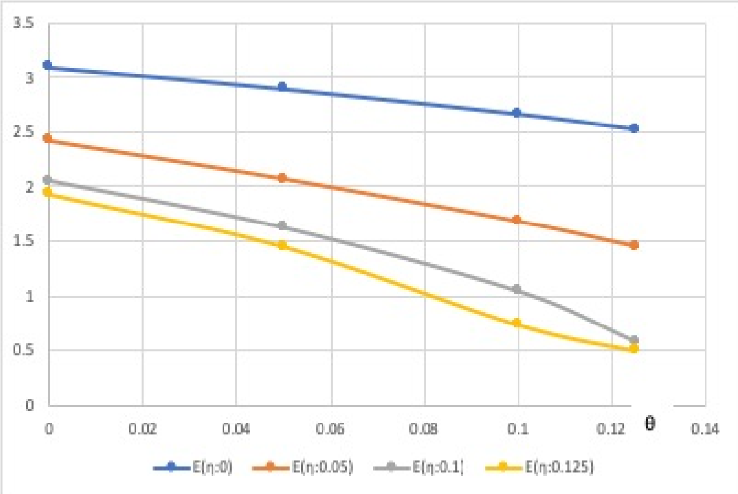
In Figures 6 and 7 we present steady-state optimal robust emissions as functions of the robustness parameters η, θ. In Figure 6 we depict changes in emissions as a function of η for given θ, while in Figure 7 we depict changes in emissions as a function of θ for given η. It is clear that increased ambiguity causes the robust regulator to reduce emissions in each region.[16] The slope in Figure 6 relative to Figure 7, and the shifts in Figure 7 relative to Figure 6, suggest that ambiguity regarding damages has a relatively stronger effect on optimal robust emissions than temperature dynamics ambiguity. The pattern is similar for the case without cross effects. If we allow asymmetry, in the sense that the regulator is more concerned about deep uncertainty in the damage function of the South relative to the North, emissions are slightly higher relative to the symmetric case but the qualitative pattern is still the same.

Figure 6: Steady-state robust emissions as a function of η



[16]The specific solution is symmetric and thus emissions are the same in each region.

Figure 7: Steady-state robust emissions as a function of θ



The emission paths toward the steady state are the usual hyperbolas, with the same qualitative behavior. An increase in η or θ will shift the robust emission paths downwards.

3.3 Policy uncertainty and decentralized implementation

In a global market economy, the representative "small" consumer takes everything regarding climate change as fixed, beyond his/her control, and has no decision to make. The representative firm, however, does have decisions to make regarding emissions. We assume that the representative firm is subject to an emission or carbon tax, and to simplify things we assume that energy has no private costs. The problem for the firm in each region is:

$$\max_{E_{it}} \left[\alpha \ln E_{it} - \tau_i E_{it} \right], \tag{23}$$

with optimality conditions

$$\frac{\alpha}{E_{it}} = \tau_{it} \Rightarrow E_{it} = \frac{\alpha}{\tau_{it}}. \tag{24}$$

Combining (24) with (58), it follows that the optimal emission tax will be:

$$\frac{\alpha}{\tau_{it}} = \frac{-\omega_i \alpha}{\sum_i \lambda_i \mu_{it}} \Rightarrow \tau^*_{it} = -\frac{1}{\omega_i} \left(\sum_i \lambda_i \mu_{it} \right) > 0. \tag{25}$$

It is clear that unless the regulator attaches different welfare weights to different regions, the optimal carbon tax will be the same across regions. The higher the welfare weight is, the lower the optimal carbon tax. Since in all numerical simulations the costates μ_{it} are negative and declining with time, the optimal carbon tax increases through time until it reaches a steady state. The time paths of the costate variable suggest that the optimal tax will be increasing and concave. This tax, however, is expressed in terms of utils. To express it in terms of consumption at date t, it should be divided by the marginal utility of consumption, which is $1/y_{it}E_{it}^{a}\exp\left(-D_i\left(T_t\right)\right)$. Since y_{it} is expected to increase over time like $\exp\left(g_i t\right)$, this would give a convex increasing tax ramp in date t consumption units. This implies that our tax ramp is compatible in consumption units with results obtained by Nordhaus or Golosov et al. Furthermore, in all numerical simulations the steady-state costate values increase as the ambiguities in terms of η_i and θ_i increase. Thus, the optimal tax increases with ambiguity from the regulator's point of view.

Optimal taxation of the form discussed above captures mainly physical risks and uncertainty associated with climate change as seen from the regulator's point of view. To capture policy risks and ambiguity associated with firms' responses to climate policy, we need to introduce ambiguity aversion and preferences for robustness in the problem of the firm which maximizes profits by taking environmental policy as exogenous to the firm but uncertain. Thus we introduce policy uncertainty or ambiguity by considering the profit maximization of a firm with preference for robustness and concerns about the size of the carbon tax which will apply to the firm's emissions under two types of robustness: (i) additive policy uncertainty, and (ii) multiplicative policy uncertainty.

Under additive policy uncertainty, the firm solves:

$$\max_{E_{it}} \min_{f_{it}} \left[\alpha \ln E_{it} - \left(\tau_{it} + f_{it}\right) E_{it} + \frac{1}{2\xi_i} f_{it}^2\right], \tag{26}$$

with optimality conditions

$$f_{it} = \xi_i E_{it} \tag{27}$$

$$\frac{\alpha}{E_{it}} = \tau_{it} + f_{it} = \tau_{it} + \xi_i E_{it} \Rightarrow \alpha = \tau_{it} E_{it} + \xi_i E_{it}^2. \tag{28}$$

Taking the total differential of (28), we obtain

$$\frac{dE_{it}}{d\xi} = -E_{it}/(\tau_{it} + 2\xi_i E_{it}) < 0. \tag{29}$$

Thus an increase in policy uncertainty will reduce emissions for a given carbon tax.

Taking the positive root of the quadratic (28), the emissions of the robust representative firm are

$$E_{it} = \frac{1}{2\xi_i}\left[-\tau_{it} + \left(\tau_{it}^2 + 4\alpha\xi_i\right)^{1/2}\right]. \tag{30}$$

Combining (30) with (58), it follows that the optimal emission tax, if the regulator takes into account the firms concerns about policy uncertainty, is the solution of:

$$\left[-\tau_{it} + \left(\tau_{it}^2 + 4\alpha\xi_i\right)^{1/2}\right] = \frac{-2\xi_i\omega_i\alpha}{\sum_i \lambda_i\mu_{it}}. \tag{31}$$

Under multiplicative policy uncertainty, the firm solves

$$\max_{E_{it}} \min_{f_{it}} \left[\alpha \ln E_{it} - \left(\tau_{it}(1+f_{it})\right) E_{it} + \frac{1}{2\zeta_i} f_{it}^2\right], \tag{32}$$

with optimality conditions

$$f_{it} = \zeta_i \tau_{it} E_{it} \tag{33}$$

$$\frac{\alpha}{E_{it}} = \tau_{it}(1+f_{it}) = \tau_{it}\left(1 + \zeta_i\tau_{it}E_{it}\right) \Rightarrow \alpha = \tau_{it}E_{it} + \zeta_i\tau_{it}^2 E_{it}^2. \tag{34}$$

Solving for E_{it}, we obtain

$$E_{it} = \frac{1}{2\tau_{it}\zeta_i}\left[-1 + (1 + 4a\zeta_i)^{1/2}\right], \tag{35}$$

and the optimal tax, if the regulator takes into account the firm's concerns about policy uncertainty, is the solution of

$$\frac{1}{2\tau_{it}\zeta_i}\left[-1 + (1 + 4a\zeta_i)^{1/2}\right] = \frac{-\omega_i\alpha}{\sum_i \lambda_i\mu_{it}}. \tag{36}$$

Taking the total differential of (34), we obtain

$$\frac{dE_{it}}{d\zeta} = -\tau_{it}^2 E_{it}^2/(\tau_{it} + 2\zeta_i \tau_{it}^2) < 0. \tag{37}$$

Thus, as in the case of additive uncertainty, an increase in policy uncertainty will reduce emissions for a given carbon tax.

If the regulator does not consider the possibility that the firm is concerned about policy uncertainty and sets the optimal carbon tax in the way described in the previous section, then conditions (28) or (34) suggest that the robust equilibrium for the decentralized firm is more "conservative" in emissions than the robust planner. Thus, because of policy uncertainty, it may be optimal to set the tax rate a bit below the optimal "Pigouvian" rate.

4 Noncooperative regional climate policies

We move now from the situation in which a global social planner exists and we consider the case in which there is a social planner in each region that decides the optimal emission path by taking into account damages in own region. Thus each region determines its own desired emission path. This behavior leads to a Nash equilibrium outcome, either open loop or feedback.

We find it reasonable to assume that since each regional planner knows that the total amount of emissions drives temperature dynamics in both regions, each planner must take into account the influence of its own energy policy on the temperature dynamics of the other region. Thus we provide the following definition for a robust open loop Nash equilibrium (ROLNE) for the two-region world.

Definition: *A robust OLNE (ROLNE)* $\{E_{it}^*, h_{it}^*, k_{it}^*\}$ *satisfies the fol-*

lowing conditions: $\{E_{1t}^*, h_{1t}^*, k_{1t}^*\}$ *solves the zero-sum game problem*

$$\max_{\{E_{1t}\}} \min_{\{k_{1t},h_{1t}\}}$$
$$\int_{t=0}^{\infty} \mathrm{e}^{-\rho t}[\alpha \ln E_{1t} - \left(\sum_{j=1}^{2} d_{1j} T_{jt} + \frac{1}{2} v_{1j} T_{jt}^2 + k_{1t} T_{1t} \right) + \frac{k_{1t}^2}{2\eta_1} + \frac{h_{1t}^2}{2\theta_1}] dt \tag{38}$$

subject to

$$\dot{T}_{1t} = -B_1 T_{1t} + \lambda_1 \left(E_{1t} + E_{2t}^* \right) + \sigma_1 h_{1t}, T_{10} \text{ given} \tag{39}$$
$$\dot{T}_{2t} = -B_2 T_{2t} + \lambda_2 \left(E_{1t} + E_{2t}^* \right) + \sigma_2 h_{2t}, T_{20} \text{ given }, \tag{40}$$

while $\left\{E_{2t}^*, h_{2t}^*, k_{1t}^2\right\}$ *solves the zero-sum game problem*

$$\max_{\{E_{2t}\}} \min_{\{k_{2t},h_{2t}\}}$$
$$\int_{t=0}^{\infty} \mathrm{e}^{-\rho t}[\alpha \ln E_{2t} - \left(\sum_{j=1}^{2} d_{2j} T_{jt} + \frac{1}{2} v_{2j} T_{jt}^2 + k_{2t} T_{1t} \right) + \frac{k_{2t}^2}{2\eta_2} + \frac{h_{2t}^2}{2\theta_2}] dt \tag{41}$$

subject to

$$\dot{T}_{1t} = -B_1 T_{1t} + \lambda_1 \left(E_{1t}^* + E_{2t} \right) + \sigma_1 h_{1t}, T_{10} \text{ given} \tag{42}$$
$$\dot{T}_{2t} = -B_2 T_{2t} + \lambda_2 \left(E_{1t}^* + E_{2t} \right) + \sigma_2 h_{2t}, T_{20} \text{ given.} \tag{43}$$

In this definition of ROLNE, the planner in region 1 (South) takes the choices of the planner in region 2 (North) as given, including the planner in region 2's choice of robustness distortions and choice of fossil fuel usage. Note that the planner in the South still has a bit of control over the evolution of temperature in the North because its choice of fossil fuel use in its own region (the South) goes into the atmospheric pool which influences the temperature in both regions. But only the contribution of region 1 to the atmospheric CO_2 path which governs temperature in both regions is taken into account by region 1, since all other variables have already been path chosen by region 2 and are, hence, fixed over time. Here fossil fuel reserves are infinite in both regions, so the only state variables are the temperature state variables. Since coal reserves are so large in the real world, assuming infinite reserves is probably a reasonable approximation to reality for a first cut.

Denote by $\mu_{11t}, \mu_{12t}, \mu_{21t}, \mu_{22t}$ the costate variables for region 1 and region 2 associated with the state equations (39),(40),(42),(43), respectively. The optimality conditions for regional emissions and regional robust distortions are:

$$E_{1t}^* = \frac{-\alpha}{\lambda_1\mu_{11} + \lambda_2\mu_{12}}, E_{2t}^* = \frac{-\alpha}{\lambda_1\mu_{21} + \lambda_2\mu_{22}} \tag{44}$$

$$h_{1t}^* = -\theta_1\sigma_1\mu_{11t}, h_{2t}^* = -\theta_2\sigma_2\mu_{22t} \tag{45}$$

$$k_{it}^* = \eta_i T_{it}, i = 1, 2. \tag{46}$$

Let $\{T_{2t}^{*,1}\}$, $\{T_{1t}^{*,2}\}$ be the best reply paths of temperatures in region 2 "chosen" by region 1, and temperatures in region 1 "chosen" by region 2, respectively. We assume that $\{T_{2t}^{*,1}\} = \{T_{2t}^*\}$, and $\{T_{1t}^{*,2}\} = \{T_{1t}^*\}$ where $\{T_{it}^*\}$ is the temperature path in each region. If the equalities do not hold, then there is a nonexistence problem for Nash equilibrium. If a Nash equilibrium exists, the state and costate equations satisfy the following system of ODEs:

$$\dot{T}_{1t} = -B_1 T_{1t} + \lambda_1 \left(E_{1t}^* + E_{2t}^*\right) + \sigma_1 h_{1t}^*, T_{10} \text{ given} \tag{47}$$

$$\dot{T}_{2t} = -B_2 T_{2t} + \lambda_2 \left(E_{1t}^* + E_{2t}^*\right) + \sigma_2 h_{2t}^*, T_{20} \text{ given} \tag{48}$$

$$\dot{\mu}_{11t} = \left(\rho + B_1\right)\mu_{11t} + \left(d_{11} + v_{11}T_{1t} + k_{1t}^*\right) \tag{49}$$

$$\dot{\mu}_{12t} = \left(\rho + B_2\right)\mu_{12t} + \left(d_{12} + v_{12}T_{2t}\right) \tag{50}$$

$$\dot{\mu}_{21t}. = (\rho + B_1)\mu_{21t} + (d_{21} + v_{21}T_{1t}) \tag{51}$$

$$\dot{\mu}_{22t} = \left(\rho + B_2\right)\mu_{22t} + \left(d_{22} + v_{22}T_{2t} + k_{2t}^*\right). \tag{52}$$

The existence of a steady state satisfying the conditions of OLNE can be verified by setting the η, θ, v parameters equal to zero in the system (47)-(52). The linearization matrix of this system is diagonal with two negative real eigenvalues $\{-B_1, -B_2\}$ and four positive eigenvalues $\{\rho + B_1, \rho + B_2\}$ with multiplicity 2 for each one. Then, using the implicit function theorem as in proposition 1, the existence of a ROLNE steady state can be shown for small robustness parameters.

To compare the cooperative steady-state solution with the corresponding noncooperative OLNE, we first start by comparing the deterministic solutions for which $\eta_i = \theta_i = 0, i = 1, 2$.

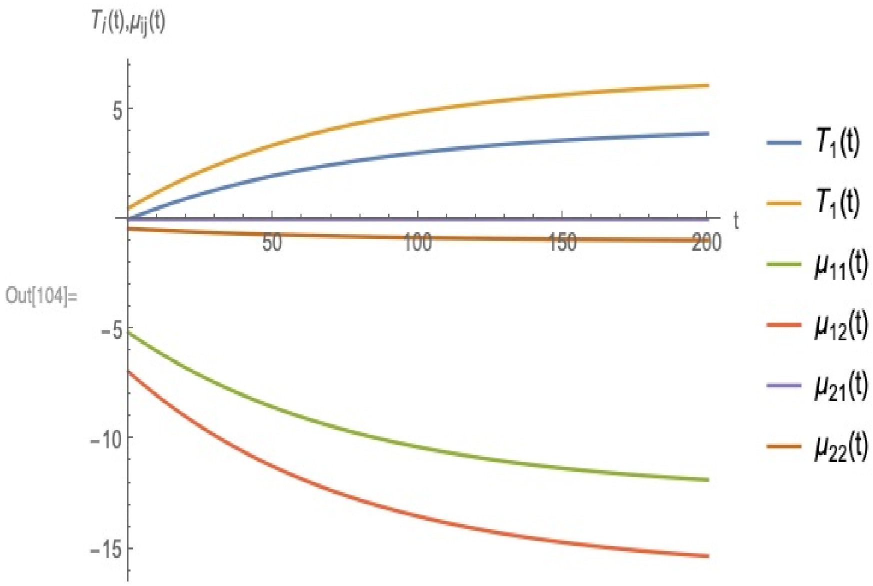
Table 2: Noncooperative steady state – no ambiguity and misspecification concerns*, $\eta_i = \theta_i = 0$, $i = 1, 2$.

- NCE: $\bar{T}_1 = 4.43(61\%)$ $\bar{T}_2 = 6.81(61\%)$
- CE: $\bar{T}_1 = 4.25(421\%)$ $\bar{T}_2 = 6.54(421\%)$

(*) Proportional deviations relative to the cooperative solution shown in parentheses.

As expected, a noncooperative solution will provide higher anomalies in equilibrium. By setting the constants associated with positive eigenvalues equal to zero, there exist initial emissions for each region such that the system will converge to the steady state which satisfies the conditions for an OLNE. In Figure 8 we present the approach toward the steady state with cross effects of the linearized system (47)-(52) at the steady state.

Figure 8: Convergence to the OLNE



4.1 The noncooperative carbon tax

Assume that each region imposes a regional carbon tax. Then (24) implies that the optimal regional tax will be, assuming no policy uncertainty,

$$\tau_{1t}^{NC} = \frac{-1}{\alpha} \left[\lambda_1 \mu_{11t} + \lambda_2 \mu_{12t} \right], \tau_{2t}^{NC} = \frac{-1}{\alpha} \left[\lambda_1 \mu_{21t} + \lambda_2 \mu_{22t} \right]. \tag{53}$$

Thus regional taxes in the two regions are different, and furthermore they are different (lower) than the optimal global carbon tax. If a cooperative solution is not feasible, then carbon leakage issues could be avoided by imposing, for example, tariffs between regions equal to $\left|\tau_{1t}^{NC} - \tau_{2t}^{NC}\right|$ applied to the lower tax region, which might attract activities from the higher tax region.

Policy uncertainty in each region will result in further differentiation of regional carbon policies. If the representative regional firm is concerned about policy uncertainty but the regional regulator is not, then the decentralized firm-robust equilibrium is more "conservative" in emissions than the robust regional planner, which is a result qualitatively similar to the one obtained for the social planner. This might also generate carbon leakage issues, since it will introduce regional differences in carbon taxes.

4.2 Noncooperative policies under deep uncertainty

To explore the effects of ambiguity on the ROLNE, we run similar simulations as in the case of the cooperative solution for different values of η_i, θ_i, $i = 1, 2$. Since each region decides about own emissions, it is reasonable to assume that differences in the robustness parameters might exist. Furthermore, the maximization of own welfare implies that cross effects of temperature in regional damage functions might be important. This is because without cross effects neither region has an incentive to take into account the evolution of temperature in the other region.

In this context we run our simulations with and without cross effects and with asymmetries in the robustness parameters. In particular, we set $\eta_2 = \theta_2 = 0.5$, and allow η_1, θ_1 to take values $\{0, 0.05, 0.1, 0.125\}$. The results for regional steady-state robust emissions can be summarized as follows:

- The qualitative behavior, when ambiguity increases, is the same as in the cooperative case. Higher ambiguity implies lower regional emissions. Also, as expected, emissions are higher relative to the cooperative case since the climate externality is not fully internalized at the OLNE.

If both regions have the same robustness preferences, then:

- When temperature cross effects from the North to the South are not included, emissions in the South are higher than emissions in the North.

This can be explained by the fact that although damages as a proportion of GDP are higher in the South, the temperature anomaly is lower. This might provide some space for increased emissions. As ambiguity increases, the gap between emissions in the South and the North is reduced.

- When temperature cross effects from the North to the South only are included, the picture is reversed. Since the anomaly in the North increases damages in the South, this induces the South to reduce emissions in order to slow down anomalies in both regions. Emissions in the North are relatively higher since the North is affected only by the North's regional anomaly. As ambiguity increases, the gap between emissions in the South and the North is reduced.

When robustness preferences are asymmetric in the way described above, then:

- As robustness in the South exceeds robustness in North, emissions in the South are lower than in the North irrespective of whether temperature cross effects exist or not.

- When regional robustness parameters increase sufficiently, there are no noncooperative emission paths which converge to the steady state satisfying OLNE. This result can be shown by considering the linearization matrix (47)-(52) at a steady state. Since this is a 6x6 matrix, Dockner's theorem does not apply. However, as shown in Appendix 3, the application of Routh's stability theorem suggests that the ROLNE steady state could lose the saddle point property. This is confirmed by numerical simulations in which, as the robustness parameters increase, the eigenvalues of the linearization matrix become positive. These results suggest that the possible breakdown of regulation under strong ambiguity emerges both in cooperative and noncooperative solutions.

Having characterized the ROLNE, a question which emerges is whether the appropriate solution concept is a feedback Nash equilibrium which does not imply an infinite period of commitment. To see whether this is a realistic approach, we consider the Paris agreement in which countries agreed to intended nationally determined contributions (NDCs), which can be regarded as nationally determined emission paths. These paths will be reviewed every five years. Since the emission paths have been determined by each individual country, this might be regarded as a set up in which each country decides intended emissions by maximizing own welfare, taking the action of other countries as fixed, and then all countries agree to implement the solution. The agreement is stable since it implements an OLNE and each country is committed to the Nash path.

According to the Paris agreement, after five years there will be an evaluation.[17] If countries do not follow their agreed paths, or temperature has increased more than expected, there will be adjustments. This could mean re-optimization with different initial conditions and commitment to different paths. Thus, given the existing international set up, the OLNE might fit the real world better than a feedback Nash equilibrium in which each country conditions current emissions to the current temperature anomaly.

5 The Cost of Robustness

An issue which arises when a robust climate policy is pursued is whether the policy incurs additional costs relative to the case in which no misspecification concerns are involved. To obtain an approximation of these costs, we calculate the welfare indicator

$$J_i^\nu = \int_{t=0}^{\infty} e^{-\rho t}[\alpha \ln E_{it}^\nu - \left(\sum_{j=1}^{2} d_{ij} T_{jt}^\nu + \frac{1}{2} v_{ij} T_{jt}^{\nu 2}\right)]dt, i = 1, 2, \nu = D, R, \tag{54}$$

where D, R correspond to emissions and temperature paths for no misspecification concerns (D) and robust control (R) respectively. The indicator corresponds to the case in which the global regulator or the regional regulators are committed to their emissions paths obtained through the relevant optimizations. The indicators were calculated numerically for the deterministic and robust cooperative solution and OLNE. In tables 1a-1b some numerical results are presented. Temperature spillover effects from the North to the South and symmetric/asymmetric robustness parameters have been considered. In the robust case the welfare penalties imposed by the adversarial

[17] In the most recent COP24 in Katowice, one of the outcomes was the agreement that from 2024 on, countries will have to report their emissions every two years.

agent are not included because they are fictitious. The results are shown in table 1a-1b, for symmetric and asymmetric robustness parameters between the two regions.

Table 1a: Welfare indicators for different robustness parameters (RP): Cooperative Solution

Region	RP(0.00,0.00)	RP(0.05,0.05)	RP(0.10,0.10)
1	-19.11	-19.75	-21.21
2	-14.39	-15.12	-15.92
	RP(0.00,0.00)	RP(0.05,0.00)	RP(0.10,0.05)
1	-19.11	-18.84	-18.89
2	-14.39	-14.93	-15.06

Table 1b: Welfare indicators for different robustness parameters (RP): OLNE

Region	RP(0.00,0.00)	RP(0.05,0.05)	RP(0.10,0.10)
1	-50.14	-20.79	-19.85
2	-9.52	-12.22	-13.44
	RP(0.00,0.00)	RP(0.05,0.00)	RP(0.10,0.05)
1	-50.14	-44.49	-21.39
2	-9.52	-9.52	-12.13

In the cooperative case, robust control is costly to both regions and an increase in preferences for robustness reduces welfare. In robust OLNE, however, region 1 benefits from robustness while for region 2 robustness is costly both in the symmetric and asymmetric cases. The result suggests that the more vulnerable region has an incentive to follow robust policies even if the other region does not follow robust policies.

6 Learning and Robust Control

The numerical results of the previous section indicate that robustness could be costly, relative to the case of no concerns about model misspecification, especially in the cooperative solution. This raises the issue of whether it is possible to avoid this cost by learning. In Anderson et al. (1998, page 2), it is stated that "Superficially at least, the perspective of the 'robust' controller differs substantially from that of a 'learner'." In our dynamic settings, the robust decision maker accepts the presence of model misspecification as a permanent state of affairs, and devotes his thoughts to designing robust controls, rather than, say, thinking about recursive ways to use data to improve his model specification over time. The idea here is that the 'learner' cannot improve against model misspecification even with a large amount of data.

However, when dealing with climate change issues the stakes are high, and learning through scientific research is an ongoing process which might remove some concerns regarding damages or temperature dynamics. To model such a process we will consider the case in which part of the labor force, that in the model without learning is fully employed in the output producing sector, could be employed in the 'learning' sector. Employment in the learning sector reduces misspecification concerns, since it allows the regulator to learn about the processes for which there is ambiguity. We assume that the robustness parameter can be expressed as the function:

$$\phi\left(L_i - l_{it}\right), \phi' > 0, i = 1, 2, \tag{55}$$

where l_{it} is labor input used in region i to produce output at time t, and $L_i - l_{it}$ is labor input allocated to the learning sector for climate damages. Assume that learning activities take place only with respect to damages from climate change and that the world is changing rapidly so that no learning stock is accumulated.[18] The regulator's objective associated with the multiplier representation of the robust control problem can be written as:

$$\max_{\{E_{it}, l_{it}\}} \min_{\{k_{it}, h_{it}\}} \tag{56}$$
$$\int_{t=0}^{\infty} \mathrm{e}^{-\rho t} \sum_{i=1}^{N} \omega_i \left[\alpha \ln E_{it} - \left(\sum_{j=1}^{N} d_{ij} T_{jt} + \frac{1}{2} v_{ij} T_{jt}^2 + k_{it} T_{it}\right) + \frac{1}{2} \phi\left(L_i - l_{it}\right) k_{it}^2 + \frac{h_{it}^2}{2\theta_i}\right] dt$$

[18]Both assumptions are simplifying. Accumulation of learning stock requires introduction of learning dynamics of the form $\dot{S}_t = \eta\left(L_i - l_{it}\right) - \delta S_t$,S_0 given. The introduction of more dynamic constraints would have complicated the solution even more. However, the use of a 'flow concept' for learning at this stage provides intuition to the problem and the use of learning dynamics is left for future research.

subject to (3). For this problem, the current value Hamiltonian is:

$$\mathcal{H}^C = \{\sum_{i=1}^{2}\omega_i\left[\ln(y_{it}l_{it}^{\beta}E_{it}^{\alpha}) - \left(\sum_{j=1}^{N}d_{ij}T_{jt} + \frac{1}{2}v_{ij}T_{jt}^2 + k_{it}T_{it}\right) + \frac{1}{2}\phi\left(L_i - l_{it}\right)k_{it}^2 + \frac{h_{it}^2}{2\theta_i}\right]$$

$$\sum_{i=1}^{N}\mu_i[.\lambda_i\mathbb{E}_t - B_iT_{it} + \sigma_i h_{it}]\}. \quad (57)$$

Optimality conditions for $i = 1, 2$ imply

$$\frac{\alpha\omega_i}{E_{it}} + \sum_{i=1}^{N}\mu_{it}\lambda_i = 0 \Rightarrow E_{it}^{*} = \frac{-\alpha\omega_i}{\sum_i \mu_{it}\,\lambda_i} \quad (58)$$

$$\frac{\beta}{l_{it}} = \frac{1}{2}\phi'\left(L_i - l_{it}\right)k_{it}^2 \quad (59)$$

$$k_{it} = \frac{T_{it}}{\phi\left(L_i - l_{it}\right)}, h_{it} = -(\theta_i/\omega_i)\sigma_i\mu_{it} \quad (60)$$

$$\dot{\mu}_{it} = \left(\rho + B_i\right)\mu_{it} + \sum_{j=1}^{N}\omega_j(d_{ji} + v_{ji}T_{it}) + \frac{T_{it}}{\phi\left(L_i - l_{it}\right)} \quad (61)$$

$$\dot{T}_{it} = \lambda_i\mathbb{E}_t - B_iT_{it} - \sigma_i^2\left(\theta_i/\omega_i\right)\mu_{it}. \quad (62)$$

This formulation implies that learning about climate damages from the regulator's point of view raises that cost of the 'adversarial agent' who tries to minimize the regulator's welfare. Since $\phi' > 0$, an increase in the amount of labor allocated to learning will reduce the penalty that the adversarial agent could impose on the regulator. To put it differently, increase in learning reduces misspecification concerns and ambiguity. Under this setup the following result can be stated.

Proposition 3 *An increase in the regional temperature will always increase the labor input allocated to the learning sector. If this impact is sufficiently strong then learning could reduce the possibility of robust regulation breakdown.*

For the proof see Appendix 1.

Robust control could be infeasible if the steady state loses its saddle point property and, because of this, the regulator cannot control the system to the optimal steady state. Saddle point stability could be lost, as shown in Proposition 2, if the robustness parameters reflecting ambiguity are sufficiently high. This proposition suggests that learning could act as a stabilizing force which will make robust regulation feasible and will reduce the possibility of hot spot emergence.

7 Concluding Remarks

Deep uncertainties are associated with both the natural and the economic characteristics of climate change. These uncertainties are amplified by the fact that in reality the temperature anomaly evolves differently across the globe, with a faster increase in the area of the North Pole relative to the equator because of natural mechanisms. In this context, we study climate change policies by using the novel pattern scaling approach of RTCREs and develop an economy-climate model under conditions of deep uncertainty associated with temperature dynamics, regional climate change damages and policy in the form of carbon taxes. The regional structure of the model allows us to analyze both cooperative outcomes emerging when a social planner chooses carbon emission policies to maximize global regional welfare, and noncooperative outcomes where each region decides its carbon emission policy by maximizing own welfare.

We applied robust control methods to derive optimal emission policies and the associated price of the climate externality under the different sources of deep uncertainty. We characterize the cooperative solution and the robust OLNE. Our results indicate that in general robust policies under deep uncertainty lead to more conservative emission policies both in cooperative and noncooperative solutions relative to a deterministic situation. Furthermore, ambiguity related to the damage function tends to produce more conservative policies than ambiguity in temperature dynamics, while robust control with high concerns about model misspecification is relatively more costly in a cooperative solution, but this could depend on the vulnerability of a region in noncooperative solutions. The more vulnerable region, in our model and parametrization, benefits in welfare terms from robust policies. We also show that competitive firms when facing ambiguity regarding carbon taxes tend to be more conservative and use smaller amounts of fossil fuels relative to the case of no policy uncertainty. Policy uncertainty could be important in practice because it relates to uncertainties in the transition to a low carbon economy.

An important aspect of the regional model is that differences in aversion to ambiguity across regions could produce big deviations in regional emissions policies in the noncooperative solutions. These deviations are amplified if damages in a region are affected by changes in temperature in another region, as in the case of Polar amplification. Finally, we show that when aversion to ambiguity in a region increases sufficiently, robust control breaks down in the sense that that there are no robust emission paths which could drive the climate-economy system to an equilibrium steady state. This introduces the idea of regional hot spots where uncertainty about damages and/or temperature dynamics is so severe that robust control is not possible. Since, as indicated in the introduction, recent studies suggest that hot spots exist in reality, this result could be important for policy purposes. Regulation breakdown could be regarded as a signal for reducing deep uncertainty. We show that this could be possible through learning.

8 Appendix

8.1 Appendix 1

Proof of Proposition 1

Proof. The proof follows directly from the application of the implicit function theorem. The system (61)-(62) has a straightforward solution at point **o**,

$$\bar{\mu}_i = -\frac{1}{(\rho + B_i)} \left[\sum_{j=1}^{N} \omega_j \, d_{ji} \right] \tag{63}$$

$$\bar{T}_i = \frac{1}{B_i} [\frac{\theta_i}{\omega_i} \frac{\sigma_i^2}{(\rho_i + B_i)} \Gamma_i + \frac{\omega_i}{\sum\limits_{i=1}^{N} \frac{\lambda_i}{(\rho_i + B_i)} \Gamma_i}], \tag{64}$$

and the Jacobian determinant of (61)-(62) is nonzero at **o**. ■

Proof of Proposition 2

Proof. The linearization matrix for the 2-region problem (61)-(62) with

$v_{ij} = 0, i, j = 1, 2, j \neq i$, is

$$J = \begin{pmatrix} -B_1 & 0 & \lambda_1 \frac{\partial \mathbb{E}}{\partial \mu_1} - (\sigma_1^2 \theta_1)/\omega_1 & \lambda_2 \frac{\partial \mathbb{E}}{\partial \mu_2} \\ 0 & -B_2 & \lambda_2 \frac{\partial \mathbb{E}}{\partial \mu_1} & \lambda_2 \frac{\partial \mathbb{E}}{\partial \mu_2} - (\sigma_2^2 \theta_2)/\omega_2 \\ \omega_1 v_{11} + \eta_1 & 0 & \rho + B_1 & 0 \\ 0 & \omega_2 v_{22} + \eta_2 & 0 & \rho + B_2 \end{pmatrix}. \tag{65}$$

If we set $\theta_i = 0, \eta_i = 0, v_{ii} = 0, i = 1, 2,$ then the linearization matrix is diagonal and the eigenvalues can be read from the principal diagonal $\{-B_1, -B_2, \rho + B_1, \rho + B_2\}$. It is clear that we have saddle point stability. Assume now that $\theta_i > 0, \eta_i > 0, v_{ii} > 0$ and note that

$$\frac{\partial \mathbb{E}}{\partial \mu_i} = \frac{a\lambda_i}{\left(\sum_i \lambda_i \mu_i\right)^2} > 0, i = 1, 2. \tag{66}$$

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Following Dockner (1985), the eigenvalues of the linearization matrix J are

$$e_{1,2,3,4} = \frac{\rho}{2} \pm \sqrt{\left(\frac{\rho}{2}\right)^2 - \frac{K}{2} \pm \frac{1}{2}\sqrt{K^2 - 4 \det J}} \tag{67}$$

where

$$K = \begin{vmatrix} -B_1 & \lambda_1 \frac{\partial \mathbb{E}}{\partial \mu_1} - (\sigma_1^2 \theta_1)/\omega_1 \\ v_{11} + \eta_1 & \rho + B_1 \end{vmatrix} + \begin{vmatrix} -B_2 & \lambda_2 \frac{\partial \mathbb{E}}{\partial \mu_2} - (\sigma_2^2 \theta_2)/\omega_2 \\ v_{22} + \eta_2 & \rho + B_2 \end{vmatrix} + 0 \tag{68}$$

or

$$\begin{aligned} K = {} & -B_1 (\rho + B_1) - \left(\lambda_1 \frac{\partial \mathbb{E}}{\partial \mu_1} - (\sigma_1^2 \theta_1)/\omega_1\right)(v_{11} + \eta_1) \\ & + [-B_2 (\rho + B_2) - \left(\lambda_2 \frac{\partial \mathbb{E}}{\partial \mu_2} - (\sigma_2^2 \theta_2)/\omega_2\right)(v_{22} + \eta_2). \end{aligned} \tag{69}$$

According to Dockner (1985, Theorem 3), necessary conditions for the eigenvalues to be real, two being positive and two being negative, are:

$$\begin{array}{ll} (i) & K<0 \\ (ii) & 0<\det J<\left(\frac{K}{2}\right)^2. \end{array} \tag{70}$$

From the definition of K in (68), it can be seen that for sufficiently large robustness parameters, K could be positive. Thus the necessary conditions for saddle point stability with two negative real eigenvalues is violated. For a numerical example, see Appendix 2. ■

Proof of Proposition 3

Proof. If we substitute the minimizer for k_{it} from the optimality conditions above the reduced form Hamiltonian becomes:

$$\mathcal{H}^C = \max_{E_i},_{l_i} \{\sum_{i=1}^{2}\omega_i \left[\ln(y_{it}l_{it}^{\beta}E_{it}^{\alpha}) - \left(\sum_{j=1}^{N} d_{ij}T_{jt} + \frac{1}{2}v_{ij}T_{jt}^2\right) - \frac{1}{2}\frac{T_{it}^2}{\phi\left(L_i - l_{it}\right)} + \frac{h_{it}^2}{2\theta_i}\right]$$

$$\sum_{i=1}^{N}\mu_i[.\lambda_i\mathbb{E}_t - B_iT_{it} + \sigma_i h_{it}]\}. \tag{71}$$

the optimal choice for the labour input is given by

$$\frac{\beta}{l_{it}} = \frac{1}{2}\frac{\omega_i^2\phi'\left(L_i - l_{it}\right)}{\phi\left(L_i - l_{it}\right)^2}T_{it}^2 \tag{72}$$

Assuming that the learning function can be specified as: $\phi\left(l\right) = \left(A/q\right)l^q$ we obtain

$$\beta\left(L_i - l_{it}\right)^{1+q} = \frac{q^2}{2A}l_{it}T_{it}^2, l_{it} = h_i\left(T_{it}\right) \tag{73}$$

which implies that the optimal allocation of labor to production and learning is a function of temperature. Implicit differentiation results in

$$\frac{dl_{it}}{dT_{it}} \equiv h_{it}'\left(T_{it}\right) = -\frac{\left(q^2/A\right)T_{it}l_{it}}{\left(1+q\right)\beta\left(L_i - l_{it}\right)^q + \left(q^2/2A\right)T_{it}^2} < 0 \tag{74}$$

Thus an increase in temperature in region i will reduce the labor input to production and increase the labor input allocated to learning and to reducing ambiguity about climate change damages. The Hamiltonian system associated with (71) is:

$$\dot{T}_{it} = \lambda_i \mathbb{E}_t^* - B_i T_{it} - (\theta_i/\omega_i)\sigma_i^2 \mu_{it} \tag{75}$$

$$\dot{\mu}_{it} = (\rho + B_i)\,\mu_{it} + \sum_{j=1}^{N} \omega_j (d_{ji} + v_{ji} T_{it}) + \frac{T_{it}}{\phi\,(L_i - h_i\,(T_{it}))}. \tag{76}$$

Following the proof of proposition 2 and assuming that a steady state exists, the linearization matrix for the 2-region problem (75)-(76) with $v_{ij} = 0, i, j = 1, 2, j \neq i$, is

$$J = \begin{pmatrix} -B_1 & 0 & \lambda_1 \frac{\partial \mathbb{E}}{\partial \mu_1} - (\sigma_1^2 \theta_1)/\omega_1 & \lambda_2 \frac{\partial \mathbb{E}}{\partial \mu_2} \\ 0 & -B_2 & \lambda_2 \frac{\partial \mathbb{E}}{\partial \mu_1} & \lambda_2 \frac{\partial \mathbb{E}}{\partial \mu_2} - (\sigma_2^2 \theta_2)/\omega_2 \\ \omega_1 v_{11} + G_1 & 0 & \rho + B_1 & 0 \\ 0 & \omega_2 v_{22} + G_2 & 0 & \rho + B_2 \end{pmatrix}. \tag{77}$$

where

$$G_i = \frac{\partial}{\partial T_i}\left(\frac{T_{it}}{\phi\,(L_i - h_i\,(T_{it}))}\right) = \frac{\phi\,(L_i - h_i\,(T_{it})) + \phi'\,(L_i - h_i\,(T_{it}))\,h_i'\,(T_{it})}{[\phi\,(L_i - h_i\,(T_{it}))]^2}, i = 1, 2 \tag{78}$$

In the definition of G_i the term $\phi' h_i'$ is negative since $h_i' < 0$. If this term is sufficiently negative – which means that an increase in temperature in region i will cause a large diversion of labor to research and learning about climate change damages – then $G_i < 0$. If the robustness parameters (i.e. misspecification concerns) associated with temperature dynamics tend to destabilize the steady state, in the sense of proposition 2, then a sufficiently negative G_i could have a stabilizing effect on the steady state.

Again following again Dockner (1985), the quantity K, which should be negative in order to have saddle point stability, is

$$\begin{aligned} K = {} & -B_1\,(\rho + B_1) - \left(\lambda_1 \frac{\partial \mathbb{E}}{\partial \mu_1} - (\sigma_1^2 \theta_1)/\omega_1\right)(v_{11} + G_1) \\ & + [-B_2\,(\rho + B_2) - \left(\lambda_2 \frac{\partial \mathbb{E}}{\partial \mu_2} - (\sigma_2^2 \theta_2)/\omega_2\right)(v_{22} + G \end{aligned} \tag{79}$$

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If the robustness parameters θ_i are sufficiently high, so that the terms $\left(\lambda_i \frac{\partial \mathbb{E}}{\partial \mu_i} - (\sigma_i^2 \theta_i)/\omega_i\right)$ are negative, then highly negative G_i terms can make $K < 0$. ■

8.2 Appendix 2

The parameter values used in simulations are shown below.

Table A1: Simulation parameters

$\omega_i, i = 1, 2$	0.5
α	0.05
d_{11}	0.05425
d_{12}	$0.0(NCE), 0.04613(CE)$
d_{21}	0.0
d_{22}	0.04513
v_{11}	0.06995
v_{12}	$0.0(NCE), 0.06151(CE)$
v_{21}	0.0
v_{22}	0.00381
$b_i, i = 1, 2$	0.008
$\sigma_i, i = 1, 2$	0.1
ρ	0.02
λ_1	1.3
λ_2	2

8.3 Appendix 3

Let J^N be the (6×6) linearization matrix that satisfies the conditions for a ROLNE evaluated at a steady state. The characteristic polynomial of this matrix will be of the form

$$x^6 - trace J^N x^5 + A_4 x^4 + A_3 x^3 + A_2 x^2 + A_1 x + \det J^N = 0. \tag{80}$$

The trace of J^N is positive, while the A coefficients are determined by the minors of J^N. Since the trace is positive, matrix J^N has at least one positive eigenvalue. Given the size of the matrix, it is not possible to derive closed form expressions for the A coefficients. However, numerical simulations consistently show that as robustness parameters increase, the eigenvalues become positive.

A similar reasoning applies for the cooperative solution, with linearization matrix J and characteristic polynomial:

$$x^4 - traceJx^3 + A_2x^2 + A_1x + \det J = 0. \tag{81}$$

In numerical simulations:

- For $\eta_1 = 0.125$; $\eta_2 = 0.125$; $\theta_1 = 0.05$; $\theta_2 = 0.05$
  the eigenvalues are: $0.0278958, 0.0258818, -0.00789579, -0.00588179$
- For $\eta_1 = 0.4$; $\eta_2 = 0.4$; $\theta_1 = 0.05$; $\theta_2 = 0.05$
  the eigenvalues are: $0.0212927, 0.0199994, -0.00129273, 6.07519 \times 10^{-7}$
- For $\eta_1 = 0.5$; $\eta_2 = 0.5$; $\theta_1 = 0.05$; $\theta_2 = 0.$
  the eigenvalues are: $0.0199935, 0.0177531, 0.00224689, 6.52477 \times 10^{-6}$

References

Alexeev, V.A. and C.H. Jackson (2013), "Polar amplification: Is atmospheric heat transport important?", *Climate Dynamics*, 41, 2, 533–554.

Anderson, E., L. Hansen and T. Sargent (1998), "Risk and robustness in general equilibrium," Preprint University of Chicago.

Anderson, E., W. Brock, L. Hansen and A. Sanstad (2014), "Robust analytical and computational explorations of coupled economic-climate models with carbon-climate response", RDCEP Working paper No. 13-05.

Anderson, E., L. Hansen and T. Sargent (2012), "Small noise methods for risk sensitive/robust economics", J*ournal of Economic Dynamics and Control*, 36, 468–500.

Anthoff, D. and R. Tol (2013), "The uncertainty about the social cost of carbon: A decomposition analysis using FUND," *Climatic Change*, 117, 515–530, DOI 10.1007/s10584-013-0706-7.

Athanassoglou, S. and A. Xepapadeas (2012), "Pollution control with uncertain stock dynamics: When, and how, to be precautious", *Journal of Environmental Economics and Management*, 63, 3, 304–320.

Bevis, M., C. Harig, S.A. Khac, A. Brown, F.J. Simons, M. Willis, X. Fettweis, M.R. van den Broeke, F.B. Madsen, E. Kendrick, D.J. Caccamise II, T. van Dam, P. Knudsen and T. Nylen (2018), "Accelerating changes in ice mass within Greenland,and the ice sheet's sensitivity to atmospheric forcing," *PNAS*, www.pnas.org/cgi/doi/10.1073/pnas.1806562116.

Brock, W.A., G. Engström, D. Grass and A. Xepapadeas (2013), "Energy balance climate models and general equilibrium optimal mitigation policies", *Journal of Economic Dynamics and Control*, 37, 12, 2371–2396.

Brock, W.A., G. Engström and A. Xepapadeas (2014a), "Spatial climate–economic models in the design of optimal climate policies across locations", *European Economic Review*, 69, 78–103.

Brock, W.A. and L.P. Hansen (2017), "Wrestling with uncertainty in climate economic models", Working Paper, Available at SSRN, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3008833. Forthcoming in V. V. Chari and R. Litterman (eds.) *Climate Change Economics: the Role of Uncertainty and Risk*.

Brock, W.A. and A. Xepapadeas (2017), "Climate change policy under polar amplification", *European Economic Review*, 94, 263–282.

Brock, W.A. and A. Xepapadeas (2018), "Regional climate change policy under positive feedbacks and strategic interactions", *Environmental and Resource Economics*. Available at https://doi.org/10.1007/s10640-018-0254-8.

Brock, W.A., A. Xepapadeas and A.N. Yannacopoulos (2014b), "Robust control and hot spots in spatiotemporal economic systems", *Dynamic Games and Applications*, 4, 257–289.

Cai, Y., W. Brock, A. Xepapadeas and K. Judd (2018), "Climate policy under cooperation and competition between regions with spatial heat transport", NBER Working Paper 24473, NBER.

Carney, M. (2015), "Breaking the tragedy of the horizon - climate change and financial stability", Speech at Lloyd's of London.

Castruccio, S., D.J. McInerney, M.L. Stein, F. Liu Crouch, R.L. Jacob and E.J. Moyer (2014), "Statistical emulation of climate model projections based on precomputed GCM runs", *Journal of Climate*, 27, 5, 1829–1844.

Desmet, K. and E. Rossi-Hansberg (2015), "On the spatial economic impact of global warming", *Journal of Urban Economics*, 88, 16–27.

Diffenbaugh, N.S., D. Singh, J.S. Mankin, D.E. Horton, D.L. Swain, D. Touma, A. Charland, Y. Liu, M. Haugen, M. Tsiang and B. Rajaratnam (2017), "Quantifying the influence of global warming on unprecedented extreme climate events", *PNAS*, 114, 4881-4886.

Dockner, E. (1985), "Local stability analysis in optimal control problems with two variables," in G. Feichtinger (ed.) *Optimal Control Theory and Economic Analysis 2*, North-Holland, Amsterdam.

Easterling, W.E. (1997) "Why regional studies are needed in the development of full scale integrated assessment modelling of global change processes", *Global Environmental Change*, 1, 4, 337-356.

Francis, J.A. and N. Skific (2012), "Evidence linking rapid Arctic warming to mid-latitude weather patterns", *Philosophical Transactions of the Royal Society*, A 273, dx.doi.org/10.1098/rsta.2014.0170.

Francis, J.A., N. Skific and S.J. Vavrus (2018), "North American weather regimes are becoming more persistent: Is Arctic amplification a factor?", *Geophysical Research Letters*, 45.

Francis, J. and S. Vavrus (2014), "Evidence for a wavier jet stream in response to rapid Arctic warming", *Environmental Research Letters*, 10, 1–12.

Gillingham, K., W. Nordhaus, D. Anthoff, G. Blanford, V. Bosetti, P. Christensen, H. McJeon, J. Reilly and P. Sztorc (2015), "Modeling uncertainty in climate change: A multi-model comparison", NBER Working Paper 21637, National Bureau of Economic Research.

Golosov, M., J. Hassler, P. Krusell and A. Tsyvinski (2014), "Optimal taxes on fossil fuel in equilibrium", *Econometrica*, 82, 1, 41–88.

Hansen, L.P. and T.J. Sargent (2001), "Acknowledging misspecification in macroeconomic theory", *Review of Economic Dynamics*, 4, 3, 519–535.

Hansen, L.P. and T.J. Sargent (2008), *Robustness in Economic Dynamics*, Princeton University Press, Princeton, NJ.

Hansen, L.P., T.J. Sargent, G. Turmuhambetova and N. Williams (2006), "Robust control and model misspecification", *Journal of Economic Theory*, 128, 1, 45–90.

Hassler, J. and P. Krusell (2012), "Economics and climate change: Integrated assessment in a multi-region world", *Journal of the European Economic Association*, 10, 5, 974–1000.

Hassler, J. and P. Krusell (2018), "Environmental Macroeconomics: The Case of Climate Change".
Available at http://hassler-j.iies.su.se/PAPERS/Handbookenvironment.pdf.

Hassler, J., P. Krusell and C. Olovsson (2018), "The consequences of uncertainty: Climate sensitivity and economic sensitivity to the climate", *Annual Review of Economics*, 189-205.

Hassler, J., P. Krusell and T. Smith (2016), "Environmental macroeconomics", in Taylor J. and H. Uhlig (eds) *Handbook of Macroeconomics*, Vol. 2b, North-Holland, Amsterdam, pp. 1893–2008.

Heal, G. and A. Millner (2014), "Uncertainty and decision making in climate change economics", *Review of Environmental Economics and Policy*, 8, 1, 120–137.

Hennlock, M. (2009), "Robust control in global warming management: An analytical dynamic integrated assessment", Technical report, RFF.

Heutel, G., J. Moreno-Cruz and S. Shayegh (2016), "Climate tipping points and solar geoengineering", *Journal of Economic Behavior and Organization*, 132, 19–45.

Hope, C. (2006), "The Marginal Impact of CO2 from PAGE2002: An Integrated Assessment Model Incorporating the IPCC's Five Reasons for Concern", *The Integrated Assessment Journal, Bridging Science & Policy,* 6, 1, 19–56.

IPCC (2013), *Climate Change 2013: The Physical Science Basis*, Working Group I's Contribution to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, Stocker T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley (eds), Cambridge University Press, Cambridge, UK and New York.

Knutti, R. (2013), "Relationship between global emissions and global temperature rise". Available at
https://https://unfccc.int/sites/default/files/7_knutti.reto.3sed2.pdf.

Knutti, R. and J. Rogelj (2015), "The legacy of our CO2 emissions: A clash of scientific facts, politics and ethics", *Climatic Change*, 133, 3, 361–373.

Krusell, P. and A. Smith (2017), "Climate change around the world", Paper presented at the conference, The Macro and Micro Economics of Climate Change, University of California, Santa Barbara, May 22-23, 2017.

Langen, P.L. and V.A. Alexeev (2007), "Polar amplification as a preferred response in an idealized aquaplanet GCM", *Climate Dynamics*, 29, 305–317.

Leduc, M., H.D. Matthews and R. de Elía (2016), "Regional estimates of the transient climate response to cumulative CO2 emissions", *Nature Climate Change*, 6, 474–478.

Lemoine, D.M. (2010), "Climate sensitivity distributions depend on the possibility that models share biases", *Journal of Climate*, 23, 4395–4415.

Lemoine, D.M. and I. Rudik (2014), "Steering the climate system: Using inertia to lower the cost of policy", Working Paper 14-03, University of Arizona.

Liu, Z., B. Anderson, K. Yan, W. Dong, H. Liao and P. Shi (2017), "Global and regional changes in exposure to extreme heat and the relative contributions of climate and population change", *Scientific Reports*, 7, article 43909, DOI: 10.1038/srep43909.

MacDougall, A.H., N.C. Swart and R. Knutti (2017), "The uncertainty in the transient climate response to cumulative CO2 emissions arising from the uncertainty in physical climate parameters", *American Meteorological Society*, DOI:10.1175/JCLI-D-16-0205.1

Matthews, H.D., N.P. Gillett, P.A. Stott and K. Zickfield (2009), "The proportionality of global warming to cumulative carbon emissions", *Nature*, 459, 829–833.

Matthews, H.D., S. Solomon and R. Pierrehumbert (2012), "Cumulative carbon as a policy framework for achieving climate stabilization", *Philosophical Transactions of the Royal Society* A, 370, 4365–4379.

Nævdal, E. and M. Oppenheimer (2007), "The economics of the thermohaline circulation – a problem with multiple thresholds of unknown locations", *Resource and Energy Economics*, 29, 4, 262–283.

Nordhaus, W.D. (2011), "Estimates of the social cost of carbon: Background and results from the RICE-2011 model", Cowles Foundation Discussion paper No. 1826, Yale University, New Haven, CT.

Nordhaus, W.D. (2014), "Estimates of the social cost of carbon: Concepts and results from the DICE-2013R model and alternative approaches", *Journal of the Association of Environmental and Resource Economists*, 1, 1/2, 273–312.

Nordhaus, W. and A. Moffat (2017), "A survey of global impacts of climate change: Replication, survey methods, and a statistical analysis", Cowles Foundation Discussion Paper no. 2096, Yale University, New Haven.

Nordhaus, W.D. and P. Sztorc (2013), *DICE 2013-R: Introduction and User's Manual*, Yale University, New Haven, CT.

North, G.R., R. Cahalan and J. Coakely (1981), "Energy balance climate models", *Reviews of Geophysics and Space Physics*, 19, 1, 91–121.

Pindyck, R.S. (2017), "The use and misuse of models for climate policy", *Review of Environmental Economics and Policy*, 11, 1, 100-114.

Ricke, K., L. Grouet, K. Caldeira and M. Tavoni (2018), "Country-level social cost of carbon", *Nature Climate Change*, 8, 895-900.

Tol, R.S.J. (2018), "Economic impacts of climate change", *Review of Environmental Economics and Policy*, 12, 1, 4–25.

Weitzman, M.L. (2010), "What is the 'damages function' for global warming – and what difference might it make?", *Climate Change Economics*, 1, 1, 57–69.

Wu, B. and J.A. Francis (2019), "Summer Arctic cold anomaly dynamically linked to East Asian heat waves", *Journal of Climate*, doi: 10.1175/JCLI-D-18-0370.1.

Xepapadeas, A. and A.N. Yannacopoulos (2017), "Spatially structured deep uncertainty, robust control, and climate change policies", Paper presented at the 23rd Annual Conference of the EAERE, Athens, Greece, 28 June 1 July 2017.