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Abstract

We study the evolution of compliance and regulation in a common pool resource setup with myopic appropriators whose decision on whether to comply or not with the harvesting rule is a result of imitation as described by a proportional rule. The regulator first sets the optimal quota and then harvesters can choose between compliance and violation. We investigate myopic regulation and optimal regulation regimes with both proportional and non-proportional fine formulation and and an endogenized probability of audition. The equilibria are then characterized in terms of their stability properties.

JEL classification:

Keywords: Common pool resources, replicator dynamics, optimal regulation, compliance

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1 Introduction

Sustainable resource management, environmental policy and enforcement mechanisms have long been the subject of great debate in the literature. Beyond the neoclassical approach that treats economic agents as fully rational agents that have full information and solve maximization problems, there has been a significant growth in the use of evolutionary game theory to tackle the problem. This can be viewed more as an extension of the standard framework rather than as an opposition to the classical game theoretic tools. The main advantages of the evolutionary method is that it can include behavioral norms that arise through the interactions of agents. In this context, it is used to model cooperation and enforcement in common pool resources as in Osés-Eraso & Viladrich-Grau (2007), where behavioral norms are developed through other-regarding preferences. Alternatively, the notion of self-enforcement and endogenous sanctions among agents in a common pool resource game has been introduced by Sethi & Somanathan (1996). In their work, whenever the resource is overexploited, i.e. when there is harvesting beyond the norm, the appropriators can effectively enforce harvesting restrictions. Related works include the work of Bischi *et al.* (2004), which uses discrete time dynamics and investigates the promotion of cooperation through self-enforcement based on endogenous sanctions. The works of Noailly *et al.* (2003) and Xepapadeas (2005) provide a combined interaction of resource dynamics and replicator dynamics in a single framework. In the first, the price can be regarded as an instrument of environmental policy as it focuses on extraction technologies, while the latter introduces the notion of an exogenous regulator rather than the self-enforcing nature of appropriators and focuses on compliance behavior and violation and the role of auditing probability. The works of Noailly *et al.* (2007) and Noailly *et al.* (2009) provide spatial analysis with a sanction mechanism enforced by the cooperating individuals with local interactions. Lamantia & Radi (2015) provide a technological adoption model in an analogous evolutionary framework of renewable resource exploitation and investigate the impact of technology switch has on the resource, in both discrete and continuous time. All the works described above focus on the use of imitation mechanism as seen in Schlag (1998), also known as the replicator dynamics equation.

This is one the most widely used evolutionary mechanisms providing a bounded rationality setup, due to its simple form that allows for analytical solutions. The replicator dynamics is in a sense a proportional rule through which the strategies that exhibit above average payoff, i.e. they are fittest payoff-wise, will spread in the population of strategies and have a tendency to dominate over others. An extensive analysis of the properties, and the advantages and disadvantages of this method can be found in many textbooks such as in Weibull (1997), Hofbauer & Sigmund (2003) and Schlag (1998), to name a few. Modeling compliance behavior in an evolutionary way, as implied by imitative dynamics, can be justified if we assume that agents have bounded ability to analyze certain aspects of the regulation and enforcement scheme as a whole. This bounded ability assumption can be encountered in a number of cases. In the evolutionary study of technical change (e.g. Nelson & Winter (2009), Silverberg *et al.* (1988), Conlisk (1989)) agents typically show bounded ability to perceive and explore the advantages of new technologies. In compliance related issues, taxpayers show bounded ability to calculate auditing probabilities (see Andreoni *et al.* (1998)), while fishermen seem to observe the activities of enforcing agents and communicate with each other in the process of deciding harvesting strategies which involve decisions regarding compliance with regulation (see Nielsen & Mathiesen (2003)).

The work of Xepapadeas (2005) sets the general framework from which we start off, which describes a renewable resource setup that is governed by two opposing forces. On the one hand we have the stock accumulation and on the other hand we have the imitative behavior of agents choosing whether to comply with the harvesting rule or to violate by harvesting more than they are allowed to. The coevolution of those two constitutes a dynamical system, whose equilibria describe the steady state of both resource level and harvesting rule. This is not an optimal regulation setup, but rather a descriptive long-run state after all environmental policy has been implemented. In this work we use the same principle used in Xepapadeas & Petrohilos-Andrianos (2012), i.e. allowing for myopic and optimal regulation that takes into account the evolutionary process governing the behavior of harvesters. The regulator knowing that the resource moves in fast time, calculates the optimal quota, and then seeks to regulate the resource given the behavior of harvesters. This is done in two ways; the first is a myopic regulation, where the regulator directly interferes with the replicator dynamics equation, and the second is the optimal regulation setup, where the regulator minimizes an objective function with respect to the replicator dynamics equation. We address the same problem with both a proportional and non-proportional fine and draw conclusions about the stability properties of the steady states. It is shown that polymorphic equilibria exist, i.e. both cooperative and non-cooperative rules are present in the long-run equilibrium under myopic and optimal regulation.

The structure of this work is to provide the general setup as a whole at first and then use specific functional forms in order to derive analytical solutions. The first section describes the bioeconomic model and the way the optimal quota is determined. In the second section we provide the general evolutionary framework which is the replicator dynamics equation and its properties. The third section proposes the model specifications and functional forms concerning the subjective probability of audition, the ways the stock is affected by harvesting rules and the different regulation regimes that we consider. The next section provides the model solution until the step that an analytical solution cannot be obtained. A numerical simulation is proposed in order to derive the solutions and stability properties of the steady states of the model.

2 Bioeconomic Model

Consider a renewable resource with common-pool characteristics, whose stock at any point in time is given by $S(t)$.[1] The stock regenerates in time with a rate given by $G(S)$, with inverted "U" properties, i.e. $G''(S) < 0$.

We consider that there is a fixed number of n harvesters having access to the natural resource, leading to an aggregate harvesting rate of $H(S) = \sum_{i=1}^{n} h_i(S)$, where h_i is the individual harvesting level, for $i = 1, \ldots, n$. Thus, the stock dynamics is the net difference

[1] For notation simplicity and without loss of generality, the time parameter will be intentionally omitted, i.e. $S(t) \equiv S$, etc.

between stock accumulation due to regeneration and stock depletion due to harvesting, namely:

$$\dot{S} = G(S) - \sum h_i. \tag{2.1}$$

We assume that the individual harvesting level is a function of effort, E, and stock level, S, at any given point in time, given by:

$$h_i(E_i, S) \tag{2.2}$$

The extraction of the resource induces an individual cost of $C_i(E_i)$, with $C_i'(E_i) > 0$ and $C_i''(E_i) \leq 0$.

Consider that the resource is regulated, since it needs to be managed for its sustainability in the face of the possible danger of the "tragedy of the commons". The regulator will first announce the allowed harvesting quota for the appropriators and then perform audits in order to counter any non-compliance. Assume that the regulator is interested in maximizing the total profits produced by the resource, having a clear picture of the stock biomass reserve at the time of intervention. Therefore, she takes into account dynamics, but assumes that the natural system evolves fast in time so that the dynamic equilibrium condition for the biomass can be regarded as a good approximation of the natural system, i.e. setting $\dot{S} = 0$.[2] The equilibrium stock can now be expressed explicitly as a function of harvesting from (2.1):

$$S = \phi\left(\sum h_i\right) \tag{2.3}$$

The optimal quota can then be calculated by the following maximization:

$$h^* = \arg\max_{h_i} \left[P \sum h_i(E_i, S) - \sum C_i(E_i)\right] \tag{2.4}$$

where P is the world price, taken as given. In order to solve this maximization problem, we first need to express everything in terms of the harvest and stock level.

[2] There is no problem to model the full dynamics for the natural system, but this will affect the tractability of the optimal problem because it will increase its dimensionality. Since we are interested in the behavior of the cheating/non-cheating strategies in the population, the simplifying assumption about equilibrium in resource dynamics will not affect out main results.

From (2.2), we solve for effort, such as:

$$E_i = E_i\left(h_i, S\right)$$

and consequently, cost can also be expressed as:

$$C_i = C_i\left(h_i, S\right)$$

The regulator's problem becomes:

$$h^* = \arg\max_{h_i}\left[P\sum h_i - \sum C_i\left(h_i, S\right)\right] \tag{2.5}$$

and since the regulator has accurate information about the stock reserve, at the time of intervention, this is equivalent to:

$$h^* = \arg\max_{h_i}\left[P\sum h_i - \sum C_i\left(h_i, \phi\left(\sum h_i\right)\right)\right] \tag{2.6}$$

In the next section we shall proceed with the solution for the optimal quota.

2.1 Optimal Quota

The first order conditions of problem in (2.6) for each $i = 1, \ldots, n$ are:

$$P - \underbrace{\frac{\partial C_1}{\partial h_1} - \frac{\partial C_1}{\partial \phi}\frac{\partial \phi}{\partial h_1}}_{\frac{\partial C_1}{\partial h_1}} - \sum_{j \neq 1}^{n}\underbrace{\frac{\partial C_j}{\partial \phi}\frac{\partial \phi}{\partial h_1}}_{\text{Cross Derivative}} = 0$$

$$P - \underbrace{\frac{\partial C_2}{\partial h_2} - \frac{\partial C_2}{\partial \phi}\frac{\partial \phi}{\partial h_2}}_{\frac{\partial C_2}{\partial h_2}} - \sum_{j \neq 2}^{n}\underbrace{\frac{\partial C_j}{\partial \phi}\frac{\partial \phi}{\partial h_2}}_{\text{Cross Derivative}} = 0$$

$$\cdots$$

$$P - \underbrace{\frac{\partial C_n}{\partial h_n} - \frac{\partial C_n}{\partial \phi}\frac{\partial \phi}{\partial h_n}}_{\frac{\partial C_n}{\partial h_n}} - \sum_{j \neq n}^{n}\underbrace{\frac{\partial C_j}{\partial \phi}\frac{\partial \phi}{\partial h_n}}_{\text{Cross Derivative}} = 0$$

Imposing symmetry across agents, i.e. $h_i = h$ and $C_i = C$, for all $i = 1, ..., n$ yields:[3]

$$
\begin{aligned}
P - \underbrace{\frac{\partial C}{\partial h} - \frac{\partial C}{\partial \phi}\frac{\partial \phi}{\partial h}}_{\frac{\partial C}{\partial h}} - \underbrace{\sum_{i=1}^{n-1}\frac{\partial C}{\partial \phi}\frac{\partial \phi}{\partial h}}_{\text{Cross Derivative}} &= 0 \\
P - \frac{\partial C}{\partial h} - \frac{\partial C}{\partial \phi}\frac{\partial \phi}{\partial h} - (n-1)\frac{\partial C}{\partial \phi}\frac{\partial \phi}{\partial h} &= 0 \\
P - \frac{\partial C}{\partial h} - n\frac{\partial C}{\partial \phi}\frac{\partial \phi}{\partial h} &= 0 \qquad (2.7)
\end{aligned}
$$

By substituting any analytical solution for $\phi = \phi\left(\sum h_i\right)$, we derive the optimal bioeconomic harvesting level h^*. Dividing h^* by the number of harvesters, n, the regulator sets the optimal per capita quota level, $\hat{h}$, as:

$$\hat{h} = \frac{h^*}{n}$$

Notice that, optimal per capita quota, $\hat{h}$, is now a function of the parameters of the model, and can thus be calculated explicitly.

2.2 Harvesting with Myopic Appropriators

After the optimal quota has been announced, the harvesters set the level of their harvesting. Assume that each harvester can only choose between two available strategies. The first strategy will be to comply with the optimal quota and harvest exactly the prescribed quantity from the natural resource. We denote the complying harvesting level by h^c, for which it holds that $h^c \equiv \hat{h}$. The alternative strategy will be to harvest over and above the allowed catch by violating the harvesting rule. The level of harvest in the non-complying

[3] If symmetry was imposed before the first order conditions we would get:

$$h^* = \arg\max_{h_i} \left[nph - nc\left(h, \phi\left(nh\right)\right)\right]$$

First order conditions would yield:

$$
\begin{aligned}
np - n\frac{\partial c}{\partial h} - n^2\frac{\partial c}{\partial \phi}\frac{\partial \phi}{\partial h} &= 0 \\
p - \frac{\partial c}{\partial h} - n\frac{\partial c}{\partial \phi}\frac{\partial \phi}{\partial h} &= 0
\end{aligned}
$$

leading to the same result.

case will be denoted by h^{nc}, with $h^{nc} > h^c$.

We will investigate the case of competitive harvesting, in which the non-complying harvesting level will result from myopic and unconstrained optimization of the profits of the violating representative harvester.

2.2.1 Non-cooperative Harvesting Rule

The representative harvester will maximize own profits $\pi(h_i, S) = Ph_i - C_i(h_i, S)$, by not taking the self-reproducible character of the resource and any possible fine-inducing inspection from the authorities into account. Thus, during this process, the stock level S will be treated as fixed and the optimization will be unconstrained. The maximization will produce the optimal non-complying Nash equilibrium level of harvest performed by the violating representative harvester:

$$h^{nc} = \arg\max_{h_i} \pi(h_i, S) \tag{2.8}$$

The first order condition $\frac{\partial \pi(h_i,S)}{\partial h_i} = 0$ and the second order condition $\frac{\partial^2 \pi(h_i,S)}{\partial h_i^2} < 0$ ensure sufficiency.[4]

3 Replicator Dynamics

Until now, we have seen the way the optimal quota is set and announced by the regulator. As far as the harvesters are concerned, they have two available strategies, i.e. comply or not with the harvesting rule.

The mechanism through which the harvesters switch between compliance and non-compliance is assumed to be described by the replicator dynamics equation, implying bounded rationality and imitative behavior. The harvesters will have the tendency to switch to the most profitable strategy according to the evolutionary game played in the population (see Hofbauer & Sigmund (1998), Weibull (1997), Gintis (2000)). In order to

[4] This optimization is used only to determine the harvesting level of non-cooperative harvesting rule. We assume that violators will choose this amount of harvesting rather than any other random harvest level that is over and above the per capita quota h^c.

formulate the replicator equation we first need to define the notion of profitability for a strategy switch.

Given that the natural resource is being regulated, each harvester faces a probability of being inspected and pay a fine in case she is caught deviating, i.e. if caught harvesting h^{nc} instead of h^{c}. The probability of being audited did not enter in the decision for setting the non-complying harvesting level, since this was an unconstrained myopic process. On the contrary, when expected profits are formed, it is critical to include this part. More specifically, after the two levels of harvesting are set, i.e. h^{nc} and h^{c}, determining the two available strategies, the expected profits after inspection become:

For the non-complying harvester:

$$\Pi^{nc} = \pi\left(h^{nc}, S\right) - p\mathcal{F}\left(h^{nc} - h^{c}\right) = \pi^{nc} - p\mathcal{F}\left(h^{nc} - h^{c}\right)$$

where p is the subjective probability of being inspected, and $\mathcal{F}$ is a fixed fine imposed on the level of deviation $\left(h^{nc} - h^{c}\right)$.

For the complying harvester:

$$\Pi^{c} = \pi\left(h^{c}, S\right) = \pi^{c}$$

The complying harvester, fully commits to the quota regime and therefore pays no fine whatsoever.

We denote by n_{nc} the number of non-complying harvesters and by n_{c} the number of complying harvesters in the population, with $n_{nc} + n_{c} = n$.[5] Thus, we express the fraction of each type of harvester in the population by $x = n_{nc}/n$ and $(1 - x) = n_{c}/n$. The average profit or pay-off flow for the population of n agents will be:

$$\bar{\Pi} = \frac{n_{nc}\Pi^{nc} + n_{c}\Pi^{c}}{n} = x\Pi^{nc} + (1 - x)\,\Pi^{c} \tag{3.1}$$

Suppose that in every time period dt a harvester, say i, following a certain strategy

[5] Aggregate realized harvest is now $H(S) = n_{nc}h^{nc} + n_{c}h^{c}$ and is assumed to be fully supplied in the market.

h^{nc} or h^c, learns the profit, and consequently the harvesting strategy, of another randomly chosen harvester, say j, with probability $\alpha dt > 0$. The agent will change her strategy to the other strategy if she perceives that the other's profit is higher because she follows the other harvesting strategy. If the probability that the harvester i will change her harvesting rule and follow the strategy of harvester j which leads to a higher pay-off, is proportional to the payoff difference between the two harvesters, then the evolution of the proportion of non-complying harvesters x in the population, can be described by the replicator dynamics equation. This is the proportional imitation rule applied to regulation compliance and leads to the following replicator dynamics equation.[6]

$$\begin{aligned}
\dot{x} &= x\left(\Pi^{nc} - \bar{\Pi}\right) = x\left(\Pi^{nc} - x\Pi^{nc} - (1-x)\,\Pi^{c}\right) \\
&= x(1-x)\left(\Pi^{nc} - \Pi^{c}\right) \\
&= x(1-x)\left(\pi^{nc} - p\mathcal{F}\left(h^{nc} - h^{c}\right) - \pi^{c}\right) \\
&= x(1-x)\left(\pi^{nc} - \pi^{c} - p\mathcal{F}\left(h^{nc} - h^{c}\right)\right) \\
&= x(1-x)\left(\pi\left(h^{nc}, S\right) - \pi\left(h^{c}, S\right) - p\mathcal{F}\left(h^{nc} - h^{c}\right)\right) \qquad (3.2)
\end{aligned}$$

To obtain a more clear picture of the new structure, let

$$\begin{aligned}
\mathcal{R} &= \pi\left(h^{nc}, S\right) - \pi\left(h^{c}, S\right) - p\mathcal{F}\left(h^{nc} - h^{c}\right) \qquad (3.3) \\
\text{if } \mathcal{R} &> 0 \Rightarrow \dot{x} > 0 \\
\text{if } \mathcal{R} &< 0 \Rightarrow \dot{x} < 0
\end{aligned}$$

Thus for any expected fine $F_E = p\mathcal{F}\left(h^{nc} - h^{c}\right)$, and resource stock S, $\mathcal{R}$ is independent of x and (3.2) does not have an interior rest point (or steady state) but two boundary rest points $x_1^* = 0$ and $x_2^* = 1$. If the expected fine is such that $\mathcal{R} < 0$ the proportion of non-complying harvesters converge to $x_1^* = 0$ and full compliance is attained. The full compliance steady state is stable since $\partial\dot{x}/\partial x|_{x_1^*=0} = \mathcal{R} < 0$. We say, in this case,

[6]For a detailed analysis of the proportional imitation rule see Schlag (1998), also Hofbauer & Sigmund (2003) for more general imitation dynamics. For the derivation of the specific replicator dynamics equation (3.2) of compliance imitation see Xepapadeas & Petrohilos-Andrianos (2012).

that non-compliance is *dominated* by compliance, and that the compliance steady state is attracting. On the other hand, if $F_E : \mathcal{R} > 0$, the proportion of non-complying harvesters converges to $x_2^* = 1$ and the non-compliance strategy dominates. The non-compliance steady state is locally stable since $\partial \dot{x} / \partial x|_{x_2^*=1} = -\mathcal{R} < 0$. The dominating, or the attracting strategy is a Nash equilibrium (see Hofbauer & Sigmund (2003)). It should be noted that $\mathcal{R}$ reflects a full compliance constraint for any expected fine F_E, since its sign determines the direction of the strategy switch.

4 Model Specifications

4.1 Endogenized Probability of Audition

Before we proceed with regulation, we will endogenize the subjective probability of being audited. We let p be a function of perceived level of violation size, as indicated by x . Assume that the regulator forms through the auditing process an unbiased estimate of the size of violation and then publicly announces it. Since the harvesters are considered to be able to communicate, by the nature of the replicator dynamics equation and the way it works, we can make such a hypothesis, i.e. the subjective probability of being audited will be a function of the violation level:

$$p = p(x)$$

It is reasonable to assume that if x is high, monitoring effort will be expected to increase in the future, thus leading to a higher subjective probability formed by the harvester, i.e. $p_x > 0$. The opposite is expected when x is low, that is high compliance is expected to induce a reduction in monitoring effort and consequently a lower subjective probability of being audited is formed by the harvester.

The exact form of $p(x)$ used for parameterization in later sections of the paper, will be the Kumaraswamy distribution, with a c.d.f. of $p(x) = 1 - (1 - x^{\gamma})^{\delta}$ and parameters $(\gamma, \delta) = (2, 5)$, exhibiting sigmoid properties, both intuitive and desirable in our setup.

4.2 Actual Stock and Harvest Levels

The aggregate realized harvest, $\tilde{H}$ is the harvest level that occurs, when both compliance and violation are present in the population, weighted by their respective shares:

$$\begin{aligned}\tilde{H} &= n_{nc}h^{nc} + n_c h^c \\ &= n\left(xh^{nc} + (1-x)\,h^c\right)\end{aligned}$$

Therefore the actual stock level at any point in time, denoted by $\tilde{S}$, will be a function of the realized harvest:

$$\tilde{S} : \tilde{S}\left(\tilde{H}\right) \tag{4.1}$$

We have seen that the non-complying harvester commits a violation of size $h^{nc} - h^c$, over the optimal per capita quota level. The violating harvesting level is an equation of S because each individual harvester has a myopic understanding of the stock. However, while the evolutionary game takes place as described by the replicator dynamics equation, the harvest is already taking place, so that each harvest level h^{nc} and h^c will directly affect the size of the stock, depending on the level of violation. Thus, the resulting violating harvest level can be expressed as a function of the parameters and the ratio of non-compliers, x , i.e. by replacing S with $\tilde{S}$ in the solution of (2.8) and solving for h^{nc}. Having found the violating harvest level, h^{nc}, we can substitute it back into (4.1) and find the actual stock as a function of x and the parameters, i.e. $\tilde{S} = \tilde{S}\left(x; \{...\}\right)$. Finally, we can calculate the profits of the non-complying type, $\pi\left(h^{nc}, \tilde{S}\right)$, by simple substitutions of h^{nc} and $\tilde{S}$. Throughout this work, we will be working only with the solutions that ensure intuitive results in the economic sense, namely the ones that produce positive associated profits.

After the profits have been determined, the replicator dynamics equation, can take its final form as a function of only x, $\mathcal{F}$ and the parameters.

$$\dot{x}\left(x, \mathcal{F}; \{...\}\right) = x\left(1-x\right)\left(\pi\left(h^{nc}, \tilde{S}\right) - \pi\left(h^c, \tilde{S}\right) - p\mathcal{F}\left(h^{nc} - h^c\right)\right) \tag{4.2}$$

4.3 Regulation Regimes

4.3.1 Myopic Regulation - Benchmark Case

In the first case of regulation we want to see how arbitrary changes in the policy instrument, i.e. the fine $\mathcal{F}$, affect the behavior of harvesters in the stage game, as described in the replicator dynamics equation in (4.2). In such a context, the regulator can be regarded as myopic, in the sense that there is no optimization involved during that process, but on the contrary she arbitrarily tampers with the policy instrument, in order to minimize the perceived violation. This will only serve as a benchmark case and a landmark for comparisons with the optimal regulation alternative.

4.3.2 Optimal Regulation under Endogenous Auditing

In the second and most important case of regulation, we will be investigating the optimal regulation policy for the natural resource, given that harvesters act as imitators rather than rational optimizers and that the subjective probability of audition is endogenized. The regulator has full information about the capacity of the stock and is aware of the incentives for over-harvesting. She is also aware of the fact that harvesters are myopic imitators and will fully account for all these characteristics in the process of setting the optimal policy instruments.

The regulator's objective is to determine the optimal fine, in order to keep the stock of the natural resource within viable levels for future sustainable harvesting, taking into account the proportional rule governing the harvesters' choice. The ideal stock level, denoted by $\hat{S}$, that keeps the stock compatible with the original quota level configuration, is the one formed under full compliance and will be the goal set by the regulator. This will be the stock level resulting when all harvesters comply with the quota, i.e. when total harvest is $H(S) = nh^c$. Notice that $\hat{S}$ will be a function of parameters, since h^c is fixed.

The explicit form of the problem will be given by:

$$\min_{\mathcal{F}} \int_0^\infty e^{-rt}[\frac{1}{2}\left(\tilde{S}-\hat{S}\right)^2 + C_{\mathcal{F}}]dt \tag{4.3}$$

s.t.

$$\dot{x} = x(1-x)\left(\pi\left(h^{nc},\tilde{S}\right) - \pi\left(h^{c},\tilde{S}\right) - p\mathcal{F}\left(h^{nc}-h^{c}\right)\right)$$

where $C_{\mathcal{F}}$ is the cost of policy enforcement.

The current value Hamiltonian of the problem is stated as follows:

$$\mathcal{H} = \frac{1}{2}\left(\tilde{S}-\hat{S}\right)^2 + C_{\mathcal{F}} + \lambda x(1-x)\left(\pi\left(h^{nc},\tilde{S}\right) - \pi\left(h^{c},\tilde{S}\right) - p\mathcal{F}\left(h^{nc}-h^{c}\right)\right) \tag{4.4}$$

Profits $\pi\left(h^{nc},\tilde{S}\right)$ and $\pi\left(h^{c},\tilde{S}\right)$ are the actual profits generated for each harvester type.

The optimality conditions for (4.4) include:

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial \mathcal{F}} &= 0 \\ \frac{\partial \mathcal{H}}{\partial x} &= r\lambda - \dot{\lambda} \end{aligned}$$

and will yield the optimal path of fine, $\mathcal{F}^*$.

The behavior of the steady states will then be determined by the Hamiltonian system, calculated for $\mathcal{F}^*$:

$$\begin{aligned} \dot{x}(x,\lambda;\mathcal{F}^*) &= x(1-x)\left(\pi\left(h^{nc},\tilde{S}\right) - \pi\left(h^{c},\tilde{S}\right) - p\mathcal{F}^*\left(h^{nc}-h^{c}\right)\right) \\ \dot{\lambda}(x,\lambda;\mathcal{F}^*) &= r\lambda - \frac{\partial \mathcal{H}}{\partial x} \end{aligned}$$

5 Model Solution

In this section we impose specific function forms according to the recent literature, in an attempt to analytically solve the model in the previously described steps. We assume that:

1. Stock regenerates following the logistic growth:

$$G(S) = gS\left(1 - \frac{S}{K}\right) \tag{5.1}$$

2. Individual harvest is described by a Cobb-Douglas function with constant returns to scale:

$$h_i(S) = qE_i^{1-a}S^b \tag{5.2}$$

where q is the catchability coefficient, a and b are parameters which lie strictly in the $[0, 1]$ range. We will examine the case where $a = b = \frac{1}{2}$ for the sake of result tractability:[7]

$$h_i(S) = \sqrt{qE_iS} \tag{5.3}$$

3. The cost induced by the extraction of the natural resource is considered to be a linear function of the effort:

$$C_i = wE_i \tag{5.4}$$

where w is a scaling parameter, also serving as the marginal cost.

5.1 Quota

Starting by (5.3) and solving for effort, we obtain:

$$E_i(h_i, S) = \frac{h_i^2}{qS} \tag{5.5}$$

and

$$C_i(h_i, S) = w\frac{h_i^2}{qS} \tag{5.6}$$

[7]Note that it is indifferent in terms of result tractability whether the catchability coefficient q is placed inside or outside the square root. We choose the former for notation simplicity.

Next step is to calculate the stock level at the moment of intervention by solving:

$$\begin{aligned}\dot{S} &= G(S) - \sum h_i \\ 0 &= gS\left(1 - \frac{S}{K}\right) - \sum h_i\end{aligned}$$

The solutions to this quadratic polynomial are:

$$S_{1,2} = \phi\left(\sum h_i\right) = \frac{Kg \pm \sqrt{-4Kg\sum h_i + (Kg)^2}}{2g} \tag{5.7}$$

provided that $\sum h_i \leq \frac{Kg}{4}$, in order to have a real solution. The only acceptable solution will be $S_1 = \frac{Kg+\sqrt{-4Kg\sum h_i + K^2g^2}}{2g}$, since the relationship between stock and harvest is negative, as dictated by the nature of the problem.[8]

The problem of the regulator in (2.6) can now be solved analytically:

$$h^* = \arg\max_{h_i}\left[P\sum h_i - \sum C_i\left(h_i, \phi\left(\sum h_i\right)\right)\right]$$

Substituting (5.6) and $S_1 = \frac{Kg+\sqrt{-4Kgnh+K^2g^2}}{2g}$ in (2.7) we get two solutions for the optimal

[8] If the non-linear relationship between stock and harvest level causes computational problems, one could resort to a linear approximation around some point given a set of parameters, e.g. around a point $\bar{H}$, for which it holds that:

$$\rho^0 S^0\left(1 - \frac{S^0}{K^0}\right) - \bar{H} = 0$$

for some calibrated values (S^0, K^0, ρ^0). The Taylor expansion will yield the linearized function of the stock, for which there will be an equilibrium in the stock dynamics, i.e. $\dot{S} = 0$.

$$\begin{aligned}S(H) &= S(\bar{H}) + \left.\frac{\partial S(H)}{\partial H}\right|_{H=\bar{H}}(H - \bar{H}) \\ S(H) &= \frac{K^0\rho^0 + \sqrt{-4K^0\rho^0\bar{H} + (K^0\rho^0)^2}}{2\rho^0} + \frac{K^0}{\sqrt{-4K^0\rho^0\bar{H} + (K^0\rho^0)^2}}(H - \bar{H})\end{aligned}$$

The expression is a function of $H = \sum h_i$ and can be rewritten as such:

$$\phi\left(\sum h_i\right) = A + B\left(\sum h_i - \bar{H}\right)$$

where $A = \frac{K^0\rho^0+\sqrt{-4K^0\rho^0\bar{H}+(K^0\rho^0)^2}}{2\rho^0}$ and $B = \frac{K^0}{\sqrt{-4K^0\rho^0\bar{H}+(K^0\rho^0)^2}}$. The linearized solution could replace the original, offering algebraic comfort in cases where a closed form solution by using the former is unfeasible.

bioeconomic harvesting level:

$$\begin{aligned}
h_1^* &= K\frac{(w^2g^2 + 2wgnPq - 2n^2P^2q^2) + |wg - 2nPq|\sqrt{w^2g^2 - wgnPq + n^2P^2q^2}}{9ngw^2} \\
&= K\frac{(wg + nPq)^2 - 3n^2P^2q^2 + |wg - 2nPq|\sqrt{(wg + nPq)^2 - 3wgnPq}}{9ngw^2}
\end{aligned}$$

and

$$\begin{aligned}
h_2^* &= K\frac{(w^2g^2 + 2wgnPq - 2n^2P^2q^2) - |wg - 2nPq|\sqrt{w^2g^2 - wgnPq + n^2P^2q^2}}{9ngw^2} \\
&= K\frac{(wg + nPq)^2 - 3n^2P^2q^2 - |wg - 2nPq|\sqrt{(wg + nPq)^2 - 3wgnPq}}{9ngw^2}
\end{aligned}$$

We need both solutions to be positive and real-valued. The radicand $(wg + nPq)^2 - 3wgnPq$ is always positive for all $w, g, n, P, q > 0$. The first solution h_1^* is always positive, since $(wg + nPq)^2 - 3n^2P^2q^2 > 0$, for all $w, g, n, P, q > 0$. The solution h_2^* is positive as long as $wg > nPq$. Dividing the optimal bioeconomic harvesting level by the number of harvesters, n, the regulator finds the optimal per capita quota level, denoted by $\hat{h}$:

$$\hat{h}_1 = K\frac{(wg + nPq)^2 - 3n^2P^2q^2 + |wg - 2nPq|\sqrt{(wg + nPq)^2 - 3wgnPq}}{9n^2gw^2}$$

and

$$\hat{h}_2 = K\frac{(wg + nPq)^2 - 3n^2P^2q^2 - |wg - 2nPq|\sqrt{(wg + nPq)^2 - 3wgnPq}}{9n^2gw^2}$$

For the sake of result refinement and tractability, we shall continue with the quota $\hat{h}_1$, that is always positive.

5.2 Harvester Side

On the harvesters side, we calculate the myopic levels of unconstrained harvesting h^{nc} from (2.8). More specifically:

$$\begin{aligned} h^{nc} &= \arg\max_{h_i} \pi\left(h_i, S\right) \\ &= \arg\max_{h_i} \left[P h_i - C_i\left(h_i, S\right)\right] \end{aligned}$$

yielding the non-cooperative Nash harvesting level:

$$h^{nc} = \frac{PqS}{2w} \tag{5.8}$$

In order to proceed with the formulation of the replicator dynamics equation, the first step is to calculate the actual stock as shown in (4.1). The aggregate harvest level $\tilde{H}$ will be given by:

$$\begin{aligned} \tilde{H} &= n_{nc} h^{nc} + n_c h^c \\ &= n\left(x h^{nc} + (1-x)\, h^c\right) \end{aligned}$$

Therefore, the actual stock can now be directly calculated from (5.7) as follows:

$$\begin{aligned} \tilde{S} &= \frac{Kg + \sqrt{-4Kg\tilde{H} + K^2 g^2}}{2g} \\ &= \frac{Kg + \sqrt{-4Kgn\left(x h^{nc} + (1-x)\, h^c\right) + K^2 g^2}}{2g} \end{aligned} \tag{5.9}$$

The final step is to solve for the violation level, expressing it as a function of the ratio of non-compliers x, and the parameters. In order to achieve that, we replace S with $\tilde{S}$ in (5.8) ending up with h^{nc} appearing in both sides of the equations. Solving for h^{nc} in each contingency will produce the violation level that we need, as follows:

$$
\begin{aligned}
h^{nc} &= \frac{PqS}{2w} \\
h^{nc} &= \frac{Pq\tilde{S}}{2w} \\
h^{nc} &= \frac{Pq\frac{Kg+\sqrt{-4Kgn(xh^{nc}+(1-x)h^{c})+K^2g^2}}{2g}}{2w} \\
h^{nc} &= h\left(x;\{K,w,g,n,P,q\}\right) \qquad (5.10)
\end{aligned}
$$

There are two solutions in (5.10). We keep the solution that produces positive profits and well behaving actual stock. We can now substitute the violation levels back into (5.9) and find the actual stock as a function of x and the parameters, i.e. $\tilde{S} = \tilde{S}\left(x;\{K,w,g,n,P,q\}\right)$. The same holds for the profits of the non-complying type, $\pi\left(h^{nc},\tilde{S}\right)$. Having expressed everything in terms of x and the parameters, the replicator dynamics equation in (4.2) can now be written as:

$$
\dot{x}\left(x,\mathcal{F};\{K,w,g,n,P,q\}\right) = x\left(1-x\right)\left(\pi\left(h^{nc},\tilde{S}\right)-\pi\left(h^{c},\tilde{S}\right)-p\mathcal{F}\left(h^{nc}-h^{c}\right)\right) \qquad (5.11)
$$

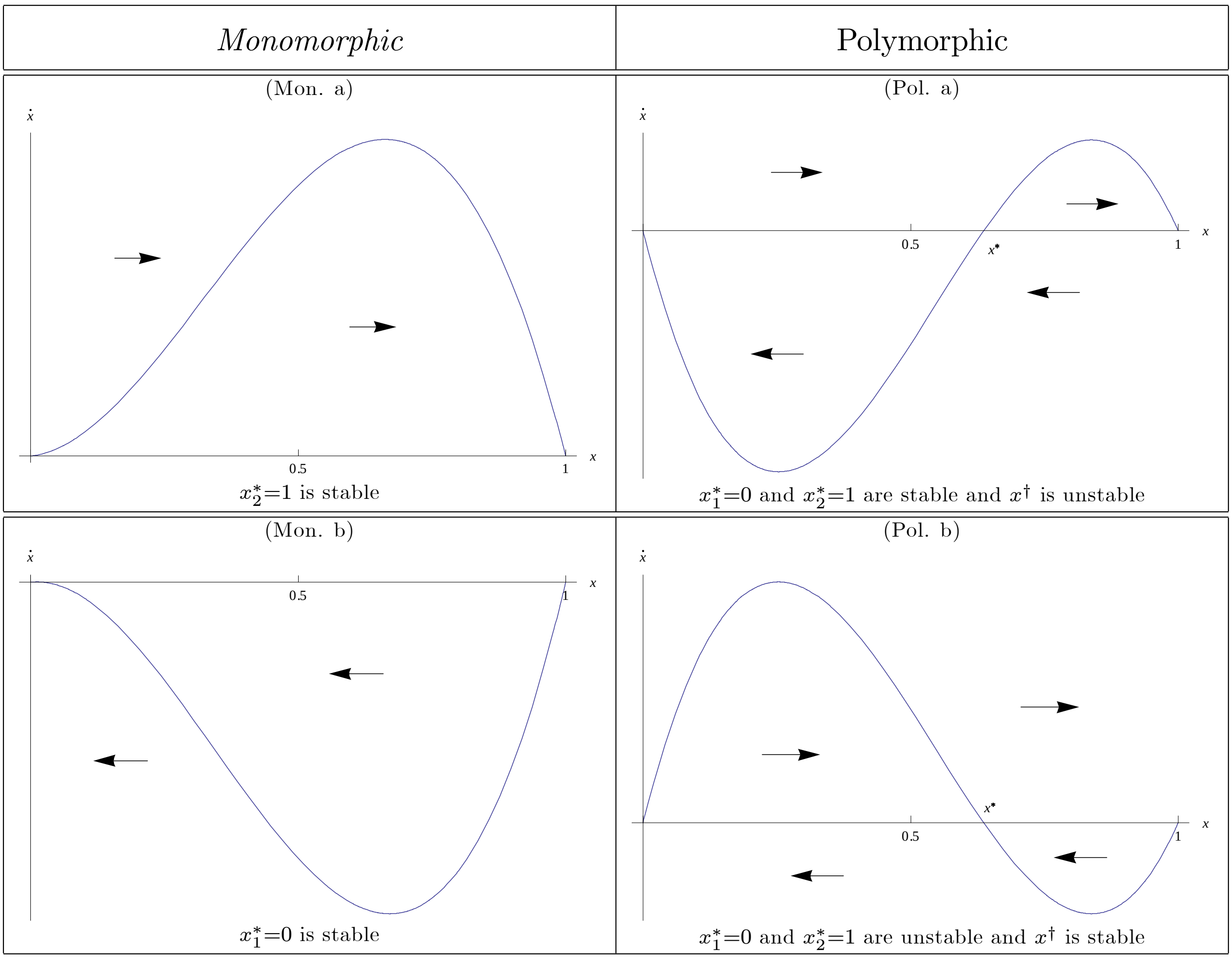
5.3 Regulation

The next step is to perform regulation in the two regimes, i.e. myopic and optimal setup. In the myopic regime, the regulator tampers with the fine, which is the only policy instrument at her disposal, in an attempt to drive the non-compliance level as low as possible, through the direct impact that the fine has on the behavior of harvesters as described in (5.11). This is obviously a non-optimal behavior since the regulator does not take into consideration any costs induced by the policy instrument and is therefore not driven to the optimal choice. In the optimal regime, the regulator has a clear view of the policy objective and the cost it induces. As a result, the fine selected will be optimal intertemporarily and will lead to the same compliance level as it would have led in the myopic regulation case, if its level was known.[9]

[9]According to the Pontryagin principle, the optimal fine should give the same state variable solution for the state equation.

5.3.1 Myopic Regulation

In the first regime, the replicator equation is the relationship that describes the dynamics of the behavior in the population of harvesters. Depending on the parameters we end up having monomorphic or polymorphic equilibria, as depicted in Figure 1. In the monomorphic case we can attain a stable full compliance level at once, if the dynamics permit it, whereas for the polymorphic case of one interior steady state, full compliance and full non-compliance are both present in the same contingency. In (Pol. a) a small perturbation from the unstable interior steady state $x^\dagger$ leads to either total conformity or total non-compliance. The desirable polymorphic case is, therefore, the one that is characterized by unstable corner solutions and a steady interior $x^\dagger$, i.e. the one depicted in (Pol. b). This is also the most realistic of all four contingencies, not mentioning cases with multiple interior steady states that outperform all of the above, in terms of diversity.



-Figure 1-

In the numerical approximation section, we end up with a situation of a monomorphic behavior as in (Mon. a), for low levels of fine. As the regulator increases the fine the behavior switches to polymorphic as in (Pol. b), implying that the interior steady state can be stable under certain conditions, e.g. if we have saddle path stability.

5.3.2 Optimal Regulation

In the second regime, the regulator solves the problem in (4.3) subject to the newly formed replicator dynamics shown by (5.11). The current value Hamiltonian of the problem will be:

$$\mathcal{H} = \frac{1}{2}\left(\tilde{S} - \hat{S}\right)^2 + C_{\mathcal{F}} + \lambda x\left(1-x\right)\left(\pi\left(h^{nc}, \tilde{S}\right) - \pi\left(h^{c}, \tilde{S}\right) - p\mathcal{F}\left(h^{nc} - h^{c}\right)\right)$$

Notice that the stock target is be given by:

$$\hat{S} = \frac{Kg + \sqrt{-4Kgnh^c + K^2 g^2}}{2g}$$

that is when all harvesters comply with the harvesting rule.

The first order conditions imply:

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial \mathcal{F}} &= 0 \\ \frac{\partial \mathcal{H}}{\partial x} &= r\lambda - \dot{\lambda} \end{aligned}$$

Notice that the probability of audition p, and the level of violation h^{nc}, are now functions of the ratio of non-compliers x. The first order conditions yield the optimal fine as a function of the state and costate variables, $\mathcal{F}^* (x, \lambda)$. The hamiltonian system calculated at $\mathcal{F}^*$ will be:

$$\begin{aligned} \dot{x} (x, \lambda; \mathcal{F}^* (x, \lambda)) &= x (1 - x) \left(\pi \left(h^{nc}, \tilde{S} \right) - \pi \left(h^c, \tilde{S} \right) - p\mathcal{F}^* (h^{nc} - h^c) \right) \\ \dot{\lambda} (x, \lambda; \mathcal{F}^* (x, \lambda)) &= r\lambda - \frac{\partial \mathcal{H}}{\partial x} \end{aligned}$$

The solution of the system for $\dot{x} = 0$ and $\dot{\lambda} = 0$ will yield the optimal x^* and λ^* which will in turn identify the optimal fine $\mathcal{F}^* (x^*, \lambda^*)$.

5.3.3 Special Case: Non-proportional Fine

As a special case, we investigate the non-proportional fine scenario, in which the fine is not imposed on the difference between the violation and the quota $h^{nc} - h^c$, but is rather a fixed fine. This will change the replicator dynamics equation and thus the behavior of harvesters and the evolution of their strategies. The proportional rule describing this special case will be:

$$\dot{x} (x, \mathcal{F}; \{K, w, g, n, P, q\}) = x (1 - x) \left(\pi \left(h^{nc}, \tilde{S} \right) - \pi \left(h^c, \tilde{S} \right) - p\mathcal{F} \right)$$

We expect this fine to be less efficient in terms of compliance enforcement, as it implies a lower perceived penalty on violators.

6 Numerical Approximation

In this section we apply numerical simulations due to the nonlinearity of the hamiltonian system. The following parameterization will be used, following Da-Rocha *et al.* (2014) and Vardas & Xepapadeas (2015):

$a = b = \frac{1}{2}$, $n = 3$ or $n = 4$, $p = 10$, $w = 5$, $q = 0.045$, $g = 0.45$, $K = 7000$, $r = 0.02$.[10]

The subjective probability of audition is given by $p(x) = 1 - (1 - x^{\gamma})^{\delta}$, with $(\gamma, \delta) = (2, 5)$.

As far as the cost of the policy instrument is concerned we assume a quadratic cost function:

$$C_{\mathcal{F}} = \frac{1}{2} z \mathcal{F}^2$$

where z is the cost coefficient set at $z = 0.1$. We will focus on the interior steady states that imply the coexistence of both harvesting rules, rather than the monomorphic states of full compliance, $x^* = 0$ and full violation, $x^* = 1$. It must be noted that the system is very sensitive to changes in parameter values and we therefore present limited results, avoiding any false conclusions and dependencies.

6.1 Proportional Fine

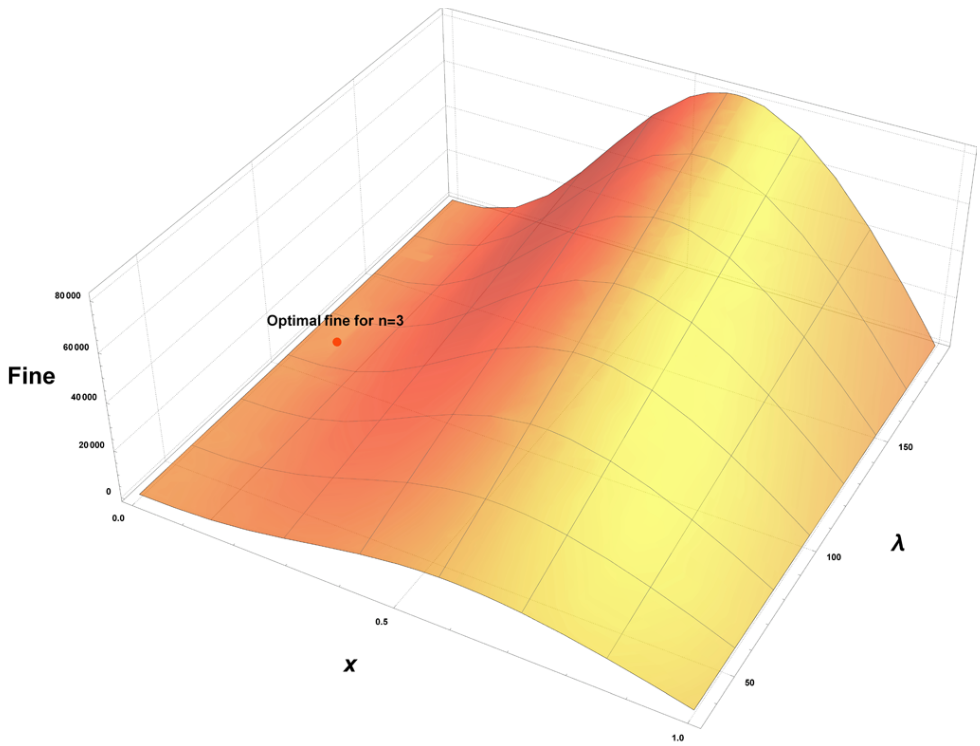
6.1.1 Optimal Regulation

In the proportional fine context we get the following optimal regulation results:

[10] The level of harvesters n, is crucial for the tractability of the results. One can think of n as a normalized number for vessels in case of a fishery, or a number of firms with many harvesters as employers.

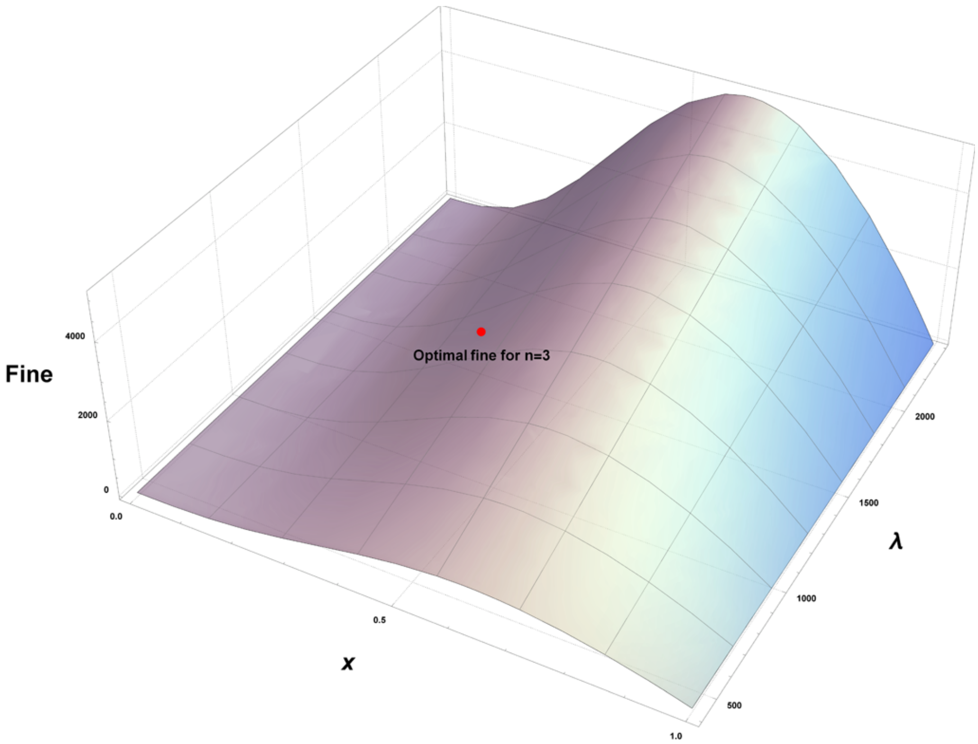
	$n=3$	$n=4$
Per capita quota h^c:	66.3614	41.8092
Compliance x^*:	0.0570834	0.0520801
Costate λ^*:	115.104	179.45
Level of violation h^{nc} :	308.02	310.219
Optimal fine $\mathcal{F}^*(x^*, \lambda^*)$:	242.345	320.732
Stock target $\hat{S}$:	6525.42	6606.21
Actual Stock $\tilde{S}$:	6417.08	6462.89
Profits of compliers $\pi\left(h^c, \tilde{S}\right)$:	592.128	389.918
Profits of violators $\pi\left(h^{nc}, \tilde{S}\right)$:	1540.1	1551.09
Probability of audition $p(x^*)$:	0.0161867	0.0134883

In the $n=3$ case, the non-complying harvesters occupy 5.7 percent of the population and the harvesting level of the violators is approximately 4.6 times the per capita quota. With an increase in the number of appropriators, we witness a decline in the per capita quota and an increase in the level of compliance. The ratio of non-compliers is now at 5.2 percent of the population and the violation is 7 times the per capita quota. The slight increase in compliance can be attributed to stock effects due to the increased number of harvesters that make extraction more costly. Since the cost of regulation remains unchanged, an increase in the profits of violators from 1540.1 to 1551.09 needs an increased fine in order for the interior steady state to be achieved. More specifically, notice that with the increase in the number of harvesters, the per capita quota has dramatically decreased from 66.3614 to 41.8092, which is detrimental to the profits of compliers which have decreased from 592.128 to 389.918. The difference between the profits of the two types of harvesters has increased and a greater intervention, i.e. a greater fine is required to keep that difference at zero level. Moreover, the costate variable λ is positive showing that a change in the constraint, namely the ratio of violators, induces a cost in the value function. The objective function in (4.3) is a cost function, therefore an increase in the number of harvesters, that has in turn increased the actual stock, makes the costate fall since the objective goal is relaxed. All steady states, i.e. $x^* = 0.0570834$, $x^* = 0.0520801$ as well as the corner steady states $x^* = 0$ and $x^* = 1$ for both n states are saddle points.



Proportional Fine, n=3

-Figure 2-



Non-proportional Fine, n=3

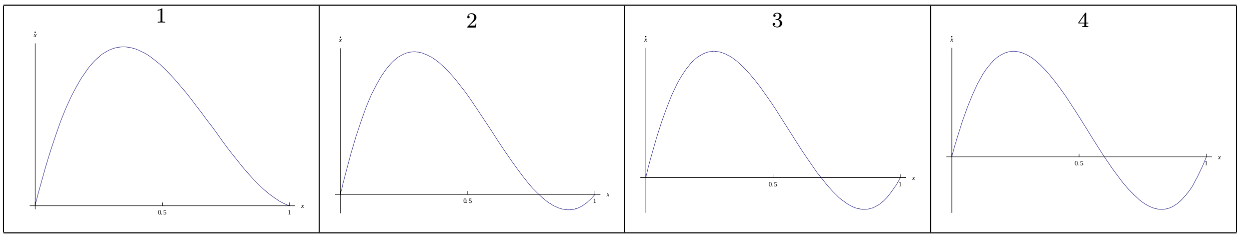
-Figure 3-

In Figures 2 and 3, we plot the optimal proportional fine as a function of the ratio of violators and the costate variable. Notice that as the ratio of non-compliers increases, an increasing fine is needed up until a certain point, in order to compensate for the cost due to the reduction of the stock. As the violation increases beyond a certain point, the probability of audition tends to unity, and the fine required is falling, since the violators profits have significantly lower payoff than the profits of the complying harvesters.

6.1.2 Myopic Regulation

In the myopic regulation context, the regulator faces the following monomorphic to polymorphic behavior as described in Figure 4. As the fine increases from zero level to higher levels [Frames 1-4 in Figure 4], more and more harvesters are switching from violation to compliance. Notice that the phase diagram of Figure 4, implies two stable steady states, i.e. the one of full violation, $x^* = 1$, and the interior steady state, as shown in Figure 1 (Mon. a) and (Pol. b) respectively. On the other hand, the full compliance equilibrium,

$x^* = 0$, is unstable. Nevertheless, the stable interior steady state is far more interesting since it implies that in equilibrium, both harvesting rules will be present at the same time.



-Figure 4-

As the fine level approaches its optimal level $\mathcal{F}^*$, the system converges to the interior steady state x^* as determined in the optimal control setup. This can be thought of as an indirect verification of the optimal control problem and is the empirical proof of Pontryagin's principle. The main difference to the optimal control setup is that in myopic regulation the level of fine may go beyond the optimal level and even lead to higher compliance levels, i.e. lower x^*. This is obviously not the optimal choice by the regulator in terms of the objective function, but in the specific context where virtually we have assumed no constraints, the regulator can drive the system to higher level of compliance to the extent that the policy can be socially, politically or financially supported.

6.2 Non-proportional Fine

6.2.1 Optimal Regulation

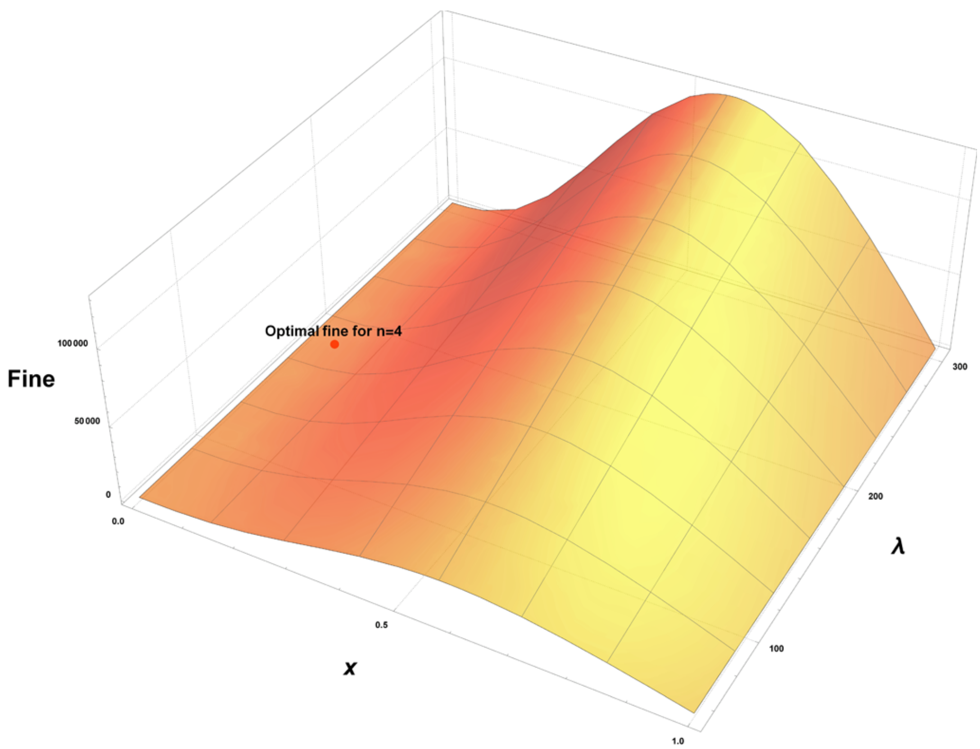
In the non-proportional fine context we get the following optimal regulation results:

	$n = 3$	$n = 4$
Per capita quota h^c:	66.3614	41.8092
Compliance x^*:	0.361323	0.337109
Costate λ^*:	1402.51	2136.72
Level of violation h^{nc} :	280.929	273.125
Optimal fine $\mathcal{F}^*(x^*, \lambda^*)$:	1628.52	2162.67
Stock target $\hat{S}$:	6525.42	6606.21
Actual Stock $\tilde{S}$:	5852.69	5690.1
Profits of compliers $\pi\left(h^c, \tilde{S}\right)$:	585.234	386.092
Profits of violators $\pi\left(h^{nc}, \tilde{S}\right)$:	1404.65	1365.62
Probability of audition $p(x^*)$:	0.503165	0.452928

In this context, the violation level increases dramatically to a 36.1 percent of the population, whereas the non-cooperative harvesting level is 4.2 times the per capita quota. We see that the less stringent penalty system of the non-proportional fine has increased the ratio of non-compliers by 600 percent from the case of the proportional fine, implying that more harvesters choose to engage into illegality. Notice that by construction the penalty system does not affect the violation level h^{nc}. The non-cooperative harvesting rule is determined by the compliance level in the population, and since the non-complying harvesters have increased notice that the absolute level of violation is lower than in the proportional fine setup (280.929 from 308.02 and 273.125 from 310.219). This is the reason the profits of harvesters have also dropped. The level of non-compliance x^* is again almost 7 times higher than in the proportional fine case, but has dropped by 3 percentage points in comparison to the case of less harvesters in the non-proportional fine case. The violation harvest level is 6 times the per capita quota.

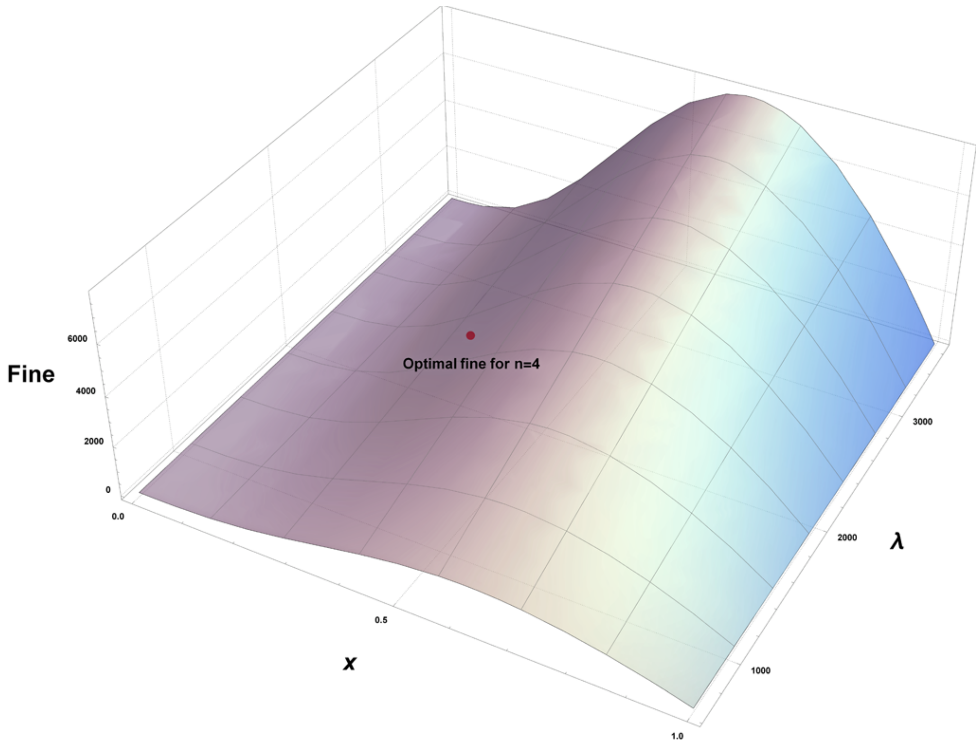
It is evident that the non-proportional fine is far less efficient than the proportional alternative. Notice that the level of non-compliance has risen significantly from a 5 percent in the proportional fine setup to a 33 percent in the non-proportional fine setup.

Moreover, the optimal fine that leads the system to rest in the interior solution x^*, has also leaped from the levels of 242 through 320, to 1628 through 2162. Notice also, the sensitivity in the number of harvesters, i.e. an increase in n from 3 to 4 comes with an increase of almost 33 percent in the fine required to achieve it. Having more non-complying harvesters implies less stock in less time, which decreases the level of h^{nc} since they share less of the resource and extraction becomes more costly. The costate λ in the non-proportional case is almost ten times larger, which shows the detrimental effect of the change in the policy instrument on the effectiveness to reduce violation. A higher tax is needed to control non-compliance and the cost of an extra violator in the population is extremely higher, as shown by the λ^* levels. All steady states, i.e. $x^* = 0.361323$, $x^* = 0.337109$ as well as the corner steady states $x^* = 0$ and $x^* = 1$ are saddle points.



Proportional Fine, n=4

-Figure 5-



Non-proportional Fine, n=4

-Figure 6-

In Figures 5 and 6, we witness the same qualitative behavior as in the proportional fine case. The only thing that changes with respect to the situation depicted in Figures 2 and 3 is the level of the optimal fine and the respective optimal levels of x^* and λ^*.

6.2.2 Myopic Regulation

The myopic regulation regime results for the non-proportional fine setup has the same qualitative characteristics presented in Figure 4 above. The only difference between the two setups is the fact that in the non-proportional fine an increase in the fine level causes a slower transition from Frame 1 to Frame 4. This is due to the lower sensitivity that a fine increase has on the profits of the non-complying harvester.

7 Discussion and further research

This work has focused on a renewable resource exploitation model in which two opposing dynamics are taken into account, namely the resource accumulation dynamic and the harvesting rules of the appropriators. The coevolution of those two forces has been investigated through a different viewpoint in both Noailly *et al.* (2003) and Xepapadeas (2005). Our work introduced an active regulator who wishes to control for the evolutionary behavior of harvesters as reflected by their strategy choice. Assuming that the regulator has information about the properties of the stock and that the natural system evolves fast in time, she chose to instantaneously regulate the resource by solving for its maximum sustainable yield. This way, she found the optimal harvesting rule that will ensure the sustainability of the resource given that all harvesters comply. The harvesters on the other hand were able to choose between compliance or non-compliance with the per capita quota introduced by the regulator. The process of strategy switching is governed by imitative dynamics as described by the replicator dynamics equation, which is a proportional rule that states that strategies with a higher payoff will tend to spread in the population, while strategies with less payoff will tend to become extinct. Without any regulation or inspections, the violating strategy is always more profitable. Therefore, we investigated the regimes of myopic and optimal regulation with endogenous auditing probability. The objective of the regulator was to minimize the square deviation from the target stock biomass, namely the stock that would be achieved if every harvester complied. The policy instrument is a fine that can be either proportional on the level of violation or remain fixed. Our analysis has shown, that optimal regulation can lead to either monomorphic states of full compliance or full violation, or a polymorphic state where harvesters are distributed between the two strategies. We have seen that the non-proportional fine is less effective and leads to equilibria with a higher share of violators in the population of harvesters. The polymorphic steady state is more interesting due to the realistic element, but the long-run equilibrium of any system like these depends on parameter values and initial conditions, on the nature of the steady states and on policy objectives.

Further research regarding the regulation of a renewable resource governed by evolutionary choice may include spatial analysis or local interactions between subgroups of agents in the population. Moreover, the inclusion of more strategies or even an infinite dimension replicator dynamics can be introduced, significantly encumbering calculations and the tractability of solutions.

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