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Spatial Environmental and Resource Economics

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Spatial Environmental and Resource Economics[1]

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Abstract

Although the spatial dimension is embedded in the vast majority of issues studied by environmental and resource economics, its incorporation into economic models – especially in the form of explicit introduction of a spatial transport mechanism – is not widespread. As a result, important aspects of these issues may not be accounted for, which could lead to regulatory inefficiencies. In this paper, the major spatial transport mechanisms are discussed, along with the way in which they can be incorporated into forward-looking optimizing economic models. Furthermore, an extension of Pontryagin's maximum principle under spatial dynamics is provided and the emergence of spatial pattern formation through optimal Turing instability is explained. A number of examples of the use of spatial dynamics illustrate why space matters in environmental and resource economics. Moreover, the differentiation of policy when spatial transport mechanisms are taken into account is presented. The tools presented in the paper, along with their applications, provide a path for future research in environmental and resource economics in which the underlying spatial dimension – which is very real – is fully taken into account.

1 Introduction

Space is a central feature in the study of the environment and natural resources: air pollutants are transported in the atmosphere from the source of their emissions via turbulent eddy motion and winds; heat is transported from the Equator toward the Poles; and resources diffuse in space, usually moving from high to low concentration locations. In terms of the natural sciences, the spatial dimension relates mainly to the study of mechanisms that explain the emergence of spatial patterns in nature, such as Polar amplification (i.e., the spatial pattern of the temperature anomaly[1]), stripes or spots on animal coats, the spatial distribution of the brown cloud in South Asia and the Indian Ocean, and many others (e.g., Ramanathan et al. 2002; Cantrell and Cosner 2003; Murray 2003; Hoyle 2006; Bekryaev, Polyakov, and Alexeev 2010).

In economics, the spatial dimension has been extensively analyzed in the context of new economic geography. There is a large body of literature studying agglomerations and clusters in various spatial scales, as a result of interactions between scale economies and spatial spillovers.[2] Moreover, a strand of literature on spatial growth theory has emerged which studies the spatiotemporal characteristics of economic growth under spatial knowledge spillovers or capital diffusion.[3]

When the spatial features characterizing the environment and natural resources are combined with the activities of economic agents who – acting as forward-looking optimizing producers or consumers – interact with the environment, a number of new issues emerge. These issues are not captured by the standard approach of environmental and resource economics which – with a number of significant exceptions which are discussed in this paper[4] –

[1]This is the change in temperature relative to a given benchmark temperature.

[2]See, for example, Krugman (1996, 1998); Fujita, Krugman, and Venables (1999); Lucas and Rossi-Hansberg (2002); Quah (2002); Baldwin et al. (2003); Baldwin and Martin (2004); Fujita and Mori (2005); Ioannides and Overman (2007); Desmet and Rossi-Hansberg (2010); Fujita and Thisse (2013); Brock, Xepapadeas, and Yannacopoulos (2014d); and Redding and Rossi-Hansberg (2017).

[3]See, for example, Quah (1996, 1997); Boucekkine, Camacho, and Zou (2009); Desmet and Rossi-Hansberg (2009); Boucekkine, Camacho, and Fabbri (2013); and Brock, Xepapadeas, and Yannacopoulos (2014a).

[4]See, for example, Kaitala, Pohjola, and Tahvonen (1992); Mäler and de Zeeuw (1998); Sanchirico and Wilen (1999, 2005); Goetz and Zilberman (2000, 2007); Smith and Wilen (2003); Xabadia, Goetz, and Zilberman (2004); Brock and Xepapadeas (2005, 2008, 2010); Sanchirico (2005); Wilen (2007); Smith, Sanchirico, and Wilen (2009); and Brock, Xepa-

does not account for the underlying spatial dimension.

Pigouvian taxes which internalize the environmental externality, or cap-and-trade policies and tradable emissions permits which try to substitute for the missing markets for environmental goods such as clean air, are the standard instruments of environmental policy. In resource management, landing taxes are used to control commercial fisheries. When space is not taken into account, the optimal price instrument (tax) or the optimal quantity instrument (permits) is derived as a solution of a social welfare maximization problem with the underlying assumption that the spatial diffusion of the externality is infinite and therefore the externality is uniform in space. This in turn implies that spatially-uniform policy instruments will be used to correct for the externality.

In reality, however, the diffusion of the environmental externalities is not infinite. This could generate spatial patterns for the externality which range from local scales such as differences in local ambient pollution, to global scales related to problems such as acid rain (with different acid depositions in different locations) or Polar amplification which induces different magnitudes of the temperature anomaly across the globe. Furthermore, the interaction of diffusive environmental externalities with mechanisms generating economic agglomerations and clustering introduces new elements which should be accounted for in policy design. In a similar way, renewable resources move in space and generate spatial patterns or clusters as their movement is taken into account in harvesting decisions.

When diffusive externalities are present, then spatially-uniform Pigouvian taxes might not be optimal. That is, instruments under the assumption of perfect mixing and spatial homogeneity might be different from the optimal instruments which take into account spatial diffusion. This could lead to the need for localized Pigouvian taxes or cap-and-trade policies. Moreover, when the diffusive externalities interact with economic centipedal or centrifugal forces, more instruments in addition to Pigouvian taxes or cap-and-trade policies might be necessary. Acemoglu et al. (2016) have shown the need for additional instruments in the context of climate change. When dirty and green technologies compete in production, a carbon tax is not sufficient to correct for the climate externality; subsidies to encourage production and innovation in green technologies are also required.

padeas, and Yannacopoulos (2013, 2014b).

An important characteristic of diffusive externalities relates to the interaction between the spatial dimension and the temporal dimension. Environmental and resource management problems are analyzed mostly in a dynamic context. When spatial diffusion is introduced, novel issues emerge. Does spatial diffusion of the environmental externality induce the evolution of spatial patterns? Is it optimal to support spatial patterns or is it optimal to suppress them and seek convergence to spatially-homogeneous outcomes? What is the appropriate policy instrument or menu of policy instruments for attaining these objectives?

Ambiguity and model misspecification concerns, along with aversion to ambiguity, are emerging as important issues in both theory and policy design (Hansen and Sargent 2001, 2008; Hansen et al. 2006). These issues are especially important in the context of climate change where large ambiguities regarding process and impacts exist and model misspecification raises issues regarding the reliability of policies derived from such models (Pindyck 2007, 2011, 2012, 2013; Brock and Hansen 2019). With a diffusive externality such as heat transport toward the Poles, ambiguity acquires a spatial structure because aversion to ambiguity could be different across locations, while misspecification concerns could be very important for high impact locations (hot spots). In this case, policy design under ambiguity and misspecification concerns, using methods such as robust control, needs to account for the spatial dimension.

The arguments presented above suggest that the study of diffusive environmental externalities and spatial spillovers is important in order to understand their interactions with the economy and the mechanisms generating endogenously spatial patterns in coupled systems of the economy and the environment, and to design the appropriate regulatory instruments to control them. The purpose of this paper is, therefore, to present ways to model spatial transport mechanisms and the emerging spatial (or diffusive) externalities and to incorporate them into dynamic, forward-looking optimizing economic models, to explore the potential endogenous emergence of optimal spatial patterns under diffusive externalities, and to present policy instruments for controlling them.

The study of diffusive externalities or resources in a dynamic optimization framework with continuous space requires the extension of standard optimal control methods in which dynamic constraints are represented by ordinary differential equations (ODEs) to the case in which the dynamic constraints are partial differential equations (PDEs) or integrodifferential equations (IDEs). We present a heuristic extension of Pontryagin's principle for solving these problems which could be a useful tool for economists studying these issues.[5]

When the analysis is extended to spatiotemporal domains, a central issue in the natural sciences is the way in which reaction-diffusion systems could generate patterns in space. Alan Turing, in his seminal paper (Turing 1952), showed how diffusion could generate spatial patterns. The Turing mechanism requires a system of at least two interacting state variables and its applications in the context of new economic geography (Krugman 1996, 1998)[6] were not directly linked to explicit dynamic optimization. The Turing mechanism has been extended to optimizing spatiotemporal systems with diffusive externalities by Brock and Xepapadeas (2008), with results suggesting that diffusion can generate spatial patterns in the quantity-shadow value space (or state-costate space), which is equivalent to pattern generation in the state-control space. This result raises issues related to optimal pattern formation, differences in spatial patterning between socially-optimal and market solutions, and the design of optimal spatially-dependent policy instruments.

In this context, our paper has three main parts. The first (sections 2 and 3) presents models of spatial transport methods and methods of spatiotemporal optimization. The second (section 4) analyzes the emergence of spatial patterns in optimizing systems with diffusive externalities. In the third (sections 5, 6 and 7), we present examples of optimal management and policy design under diffusive externalities and spatial spillovers which refer to fishery management, groundwater management, pollution control, urban economics, climate policy, and the management of spatially-structured un-

[5] It should be noted that when the space is discrete (i.e., we have patches with dispersion of populations, or pollutants across patches), then dynamic optimization involves a set of ODEs, one for each patch (see, for example, Smith, Sanchirico, and Wilen (2009) as well as section 5.1 in this paper). In continuous space the dimensionality is drastically reduced, although the difficulty of solving a dynamic optimization problem with PDEs as constraints is added. Furthermore, as will be shown in section 4, the mechanism generating spatial patterns and agglomerations is better exposed, and it is possible to incorporate more complex mechanisms for modeling spatial transport which emerge in the context of climate change.

[6] Krugman used Turing's method to explain the generation of agglomerations in the 12-region racetrack model.

certainty.

2 Modeling Spatial Transport

The spatial dimension and the transport of all kinds of things across locations – whether they are environmental variables such as pollutants, water, animals and vegetation, or economic variables such as capital, labor and knowledge – can be incorporated into models in various ways. Implicit representations could assign weights associated with a spatial scale or describe the part of the spatial domain occupied by a certain type of population. Explicit representations treat space either as a continuum or as a collection of discrete patches and define a mechanism which characterizes the movements across patches. We describe some of the more important mechanisms below.[7]

2.1 Diffusion

A central concept in any attempt to model the movement of variables associated with environmental and natural resources in continuous space is the concept of diffusion. A diffusion process describes a situation in which movements of individual objects such as pollutants or animals result in a regular macroscopic flow. Diffusion models can be derived from dispersal models of random walks, from Fick's law, or from stochastic differential equations. Fick's law states that in one-dimensional space (that is, a line), diffusion moves objects from locations of high concentration to locations of low concentration. If the concentration of a material at time $t \geq 0$ and spatial point $x \in \mathcal{O} \subseteq \mathbb{R}$, where $\mathcal{O}$ is the spatial domain, is denoted by $y(t,x)$, then Fick's law states that the flux J of the material is proportional to the gradient (that is, the derivative with respect to space $\frac{\partial y(t,x)}{\partial x}$) of the material, or $J \propto -\frac{\partial y(t,x)}{\partial x}$, $J = -D\frac{\partial y(t,x)}{\partial x}$. If the material's net growth in a spatial location is determined by $f(y(t,x), u(t,x))$, then concentration dynamics

[7] For a more detailed description of these mechanisms, see Murray (2003) and Cantrell and Cosner (2003). See also Smith, Sanchirico, and Wilen (2009) who present linear diffusion processes and dispersion processes in discrete space and discuss applications to renewable resource management.

under diffusion is given by the PDE[8]

$$\frac{\partial y(t,x)}{\partial t} = f(y(t,x), u(t,x)) + D\frac{\partial y^2(t,x)}{\partial x^2},\ y(0,x) = y_0(x). \tag{1}$$

In (1), D is called diffusivity and is a constant indicating that the diffusion is linear. Diffusivity, as we will see later on in this section, could depend on the location or the concentration itself in the context of nonlinear diffusion. This is important in the context of one- and two-dimensional models of heat diffusion toward the Poles (e.g., North 1975a, 1975b; Ghil 1976; North, Cahalan, and Coakley 1981; North and Kim 2017). The function $u(t,x)$ represents a control, such as harvesting, emissions or abatement. Equation (1), apart from the temporal initial condition, should be supplemented with spatial boundary conditions which provide information about what the concentration is expected to be at the boundary of the spatial domain $\mathcal{O}$ at all times.[9]

If we consider a vector $\mathbf{y} = (y_1, ..., y_n)$ of concentrations at time t and location x, which diffuse with diffusivities $\mathbf{D} = (D_1, ..., D_n)$ and interact among themselves, and a vector of controls $\mathbf{u} = (u_1, ..., u_n)$, then (1) will represent a reaction-diffusion system. The Fickian diffusion framework can be further extended to include an advection term which represents a drift in the process caused by external forcing such as wind or currents (Murray 2003; Wilen 2007). In this case the advection term $-V(\partial y(t,x)/\partial x)$ is added to the right-hand side of (1).

Figure 1 presents the spatiotemporal evolution of resource biomass under linear diffusion.[10]

[8] For the derivation and for extensions to higher dimension spatial domains, see Brock, Xepapadeas, and Yannacopoulos (2014b).

[9] Possible boundary conditions are: (a) periodic boundary conditions which imply that the spatial domain is a circle; (b) Dirichlet-type boundary conditions which specify the concentration y on the boundary; and (c) Neumann-type boundary conditions which specify the flux at the boundary. In one-dimensional domain $\mathcal{O} = [-L, L]$, these conditions imply for all t: (a) $y(t,-L) = y(t,L)$; (b) hostile boundaries $y(t,-L) = y(t,L) = 0$; and (c) zero flux at the boundaries or $\frac{\partial y(t,-L)}{\partial x} = \frac{\partial y(t,L)}{\partial x} = 0$.

[10] The evolution equations are shown in Appendix 1.

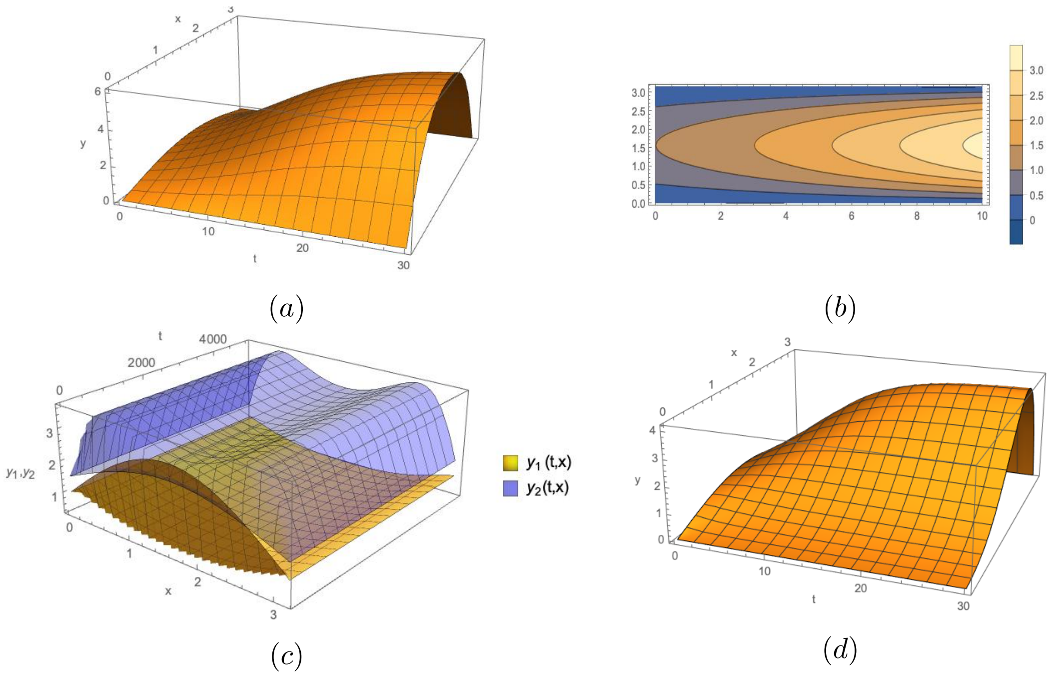


Figure 1: (a) Biomass evolution in time and space (t, x) of a population with logistic growth under linear diffusion. (b) Contours of the evolution surface. (c) Biomass evolution in time and space (t, x) of two interacting populations with logistic growth under linear diffusion (reaction-diffusion system). (d) Case (a) with advection.

Figure 1 indicates that the role of diffusion is to generate a spatially-heterogeneous pattern of concentration. In our example, this spatial pattern seems to persist over time. Persistent spatial heterogeneity raises policy questions regarding the need to design spatially-heterogeneous policies if the emerging spatial pattern is not the desired one.

When diffusivity D depends on the concentration or the spatial location, diffusion is nonlinear. Nonlinear diffusion in spatial models of climate change is modeled by the term $D\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T(t,x)}{\partial x}\right]$, where $x \in [-1, 1]$ is the sine of latitude and $T(t, x)$ is surface temperature at the latitude with sine x. More details will be presented in section 6.2.4.

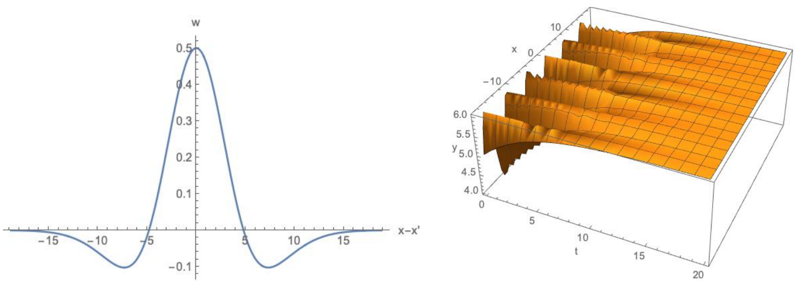
2.2 Long-Range Transport

Spatial diffusion captures local or short-range spatial interactions (Murray 2003). In economics as well as in environmental and resource management, spatial interactions and spatial effects could be long range. This means that the rate of change of the concentration at a specific location x is affected by the concentrations of all other locations $x' \in \mathcal{O}$. These long-range interactions can be expressed by the model

$$\frac{\partial y(t,x)}{\partial t} = f(y(t,x), u(t,x), Y(t,x)), \ y(0,x) = y_0(x) \tag{2}$$

$$Y(t,x) = \mathbf{K}y(t,x) := \int_{\mathcal{O}} w(x-x')\, y(t,x')\, dx', \tag{3}$$

and appropriate boundary conditions. In (3), $\mathbf{K} = \int_{\mathcal{O}} w(x-x')\, dx'$ is a linear integral operator acting on a function $y(t,x)$ and $w(x-x')$ is a kernel function which models the effect that location x' has on location x.[11] Since one of the basic premises of spatial economics is that what happens near us matters more than what happens far from us, it is reasonable to assume that the kernel is declining with the distance $|x-x'|$, and that the influence tends to zero when this distance becomes sufficiently large. Another usual assumption is that the effects are spatially symmetric. Spatial kernels could reflect positive effects such as knowledge or productivity spillovers, or negative effects such as congestion effects. Kernels are usually modeled by exponential functions. Figure 2 depicts the spatiotemporal evolution of biomass under long-range effects.[12]



(a) (b)

Figure 2. (a) Exponential kernel with positive and negative spatial effects. (b) Biomass evolution in time and space (t,x) of a population with logistic growth and nonlinear predation effects under long-range transport.

Figures 1 and 2 show an interesting qualitative characteristic of the transport mechanisms in continuous time and space. In figure 1 spatial hetero-

[11] Integrable kernel functions have been used in economics to model geographic spillovers (e.g., Krugman 1996; Lucas 2001; Lucas and Rossi-Hansberg 2002; Chincarini and Asherie 2008; Kyriakopoulou and Xepapadeas 2013, 2017;.Brock, Xepapadeas, and Yannacopoulos 2014a, 2014d).

[12] The kernel and the evolution equation are shown in Appendix 1.

geneity of the concentration is preserved with the passage of time, while in figure 2 the initial spatial heterogeneity vanishes and the concentration becomes spatially homogeneous, or returns to a "flat earth" situation. The emergence of spatial heterogeneity – or spatial pattern formation – from an initial state of flat earth, or the convergence to flat from an initial state of spatial heterogeneity, is an important issue when the system is controlled optimally and is related to the design of efficient spatial policies. These issues are examined in sections 4 and 4.1.

2.3 Discrete Space: Metapopulation Models and Dispersion

A class of spatial models – metapopulation models[13] – most often encountered in environmental and resource economics treats space as discrete, consisting of patches, and describes how populations or other objects move across patches. In bioeconomics based on metapopulation models, harvesting takes place in a patchy spatial domain and populations on different patches are connected by the dispersal process. Dispersion is modeled by a system of ODEs which, for a spatial domain with $i = 1, ..., n$ patches, can be written as:

$$\frac{dy_i(t)}{dt} = f_i(y_i(t), u_i(t)) + \sum_{j=1, j\neq i}^{n} d_{ij} y_j(t) - D_i y_i(t), \; y_i(0) = y_{i0}, \; \sum_{j=1, j\neq i}^{n} d_{ij} \leq D_i, \tag{4}$$

where $f_i(y_i(t), u_i(t))$ represent population dynamics, and individuals disperse from patch i at a rate $D_i \geq 0$ and arrive from patch j at a rate $d_{ij} \geq 0$ (Cantrell and Cosner 2003). Models like (4) can be extended to include density-dependent dispersal, multi-species interactions, bioinvasions, or pollution transportation across patches.

There is a considerable amount of literature that analyzes environmental and resource management issues using patchy environments and metapopulation models with dispersion across patches.[14]

Dispersion models have been used to study the so-called "acid rain game" (e.g., Mäler 1989; Kaitala, Pohjola, and Tahvonen 1992; Mäler and de Zeeuw 1998, Nagase and Silva 2007) in which acid deposits damage regions due to

[13] For the analysis of metapulation models, see Levin (1974, 1976), Hastings (1982) and Hastings and Harrison (1994).

[14] See, for example, Wilen (2007) or Smith, Sanchirico, and Wilen (2009) and the references therein, and section 5.1 for further analysis.

acid rain generated by sulphur emissions in other regions. In Mäler and de Zeeuw (1998), acid depositions in each of the $i = 1, ..., n$ countries are given by $A\mathbf{e}$, where $A = [a_{ij}], i, j = 1, ..n$ is a transportation matrix in which the element a_{ij} denotes the fraction of country j's emissions e_j of sulphur or nitrogen oxides that is deposited in country i, so $A\mathbf{e}$ is the vector of acid depositions in each country. The accumulation of depositions, d_i, in each country is given by the system of ODEs

$$\dot{\mathbf{d}}(t) = A\mathbf{e}(t) - \mathbf{c}, \mathbf{d}(0) = \mathbf{d}_0,$$

where $\mathbf{d}$ is the vector of depletion of each country's acid buffer stock and $\mathbf{c}$ is the vector of critical loads. An increase in depletion means damage to the country's soil. The objective is to choose emission paths to minimize the cost of reducing emissions plus damages from depletion.

Metapopulation models have also been used in the study of biological invasions. Albers, Fischer, and Sanchirico (2010) study the spread of invasive species over heterogeneous regions and compare optimal spatially-heterogeneous policy to spatially-uniform policy. Epanchin-Nieli and Wilen (2012) study optimal spatial control of biological invasions in spatiotemporal models in which the spatial domain is two-dimensional. Bioinvasion spreads from the invaded cell to adjacent cells in the absence of regulation. Epanchin-Nieli and Wilen develop optimal policy with a spatially-explicit characterization.

Brock and Xepapadeas (2002) analyze a species competition for a limited resource in a patchy environment. They show that three different equilibrium species specialization patterns emerge – undisturbed Nature with harvesting, and private optimal and social optimal with harvesting – and show that policy rules are spatially dependent.

Dispersion type of modeling has been used to study heat transport from the equator to the North Pole in the economics of climate change. This includes "two-box" models in which heat moves from the Equatorial region to the North and causes Arctic amplification.[15]

Spatial kernels can also be incorporated into spatial domains consisting

[15] See, for example, Alexeev, Langen, and Bates (2005); Alexeev and Jackson (2013); and Brock and Xepapadeas (2017, 2019, 2020).

of patches. In this case, (4) can be written as

$$\frac{dy_i(t)}{dt} = f\left(y_i(t), u_i(t), \sum_{j=1,\ j\neq i}^{n} w_{ij}y_j\right),\ y_i(0) = y_{i0},$$

where the w_{ij} element of the kernel provides a measure of the influence of the state of the system at patch j on the state of the system at patch i.

In section 5, it will be shown how the transport mechanisms presented in this section have been used to analyze specific issues in environmental and resource economics.

3 Dynamic Optimization in Space-Time: A Spatial Maximum Principle under Diffusion and Long-Range Transport

In environmental and resource economics, transition dynamics modeled by (1), (2) or (4) are typically used as constraints in optimization problems in which the objective is to maximize the present discounted value of an objective depending on state y and control u, which are defined over the entire spatial domain. In the context of continuous time and space, this can be regarded as the problem of a social planner or environmental regulator defined as:[16]

$$\max_{u\in\mathcal{U}} \int_{x\in\mathcal{O}} \int_0^\infty e^{-\rho t} U(y(t,x), u(t,x))dtdx \qquad (5)$$
$$\text{subject to (1) or (2),}$$

where $U(\cdot,\cdot)$ is a standard utility or net benefit function. Problem (5) is not the typical dynamic optimization problem encountered in economics, since the constraints are either PDEs or IDEs.

Necessary optimality conditions can, however, be stated in terms of a spatial maximum principle[17] as follows. If the path $u^*(t,x), y^*(t,x)$ solves problem (5) subject to (1), then there exists a costate $p(t,x)$ such that u^*

[16]To simplify, we assume that transition dynamics are time autonomous and that the utility function $U(y,u)$ does not explicitly depend on time t.

[17]We present only the conditions here; for details and derivations see, for example, Derzko, Sethi, and Thompson (1984); Brock and Xepapadeas (2008); and Brock, Xepapadeas, and Yannacopoulos (2014b).

maximizes the current value Hamiltonian function[18]

$$
\begin{aligned}
\mathcal{H}(y,p,u) &= U(y,u) + p\left(f(y,u) + D\frac{\partial y^2}{\partial x^2}\right), \text{ or} && (6)\\
u^*(y,p) &= \arg\max_u \mathcal{H}(y,p,u), && (7)
\end{aligned}
$$

y^* and p satisfy the system of PDEs

$$
\begin{aligned}
\frac{\partial y^*}{\partial t} &= f(y^*,u^*) + D\frac{\partial^2 y}{\partial x^2} && (8)\\
\frac{\partial p}{\partial t} &= \rho p - \frac{\partial \mathcal{H}(y^*,p,u^*)}{\partial y} - D\frac{\partial^2 p}{\partial x^2}, && (9)
\end{aligned}
$$

and a transversality condition at infinity is satisfied,

$$
\lim_{t\to\infty}\int_{\mathcal{O}} e^{-\rho\, t} y^*(t,x) p(t,x)\, dx = 0.^{19} \qquad (10)
$$

In Appendix 2 we present a sketch of a heuristic proof of result (8)-(9), while in Appendix 3 we present a method for solving a linear quadratic problem (see (14)-(15) below). It is important to note that in (9) diffusivity has a negative sign as opposed to the positive diffusivity of (8). Since y can be interpreted as quantity at spatial point x, while p can be interpreted as as the shadow value (i.e., price of a useful resource, or cost in the case of a pollutant) of this quantity at x, the opposite signs imply that quantities and prices move in opposite directions in the spatial domain.

With long-range spatial effects modeled by kernels, the extension of the maximum principle provides the following necessary conditions (Brock, Xepapadeas, and Yannacopoulos 2014a, 2014d):

$$
\begin{aligned}
u^*(y,p) &= \arg\max_u \mathcal{H}(y,p,u,Y) && (11)\\
\mathcal{H}(y,p,u) &= U(y,u) + pf(y,u,Y),
\end{aligned}
$$

where y^* and p satisfy the system of IDEs with appropriate spatial boundary

[18] We drop (t,x) to ease notation.

[19] A solution procedure for this problem is described in Appendix 2. Problems in finite terminal time can be handled by adding appropriate terminal and transversality conditions.

conditions

$$\frac{\partial y^*}{\partial t} = f(y^*, u^*, Y^*) \ , \ Y^* = \mathbf{K}y^*(t,x) \tag{12}$$

$$\frac{\partial p}{\partial t} = \rho p - \frac{\partial \mathcal{H}(y^*, p, u^*, Y^*)}{\partial y} - \mathbf{K}\frac{\partial \mathcal{H}(y^*, p, u^*, Y^*)}{\partial Y}, \tag{13}$$

along with the intertemporal transversality condition. Appendix 2 presents a sketch of a heuristic proof of this result.

4 Spatial Pattern Formation

Patterns in space refer to spatial or spatiotemporal forms or regularities which are observable as different concentrations of a quantity of interest, such as biomass, pollutants, temperature, stock of capital or knowledge at different spatial points. If there is no spatial transport, the system will remain at its initial spatial state and no new patterns will emerge. A fundamental question in this context is whether spatial transport can create the endogenous emergence of patterns from a spatially-homogeneous or flat earth state. In biology the issue of pattern formation is referred to as morphogenesis and pattern formation mechanisms try to explain classic questions such as "how the leopard got its spots."

A fundamental pattern formation mechanism under spatial diffusion is the Turing mechanism (Turing 1952). In general, a diffusion process in a system of interacting populations or materials tends to produce a spatially-uniform population density, that is, spatial homogeneity. Thus it might be expected that diffusion acts as a homogenizing force or a stabilizer in case of spatial perturbations. There is however one exception, known as diffusion-induced instability or diffusive instability. Turing suggested that under certain conditions, diffusion acting on reaction-diffusion systems can generate spatially-heterogeneous patterns.

This is the so-called Turing mechanism[20] for generating diffusion instability or just *Turing instability*, which means that a flat-earth state is destabilized by diffusion and this destabilization is a precursor to the emergence of persistent spatial patterns. The Turing mechanism requires a system of at least two interacting state variables, and its applications are not directly linked to explicit dynamic optimization. Thus a question which is

[20] See also Levin and Segel (1985) and Murray (2003).

relevant for environmental and resource economics is whether a system in which an environmental variable is transported across space through natural mechanisms, and a forward-looking agent – e.g., a regulator – is seeking to control the system optimally, can exhibit pattern formation in the space of quantities-shadow values. If the optimal control of a system with a diffusive externality like problem (5) generates an optimal spatial pattern, the important policy question then is what kind of policy can support this optimal spatial pattern.

Brock and Xepapadeas (2008) were the first to show that diffusion can destabilize a flat-earth steady state in the quantities-shadow values (state-costate) space, or equivalently in the state-control space, in a way which is similar to the Turing mechanism. The reasoning behind the optimal diffusion instability can be explained in the following way. From standard optimal control theory we know that, without diffusion (i.e., $D = 0$ in (1)), and under appropriate concavity assumptions, if a steady state defined as $(y^*, p^*) : (\dot{y} = 0, \dot{p} = 0)$ exists, then this steady state will have the local saddle point property or it will be unstable (e.g., Kurz 1968).

The steady state with the saddle point property is spatially homogeneous, or a flat optimal steady state (FOSS). This means that a stable manifold exists – which is globally stable under appropriate assumptions – such that for any initial value for the state (e.g., stock of greenhouse gases, or biomass) there is an initial value for the costate (the shadow value of the externality) and the control, such that the system will stay on the stable manifold and converge to the FOSS. If a temporal perturbation moves the system away from the steady state but the system is optimally controlled, then on the stable manifold the perturbation will die out with the passage of time and the system will return to the FOSS.

In the context of a flat-earth system without diffusion which evolves in time and space, a FOSS can be interpreted as a state $(y^*(x), p^*(x)) : \left(\frac{\partial y(t,x)}{\partial t} = 0, \frac{\partial p(t,x)}{\partial t} = 0\right)$, with $(y^*(x), p^*(x)) = (y^*(x'), p^*(x'))$, for all $x, x' \in \mathcal{O}$. The stable manifold for this FOSS indicates that for any spatially-homogeneous (flat) initial value for the state $y(0, x)$, there are flat initial values for the costate and the control such that the system will converge to the FOSS.

Suppose now that spatial diffusion occurs and that the FOSS is perturbed in the spatiotemporal domain. The optimality conditions from the extended

Pontryagin's principle suggest that spatial sinusoidal wave-like patterns will emerge at the stable manifold in the neighborhood of the FOSS. If, with the passage of time, these patterns die out, then the FOSS is stable and the system will return to this FOSS. If, however, the patterns keep growing over time, then diffusion destabilizes the stable manifold in the neighborhood of the FOSS.

We call the emergence of spatial patterns in the optimally-controlled system *optimal Turing instability*, which induces optimal spatial patterns. To obtain a better picture of optimal Turing instability, consider the linear quadratic optimal control problem

$$\max_{u(t,x)} \int_0^L \int_0^\infty \mathrm{e}^{-\rho t} \left[-\frac{A}{2} y(t,x)^2 - \frac{B}{2} u(t,x)^2 + N x(t,x) u(t,x) \right] dt dx \quad (14)$$

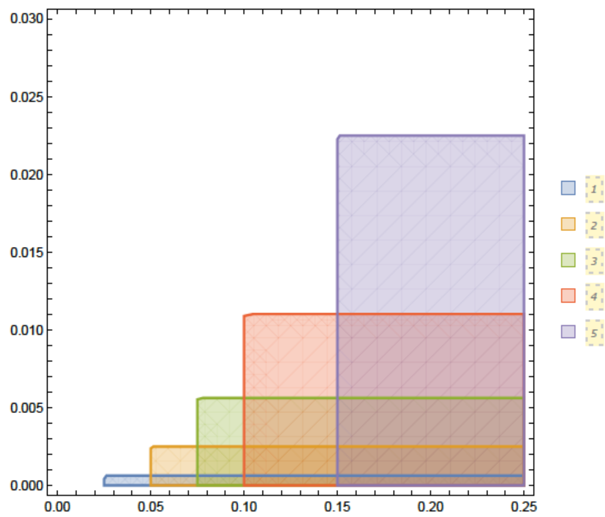
$$A, B, \rho > 0, \; AB - N^2 > 0$$

$$\text{s.t. } \frac{\partial y(t,x)}{\partial t} = F y(t,z) - G u(t,x) + D \frac{\partial^2 y(t,x)}{\partial x^2} \quad F, G > 0, \quad (15)$$

with the solution analyzed in detail in Appendix 3. Setting $\alpha \equiv \left(F - \frac{GN}{B}\right)$, $\beta \equiv \left(A - \frac{N^2}{B}\right)\left(\frac{G^2}{B}\right)$, we show in Appendix 4 (Theorem 1) that optimal Turing instability will emerge for parameter values in the non-empty set

$$T \equiv R \cap S, \left\{ (\alpha, \beta) : \alpha > \frac{\rho}{2} \right\} \cap \left\{ (\alpha, \beta) : \beta < \frac{\rho^2}{4} \right\}.$$

The Turing set T for different values of the discount rate ρ is shown as the shaded areas of figure 3. When the set is non-empty, then a $D > 0$ can be found that destabilizes the FOSS. The Turing set is empty for $\rho = 0$.



Legend: 1:$\rho = 0.05$;2:$\rho = 0.1$;3:$\rho = 0.15$;4:$\rho = 0.2$;5:$\rho = 0.3$
$\alpha \in [0, 0.25]\,, \beta \in [0, 0.03]$
Figure 3. The Turing Set for the Optimal Turing Instability for Different Values of ρ

Since in a system with diffusive externality the costate is the price of the externality, this result suggests that due to diffusion the externality should be priced differently at different spatial points or, equivalently, the optimal control should be different at different locations. If the diffusive externality has wider impacts such as productivity effects, then additional spatially-dependent instruments might be required to support the optimal spatial pattern.

The optimal diffusive instability can be regarded as a precursor to the emergence of agglomerations and clustering in optimally-controlled systems. Diffusion may, however, have stabilizing effects. Consider for example an optimal control problem such as (14), with a steady state (y^*, p^*) which is completely unstable for $D = 0$. Then, as shown in Appendix 4, there exists a Turing space T_- for the parameters of the problem and a diffusivity $D > 0$ such that the optimized system will converge to a FOSS along a stable manifold. In terms of policy, this result suggests that allowing transport of the material or resource associated with the state variable in the case of a diffusive externality will make possible regulation with a spatially-uniform policy instrument.

The analysis above focused on optimal Turing instability as a precursor to persistent spatial patterns under diffusion. A similar approach can be used to study pattern formation induced by long-range spatial effects represented by kernels. A FOSS with the saddle point property could be destabilized by spatiotemporal perturbations induced by a spatial kernel as defined in (3). Destabilization refers to the stable manifold associated with the FOSS (see Brock, Xepapadeas, and Yannacopoulos 2014a, 2014d).[21]

4.1 Optimal Pattern Formation and Policy Implications

In the case of optimized systems, the important differences between the "optimal spatial instability" – whether it is diffusion-driven or kernel-driven – and the celebrated Turing instability which explains pattern formation in biological and chemical systems are that: (a) contrary to the spirit of the Turing model, here the instability is driven by optimizing behavior, so it is the outcome of forward-looking optimizing behavior by economic agents and not the result of reaction-diffusion in chemical or biological agents; (b) the spatial patterns do not emerge between two state variables which in general reflect quantities, but with a state variable and its shadow price, thus the spatial pattern occurs in the price-quantity space; and (c) contrary again to the Turing approach, there is no need to have two or more diffusing/interacting state variables to general patterns, but only one diffusing state. Optimization induces diffusion to the price system (the costate) and the interaction of the price-quantity system generates patterns.

The optimal Turing instability is quashed when the discount rate ρ becomes zero (see also figure 3). This relation of the discount rate with Turing instability can be linked with general results from the turnpike literature and the role of the utility discount rate. In the classical turnpike theory of multisectoral capital theory (Cass and Shell 1976; McKenzie 1976), a discount rate close to zero is associated with unique steady-state equilibria and global asymptotic stability. When the discount rate is close to zero, the optimal Turing instability is expected to vanish as well. Thus the optimal Turing instability can be regarded as a new form of instability which may emerge if diffusion is present. The role of the discount rate should also be important in a potential integration of optimal Turing theory with Pareto optima in general equilibrium theory, which could be an area of future research in ex-

[21] See Appendix 5 for a sketch of the procedure.

tending the work of Bewley (1982) in the integration of equilibrium theory and turnpike theory to spatial settings with diffusion.

Optimal spatial patterns or agglomeration as persistent outcomes implies that the value of the spatially-heterogeneous system exceeds the value of the flat earth system. In terms of policy design, an interesting distinction could be made between privately optimal solutions which ignore the diffusive or the long-range externality, and the socially-optimal solution which internalizes the spatial externality. If spatial transport phenomena produce different outcomes between the social and the private optimum – e.g., the equilibrium outcome at the private optimum implies spatial heterogeneity while the social optimum implies different spatial patterns or even flat earth – then policy instruments which will promote socially-optimal patterns or suppress spatial heterogeneity should be designed. In the remainder of this paper, we use the analytical tools presented earlier to study the issue of characterizing and designing policies under spatial transport for typical environmental and resource problems.

5 Environmental and Resource Management Policy under Spatial Dynamics

The spatial characteristics of environmental policy emerge naturally when there is spatial differentiation with respect to a specific characteristic (e.g., land quality) or a flow of a pollutant or biomass across regions or spatial locations. This transportation process is often referred to as cross-border or transboundary. When pollution – that is, the externality – crosses borders,[22] there are two major types of issues related to policy design. The first type is the case in which pollution is a local "public bad" or, to put it differently, environmental quality is a local public good. This means that damages emerge from the local pollution level after local emissions and spatial dispersion of pollutants takes place. In this case, environmental policy has to correct the local externality. A typical example is upstream-downstream water pollution problems. The second type is the case of a global externality, or global public bad, in which damages in each region are associated with global pollution and are independent of the spatial point from which pollution originates. A typical example is the case of climate change.

[22] The borders could refer to nations or to subnational jurisdictions.

The flow of the pollutants or the resources implies the existence of a transportation mechanism and is the specific transportation mechanism described in section 2, which provides the link between the forward-looking optimization of economics and the natural laws governing the flow of pollutants or resources. It is this link that we are studying as the driver of spatiotemporal patterns and location specific policies which can internalize externalities, including spatial externalities.

The largest part of the existing literature on pollution control or bioeconomic analysis which considers space focuses on discrete space with a static or dynamic temporal dimension. However, aside from some notable exceptions in the literature related to fishery management – and, to a lesser extent, the literature regarding groundwater management, pollution control, bioinvasions or acid rain issues – the main body of the environmental and resource management literature does not include explicit spatial transport mechanisms across locations.

An important strand of this literature, developed mainly in the 1990s, studies the link between environmental quality and international trade. Two seminal papers by Copeland and Taylor (1994, 1995) analyze pollution and trade. In the first, pollution does not disperse across countries. In this case, free trade shifts pollution-intensive production to the country where human capital is scarce and world pollution increases. In the second, pollution crosses boundaries and is a global public bad such as climate change or ozone depletion. One of the results is that if countries are different, trade creates "pollution havens," which are countries in which pollution-intensive industries locate due to lax environmental policies. This is a result suggesting that under global pollution, spatial patterns related to the pollution intensity of production emerge. Spatial patterns are induced by international trade. In the same analytical framework, Silva and Caplan (1997) study environmental policy for a global public bad in the context of a federal system. Copeland (1996), in a two-country, static model with unidirectional cross-border pollution[23] and international trade, derives optimal tariff policy for a country which imports goods and is harmed by cross-border pollution generated in the neighboring country, while Hatzipanayotou, Lahiri, and Michael

[23] Pollution transportation in a two-region model is a special case of the general dispersion model (4) with $i = 2$. Unidirectional transportation, for example, means that $d_{12} = D$ and $d_{21} = 0$.

(2002, 2005) study multilateral policy reforms under cross-border pollution, international trade and foreign aid.[24]

There is extensive literature on dynamic models of global pollution which is accumulated in the ambient environment. The growth of global pollutants depends on aggregate emissions per unit time originating from different agents. In these models the spatial dimension is implicit, since it is natural to assume that agents emit from different spatial locations. However, since there is no transport mechanism, it is difficult to value the spillover externality. The analysis of these problems focuses on two types of solution concepts: a cooperative solution, in which a regulator maximizes aggregate welfare net of damages; and noncooperative solutions, in which each location is treated as a forward-looking agent that maximizes own welfare net of own damages by taking into account the behavior of the other forward-looking agents. Two types of behavior are in general examined: the open loop Nash equilibrium in which each agent takes the emission paths of the other agents as given, and the feedback Nash equilibrium in which the emissions of each agent depend on the current stock of pollution accumulation (Başar and Olsder 1995). The feedback solution is Markov perfect by construction. Typical examples of this modeling are the cases of transboundary pollution games (e.g., van der Ploeg and de Zeeuw 1992; Dockner and van Long 1993)[25] and the lake games (e.g., Brock and Starrett 2003; Mäler, Xepapadeas, and de Zeeuw 2003; Wagener 2003; Kossioris et al. 2008, 2011)

This discussion suggests that the spatial dimension is implicit in a large number of issues which are central to environmental and resource economics. However, the absence of explicit transport mechanisms, such as those reviewed earlier in this paper, does not allow for full exploration of the impact of spatial dynamics on environmental and resource management. In this section we present specific applications in spatially-structured environments in which flows are explicitly driven by spatial transport mechanisms. Our aim is to show how this analytical framework could be helpful in better understanding spatial heterogeneity and spatial patterns as outcomes of optimizing behavior, and also in the design of efficient space-dependent policies.

[24] For a survey on environmental policy and international trade, see Ulph (1997).

[25] For surveys, see for example Jorgensen, Martin-Herrán, and Zaccour (2010) and Calvo and Rubio (2012).

5.1 Fishery Management in Patchy Environments

In fishery management, the explicit introduction of space is implemented in the context of metapopulation models with sub-populations in patches and population dispersal among them due to natural forces (e.g., winds or currents).

Smith, Sanchirico, and Wilen (2009) present spatial dynamic processes and their applications to renewable resource management. Sanchirico and Wilen (1999) use the modeling approach in (4) to describe the evolution of fish biomass in patch $i = 1, ..., n$ under harvesting modeled by the catch function $h_i(E_i(t), y_i(t))$ with $E_i(t)$ being the level of fishing effort. In an open access patchy system, fishing effort and biomass in each patch could evolve as

$$\begin{aligned}\frac{dE_i(t)}{dt} &= s_i R_i(y_i(t), E_i(t)) + \sum_{j=1, j\neq i}^{n} s_{ij}\left[R_i(y_i(t), E_i(t)) - R_j(y_j(t), E_j(t))\right] \\ \frac{dy_i(t)}{dt} &= f_i(y_i(t))\, y_i(t) + ND_i(y_1(t), ..., y_n(t)) - h_i(E_i(t), y_i(t)),\end{aligned}$$

where $f_i(y_i)$ is per capita growth function; ND_i is the net dispersal function in patch i; $R_i(y_i, E_i)$ denote rents in patch i; s_i is entry exit rates; and $s_{ij}(R_i(y_i, E_i) - R_j(y_j, E_j))$ is fleet dispersal because of revenue differentials across patches. The biomass-effort steady state of the system is determined as $(E_i^*, y_i^*) : (dE_i/dt = 0, dy_i/dt = 0)$. Sanchirico and Wilen find the equilibrium patterns of biomass and effort across the system to be dependent upon bioeconomic conditions within each patch, and the nature of the biological dispersal mechanism between patches. In terms of policy, they conclude that optimal instruments should reflect the interplay between the spatial gradient of rents and the spatial gradient of biological dispersal.

Sanchirico and Wilen (2001) study the creation of marine reserves in a patchy environment and show that, under certain conditions, creating a reserve by closing a patch for harvesting could increase aggregate biomass and harvest.[26]

In a continuous space fishery model, Behringer and Upmann (2014) find that in atomistic equilibrium, each agent exploits one location only and

[26] For further analysis of bioeconomic models in patchy environments and issues related to marine reserves, see for example Smith and Wilen (2003); Costello and Polasky (2004, 2008); Sanchirico and Wilen (2005); and Smith, Sanchirico, and Wilen (2009).

tends to harvest the resource to extinction in this location. This result also points to spatially-structured policy interventions.

5.2 Groundwater Management

In groundwater management, the early literature such as Gisser and Sanchez (1980), Negri (1989) and Provencher and Burt (1993) considered the underground aquifer as a homogeneous single-cell "bathtub" in which abstraction by one user caused an instantaneous impact on others. More recent literature recognizes the fact that hydrological factors such as seepage or aquifer transmissivity introduce a spatial pumping externality. In this case, pumping by a farmer affects and is affected by the pumping behavior of the neighbors through the emergence of overlapping cones of depression in the aquifer. Thus optimization problems which seek to maximize benefits from the underground aquifer acquire an explicit spatial structure (see, for example, Saak and Peterson 2007; Brozovic, Sunding, and Zilberman 2010).

Pfeiffer and Lin (2012) model a "patchy" groundwater aquifer with water flowing across patches according to hydrological rules. The dynamics of water stock $y_i(t)$ in each patch are given by

$$\frac{dy_i(t)}{dt} = -u_i(t) + g_i(u_i) + \sum_{j=1, j \neq i}^{n} \theta_{ij} y_j(t), \tag{16}$$

where u_i is water pumping, $g_i(u_i)$ is recharge to patch i and flow parameters θ_{ij} are determined by Darcy's law or $\theta_{ij} = (y_i - y_j)/x_{ij}$ where x_{ij} is the distance between patches. Groundwater dynamics (16) act as a constraint to the problem of a social planner seeking to maximize discounted aquifer benefits, or

$$\max_{u_i(t)} \int_0^\infty \mathrm{e}^{-\rho t} \left[\sum_{i=1}^{n} \left[R_i(u_i) - C(y_i) u_i \right] \right] dt.$$

Results suggest that the spatial externality results in overpumping relative to the social optimum (Pfeiffer and Lin 2012), and that the spatial externality is important for large, unconfined groundwater aquifers (e.g., Brozovic, Sunding, and Zilberman 2010). Kuwayama and Brozovic (2013) consider the adoption of a spatially-differentiated groundwater permit system to efficiently regulate when groundwater pumping affects the flow of surface water.

Brock and Xepapadeas (2010) studied a semi-arid system with reaction-diffusion characteristics in which plant biomass and soil water interact and diffuse in a continuous space with linear (Fickian) diffusion. The plant-soil water dynamics are given by[27]:

$$\begin{aligned}
\partial_t P(t,x) &= g(W(t,x), P(t,x)) - bP(t,x) - h(t,x) + D_p \partial_{xx} P(t,x) \\
\partial_t W(t,x) &= f(P(t,x), R) - \upsilon(W(t,x), P(t,x)) - r_W W(t,x) + D_w \partial_{xx} W(t,x) \\
& \quad P(0,x), W(0,x) \ given, \\
P(t,0) &= P(t,L), W(t,0) = W(t,L) \ \forall t,
\end{aligned}$$

where $P(t,x)$ is plant density (biomass); $W(t,x)$ is soil water at time $t \in [0,\infty)$ and location $x \in [0,L]$; R is fixed rainfall; h is harvesting of plant biomass through grazing; $g(W,P)$ is plant growth, increasing in soil water and plant density; bP is plant senescence; $f(P,R)$ is water infiltration; $\upsilon(W,P)$ is water uptake by plants; r_W is specific rate of water loss due to evaporation and percolation; and D_P and D_W are diffusion coefficients for plant biomass (plant dispersal) and soil water, respectively. The authors consider the problem of a myopic agent who optimizes profits by ignoring spatiotemporal dynamics, and the problem of a social planner who internalizes the spatial externality. They find that at the myopic solution, spatial patterns in plant-soil water are generated through the Turing mechanism but the socially-optimal solution is spatially homogeneous. Spatially-dependent instruments are required in order to internalize the spatial externality.[28]

5.3 Pollution Control

Goetz and Zilberman (2000) consider pollution accumulation in a lake associated with the run-off from mineral fertilizers and animal manure. They employ a two-stage optimization, optimizing first across the spatial and then across the temporal dimension. The social optimum can be implemented with site-specific taxes on mineral fertilizers, manure and large animal units.

Sigman (2005) considers transboundary river pollution and examines

[27] We use ∂_z instead of $\partial/\partial z$ and similar for second derivatives to simplify notation.

[28] For a detailed numerical analysis of optimal harvesting in the semi-arid system, see Uecker (2016). For the application of this methodology in invasive species control, see Liu and Sims (2016).

whether states which, under decentralized policies, control their Clean Water Act programs, free ride on downstream states. Sigman does not include an explicit pollution transport mechanism along the river and defines water quality, WQ_{it}, in location i and time t, as

$$WQ_{it} = \sum_{h=1}^{H} \frac{P_{ht}}{F_{ht}} \delta^{h-i},$$

where P_{ht} is pollution at upstream locations h, F_{ht} is flow that dilutes pollution and $\delta < 1$ is a spatial discount term that diminishes pollution effects with distance. Using econometric estimation, Sigman concludes that there is free riding whose costs need however to be compensated with benefits derived from the flexibility introduced by decentralization.

Xabadia, Goetz, and Zilberman (2006, 2008) study policies for controlling agricultural stock pollution in a framework in which the spatial differentiation is related to the heterogeneity in the land quality of producers located at different sites. Pollution generated by the heterogeneous producers is accumulated in the environment and the optimal emission policy is site specific. Although there is no explicit spatial transportation mechanism, this research studies a spatially-distributed parameter problem in the space of qualities which are distributed in space and presents another approach to spatial issues.

Explicit pollution diffusion across space was introduced by Brock and Xepapadeas (2008) into the so-called shallow lake problem. In this case, pollution (phosphorous) accumulates in time and space according to the PDE

$$\partial_t P(t,x) = E(t,x) - mP(t,x) + f(P(t,x)) + D\partial_{xx} P(t,x), \tag{17}$$

with appropriate initial and spatial boundary conditions. In this set-up, $P(t,x)$ denotes the stock of the pollutant at t and x; E is emissions by location x; m is the pollution decay rate; and $f(P(t,x))$ is in general a convex-concave function indicating nonlinear feedbacks which underlie the lake dynamics. Using (17) as a constraint in the planner's problem for maximizing benefits over the whole spatial domain, and realistic parametrization for the lake, it is shown that a steady state can be destabilized in the context of optimal Turing instability, and spatial patterns in the accumulation of the pollutant start emerging due to pollution diffusion. Since the spatial patterns for the pollutant imply spatial patterns for its shadow cost, this result suggests spatially-differentiated emissions taxes.[29]

Camacho and Pérez-Barahona (2015) use the model of Gaussian plume to describe the spatial dynamics of pollution and consider a pollution accumulation equation at location x of the form

$$\partial_t P(t,x) = E(x,t) + \partial_{xx} P(t,x),$$

in a model of optimal land use in which the interaction between land use and the environment generates a spatially-heterogeneous solution and abatement technology is central in pollution stabilization.

De Frutos and Martín-Herran (2019) consider a spatiotemporal-pollution dynamics problem of the form shown in (17) without the nonlinear feedback to study optimal regulation in a transboundary pollution problem. By making a linear quadratic approximation and discretizing the space, they derive regional cooperative and noncooperative emission paths.

5.4 Urban Economics and Spatial Effects

Environmental externalities are prominent in the area of urban economics. Pollution from transportation or industrial activities affects the location decisions of both individuals and firms and significantly affects the spatial structure of a city. In this context, environmental policy is an important factor in the development of residential and industrial clusters, since strict environmental measures can discourage firms from polluting urban areas, while reduced pollution levels can encourage people to locate closer to industrial areas, thus reducing commuting costs.

Henderson (1977) studied a problem in which industrial pollution diffuses towards the residential/central business district boundary. The optimal policy consists of Pigouvian taxes along with regulations controlling the allocation of land between polluting firms and individuals, and potential redistribution of tax receipts between heavily-taxed and lightly-taxed communities. The combination of optimal taxes with zoning policies will prevent polluting firms from locating in residential areas. Verhoef and Nijkamp

[29] For a detailed numerical analysis of this problem, see Grass and Uecker (2017).

(2002) use a monocentric city model and study first- and second-best policies in order to control the effect of industrial pollution on residential areas. They show that environmental goals can be promoted either at the expense of or in favor of agglomeration economies.

Arnott, Hochman, and Rausser (2008) study a circular city in which firms generate pollution and households commute at a cost and receive disutility from pollution. Pollution disperses according to a function $D\left(e\left(x\right), x-x'\right)$, where $e\left(x\right)$ denotes emissions at x and $x-x'$ is the distance between location x and x'. The kernels introduced in section 2.2 could be a reasonable specification of such a function. In this set-up, the optimal allocation can be decentralized by imposing a tax per unit of area of industrial land at a particular location equal to the total damage caused by the pollution from that unit area, evaluated at the optimum. Location-specific Pigouvian taxes that do not fully internalize the total damage caused by this site are inefficient.

Rossi-Hansberg, Sarte, and Owens (2010) and Rossi-Hansberg and Sarte (2012) study housing externalities, defined as the effect that impacts of a house's characteristics have on neighbors. They find that these externalities decay fast with distance, with the impact on location x from housing services $H\left(x'\right)$ in nearby location $x' \in [-L, L]$ defined as:

$$\delta \int_{-L}^{L} \mathrm{e}^{-\delta|x-x'|} H\left(x'\right) dx'.$$

They conclude that residents in a neighborhood depend on the quality of nearby housing and propose, as policy measures, minimum maintenance requirements or zoning policies.

Kyriakopoulou and Xepapadeas (2013, 2017) consider a linear city with: (i) productivity spillovers which decline with distance, or

$$z\left(x\right) = \delta \int_{0}^{L} \mathrm{e}^{-\delta\left(x-x'\right)^2} \lambda\left(x'\right) \ln L\left(x'\right) dx',$$

where $\mathrm{e}^{-\delta\left(x-x'\right)^2}$ is a normal dispersal kernel, $\lambda\left(x'\right)$ is the proportion of land occupied by firms at the spatial point x', and $L\left(x'\right)$ is labor input; and (ii) pollution, P, which diffuses across the city and concentrates in specific locations according to

$$\ln P\left(x\right) = \int_{0}^{L} \mathrm{e}^{-\zeta\left(x-x'\right)^2} \lambda\left(x'\right) \ln E\left(x'\right) dx',$$

where $E(x')$ denotes industrial pollution. In this set-up, equilibrium and optimal solutions regarding the spatial structure of the city are compared and optimal policy is derived. In Kyriakopoulou and Xepapadeas (2013), where a first-nature advantage assumption is made, it is shown that the equilibrium outcome leads to either a monocentric city or a polycentric city with the first-nature advantage site attracting the majority of economic activity. On the contrary, the socially-optimal solution leads to a duocentric city, where neither of the two centers is formed around the natural advantage site. The authors show that sites with inherent advantages can lose their comparative advantage when the social cost of pollution is taken into account.

Kyriakopoulou and Xepapadeas (2017) consider a general equilibrium set-up where there is competition for land between industries and households, polluting industrial activity, production externalities and costly commuting. In equilibrium the center of the city is mixed residential/industrial, while at the social optimum there are distinct residential and industrial clusters across the city. The presence of spatial productivity spillovers and spatial pollution spillovers requires that the optimal policy be site-specific and consist of two instruments: pollution taxes to internalize the negative pollution externality and labor subsidies to internalize the productivity externality. Uniform instruments are suboptimal.

Regnier and Legras (2018) use a model à la Fujita and Ogawa (1982) to study the urban patterns derived in the presence of industrial pollution, which decreases the environmental quality E at the spatial point x, according to:

$$E(x) = \bar{E} - \int_X [e - \eta\,|x-y|]b(y)dy,$$

where $\bar{E}$ denotes environmental quality without pollution, e is the quantity of pollution emitted by one firm, η shows how pollution disperses in space, $x - y$ is the distance between firm and household, and $b(y)$ is the density of firms at y. The authors show that the internalization of pollution forms more specialized areas in the city, which has a negative impact on greenhouse gases from commuting.

There is also a growing literature on the internal structure of cities when pollution comes from commuting. Verhoef and Nijkamp (2003) point out the importance of space in the analysis of urban air pollution which is affected by aggregate commuting and not by the number of commuters. Schindler, Caruso, and Picard (2017) study how traffic-induced pollution affects residential choices and find that higher pollution levels reduce the size and the population of the city. Pollution (P), in that framework, increases with the traffic volume passing by r, as

$$P(r) = 1 + a + b \int_{r}^{r_f} n(r)dr,$$

where a and b measure the impacts of regional and traffic-induced pollution in the city and $n(r)$ is the number of people crossing location r. Finally, Denant-Boemont, Gaigné, and Gaté (2018) show that polycentric cities imply higher welfare and lower pollution levels.

6 Spatially-Differentiated Regulation for Transboundary and Global Externalities

In this section we focus more on the regulation of transboundary local and global externalities. We use the modeling of local transboundary externalities as a natural introduction to the explicit spatial modeling of climate change.

6.1 Regulating a Transboundary Externality

We consider a simple transboundary (or cross-border) – but not global – externality with local damages in a two-region model, which is a special case of the general dispersion models described above with $d_{12} = D > 0$ and $d_{21} = 0$. We consider a very simple model of transboundary pollution because we want to stress the fact that optimal policies should have a spatial structure even when regions are symmetric in their fundamentals. Our approach, despite its simplicity, makes clear the impact of spatial transport on optimal environmental policies, and the core model presented below can be extended along many different lines.

Let $u_i\left(c_i\right) = \ln\left(y_i E_i^{\alpha} \mathrm{e}^{-v_i(P_i)}\right), i = 1, 2,$ denote utility in region i from using emissions or energy, E_i, in production net of pollution damages, where y_i is an exogenous process incorporating the impact of other factors of production. Capital accumulation is not considered in order to simplify dynamics.

The use of E accumulates a pollutant P_i in each region. Some of the pollutant accumulated in region 1 is transported to region 2 through natural forces (e.g., river flows, winds). The accumulated pollutant in each region generates damages according to a convex damage function $\upsilon_i(P_i), \upsilon_i' > 0, \upsilon_i'' \geq 0$. Pollution dynamics, with the explicit dependence on t omitted to ease notation, can be written as:

$$\dot{P}_1 = -BP_1 - DP_1 + E_1 \ , \ P_1(0) = P_{10} \text{ given} \tag{18}$$

$$\dot{P}_2 = -BP_2 + DP_1 + E_2 \ , \ P_2(0) = P_{20} \text{ given,} \tag{19}$$

where $D > 0$ is the pollution transportation coefficient, or diffusivity, and $B > 0$ is a pollution depreciation rate. A regulator or a social planner will determine emission paths and emission taxes by maximizing the sum of discounted regional utilities subject to pollution dynamics.

The Hamiltonian representation of the regulator's problem with a quadratic damage function, $\upsilon_i(P_i) = c_{1i}P_i + (c_{2i}/2) P_i^2 \ , \ i = 1, 2$, can then be written as:

$$\max_{E_i} \mathcal{H} = \max_{E_i} \left\{ \sum_{i=1,2} w_i \left[\alpha \ln E_i - \upsilon_i(P_i)\right] + \right. \tag{20}$$
$$\left. \mu_1(-BP_1 - DP_1 + E_1) + \mu_2(-BP_2 + DP_1 + E_2) \right\},$$

where $w_i, \sum_i w_i = 1$ are regional welfare weights.

Assume that each region is populated by identical atomistic agents which we represent by a representative agent in each region. Each such agent maximizes their own utility ignoring climate impacts on their own region as well as the other region. Thus they only optimize over energy use. The socially-optimal solution resulting from problem (20) can be implemented with regional emission taxes solving the consumer's problem with Hamiltonian representation

$$\max_{E_i} \mathcal{H}_i = \max_{E_i} \{\ln(c_i)\} = \max_{E_i} \{\ln(y_i E_i^\alpha - \tau_i E_i + Tr_i)\}, \tag{21}$$

where Tr_i denotes lump sum transfers to the representative agent in region i. By combining the optimality conditions of problems (20) and (21), the optimal regional emission tax and transfers are

$$\tau_i = y_i \alpha \left(\frac{-w_i \alpha}{\mu_i} \right)^{\alpha - 1}, \; Tr_i = y_i \alpha \left(\frac{-w_i \alpha}{\mu_i} \right)^{\alpha - 1} \left(\frac{-w_i \alpha}{\mu_i} \right). \tag{22}$$

As usual, regional emission taxes, $\tau = (\tau_1, \tau_2)$, are determined by the costate variables of the current value Hamiltonian, (μ_1, μ_2), which are negative since these variables express the marginal cost of the accumulated pollution, i.e., the cost of the externality. The optimality conditions are shown in Appendix 6. For the general case in which regions are asymmetric, regional taxes will be different. However, unidirectional pollution transport induces regionally-differentiated optimal emission taxes even under full symmetry with respect to pollution damages, pollution dynamics, welfare weights and exogenous endowments y_i across regions. As shown in Appendix 6, the following results can be obtained.

Proposition 1.

1. *With constant marginal damages,* $c_2 = 0$, *optimal emissions and emission taxes at a steady state are the same in both regions.*
2. *With linear and increasing marginal damages,* $c_2 > 0$, *emission taxes are different between regions. If* $D \geq B$, *then* $\mu_1 > \mu_2$, *which implies* $0 < \tau_1 < \tau_2$. *When* $D < B$, *then* $0 < \tau_1 \lesseqgtr \tau_2$.

Result 1 follows from the fact that the amount of social cost "saved" in region 1 because pollution is transported to region 2 is equal to the amount of social cost increase in region 2 because of the transported pollution.

Result 2 implies that when the pollution flux from region 1 to region 2 is stronger than pollution depreciation, then a regulator who weights regional welfare equally and maximizes global welfare will tax emissions in region 2 relatively more than in region 1. Region 2 is a high pollution accumulation region due to the unidirectional transport, so by taxing region 2 more the regulator seeks to reduce pollution accumulation in region 2 by restricting emissions generated in 2. This is a rather unexpected result, since relatively higher taxation in region 1 – which generates the transported pollution – might have been expected. However, since the marginal social cost of pollution is now increasing in the amount of pollution, the "Coase" type argument suggesting that the reduction in social cost in region 1 from transport of some of its pollution to region 2 exactly cancels out and the social cost increase in region 2 no longer applies. With increasing marginal cost from the added pollution from transport, it is possible that at a certain point marginal cost will increase in region 2 more than the reduction in marginal cost in region 1. In this case, it is optimal to tax emissions generated in region 2 relatively more.[30]

To express the optimal taxes in consumption terms, the taxes should be divided by the marginal utility of consumption in each region. If consumption levels are different, there will be a further differentiation of the optimal pollution taxes. Thus, even in this simple two-region symmetric model, spatial transport of pollution implies that optimal taxes could, under reasonable assumptions, be spatially dependent.

In a symmetric two-region model with constant marginal damages, the steady state for the regional pollution accumulation and the corresponding social pollution cost is a global saddle point.[31] This means that for any two initial states of regional pollution accumulation, the regulator can calculate initial values for the social pollution cost, and therefore initial values and time paths for the optimal regional pollution taxes so that the regulated system will converge to the optimal steady-state regional pollution accumulation. For the proof, see Appendix 6.

The same results can be obtained under the assumption that each region can borrow and lend, b_i, at rate r. In this case the Hamiltonian representation of the representative consumer's problem in each region will be

$$\max_{E_i} \mathcal{H}_i = \max_{E_i} \left\{ \ln(c_i) + \mu_{b_i} \left(rb_i + y_i E_i^{\alpha} - \tau_i E_i + Tr_i - c_i \right) \right\}.$$

Combining the optimality conditions with the planner's problem will provide the same regional taxes and transfers as in (22).

[30] For an early analysis of uniform versus differentiated regulation in a static context, see for example Kolstad (1987).

[31] Under saddle point stability, the regulator can choose initial values $\mu_i(0) = \phi_i(P_{10}; P_{20}); i = 1, 2$, so that the trajectories $((P_1(t)); P_2(t); \mu_1(t); \mu_2(t)); t \geq 0$ converge on a two-dimensional stable manifold generated by the eigenspace of the two negative eigenvalues of the Hamiltonian system, to the socially-optimal steady state. Since $-\mu_i(0)$ determine the initial emission taxes, the regulator can calculate the optimal paths for emission taxes and emissions.

6.1.1 A Hybrid Model of Transboundary Pollution with Spatial Diffusion and Spillovers

The discussion about transboundary pollution can be combined in a model with the concepts of diffusion and spatial spillovers discussed above. Consider that in the optimization problem represented in (20), space is continuous, finite and linear $x \in [0, L]$, and that: (i) positive productivity spillovers of the type introduced by Lucas (2001) from the use of emissions or energy exist, modeled as

$$e^{z(x)} \text{ with } z(x) = \delta \int_0^L e^{-\delta(x-x')} E(x')\, dx',$$

and (ii) pollution diffuses following a Fickian diffusion process modeled by $D\partial_{xx}P(x)$. The social planner maximizes benefits over the whole spatial domain and the Hamiltonian for this problem, omitting t to ease notation, is

$$\begin{aligned} \mathcal{H} \;=\; & \int_0^L \Big\{ w(x) \Big[\ln \Big(y(x)\, E(x)^a\, \mathrm{e}^{-\upsilon(x)P(x)} \mathrm{e}^{z(x)} \Big) \Big] + \\ & \mu(x) \left[E(x) - mP(x) + D\partial_{xx}P(x) \right] \Big\}\, dx. \end{aligned}$$

Applying the maximum principle of section 3, we obtain:

$$E^*(x) = \frac{aw(x)}{-\mu(x) - \delta S^E(x)}\ ,\ S(x)^E = \int_0^L e^{-\delta(x-x')^2} dx' \tag{23}$$

$$\partial_t \mu(x) = (\rho + m)\,\mu(x) + \upsilon(x) - D\partial_{xx}\mu(x) \tag{24}$$

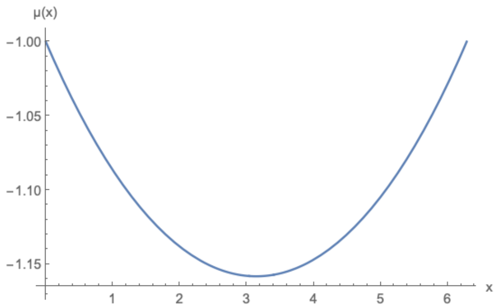
$$\partial_t P(x) = E^*(x) - mP(x) + D\partial_{xx}P(x)\,. \tag{25}$$

When atomistic representative agents in each site do not take into account the pollution and the productivity externality, the optimal site-specific pollution tax is

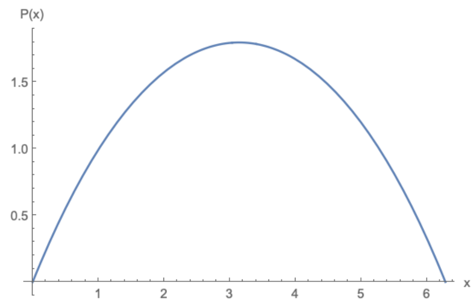
$$\tau^*(x) = y(x)\,\alpha \left(\frac{w(x)\,\alpha}{-\mu(x) - \delta S^E(x)} \right)^{\alpha - 1}. \tag{26}$$

In this case, even with flat earth $y(x) = y$ and equal weights $w(x) = w$, the pollution tax is site specific because pollution diffusion and spatial spillovers induce spatial structure in $\mu(x)$ and $S^E(x)$. A steady-state spatial distribution for $P(x), \mu(x)$ obtained from the system (23)-(25) for $\partial_t \mu(x) =$

$0, \partial_t P(x) = 0$ is shown in figure 4.[32] At the center of the spatial domain, the stock of pollution is high and its shadow cost is also high, indicating higher emissions taxes. The size of the emission tax is reduced by the spillover effect as indicated by (26), which reveals the trade-off between the productivity benefits from clustering emissions and the environmental cost of clustering pollution.



(a)



(b)

Figure 4: (a) The Shadow Cost of Pollution $\mu(x)$, (b) The Stock of Pollution $P(x)$

$L = 2\pi, \rho = 0.01, a = 0.3, m = 0.05, D = 1, \upsilon = 0.1, \delta = 0.01$

This hybrid model can be extended in many directions to become more realistic, but this example clearly indicates how different transport mechanisms which act on real spatial phenomena could be combined in modeling and provide insights for policy design.

6.2 Regulating a Global Externality and Designing Global Climate Policy

The need for regional analysis of the impacts of climate change, in contrast to the global approach taken by Integrated Assessment Models (IAMs) such as DICE (Nordhaus and Sztorc 2013; Nordhaus 2014), has been clearly recognized in the literature (see, for example, Easterling 1997). In fact, major IAMs such as RICE (e.g., Nordhaus 2011), FUND (e.g., Anthoff and Tol 2013), or PAGE (e.g., Hope 2006) explicitly include regional components. The regional aspects have been extended to both regional temperature effects and regional economic effects (e.g., FUND, PAGE), or to regional eco-

[32] The simulation is provided for illustration purposes only. It does nor refer to a real example.

nomic effects with predictions about mean global temperature (e.g., RICE). Multi-region modeling in climate change economics has been developed since RICE. Desmet and Rossi-Hansberg (2015) developed a spatial model of climate change, Krusell and Smith (2017) introduced a 20,000-region spatial model, and Hassler and Krusell (2018) discuss approaches to multi-region climate modeling.

6.2.1 Pattern Scaling

An approach which climate science uses to generate spatial temperature variation across regions is pattern or statistical downscaling, or statistical emulation methods (see, for example, Castruccio et al. 2014; Hassler, Krusell, and Smith 2016; Krusell and Smith 2017). Pattern scaling assumes that all regional temperature anomalies relative to the preindustrial temperature in region i, T_{i0} defined as $T_i(t) - T_{i0}$, are proportional to the global mean temperature anomaly $T_{GM}(t) - T_{GM,0}$. That is,

$$T_i(t) - T_{i0} = \alpha_i \left[T_{GM}(t) - T_{GM,0}\right].$$

Castruccio et al. (2014) fit the equation

$$\begin{aligned} T_t &= \beta_0 + \beta_1 \frac{1}{2}\left[\ln \frac{\mathrm{CO}_{2,t}}{\mathrm{CO}_{2,0}} + \ln \frac{\mathrm{CO}_{2,t-1}}{\mathrm{CO}_{2,0}}\right] + \beta_2 \sum_{k=2}^{k=t} \rho^k \ln \frac{\mathrm{CO}_{2,t-k}}{\mathrm{CO}_{2,0}} + \varepsilon_t \\ \varepsilon_t &= \varphi \varepsilon_{t-1} + \sigma z_t \ , \quad \{z_t\} \, \mathrm{IIDN} \ \ (0,1) \end{aligned}$$

– where T_0 is given, $\mathrm{CO}_{2,t}$ is concentration of CO_2 at t, and $\mathrm{CO}_{2,0}$ is preindustrial concentration – to regional yearly temperature data generated by their atmosphere–ocean general circulation model (AOGCM) for one scenario to “train” their emulator. They then use their estimated equation for that scenario to mimic the output of their AOGCM for another scenario. They do this procedure for 47 regions (for estimates, see Castruccio et al. (2014), table S1 in the supplementary material). They estimate regressions of the form

$$T((L,l),t) - T_0(L,l) = \alpha_{(L,l)} \left[T_i(t) - T_{i,0}\right] + \varepsilon_{(L,l)},$$

where (L, l) denote longitude and latitude respectively. Performance measures suggest that the emulator does a fairly good job of mimicking the output of the much more complicated AOGCM. Figure 6 in Castruccio et al. (2014) displays the emulated temperatures with the top of the display corresponding to the northern latitude regions and the bottom to the southern latitude regions. The pattern of higher temperatures as one moves toward the northern regions is clear.

6.2.2 Heat and Precipitation Transport: Two-Box Models

Regional aspects of climate change and associated policies have been introduced in low-dimensional IAMs in which regional temperature dynamics are driven by endogenous mechanisms of heat and precipitation transport from the Equator to the Poles (see Brock et al. 2013; Brock and Xepapadeas 2017, 2019; Cai et al. 2019). The climate science part of these models is based on one- or two-dimensional dynamic energy balance climate models (EBCMs), defined either in discrete space in the context of South-North "two-box" models (e.g., Langen and Alexeev 2007), or in continuous space (e.g., North, Cahalan, and Coakley 1981). EBCMs generate spatial variability of temperature across regions through the endogenous mechanism of heat transfer.

Regional temperature differentiation also emerges from the use of the transient climate response to cumulative carbon emissions (TCRE) on a regional basis. The TCRE embodies both the physical effect of CO_2 on climate and the biochemical effect of CO_2 on the global carbon cycle (e.g., Matthews et al. 2009; Matthews, Solomon, and Pierrehumbert 2012; Knutti 2013; Knutti and Rogelj 2015; MacDougall, Swart, and Knutti 2017). The TCRE, denoted by Λ, is defined as $\Lambda = \Delta T(t)/CE(t)$, where $CE(t)$ denotes cumulative carbon emissions up to time t and $\Delta T(t)$ the change in temperature during the same period with $\Lambda = 1.7 \pm 0.4$°C per TtC (Leduc, Matthews, and de Elía 2016). The approximate constancy of TCRE suggests an approximately linear relationship between a change in global average temperature and cumulative emissions. This roughly linear relationship has also been recognized by the IPCC (2013).

Leduc, Matthews, and de Elía (2016) identify an approximately linear relationship between cumulative CO_2 emissions and regional temperatures. This relationship is quantified by regional TCREs, or RTCREs. The RTCRE parameters range from less than 1°C per TtC for some ocean regions to 5°C per TtC in the Arctic. The authors consider their approach to be a novel application of pattern scaling.

Heat and moisture transport from the Equator to the Poles, when combined with the surface-albedo feedback, results in the observed phenomenon of Polar or Arctic amplification (IPCC 2013, p. 396).[33] Arctic amplification could cause serious detrimental environmental effects which could be diffused to other regions south of the Arctic. Thus one implication of adopting a regional representation of climate is that changes in the temperature in one region could generate damages in another region. The existence of geographical spillover damage effects across regions is supported by recent studies.[34] If the stronger anomaly growth in the Arctic relative to the Equator could cause damages in the South due to sea level rise or extreme weather phenomena, then local damages should depend on the local temperature anomalies in both regions. At the same time, heat transfer from the South to the North might benefit the South by reducing temperature levels in the relatively more vulnerable areas around the Equator. For example, if heat transfer from the Equator to the Poles did not exist, then damages from extreme heat documented by Hsiang et al. (2017) in the low latitudes might be even larger and mortalities from both extreme heat in the low latitudes and extreme cold in the high latitudes documented by Gasparrini et al. (2015) might be even larger.

Detailed work on estimating marginal temperature and damage impacts due to spatial temperature differentials is an area for further research, since it will be needed in order to compute the impacts on optimal policy. Thus the issue of explicit consideration of heat transfer mechanisms in coupled models of climate and the economy could be important for policy purposes but, as far as we know, has not been explicitly addressed by large-scale IAMs.

The two-box framework with meridional heat and moisture transport[35]

[33] Bekryaev, Polyakov, and Alexeev (2010), using an extensive data set of monthly surface air temperature, document a high-latitude ($> 60°$N) warming rate of 1.36°C/century for 1875–2008, with the trend being almost two times stronger than the Northern Hemisphere trend of 0.79°C/century. The high RTCRE in the Arctic reported by Leduc, Matthews, and de Elía (2016) is indicative of Arctic amplification.

[34] See, for example, Francis and Vavrus (2014), Francis and Skific (2015), Francis (2017), Francis, Skific, and Vavrus (2018) and Wu and Francis (2019). The main message is that further Arctic warming may favor persistent weather patterns that can lead to weather extremes.

[35] For details, see Alexeev, Langen, and Bates (2005), Langen and Alexeev (2007) and

with box $i = 1$ which is the South $(0°, 30°N)$ and box $i = 2$ which is the North $(30°N, 90°N)$, combined with the RTCRE approach, implies the following regional dynamics for the temperature anomalies:

$$\dot{T}_1 = \frac{1}{H}\left[(-B_1 - \gamma_1 - \gamma_2) T_1 + \gamma_1 T_2 + \Lambda_1 E\right],\ T_1(0) = 0 \quad (27)$$

$$\dot{T}_2 = \frac{1}{H}\left[(\gamma_1 + \gamma_2) T_1 + (-B - \gamma_1) T_2 + \Lambda_2 E\right],\ T_2(0) = 0 \quad (28)$$

$$E = E_1 + E_2, \quad (29)$$

where (E_1, E_2) are regional carbon emissions, H is heat capacity, and (Λ_1, Λ_2) are the local TCRE in the South and North respectively.[36] Note that with $\gamma_1 = \gamma_2 = 0$, the temperature dynamics model (27)-(29) is reduced to the Leduc, Matthews, and de Elía (2016) model, while for $\Lambda_1 = \Lambda_2 = \Lambda$ it is reduced to the Langen and Alexeev (2007) model.

If a social planner seeks to maximize global welfare by choosing the paths of regional carbon emissions $E_i(t)$, the planner's objective, considering a log-utility function similar to the transboundary problem, is:

$$\max_{E_1,E_2} \int_0^\infty e^{-\rho t}\left[\sum_{i=1,2} w_i \left[\ln y_i + \alpha \ln E_i - \upsilon_i(T_1, T_2)\right]\right] dt \quad (30)$$

subject to (27)-(29),

where w_i represent as before welfare weights. Damages in each region depend on the temperature anomaly in the other region. This modeling seeks to capture effects such as damages in the South for the faster temperature increase in the Arctic which may increase the frequency or/and the severity of extreme weather phenomena.

In a world with frictionless transfer of resources across regions and unlimited fossil fuels, the solution of the global externality problem (30) can be implemented, following the approach of the previous section, by carbon taxes and transfers defined as:

$$\tau_i = y_i \alpha \left(\frac{w_i \alpha H}{\mu_1 \Lambda_1 + \mu_2 \Lambda_2}\right)^{\alpha-1},\ Tr_i = \left[y_i \alpha \left(\frac{w_i \alpha H}{\mu_1 \Lambda_1 + \mu_2 \Lambda_2}\right)^{\alpha-1}\right]\left(\frac{w_i \alpha H}{\mu_1 \Lambda_1 + \mu_2 \Lambda_2}\right).$$

Alexeev and Jackson (2013).

[36] The parameter values for the climate model can be obtained from calibrations of climate science models (e.g., Langen and Alexeev 2007; Leduc, Matthews, and de Elía 2016).

The following result can easily be seen. When welfare weights are equal, regional emissions are equal, therefore $\tau_1 < \tau_2$ if $y_1 < y_2$, and in this case the poorer region should pay a lower carbon tax. The size of carbon taxes depends on the shadow cost of regional temperature anomalies (μ_1, μ_2). In Appendix 6, the Hamiltonian system for problem (30) with quadratic damage functions is presented. The impact of heat and precipitation transport can be analyzed by comparative analysis of parameters (γ_1, γ_2). Since in the poorer region $C_1 < C_2$, the poorer region will pay a lower carbon tax in consumption terms since

$$\tau_1^C = \frac{\tau_1}{u'(C_1)} < \tau_2^C = \frac{\tau_2}{u'(C_2)}.$$

In climate change policy a uniform carbon tax, or carbon price, across locations is a common result stemming from the global nature of the climate externality. This attitude seems to change, however, as more aspects of the climate and the economy are taken into account. The High-Level Commission on Carbon Prices (2017) report and Stiglitz (2019) recommend non-uniform carbon taxes, with carbon taxes being relatively higher in regions where consumers are disproportionately rich. Brock, Engström, and Xepapadeas (2014), in a continuous space model with heat transport polarward, show that optimal carbon taxes are higher in relatively richer regions in which the marginal utility of consumption is lower.

Recently Cai et al. (2019) developed a novel stochastic North-South large scale IAM, based on the DICE/RICE framework for the climate module, with meridional heat and moisture transport, sea level rise, permafrost thaw and stochastic tipping points. Cai et al. introduce adjustments costs in the economic interactions across regions and show that if these adjustment costs are zero, then the regional carbon tax is the same across regions since the marginal return of capital is equated across regions. However, with nonzero adjustment costs between regions, the regional carbon tax is different across regions.

Another issue emerging in regional models of climate change with heat and moisture transport is whether ignoring such a phenomenon introduces bias in optimal climate policies. Brock and Xepapadeas (2017, 2019) and Cai et al. (2019) show that ignoring heat and moisture transport could introduce serious bias in the optimal carbon taxes. The direction of the bias depends crucially on whether the costs to the South – from the faster increase in temperature in the North caused by the surface albedo feedback and heat flux – exceed the benefits in the South from the reduction in the regional temperature due to heat transfer.

6.2.3 Strategic Behavior

In major IAMs which involve optimization at the global or regional level, such as DICE or RICE, the objective is the maximization of a global welfare criterion (as with DICE) or the sum of welfare criteria across regions (as with RICE). In the case of RICE, the solution for the given objective corresponds to a cooperative solution in which a social planner chooses emissions paths to maximize aggregate regional welfare subject to economic and climate constraints. This assumption implies that regions or countries have agreed, through some kind of an international agreement, to follow cooperative emission paths.[37]

This approach is useful in identifying optimal cooperative emission paths and indicating policy instruments such as carbon taxes to attain these paths. However, when it comes to the real world, countries or regions might not be willing to follow a cooperative solution. Although they may recognize the impact of climate change on global welfare, a specific region or country might be willing to choose emission paths which will maximize own welfare, which will in general be gross benefits from using fossil fuels net of own climate damages. In this case the appropriate solution concept is the solution of a noncooperative dynamic game. The explicit introduction of regional temperature dynamics makes the noncooperative solution concept more realistic since each country or region will try to design optimal policies by considering own temperature dynamics and not global temperature dynamics.

A noncooperative solution in the context of climate change is an equilibrium outcome in which countries maximize own welfare subject to economic and climatic constraints and assumptions about the climate policies of other

[37] Relevant examples are the outcomes of the Conferences of the Parties, such as the Kyoto Protocol or the Paris Accord, which provide an idea of such a cooperative solution in the real world.

countries. In terms of the objective (30), this means

$$\max_{E_i} \int_0^\infty e^{-\rho t} \left[\ln y_i + \alpha \ln E_i - \upsilon_i \left(T_1, T_2\right)\right] dt \ , \ i = 1, 2, \qquad (31)$$

subject to (27)-(29),

and assumptions about the paths of $E_j(t); i \neq j$.

Nordhaus and Yang (1996), in the context of the RICE model, were the first to study noncooperative outcomes using the solution concept of the open loop Nash equilibrium in which each country sets its climate policy to maximize its own economic welfare, assuming that other countries' policies are invariant to its policies. Dutta and Radner (2006), in a game-theoretic approach to global warming, consider models with multiplicity of equilibria which allow the identification of "Pareto-improving" equilibria. Bosetti et al. (2006) also derive open loop Nash equilibrium solutions in the context of the regional WITCH model.[38]

Noncooperative solutions in general indicate that emissions will be higher and carbon taxes lower relative to the cooperative solutions. The earlier literature, although dealing with regional models, did not explicitly include heat transfer. Brock and Xepapadeas (2019) consider strategic interactions in a simple two-box model with heat transfer and damages in one region affected by the temperature in the other region, to capture impacts of Arctic amplification in the South. In addition to the open loop equilibrium, they also examine the feedback Nash equilibrium.[39] Cai et al. (2019) use a novel algorithm to determine the feedback Nash equilibrium in the stochastic two-region model described above, which contains eleven state variables and eight decision variables.

The main message from the two-region climate models with heat and moisture transfer is that in both cooperative and noncooperative solutions, ignoring the transport mechanism, which is a well-established mechanism,

[38]For a detailed exposition of cooperative and noncooperative solutions in the context of integrated assessment modeling, see Yang (2008).

[39]It is well-known that the open loop equilibrium does not possess the Markov perfect property and is not robust against unexpected changes in the state of the system. Thus a feedback equilibrium is considered to be a more satisfactory solution. With an open loop information structure, each region takes the emission path of the other region as given. In a feedback structure, each region assumes that the emissions of the other region are a function of the current temperature anomalies or $E_i(t) = h_i(T_1(t), T_2(t); t)$.

The feedback Nash equilibrium is derived in a dynamic programming framework (e.g., Başar and Olsder 1995) and by construction is Markov perfect.

could introduce serious biases in climate policy.

6.2.4 Heat and Precipitation Transport: One-Dimensional Continuous Space Models

Two-region climate models provide important insights into the role of transport mechanisms in the design of climate policy. Similar insights can also be provided by more detailed EBCMs in continuous space. EBCMs are distinguished into wet models in which temperature diffusion is replaced by moist static energy diffusion (e.g., Flannery 1984), and dry models in which the basic thermodynamic variable used to determine energy transport is temperature (e.g., Sellers 1969; North 1975a, 1975b; Ghil 1976; North, Cahalan, and Coakley 1981; Ghil and Lucarini 2019). In both models, energy transports generate Polar amplification under different assumptions.

Following Merlis and Henry (2018), and dropping t to ease notation, a wet EBCM is written as

$$C\partial_t T\left(\phi\right) = \frac{1}{4} Q S\left(\phi\right) a\left(\phi\right) - \left[A + BT\left(\phi\right)\right] - \nabla \cdot \mathbf{F}_a\left(\phi\right) + \mathcal{F}, \tag{32}$$

where C is heat capacity; T surface temperature; ϕ latitude;[40] Q the solar constant; $S(\phi)$ the insolation structure function; $a(\phi)$ coalbedo; $[A + BT(\phi)]$ outgoing long-wave radiation; $\mathcal{F}$ radiative forcing; and $\nabla \cdot \mathbf{F}_a = -\partial_x \left[\mathcal{D}\left(1 - x^2\right)\partial_x h\left(x\right)\right]$ with $x = \sin\phi$ is the divergence of the atmospheric energy flux which is governed by the diffusion of the moist static energy h measured in units of temperature, with diffusivity $\mathcal{D}$.

In dry EBCMs, the term $\nabla \cdot \mathbf{F}_a$ is replaced by $-\partial_x \left[\mathcal{D}\left(1 - x^2\right)\partial_x T\left(x\right)\right]$ with ϕ replaced by x in the rest of the functions. The temperature spatiotemporal dynamics described by (32) can be incorporated into an economic model of climate change by an appropriate specification of radiative forcing $\mathcal{F}$. Brock et al. (2013) and Brock, Engström, and Xepapadeas (2014, 2015) use (32) in a coupled model of the economy and the environment and define forcing using the standard relationship $\mathcal{F} = (\lambda/\ln 2)(\ln(S_t/S_0)$ where λ is climate sensitivity and S_t/S_0 is the ratio of the concentration of carbon dioxide in the atmosphere between period t and the preindustrial concentra-

[40] When the spatial dimension is one, that is, latitude only, the EBCMs are called one-dimensional. In contrast, models without spatial transport mechanisms are called zero-dimensional.

tion S_0. They show that if welfare weights across locations are equal, then in cooperative solutions the location with the lower per capita consumption should pay lower carbon taxes, a result which is in line with Stiglitz (2019).

In the context of the approximate proportional relationship between changes in temperature and emissions, the optimization of a welfare objective subject to (32) can be simplified by using instead of $\mathcal{F}$ the term $\Lambda \int_{x=-1}^{x=1} E(x,t)\,dx$ where Λ is the TCRE and the integral term corresponds to global emissions at time t.[41]

Using an objective similar to (30), the planner's problem when a continuous space one-dimensional dry EBCM with local TCRE is used to model climate can be written as:

$$\max_{E_1 E_2} \int_0^{\infty} e^{-\rho t} \left[\int_{-1}^{1} w(x) \left[\ln y(x) + \alpha \ln E(x) - \upsilon \left[T(x) \right]^2 \right] dx \right] dt \tag{33}$$

subject to

$$C \partial_t T(x) = \frac{1}{4} QS(x)\, a(x) - \left[A + BT(x) \right] - \tag{34}$$
$$\partial_x \left[\mathcal{D} \left(1 - x^2 \right) \partial_x T(x) \right] + \Lambda \int_{-1}^{1} E(x)\, dx.$$

Optimal local emissions are determined as

$$E(x,t) = \frac{-\alpha w(x)}{\Lambda \int_{-1}^{1} \mu(x)\, dx}. \tag{35}$$

Using the heuristic proof for the derivation of the maximum principle, the Hamiltonian system of (33) implies that the local shadow cost of changes in temperature, which determines optimal emissions, evolves according to:

$$\partial_t \mu(x) = (\rho + B)\, \mu(x) + \upsilon(x) + \partial_x \left[\mathcal{D} \left(1 - x^2 \right) \partial_x \mu(x) \right],$$

along with (34) in which $E(x)$ is replaced by (35).

If we assume again that each location x is populated by an identical representative agent who maximizes own utility ignoring climate impacts on own location as well as on the other locations, the socially-optimal solu-

[41] In the linear approximation of local temperature change,

$$T(t,x) - T(t,0) = \int_0^t \left[\Lambda \int_{-1}^{1} E(x,t)\, dx \right] dt,$$

therefore $\partial_t T(t,x) = \Lambda \int_{-1}^{1} E(x,t)\, dx$.

tion resulting from problem (33) can be implemented with latitude-specific carbon taxes of the form

$$\tau(x) = \alpha y(x) \left(\frac{\alpha w(x) C}{\Lambda \int_{-1}^{1} \mu(x) dx} \right)^{\alpha - 1}.$$

The problem which involves a Hamiltonian system in nonlinear PDEs can in principle be solved numerically. An approximation approach for solving this problem in terms of ODEs is presented in Appendix 7.

7 Uncertainty and Space

An issue that acquires importance in a spatial context is uncertainty. In recent papers, Barnett, Brock, and Hansen (2019), Brock and Hansen (2019) and Hansen and Sargent (2019) distinguish three forms of uncertainty.

- Risk: The probabilities (objective or subjective) of uncertain outcomes are known, and the decision maker is confident about the model used. Uncertainty exists within the model.
- Ambiguity: There are a large number of potential models which could be used by the decision maker. There is a question regarding the level of confidence of the decision maker on each model.
- Misspecification: The question here is how the decision maker uses models that are not perfect and may have unknown flaws.

Sometimes the last two forms are referred to as "deep uncertainty." Uncertainty could have a profound spatial structure, i.e., different forms of uncertainty or combinations of forms with spatially-heterogeneous characteristics could prevail across locations. For a regulator seeking to derive optimal policies for the whole spatial domain, the spatial structure of uncertainty presents an additional challenge, since the policy should take into account spatial heterogeneities induced by transport mechanisms and uncertainty.

The robust control approach to uncertainty introduced to economics by Hansen and Sargent (e.g., 2001, 2008) is very convenient for extensions to situations in which the regulator faces uncertainty with different regional characteristics or, to put it in terms of robust control methods, the regulator has different misspecification concerns about different locations. These concerns could refer to local damages, pollution or resource dynamics, or diffusivity.

Brock, Xepapadeas, and Yannacopoulos (2014c) study a robust control problem with site-specific misspecification concerns. Their main finding is the identification of specific sites which they call “hot spots” in which serious concerns about misspecification could lead to the inability to define efficient regulation for the whole spatial domain. The emergence of “regulatory hot spots” acquires a high level of importance in the analysis of climate change because of the existence of tipping points (Lenton et al. 2008), which are locations associated with the triggering of big damages and which are surrounded by large uncertainties.

A brief exposition of modeling a spatial robust control problem can be presented in the following way. Brock and Xepapadeas (2018) use robust control to construct a regional policy that works uniformly well over a set of alternative models surrounding a “baseline” model. Intuitively the robust control method leads to the regulator maximizing against a “worst case” model in the set of alternative models. The worst case model is chosen by an adversarial agent who is trying to minimize the regulator’s objective. Following Anderson, Hansen, and Sargent (2012) and Anderson et al. (2014), the stochastic robust control model can, under appropriate scaling, be transformed into a simpler deterministic robust control model.

Consider a multi-regional version of problem (30) and assume the regulator has concerns about misspecification in regional temperature dynamics, damages and diffusivity. This can be interpreted as the regulator having a benchmark model describing dynamics, spatial transport and damages, but he/she is not confident about the model and wants to regulate by taking into account the possibility that alternative models which represent distortions of the benchmark model could be realized as the actual model. To discipline the sensitivity analysis underlying the distortion of the parameters, the alternative models are contained in a bounded set and the adversarial agent chooses models/distortions to minimize the regulator’s objective.

The spatial deterministic robust control problem can be defined as:

$$\max_{\{E_{it}\}} \min_{\{k_{it}, h_{it}\}} \tag{36}$$

$$\int_{t=0}^{\infty} \mathrm{e}^{-\rho t} \sum_{i=1}^{N} w_i \left[\ln y_i + \alpha \ln E_{it} - \upsilon_i \left(T_1, ..., T_n, k_i\right) + \frac{k_{it}^2}{2\eta_i} + \frac{h_{it}^2}{2\theta_i} \right] dt,$$

subject to

$$\dot{T}_{it} = \Lambda_i \sum_{i=1}^{N} E_{it} - B_i T_{it} + \sigma_i h_{it}, \; T_{i0} \geq 0. \tag{37}$$

In (37), the parameter σ_i represents volatility of regional temperature dynamics, h_{it} the corresponding drift distortion reflecting deep uncertainty, and θ_i the concerns about misspecification of temperature dynamics. The initial conditions reflect that T_{it} represents the temperature anomaly relative to a given base period. We assume that concerns about regional temperature dynamics are specific to the region, and therefore embody concerns about the RTCRE, which could also be an uncertain parameter. The k_i represents ambiguity about damages in region i, and η_i the concerns about misspecification of the damage function. Note that misspecification concerns are site specific and thus embody the different degree of uncertainty that the regulator faces across locations. Note also that the damage function in region i embodies geographical damage spillovers, or cross effects, which are damages caused by temperature increases in other regions. For example, the larger anomaly in the high northern latitudes may generate damages in terms of sea level rise or greenhouse gases emitted by permafrost melting in southern regions.

Optimal emissions are given by

$$E_{it}^{*} = \frac{-\alpha w_i}{\sum_i \Lambda_i \mu_{it}}.$$

The regional temperature shadow costs and the optimal emissions in this case are determined – through the Hamiltonian system – by the local misspecification concerns (η_i, θ_i). The same holds for the optimal site-specific taxes.

Spatial robust control methods can be extended to stochastic control problems, continuous spatial domain and noncooperative solutions. Given the spatial differentiation of uncertainty across locations, this is a very interesting area of further research.

8 Concluding Remarks

Although the spatial dimension is embedded in the vast majority of issues studied by environmental and resource economics, its incorporation into economic models – especially in the form of explicit introduction of a spatial transport mechanism – is not widespread. There are a number of important exceptions, many of which are discussed in this article. Failure to explicitly incorporate the spatial dimension means that important aspects of the problem may not be accounted for, which could result in regulatory inefficiencies.

Furthermore, when it comes to policy design, accounting for the spatial dimension implies spatially-dependent instruments and possibly the need for menus of instruments to deal with the potential emergence of various spatial externalities. Again, the lack of such instruments may result in inefficient policies.

The purpose of this paper, therefore, is threefold: to present the evolution of spatial methods in environmental and resource economics; to emphasize that space matters in the design of efficient policies; and to indicate research areas where spatial methods could provide new and useful insights.

In this context, we presented the major spatial transport mechanisms and the way in which they can be incorporated into forward-looking optimizing economic models. We provided an extension of Pontryagin's maximum principle under spatial dynamics and explained how optimal Turing instability may emerge in this set-up. Optimal Turing instability is a precursor of spatial pattern formation in the quantity-shadow price domain and provides the basis for introducing spatially-dependent policies.

Moreover, we presented examples of the use of the framework of spatial dynamics, which illustrate why space matters in environmental and resource economics, and how policy is differentiated when spatial transport mechanisms are taken into account. These examples include topics related to fishery management, groundwater management, pollution control, urban economics, climate policy, and the management of spatially-structured uncertainty.

The tools presented in the paper, along with some examples of their application, provide a path for future research in spatial environmental and resource economics in which the underlying spatial dimension – which is very real – is fully taken into account.

9 Appendix

9.1 Appendix 1

The PDE whose solution is shown in figure 1a is:

$$\begin{aligned}
\frac{\partial y(x,t)}{\partial t} &= ry(t,x)\left(1-\frac{y(t,x)}{K}\right)-uy(t,x)+D\frac{\partial^2 y(t,x)}{\partial x^2}\\
t &\in [0,5000],\ x\in[0,\pi],\ y(0,x)=\sin(x)\\
y(t,0) &= y(t,\pi)=0,\ \text{hostile boundary}\\
r &= 0.3, K=10, u=0.1, D=0.03.
\end{aligned}$$

The reaction-diffusion system whose solution is shown in figure 1c is:

$$\begin{aligned}
&\frac{\partial y_1(x,t)}{\partial t} = r_1 y_1(t,x)\left(1-\frac{y(t,x)}{K_1}\right)-\frac{\beta_1 y_2(t,x)}{y_1(t,x)+K_2}-u_1 y_1(t,x)+D_1\frac{\partial^2 y_1(x,t)}{\partial x^2}\\
&\frac{\partial y_2(x,t)}{\partial t} = r_2 y_2(t,x)\left(1-\frac{\beta_2 y_2(t,x)}{y_1(t,x)}\right)-u_2 y_2(t,x)+D_2\frac{\partial^2 y_2(x,t)}{\partial x^2}\\
&t\in[0,5000],\ x\in[0,\pi],\ y_1(0,x)=1+2\sin(x),\ y_2(0,x)=1.2+\sin(x)\\
&y_1(t,0)=y_1(t,\pi)=1,\ y_2(t,0)=y_2(t,\pi)=1.5\\
r_1 &= 0.3, r_2=0.35, K_1=10, K_2=1, \beta_1=\beta_2=1, u_1=u_2=1, D_1=0.03, D_2=0.02.
\end{aligned}$$

The composite kernel shown in figure 2a is

$$\begin{aligned}
w(x-x') &= \exp\left(\alpha(x-x')^2\right)-\gamma\exp\left(\beta(x-x')^2\right),\ x,x'\in[-6\pi,6\pi]\\
\alpha &= -0.05, \gamma=0.5, \beta=-0.02.
\end{aligned}$$

The integrodifferential equation whose solution is shown in figure 2b is:

$$\begin{aligned}
\frac{\partial y(x,t)}{\partial t} &= ry(t,x)\left(1-\frac{y(t,x)}{K}\right)-uy(t,x)-\frac{\delta y(t,x)^2}{1+y(t,x)^2}+\phi\int_{-6\pi}^{6\pi} w(x-x')\,y(x-x')\,dx'\\
t &\in [0,5000],\ x\in[-6\pi,6\pi],\ y(0,x)=5+\sin(x)\\
y(t,-6\pi) &= y(t,6\pi),\ \text{periodic boundary conditions (circle)}\\
r &= 0.475, K=10, u=2, \delta=0.1, \phi=1.1.
\end{aligned}$$

All numerical solutions and their plots were obtained using the solver *NDSolve* of Mathematica 11.

9.2 Appendix 2

A sketch of a heuristic proof of the maximum principle under diffusion can be presented by using a variational argument along the lines of Kamien and Schwartz (1991, pp. 124–127). Considering a finite time horizon $t \in [t_0, t_1]$, problem (5) subject to (1) with appropriate spatial boundary conditions can be written as:

$$\begin{aligned} J &= \int_{\mathcal{O}} \int_0^{\infty} \mathrm{e}^{-\rho t} U\left(y\left(t,x\right), u\left(t,x\right)\right) dt dx = \int_{\mathcal{O}} \int_0^{\infty} \mathrm{e}^{-\rho t} \left\{ U\left(y\left(t,x\right), u\left(t,x\right)\right) \right. \\ &\quad \left. p\left(t,x\right) \left[f\left(x\left(t,x\right), u\left(t,x\right)\right) + D \frac{\partial^2 y}{\partial x^2} - \frac{\partial y}{\partial t} \right] \right\} dt dx. \end{aligned} \tag{38}$$

Integrate by parts the last two terms of (38) to express the terms $\mathrm{e}^{-\rho t} p\left(t,x\right) \frac{\partial y}{\partial t}$ and $\mathrm{e}^{-\rho t} p\left(t,x\right) D \frac{\partial^2 y}{\partial x^2}$ in terms of $y \frac{\partial p}{\partial t}$ and $\frac{\partial y}{\partial x} \frac{\partial p}{\partial t}$, integrate by parts once more to express the last term in terms of $y\left(t,x\right) \frac{\partial^2 p}{\partial x^2}$, and use spatial boundary and limiting intertemporal transversality conditions to eliminate constants. Introduce a one parameter family of comparison controls $u^*\left(t,x\right) + \epsilon \eta\left(t,x\right)$, where $u^*\left(t,x\right)$ is the optimal control, $\eta\left(t,x\right)$ is a fixed function and ϵ is a small parameter. Let $y\left(t,x,\epsilon\right)$, $t \in [t_0, t_1]$, $x \in \mathcal{O}$ be the smooth state variable generated by the diffusion process with control $u^*\left(t,z\right) + \epsilon \eta\left(t,z\right)$. Write $J\left(\epsilon\right)$ in terms of $y\left(t,x,\epsilon\right)$ and $u^*\left(t,x\right) + \epsilon \eta\left(t,x\right)$, and note that since u^* is a maximizing control the function $J\left(\epsilon\right)$ assumes the maximum when $\epsilon = 0$ or $\left.\frac{dJ(\epsilon)}{d\epsilon}\right|_{\epsilon=0} = 0$. Performing the maximization and using transversality and spatial boundary conditions, we obtain the maximum principle.

For a sketch of a heuristic proof of the maximum principle under spatial kernels, write (5) subject to (2) as:

$$\begin{aligned} J &= \int_{\mathcal{O}} \int_0^{\infty} \mathrm{e}^{-\rho t} U\left(y\left(t,x\right), u\left(t,x\right)\right) dt dx = \int_{\mathcal{O}} \int_0^{\infty} \mathrm{e}^{-\rho t} \left\{ U\left(y\left(t,x\right), u\left(t,x\right)\right) \right. \\ &\quad \left. p\left(t,x\right) \left[f\left(x\left(t,x\right), u\left(t,x\right), \mathbf{K} y\left(t,x\right)\right) - \frac{\partial y}{\partial t} \right] \right\} dt dx. \end{aligned} \tag{39}$$

Integrate by parts the term $\mathrm{e}^{-\rho t} p \frac{\partial y}{\partial t}$ and use spatial boundary and limiting intertemporal transversality conditions to eliminate constants. Introduce comparison controls $u^*\left(t,x\right) + \epsilon \eta\left(t,x\right)$, as before, with $Y\left(t,x,\epsilon\right) = \mathbf{K} y\left(\epsilon\right)$, define $J\left(\epsilon\right)$, calculate $\left.\frac{dJ(\epsilon)}{d\epsilon}\right|_{\epsilon=0} = 0$ and then use the linearity of the kernel operator to obtain the necessary conditions.

9.3 Appendix 3

Solution of a linear quadratic problem (LQP) under spatial diffusion.

Consider the following LQP with $\mathcal{O} = [0, L]$:

$$\max_{u(t,x)} \int_0^L \int_0^\infty \mathrm{e}^{-\rho t} \left[-\frac{A}{2} y(t,x)^2 - \frac{B}{2} u(t,x)^2 + N y(t,x) u(t,x) \right] dt dx \quad (40)$$

$$A, B, \rho > 0, \;\; AB - N^2 > 0$$

$$\text{s.t. } \frac{\partial y(t,x)}{\partial t} = F y(t,z) - G u(t,x) + D \frac{\partial^2 y(t,x)}{\partial z^2} \quad F, G > 0 \quad (41)$$

$$y(0,x) = y_0(z) \text{ given, } x \text{ in a circle } z \in [0, L] \quad (42)$$

$$y \text{ in a circle } x \in [0, L], y(t,0) = x(y,L) \text{ for all } t. \quad (43)$$

For the control problem (40)-(43), consider a set of controls $\mathcal{U}$ which have Fourier series expansions with piecewise continuous coefficients in t for nodes $n = 0, 1, 2, ...$, or

$$\mathcal{U} = \left\{ u(t,x) : u(t,z) = \sum_n \left[u_{1n}(t) \cos\left(\frac{2\pi n x}{L}\right) + u_{2n}(t) \sin\left(\frac{2\pi n x}{L}\right) \right] \right\}. \quad (44)$$

In this case the solution of (41), under appropriate regularity assumptions, for any $u(t,z) \in U$ will have a Fourier series expansion with piecewise continuously differentiable coefficients in t, or

$$y(t,x) = \sum_n \left[y_{1n}(t) \cos\left(\frac{2\pi n x}{L}\right) + y_{2n}(t) \sin\left(\frac{2\pi n x}{L}\right) \right] \quad (45)$$

$$p(t,x) = \sum_n \left[p_{1n}(t) \cos\left(\frac{2\pi n x}{L}\right) + p_{2n}(t) \sin\left(\frac{2\pi n x}{L}\right) \right]. \quad (46)$$

Substituting the $y(t,x)$ and $u(t,x)$ in (41), we obtain a set of transition equations parametrized by $n = 0, 1, 2, .$

As shown in Brock and Xepapadeas (2008), using the fact that the set of functions $\left\{ \cos\left(\frac{2\pi n z}{L}\right), \sin\left(\frac{2\pi n z}{L}\right) \right\}$, $n = 0, 1, 2, ...$ is a complete orthogonal set over $[0, L]$, the following countable set of spatially-independent optimal control problems for each $n = 0, 1, 2, ...$ can be obtained:

$$\max_{u_0(t)} \int_0^\infty e^{-\rho t} \left[-\frac{A}{2} y_0^2 - \frac{B}{2} u_0^2 + N y_0 u_0 \right] dt \tag{47}$$

$$\text{subject to } \dot{y}_0 = F y_0 - G u_0 \ , \ n = 0 \tag{48}$$

$$\max_{u_{1n}(t)} \frac{L}{2} \int_0^\infty \mathrm{e}^{-\rho t} \sum_n \left[- \left(\frac{A}{2} y_{1n}^2 + \frac{B}{2} u_{1n}^2 \right) + N y_{1n} u_{1n} \right] dt \tag{49}$$

$$\text{subject to } \dot{y}_{1n} = S(k)\, y_{1n} - G u_{1n} \ , n = 1, 2, ... \tag{50}$$

$$\max_{u_{2n}(t)} \frac{L}{2} \int_0^\infty e^{-\rho t} \sum_n \left[- \left(\frac{A}{2} y_{2n}^2 + \frac{B}{2} u_{2n}^2 \right) + N y_{2n} u_{2n} \right] dt \tag{51}$$

$$\text{subject to } \dot{y}_{2n} = S(k)\, y_{2n} - G u_{2n} \ , n = 1, 2, ... \tag{52}$$

$$S(k) = F - D \left(\frac{2\pi n}{L} \right)^2 = F - Dk^2 \ , k = \frac{2\pi n}{L}. \tag{53}$$

The solutions $\{y_{in}^*, p_{in}\}$, $i = 1, 2$ and finite number of nodes n should be substituted back into (44) (45), (46) to obtain the optimal spatiotemporal paths for $y(t,x)$,$p(t,x)$ and $u(t,x)$.

Nonlinear quadratic problems require numerical solutions. For a solution of the LQP using dynamic programming, see Boucekkine et al. (2019).

9.4 Appendix 4

Emergence of the optimal diffusion instability.

The maximized current value Hamiltonian or pre-Hamiltonian for the flat LQ system ($D = 0$), where we drop subscripts and superscripts to simplify notation, is

$$H^0(y,p) = \max_u \left\{ -\frac{A}{2} y^2 - \frac{B}{2} u^2 + Nyu + p\,[Fy - Gu] \right\}. \tag{54}$$

The Jacobian of the Hamiltonian system at the flat optimal steady state (y^*, p^*) is defined as:[42]

$$J^0(y^*, p^*) = \begin{bmatrix} H_{yp}^0(y^*, p^*) & H_{pp}^0(y^*, p^*) \\ -H_{yy}^0(y^*, p^*) & \rho - H_{py}^0(y^*, p^*) \end{bmatrix} = \begin{bmatrix} F - \frac{GN}{B} & \frac{G^2}{B} \\ A - \frac{N^2}{B} & \rho - F + \frac{GN}{B} \end{bmatrix}. \tag{55}$$

The theorem below gives one set of sufficient conditions for diffusion-

[42] Subscripts associated with functions denote partial derivatives.

induced instability of optimal control.

Theorem 1 (Optimal Turing Instability) *Assume that in the LQP with $D = 0$, the optimal flat steady state (y^*, p^*) associated with the Jacobian matrix $J^0(y^*, p^*)$ has the local saddle point property. Then, if*

$$\alpha \equiv \left(F - \frac{GN}{B}\right) > \frac{\rho}{2} \tag{56}$$

$$\frac{\rho^2}{4} > \left(A - \frac{N^2}{B}\right)\left(\frac{G^2}{B}\right) \equiv \beta, \tag{57}$$

there is a $D > 0$ such that the negative eigenvalue of the linearization

$$\mathbf{w}_t = J^0\mathbf{w} + \tilde{D}\mathbf{w}_{zz}\,, \tilde{D} = \begin{pmatrix} D & 0 \\ 0 & -D \end{pmatrix}, \tag{58}$$

where $\mathbf{w} = (y(t,x) - y^*, p(t,x) - p^*)$, becomes positive. That is, both eigenvalues of the Jacobian matrix in (58) have positive real parts. Thus diffusion locally destabilizes the optimal flat steady state, and optimal dynamics are unstable in the spatiotemporal domain.

For the proof, see Brock and Xepapadeas (2008, Theorem 1).

The eigenvalues of the Jacobian matrix in (58) are given by

$$\lambda_{1,2}\left(k^2\right) = \frac{1}{2}\left(\rho \pm \sqrt{\rho^2 - 4h\left(k^2\right)}\right)\ ,\ k = \frac{2n\pi}{L} \tag{59}$$

$$h\left(k^2\right) = -D^2k^4 + D\left(2H^0_{yp} - \rho\right)k^2 + \det J^0. \tag{60}$$

The conditions of the theorem state that in the parameter space – the Turing space – in which these conditions are satisfied, there exists a diffusivity $D > 0$ and a node n such that both $\lambda_{1,2}\left(k^2\right)$ are positive, while for $D = 0$, the eigenvalues have opposite signs, since $\det J^0 < 0$. The emergence of optimal Turing instability requires that $h\left(k^2\right) > 0$ for some k.

From (59)-(60), diffusion will act as a stabilizer if $\det J^0 = \alpha(\rho - \alpha) - \beta > 0$ and

$$h\left(z^2\right) = -z^2 + (2a - \rho)z + \det J^0 < 0.$$

D is chosen such that $z = D(2\pi/L)$. This choice will stabilize all nodes $n > 1$.

9.5 Appendix 5

Consider a LQP like (40)-(43) with the following structure:

$$\max_{u(t,x)} \int_0^L \int_0^\infty \mathrm{e}^{-\rho t} \left[-\frac{A}{2} y(t,x)^2 - \frac{B}{2} u(t,x)^2 + N y(t,x) u(t,x) + \gamma \mathbf{K} y(t,x) \right] dt dx \tag{61}$$

$$A, B, \rho > 0,\ AB - N^2 > 0,\ \mathbf{K} y(t,x) := \int_{\mathcal{O}} w(x - x') y(t, x') dx'$$

$$\text{s.t. } \frac{\partial y(t,x)}{\partial t} = F y(t,z) - G u(t,x), \quad F, G > 0 \tag{62}$$

$$y(0,x) = y_0(z) \text{ given}, \quad x \in [0, L] \tag{63}$$

$$y \text{ in a circle } x \in [0, L], y(t,0) = y(x,L) \text{ for all } t. \tag{64}$$

When $\gamma = 0$, the problem is spatially homogeneous and admits a FOSS. Assume that, as in (40)-(43), the FOSS has the saddle point property. When the spatial effect is introduced by allowing for $\gamma \neq 0$, the Jacobian matrix for the linearization of the Hamiltonian system becomes:

$$J^0(y^*, p^*) = \begin{bmatrix} F - \frac{GN}{B} & \frac{G^2}{B} \\ A - \frac{N^2}{B} & \rho - F + \frac{GN}{B} - \gamma \mathbf{K} \end{bmatrix}, \mathbf{K} = \int_{\mathcal{O}} w(x - x') dx'. \tag{65}$$

The saddle point property for the FOSS implies that $\det J^0(y^*, p^* | \gamma = 0) < 0$. If, for $\mathbf{K} > \mathbf{0}$,$\gamma \neq 0$, trace $J^0(y^*, p^* | \gamma \neq 0) > 0$ and $\det J^0(y^*, p^* | \gamma \neq 0) > 0$, then this spatial effect – realized though the kernel – will destabilize the stable manifold of the flat earth and patterns will emerge.

9.6 Appendix 6

Given problem (20), the maximum principle under the symmetry assumptions implies the following first-order necessary conditions for $i = 1, 2$:

$$E_i = \frac{-\alpha w_i}{\mu_i}, i = 1, 2 \tag{66}$$

$$\dot{P}_1 = -BP_1 - DP_1 - \frac{\alpha w_1}{\mu_1} \tag{67}$$

$$\dot{P}_2 = -BP_2 + DP_1 - \frac{\alpha w_2}{\mu_2} \tag{68}$$

$$\dot{\mu}_1 = (\rho + B + D)\,\mu_1 + c_1 + c_2 P_1 - D\mu_2 \tag{69}$$

$$\dot{\mu}_2 = (\rho + B)\,\mu_2 + c_2 + c_2 P_2. \tag{70}$$

Proof of Proposition 1.

A steady state $(P_1^*, P_2^*, \mu_1^*, \mu_2^*)$, if it exists, will satisfy the nonlinear system (67)-(70) for $\dot{P}_1 = \dot{P}_2 = \dot{\mu}_1 = \dot{\mu}_2 = 0$. Solving at a steady state for μ_1^*, μ_2^* from (69), (70), replacing P_1, P_2 from (67),(68), and subtracting, we obtain:

$$(\mu_1^* - \mu_2^*) = \frac{\alpha c_2}{(\rho + B + D)\,\mu_1 B}\left[-\frac{w_2}{\mu_2} - \frac{w_1\,(D - B)}{\mu_1\,(D + B)}\right]. \tag{71}$$

If $c_2 = 0$, then $\mu_1^* = \mu_2^*$ and $\tau_1 = \tau_2$ for $w_1 = w_2$. If $D \geq B$, $c_2 > 0$ and $w_1 = w_2$, *then* $\mu_1 > \mu_2$, *which implies* $0 < \tau_1 < \tau_2$■

Proof of saddle point stability.

The Jacobian matrix of the linearization of the Hamiltonian system (67)-(70) is

$$J = \begin{pmatrix} -B - D & 0 & \frac{aw}{\mu_1^2} & 0 \\ D & -B & 0 & \frac{aw}{\mu_2^2} \\ 0 & 0 & B + D + \rho & -D \\ 0 & 0 & 0 & B + \rho \end{pmatrix}.$$

The trace is $\mathrm{tr}J = 2\rho$. Following Dockner (1985), the quantity K is defined as:

$$\begin{aligned} K &= \begin{vmatrix} \frac{\partial \dot{P}_1}{\partial P_1} & \frac{\partial \dot{P}_1}{\partial \mu_1} \\ \frac{\partial \dot{\mu}_1}{\partial P_1} & \frac{\partial \dot{\mu}_1}{\partial \mu_1} \end{vmatrix} + \begin{vmatrix} \frac{\partial \dot{P}_2}{\partial P_2} & \frac{\partial \dot{P}_2}{\partial \mu_2} \\ \frac{\partial \dot{\mu}_2}{\partial P_2} & \frac{\partial \dot{\mu}_2}{\partial \mu_2} \end{vmatrix} + 2 \begin{vmatrix} \frac{\partial \dot{P}_1}{\partial P_2} & \frac{\partial \dot{P}_1}{\partial \mu_2} \\ \frac{\partial \dot{\mu}_1}{\partial P_2} & \frac{\partial \dot{\mu}_1}{\partial \mu_2} \end{vmatrix} = \\ &= \begin{vmatrix} -(B + D) & \frac{\alpha w}{\mu_1^2} \\ 0 & B + D + \rho \end{vmatrix} + \begin{vmatrix} -B & \frac{\alpha w}{\mu_2^2} \\ 0 & (\rho + B) \end{vmatrix} + 2 \begin{vmatrix} 0 & 0 \\ 0 & -D \end{vmatrix} = \\ &= -(B + D))(\rho + B + D) - B\,(\rho + B) < 0. \end{aligned} \tag{72}$$

From Dockner's theorem 3, under the assumption $\rho \geq 0$, the conditions (i) $K < 0$ and (ii) $0 < \det J \leq \left(\frac{K}{2}\right)^2$ are necessary and sufficient for the eigenvalues of the Hamiltonian (67)-(70) system to be real, with two being positive and two being negative. Since

$$\left(\frac{K}{2}\right)^2 - \det J = \frac{1}{4}\left[(B+D))(\rho+B+D) - B(\rho+B)\right]^2 > 0,$$

both conditions are satisfied and the steady state is a saddle point. The result can be extended to increasing marginal damages.■

The Hamiltonian system for problem (30).

Consider a quadratic damage function for each region with spillover temperature effects

$$\begin{aligned} D_1(T_1, T_2) &= c_1^1 T_1 + \frac{1}{2} c_2^1 T_1^2 + \zeta_1^1 T_2 + \frac{1}{2} \zeta_2^1 T_2^2, \\ D_2(T_1, T_2) &= c_1^2 T_2 + \frac{1}{2} c_2^2 T_2^2 + \zeta_1^2 T_1 + \frac{1}{2} \zeta_2^2 T_1^2. \end{aligned}$$

Then

$$E_i = \frac{-\alpha w_i}{\mu_1 \Lambda_1 + \mu_2 \Lambda_2}\ , i = 1, 2, \tag{73}$$

and

$$\begin{aligned} \dot{\mu}_1 &= (\rho + B + \gamma_1 + \gamma_2)\mu_1 - (\gamma_1 + \gamma_2)\mu_2 + c_1^1 + c_2^1 T_1 + \zeta_1^2 + \zeta_2^2 T_1 \\ \dot{\mu}_2 &= (\rho + B + \gamma_1)\mu_2 - \gamma_1 \mu_1 + c_1^2 + c_2^2 T_2 + \zeta_1^1 + \zeta_2^1 T_2. \end{aligned}$$

The temperature dynamics of the Hamiltonian system are obtained by replacing E_i in (27)-(29) with (73). This is a dynamical system which can be analyzed using standard methods.

9.7 Appendix 7

An approximate two-mode solution of problem (33).

Following North (1975a), we consider the two-mode approximation of the temperature function in terms of even-numbered Legendre polynomials

$$\hat{T}(x,t) = T_0(t) + T_2(t) P_2(x)\ ,\ P_0(x) = 1, P_2(x) = \frac{(3x^2 - 1)}{2}.$$

Note that

$$\begin{aligned}\int_{-1}^{1} P_n(x) P_m(x)\, dx &= \langle P_n(x), P_m(x)\rangle = \frac{\delta_{nm}}{2n+1}\ ,\ \delta_{nm} = \begin{cases} 0 \text{ for } n \neq m \\ 1 \text{ for } n = m \end{cases} \\ \langle P_0(x), P_0(x)\rangle &= 2, \langle P_0(x), P_2(x)\rangle = 0, \\ \int_{-1}^{1} P_2(x)\, dx &= 0, \langle P_2(x), P_2(x)\rangle = \frac{2}{5}.\end{aligned}$$

In the temperature dynamics (34), make the substitutions

$$\begin{aligned}\partial_t \hat{T}(x,t) &= \dot{T}_0(t) + \dot{T}_2(t) P_2(x) \\ \partial_x \hat{T}(x,t) &= T_2(t) \frac{dP_2(x)}{dt} = T_2(t)\, 3x,\end{aligned}$$

then set $C = 1$ to simplify notation and multiply first by $P_0(x)$ and then by $P_2(x)$ and integrate both sides from -1 to 1, to obtain the two-mode temperature dynamics

$$\begin{aligned}\dot{T}_0 &= Z_0 - (A + BT_0) + \frac{1}{2}\Lambda \int_{-1}^{1} E(x)\, dx\ ,\ Z_0 = \int_{-1}^{1} \frac{1}{4} QS(x)\, a(x)\, dx \\ \dot{T}_2 &= \frac{5}{2} Z_2 - (B + 6D) T_2\ ,\ Z_2 = \int_{-1}^{1} \frac{1}{4} QS(x)\, a(x)\, P_2(x)\, dx.\end{aligned}$$

The current value Hamiltonian for the transformed problem will be

$$\begin{aligned}&\int_{x=-1}^{x=1} \Big\{ w(x) \Big[\ln y(x) + \alpha \ln E(x) - \upsilon \left[T_0 + T_2 P_2(x)\right]^2 \Big] \\ &+\mu_0 \left[Z_0 - (A + BT_0) + \frac{1}{2}\Lambda \int_{-1}^{1} E(x)\, dx \right] + \mu_2 \left[\frac{5}{2} Z_2 - (B + 6D) T_2 \right] \Big\}\, dx.\end{aligned}$$

From the optimality conditions, we obtain

$$\begin{aligned}E(x) &= \frac{-w(x)\, a}{\mu_0} \\ \dot{\mu}_0 &= (\rho + B)\, \mu_0 + \upsilon T_0 \\ \dot{\mu}_2 &= (\rho + B + 6D)\, \mu_2 + \frac{2}{5} \upsilon T_2 \\ \dot{T}_0 &= Z_0 - (A + BT_0) + \Lambda \frac{\alpha}{\mu_0} \\ \dot{T}_2 &= \frac{5}{2} Z_2 - (B + 6D)\, T_2.\end{aligned}$$

Solution of this system will provide the temperature and its shadow cost function across locations as

$$\hat{T}\left(x,t\right)=T_0\left(t\right)+T_2\left(t\right)P_2\left(x\right)\ ,\ \hat{\mu}\left(x,t\right)=\mu_0\left(t\right)+\mu_2\left(t\right)P_2\left(x\right).$$

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