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The bioeconomics of migration: A selective review towards a modelling perspective

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Abstract We present a selective review of migration and its connection with the economy, focusing on issues leading towards a modelling perspective. We introduce a class of models based on difference equations on directed graphs that may provide a quantitative and qualitative description of human migration and present some of their bioeconomic, mathematical and simulation challenges.

1 Introduction

Migration develops different spatio-temporal patterns depending on specific historical social and economic conditions. Furthermore, it is an issue of paramount importance in social, political and economic theory, that infiltrates many aspects of everyday life.

The modelling of migration is a very challenging issue from both the theoretical and practical point of view and nowadays its study involves a variety of disciplines ranging from the social sciences and economics to science, mathematics and statistics and extending to the arts and philosophy, since migration is a multi-dimensional and multi-faceted complex phenomenon and the mobilities of people have intensified and have become a 'permanent' feature of social reality.

It is the purpose of this expository chapter to (a) present a brief account of the theoretical study on migration as has been developed since the 1970s placing the

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emphasis on the economic aspects of migration (b) develop a bioeconomic type framework for the monitoring of mobility within different geographical regions, incorporating various socio-economic factors in the decision to migrate using a discrete choice model and (c) coupling the population dynamics to a simple growth model and thus creating a spatio-temporal dynamical system that can monitor the complex interaction between the economy and migration and discuss the mathematical, economic and simulation challenges of such models. It must be admitted that there exists a huge literature on the field ranging from econometric studies to studies in economic growth theory, therefore our references to the literature are very selective.

2 Theories and models of migration

After the 19th century the emergence of modern state with territorial borders turned population into an important concept made it a fundamental issue for political interest and introduced migration alongside with fertility and mortality as one of the main subjects of the newly developed science of demography. As a result, the first attempts to construct a migration theory, or better not only to find out the rules and laws under which migration occurs but also changes in time, were in demographic terms and were influenced by the empirical observations of that time. Simultaneously, the emergence and establishment of the modern state and its national borders meant that movements of people were divided into different types i.e., internal-international/migrants-refugees etc. Specific importance was attached in migration studies to the so called immigration or settler countries such as the USA, Canada and Australia.

Later on, due to the fact that the Second World War triggered ‘huge’ migration labour movements within Europe and also to Europe to assist rebuilding of Western Europe after the War, scholars’ interest in migration expanded considerably, turning their attention from just the traditional immigration states to the European states and to almost all states either as immigration or emigration ones.

An important group of theories of migration are called ‘equilibrium’ (or functional or orthodox theories) and have as their unit of analysis the individual and their explanation for the migration movements is based on the grounds of free rational choice. Migration is seen as caused by wage or income differentials between geographical areas. The motivation and decision making to migrate is seen exclusively from the point of migrant’s perceptions and their interests. The presupposition is that the individual is relatively free and well-informed about wage differentials or generally about the labour market situation between countries and decides to migrate, on the basis of cost-benefit analysis. In these theories migration is seen as a means for individuals’ ends.

Equilibrium theories are based on elements of neoclassical economic, modernisation theories and microeconomic perspective. In these theories, the perspective and the logic of the push-pull factors are predominant, focusing on the starting, continuing and stopping points of migration, (rural-urban. international migration) while its essential assumption is that remittances and the return of skilled migrants to the source region will stimulate economic growth.

According to a general theory developed by Lee [22] both origin and destination places are characterised by sets of plus and minus factors while between them there are sets of intervening obstacles such as distance, actual physical barriers and immigration laws, and all the above factors are causes for migration. Migration is thus seen as an almost rational individual decision in the terms of balancing costs and benefits.

In a more precise way than Lee's obstacles and sets of plus and minus factors, equilibrium theories especially in the neoclassical context, explain causes for migration on the basis of market disequilibrium between geographical areas. Traditional agricultural areas are characterised by low productivity, a supply of labour and low wages, while modern industrial urban areas represent a high productivity, a demand of labour and high wages. Thus, there is a rational transference of labour from the rural sector to the urban sector. In this perspective, Todaro' s model [17] is the most developed one, because it takes into account probabilistic factors in explaining the continuing migration in the urban areas, even in the case of high unemployment. He points out that the decision to migrate depends on 'expected' rather than actual urban-rural real wage differentials and does not assume urban full employment.

Generally, equilibrium theories argue that the equilibrium between wage differences is achieved by the aggregation of individuals' decisions to migrate and focus on geographic differences in wages and unemployment, underlying the importance of push factors in migration decision making. It can be said that these theories describe the ways that actual migration is considered and functions. Both their units of analysis, the individual, and the social context in which migration takes place are taken for granted and are seen as natural and stable. The market rationality is identified with individual rationality so, the exclusive definition of individual interest is maximisation of income.

A last point is that in the logic of cost-benefit analysis for individuals and markets, other aspects of social life, such as political, ideological, social, cultural, which constitute migration are ignored. Although, it can be argued that Lee's model incorporates these factors, they are concealed in sets of plus and minus factors and intervening obstacles, in which political and legal frameworks are described as distance or actual physical barriers or just immigration laws.

A second group of theories are the so called structural or historical-structural or conflict theories. These theories emphasise structural factors which force individuals, mainly as members of the working class, to migrate according to the needs of capital and as a result labour migration intensifies the existing exploitative relations and uneven development among countries. Migration is explained in the terms of social forces and constraints posed on migrants, that is migration is seen as a structural response and a manipulated behaviour.

The historical-structuralist perspective has been founded on different versions of theories such as dependency, world system and articulation or modes of production developed by Amin, Wallerstein and the neo-Marxists (eg Meillassoux). Each theorist relates migration - mainly labour migration- with other broader social phenomena. For example, Piore [28] pays attention to the dual labour market at the global level, Nikolinakos [26] focuses on dependency between periphery and centre, Castells [7] on uneven development inherent in the capitalist mode of production. and finally, Portes [29] shows that migration is a link between spatially separate modes of production due to the coexistence of modes of production.

Although structuralist theories include interesting elements of migration. they also have inconsistencies in a sense similar to those of individualistic ones, since they view capitalism as merely an economic system which develops according to some objective laws and they perceive migrants as functional elements, or mere economic units, whose actions and practice are responses to the requirements of capital such as the need for a reserve army or to split indigenous labour movement. Later research on migration, in order to avoid the above dichotomy and fragmentation in the explanation of migration, attempted to integrate both approaches in a migration theory. In order to connect the two opposite approaches researchers introduce additional units such as the 'household', 'social systems' and 'social networks'.

The third perspective of migration which attempts to combine both individual and structural approaches in the explanation of migration includes a variety of theories. In this logic, Wood argues that the unit of analysis should shift to 'household', because a migration decision may not be made merely by an individual but made by a group of individuals. This approach is similar to that adopted in the world-system approach, in which groups adopt strategies in order to overcome the limitations imposed by their socio-economic and physical environment. He argues that household behaviour is based on a series of 'sustainance dynamic strategies including geographical mobility by which household actively strives to achieve a fit between its consumption necessities, the labour power at its disposal and the alternatives for generating monetary and non-monetary income'. The 'household' must adopt dynamic, flexible and innovative strategies in order to respond to the structural factors. Both seasonal or permanent migration are seen as such strategies, permitting the household to achieve its goal which is attaining a desired level of consumption and investment.

Moreover, in the process of integrating individual and structural migration perspectives and of explaining the continuation of migration, some researchers apply the notion of networks in migration theories. In these terms, Kearney [18] introduces the concept of the 'Articulator Migrant Network' (AMN) in order to capture the complex processes of micro differentiation that occur in 'traditional' communities (characterised by the non capitalist mode of production) as they become articulated with the developed world' (characterised by the capitalist mode of production). Kearney's theory links household and migrant networks in anthropological research as efficient units of analysis of migration in connection with economic development. Owing to the fact that return migration has a negative or neutral influence on modernisation of underdevelopment countries, he identified 'household' as the appropriate unit of analysis for migration, since it reveals the role of women in production and marketing activities in the households that derive income for migrants. Moreover, the concept of the AMN can address the movements of migrants into various 'spaces' 'not only geographic but also economic and social niches and also the flow of surplus and goods within the migrant community'.

Another theory is developed by Portes and Kelly [29] who adopt the 'social network' perspective in order to explain the stability of migrant flows after the original causes have disappeared. Social networks are links between the domestic unit, and the global economic system, which includes household and extending to the family and community levels. Social networks construct and are also constructed by collective relationships across time and space. They argue that migration flows do not respond automatically to economic or political changes due to the mediation of social networks which become key structures and stabilise migration flows.

Massey and his collaborators in their exploration of migration within the context of network theory, argue that migration can be seen as 'a self-sustaining diffusion process: in which migration networks are sets of interpersonal ties between migrants, former migrants and non migrants in origin and destination areas' and these networks regulate migration behaviour. Migration networks, when expanding, assist to reduce the risks and costs of migration and 'once the number of network connections in country of origin reaches a critical level migration becomes self-perpetuating because migration itself creates the social structure to sustain it'.

The main proposition of network theories is that migration is continued almost constantly, even though not with the same volume, after the constitution of migration networks among coethnics and consequently migration becomes 'a self perpetuating social structure', independent from social and economic causes from which the migration movement started. This happened because the development of migration networks reduces costs and risks for new migrant movements and eventually migration spreads to all socioeconomic segments of the sending society. Due to well developed networks, governments have great difficulty in controlling migration while migration continues irrespectively of the kind of migration policies that governments apply.

Another theory which refers to the causes for the continuation of migrant flows is the so called institutional theory. Its initial assumption is that there is not a correspondence between the numbers of people who seek entry into rich countries and the number of visas that these countries issue. Due to this fact, an underground market for migration is developed exploiting migrants. A similar argument to those of network theories is developed by proponents of institutional theory who assume that international migration movements continue and liberate themselves from the initial causes of migration while governments are unable to curb further migration due to the pressures which both underground market and humanitarian groups put on the governments.

Fawcett [10] associates 'migration systems' with the notion of social networks in order to stress linkages between people (i.e. personal or family networks) and the economic and political linkages among sending and receiving countries and the relation between personal and non personal links. Under the same perspective, Kritz and Zlotnik [21] argue that the systems approach includes a group of countries linked by migration flows which can capture structural conditions in these countries and also economic and social ties between them. Moreover, with the incorporation of networks in the systems approach, it is possible for a migration theory to explain who is likely to become a migrant and who actually migrates, that is, to connect structures with potential migrants.

In fact, the attempts to integrate the individual and structural migration theories seem to be based on the assumption that these two elements of migration, that is, individuals and structures exist as independent but they acquire a cause-effect relationship in the case of migration. On these grounds, the above segmentation in migration research is taken for granted and these theories search for an intermediate unit of analysis in order to connect these independent but related units of analysis in migration, and consequently to explain previous inconsistencies of theories related to stability of migration flows but not in the terms of labour migration. Thus, the household and networks have been the concepts which replace the individual or migrant workers as units of analysis.

As Bach and Schraml [3] in their study of migration emphasise, migration decision making is organised by a set of dynamic as well as pre-established social relations. Another attempt to reconcile competing migration theories and evaluate them on empirical grounds is made by Massey and his collaborators [25]. The first part of their work focuses on examining assumptions and propositions of the leading but competing migration theories, dividing them into theories which explain origins of migration and theories which provide support for the continuation of international migration movements. Their stated goal is the usage of some propositions from each migration theory because each one can offer useful insights into the understanding of the multi-dimensionality of migration and in analysing various levels of migration. In the second part of their work they review empirical migration studies as the basis for the evaluation of existing theoretical propositions in order to show deficiencies and inconsistencies of migration theories and choose the correct assumptions of theoretical elaborations which are connected with dimensions of migrations.

In order to overcome the polarisation of individual and structural perspectives in migration theory. other researchers introduce Gidden's 'structuration theory' into migration analysis. In this framework. Goss and Lindquist [15] explain international migration as 'the result of knowledgeable individuals undertaking strategic action within institutions - specifically the institution of migration - which operate according to recognisable rules and which attribute resources accordingly'.

In this theory, structures are defined as rules and resources which both constraint and enable individuals' action. Then, individuals' actions and practices produce. reproduce and change the social structures. In the case of international labour migration, international migrant institutions have emerged by practices of knowledgeable individuals - potential migrants, return migrants - and the agents of organisations (from migrant associations to multinational corporations) and other institutions (from kinship to the state). Both individuals and agents draw upon sets of rules in order to increase access to resources.

The preceding selective review of migration theories shows that theoretical elaborations of migration are expanding and including various perspectives influenced by both broader theoretical issues, and contemporary issues. In fact, it cannot be said that there is a theory which can explain the multi-dimensionality and complexity of migration, but each theory reveals important aspects which can lead to a fertile and ongoing discussion of the subject.

It can be said that existing migration theories give the general directions for a new theoretical construction, in these terms, migration is an ongoing social phenomenon, whose understanding depends on the exploration of the social historical context globally. Migration incorporates social, economic, political, cultural, ideological and ethical dimensions and a theory of migration cannot exhaust its understanding focusing exclusively on states, economics and migrants but it also has to take into account the general social framework. Therefore, migration should be situated in a global context and a theory of migration must study the broader ways that societies are organised in order to explain migration as a contemporary social phenomenon.

3 Modelling the population dynamics

Consider a set of countries or geographical regions $\mathscr{G} = \{1, \cdots, N\}$. Each of these regions i supports a population u_i. For the time being we do not specify the type of the population. Depending on the use of the model, the population may refer to labour force of a particular type (e.g. specialized or unspecialized labor), migrant population of specific characteristics or the general population.

The population may change with respect to time thus leading to a real valued function $u_i : [0,T] \to \mathbb{R}$; $u_i(t)$ being the population of region i at time $t \in [0,T]$. We will focus on migrant population and consider $u_i(t)$ as the total population of migrants in a region i at time t. For the time being we will consider the migrant community as homogeneous, i.e., we will not distinguish between migrants of various characteristics, such as nationality, gender, occupation, high or low skilled workers etc.

We next consider a connectivity structure on $\mathscr{G}$. Let $\mathscr{E} \subset \mathscr{G} \times \mathscr{G}$ be the set of edges of $\mathscr{G}$. The pair $(i, j) \in \mathscr{E}$ if and only if there exists a migration flow from i to j. The pair $(\mathscr{G}, \mathscr{E})$ constitutes a graph, which will be called the *migration graph* $\mathscr{M}$. This graph is considered as a directed graph, the pairs (i, j) and (j,i) being considered as different; it is not necessary that $(i, j) \in \mathscr{E}$ implies that also $(j,i) \in \mathscr{E}$ unless there is also a possibility of return migration from region j to region i.

Through the connectivity structure of the graph, there may be secondary connections between different regions. For instance even if $(i, j) \notin \mathscr{E}$, therefore, not being a direct connection between regions $i = i_1$ and $j = i_n$ one may find a path to go from i_1 to i_n *indirectly*. That means that there may exist a sequence $i = i_1 \to i_2 \to \cdots i_{n-1} \to i_n = j$, with the property $(i_r, i_{r+1}) \in \mathscr{E}$, $r = 1, \cdots n-1$. This is a path, connecting i to j in n moves. This indirect connectivity is of particular importance to our model, since it implies that regions what are apparently not connected directly may be indirectly connected; therefore what happens in one of them may have an effect (even in long term) to what happens to the other. A graph such that for any $i, j \in \mathscr{G}$ there exists a path connecting i with j is called a connected graph.

Clearly, the direct connectivity between two vertices of the graph i and j is not enough to provide a complete description of the migration flow between i and j. Some notion of intensity of migration between these two vertices must be defined, which will quantify whether there is a pronounced migration movement from i to j or not. We will then consider a positive weight $w(i, j)$, quantifying this. It is convenient to scale this weight so that $w(i, j) \in [0,1]$. Then, this positive weight can be interpreted as the probability of migrating from region i to region j. The larger $w(i, j)$ the more likely is it for an agent to migrate from i to j. First of all we emphasize that in general $w(i, j) \neq w(j,i)$. Furthermore, because of the adopted scaling it must hold that

$$\sum_{(i,j)\in\mathscr{E}} w(i, j) = 1, \;\; \forall\, i \in \mathscr{G},$$

meaning that an agent originating from region i will end up to one of the regions of $\mathscr{G}$, directly connected with i with probability 1, allowing of course for the possibility of the agent staying in i. This is the probability of a single migration (single transition) so only the set of edges $\mathscr{E}$ is considered.

From the geographical point of view some comments are in order. The set of edges, which describes the "direct" connectivity of the migration graph, in some sense defines the geography of the regions. One could dare to use the word topology here, in the sense that $\mathscr{E}$ defines a system of neighbourhoods on the migration graph $\mathscr{M} = (\mathscr{G},\mathscr{E})$, describing affinity of certain regions with others. Geographical distance (physical distance) on the globe plays no particular role in $\mathscr{E}$. Regions that may be miles apart (such as India and UK for example) may present very strong connectivity properties (indicated both by $\mathscr{E}$ as well as the corresponding weight structure $w(i, j)$) because of bilateral agreements etc.

The introduction of the weights may now simplify the exposition. Define a generalized weight function $w : \mathscr{G}\times\mathscr{G} \to [0,1]$, with the property $w(i, j) = 0$ if $(i, j) \neq \mathscr{E}$. Therefore, the knowledge of the weight function w automatically provides knowledge of the edge structure $\mathscr{E}$, clearly

$$\mathscr{E} := \{(i, j) \in \mathscr{G}\times\mathscr{G} \,:\, w(i, j) \neq 0\}.$$

We will then build our model on the directed weighted graph $\mathscr{M} = (\mathscr{G},w)$. We may further define the space of all functions u on $\mathscr{G}$, $u(i) =: u_i$, $i \in \mathscr{G}$, such that $\sum_{i\in\mathscr{G}} |u(i)|^2 < \infty$ which is a Hilbert space, $L^2(\mathscr{G})$.

Suppose that the population on $\mathscr{G}$ is given by the function $u : \mathscr{G} \to \mathbb{R}^N$. We will try to compare the population at a site $i \in \mathscr{G}$ between two times t and $t+1$ (time is considered discrete without loss of generality). If $w(i, j)$ is considered as the probability of migrating from $i \in \mathscr{G}$ to $j \in \mathscr{G}$ then the total number of agents to move from i to j from t to $t+h$ is $w(i, j)u_i(t)h$. Since they may choose to move to any of the possible destinations the total outflow of agents from i in $[t,t+h]$ is

$$\mathscr{O}_i = \sum_{\{j:(i,j)\in\mathscr{E}\},j\neq i} w(i,j)u_i(t) = \left(\sum_{\{j:(i,j)\in\mathscr{E}\}} w(i,j)\right) u_i(t)h,$$

where the sum is taken over all the edges $\mathscr{E}$. Similarly, agents from region $j \in \mathscr{G}$ may choose to migrate to region $i \in \mathscr{G}$ with probability $w(j,i)$ in the time interval $[t,t+h]$, so that the total number of agents to move from j to i in this time interval is $w(j,i)u_j(t)h$. Then the total inflow of migrant population to i in $[t,t+h]$ is

$$\mathscr{I}_i = \sum_{\{j:(i,j)\in\mathscr{E}\},j\neq i} w(j,i)u_j(t)h. \tag{1}$$

Of course there is local changes of the population at i (birth-death) with the total rate of change being $a(i)u_i(t)$, where $a(i)$ will be assumed to be either positive or negative depending on whether local population increases or decays.

The total balance of the population at $i \in \mathscr{G}$ then yields

$$u_i(t+h) = u_i(t) - (\alpha_i u_i(t) + \mathscr{I}_i - \mathscr{O}_i)h$$

which is conveniently written as

$$\frac{u_i(t+h)-u_i(t)}{h} = -a_i u_i(t) + \sum_{\{j:(i,j)\in\mathscr{E}\},j\neq i} w(j,i)(u_j(t)-u_i(t)),$$

where

$$a_i = \alpha_i - \sum_{\{j:(i,j)\in\mathscr{E}\}} (w(j,i)-w(i,j)).$$

This equation is in the general form of a master equation (see e.g. [16] for an example of the use of a master equation in the study of interregional migration).

Define the operators

$$(\mathsf{A}u)_i = -a_i\,u_i + \sum_j w(j,i)(u_i - u_j)$$

and its adjoint operator

$$(\mathsf{A}^* v)_i = -a_i^*\,v_i + \sum_j w(i,j)(v_i - v_j)$$

where $a_i^* = a_i + \sum_j (w(j,i)-w(i,j))$. The adjoint operator A^* is defined through the standard property $\langle \mathsf{A}u, v\rangle = \langle u, \mathsf{A}^* v\rangle$, for all $u,v \in L^2(\mathscr{G})$. The operator $\mathsf{A} : L^2(\mathscr{G}) \to L^2(\mathscr{G})$ is called the weighted graph Laplacian corresponding to the weight function w. The graph Laplacian is a symmetric operator if the weights are symmetric, i.e., if $w(i,j) = w(j,i)$ for all $i,j \in \mathscr{G}$.

The total balance equation is then written in compact operator form as

$$\frac{u(t+h)-u(t)}{h} = \mathsf{A}u(t), \tag{2}$$

or equivalently as

$$u(t+h) = (I+h\mathsf{A})u(t).$$

One may either assume that $h \to 0$ and obtain an ordinary differential equation on the graph for the evolution of the population, or assume that the time unit of interest is finite and set without loss of generality $h = 1$ and obtain a difference equation for the evolution of the population. We will choose this option here for simplicity.

An alternative form of writing the model is the following. As before, let $w(i,j)$ be the probability that somebody moves from location i to location j, so that the inflow of immigrants into country i is given by $\mathscr{I}_i$ as in equation (1). Also let $w(i,i)$ be the joint probability that an individual at i survives from t to $t+1$ and decides to stay at location i. Then the agents from location i that remain in this location (between the periods t and $t+1$) are $w(i,i)u_i(t)$. To the new comers we must include the newborns which are $f_i u_i(t)$ where f_i is a local growth rate. Then the population balance equation is

$$u_i(t+1) = (f_i + w(i,i))u_i + \sum_{\{j:(i,j)\in\mathscr{E}\}, j\neq i} w(j,i)u_j(t),$$

or in matrix form as

$$u(t+1) = \mathsf{T}u(t)$$

where

$$\mathsf{T} = \begin{pmatrix} f_1 + w(1,1) & w(2,1) & \cdots & w(N,1) \\ w(1,2) & f_2 + w(2,2) & \cdots & w(N,2) \\ \cdots & \cdots & \cdots & \cdots \\ w(1,N) & w(2,N) & \cdots & f_N + w(N,N) \end{pmatrix}$$

which clearly is a positive matrix.

To make the above discussion useful as a model for migration we need to complete the following important steps:

(1) Construct the relevant migration graph $\mathscr{M} = (\mathscr{G}, \mathscr{E})$ by identifying the edge structure $\mathscr{E}$.
(2) Find the relevant weight function w. This will in principle depend on a number of factors and parameters, which when modelled properly will lead to a well versed decision theoretic tool to assist policy making.
(3) Study the quantitative and qualitative properties of the time dependent and steady state equations (2) and how these depend on the connectivity properties of the migration graph. Then, one can monitor how changes in the connectivity of the migration graph, that may be influenced by policies, bilateral agreements etc. may affect the long term behaviour of the population distribution on $\mathscr{G}$. To this

point this is a linear equation, but in the next section will be turned into a non-linear equation through coupling with an model for economic growth (a fact that will internalize the migration dynamics and make the model more interesting).

The above steps require also the employment of statistical techniques in order to infer the important quantities of the model, topological (e.g. $\mathscr{E}$) or quantitative (e.g., w) from available data. Furthermore, our study will highlight directions that may be needed in data collection so as to acquire data that are sufficient so as to reveal the desired quantities.

4 A fine tuning of the model: Calculation of w

An important feature of the model, which so far is a generic book counting model of population movement into different spatial locations (countries) is the matrix of migration probabilities (w_{ij}). The model will only be of use in the understanding of migration if the migration probabilities from location to location are specified in a realistic fashion. Our modelling of the migration probabilities will be a combination of the neoclassical (equilibrium) theories, which assume that the migration decision is based on economic grounds as well as on theories based on individual and structuralist approaches (which argue for the importance of effects such as networks) as presented in Section 1.

For example, let us consider the probability $w(i,j)$ of an agent at location i to migrate to location j. This can be modelled in terms of a discrete choice model. Discrete choice models have been very useful tools in modelling a decision maker's choice between mutually exclusive and collectively exhaustive alternatives (for an introduction to discrete choice models see e.g., [34]). These models are related to the maximization of random utility functions [24]. Discrete choice models have been used by various authors and in various contexts to model the migration decision (see e.g. [27], [19] and references therein).

According to this model consider a representative agent residing at time t in region i and facing the decision (lottery) to migrate to another region j (the case where $j = i$ is included and corresponds to staying in the region where the agent is originally situated). Let $V_{ij}(t)$ be the utility of this agent that decides to realize a move from region i to region j between the times instances t and $t+1$. This utility is subject to a random term ε_{ij}. This random term models either uncertainty with respect to the utility that an agent is likely to face, on account of incomplete knowledge of the situation she is likely to face or stochastic changes in the overall underlying situation. Alternatively, this term may model possible deviations of the agent from rationality (it can easily be argued that migrants may not always act rationally, for a variety of reasons). Therefore, the overall utility of an agent that decides to migrate from region i to region r will be a random function $V_{ir}(t) + \varepsilon_{ir}$. The probability that this agent initially located at region i prefers to move to location r rather than to location k is

$$p_{i,r>k} := P(V_{ir}(t) + \varepsilon_{ir} > V_{ik}(t) + \varepsilon_{ik}) = P(V_{ir}(t) - V_{ik}(t) > \varepsilon_{ik} - \varepsilon_{ir})$$

The probability that an agent decides to move from site i to site j is the probability that $p_{i,j>k}$ for all $k \neq j$. This can be expressed in terms of utilities as

$$\begin{aligned} w(i,j) &= P(V_{ij}(t) - V_{ik}(t) > \varepsilon_{ik} - \varepsilon_{ij},\ \forall j \neq k) \\ &= P(\varepsilon_{ik} < V_{ij}(t) - V_{ik}(t) + \varepsilon_{ij},\ \forall k \neq j) \\ &= \prod_{k=1}^{N-1} P(\varepsilon_{ik} < V_{ij}(t) - V_{ik}(t) + \varepsilon_{ij}) \end{aligned}$$

where in the last expression the independence of the ε_{ik} for different values of k has been used.

This probability can be calculated numerically, or even analytically for certain choices of functional form for the distribution of the error terms ε_{ij}. For example, under the generic choice of Gumbel type distributions for this error term, one can complete the calculation and derive a multinomial type model for the probability of a representative agent to migrate from i to j of the form

$$w(i,j) = \frac{e^{V_{ij}}}{\sum_{k=1}^{N} e^{V_{ik}}}$$

where clearly the denominator in the above expression serves as a normalization factor. The choice of the Gumbel family of distributions for the error term is not accidental or simply in order to simplify the algebra. It derives from a deep result in probability theory related to the asymptotic behaviour of independent errors, the celebrated Fisher-Tippet-Gnedenko theorem (see e.g., [20] or [9]). According to this theorem, if the error terms are drawn from a wide variety of distributions (including the normal, lognormal, exponential, gamma and the logistic) the distribution of their maximum follows the Gumbel distribution.

To complete the model, and fully specify the probability of moving from country i to country j, it remains to specify the utility levels V_{ik} which in some sense corresponds to the average (mean) utility and is the deterministic part of the the random utility. This captures the average 'rational' behaviour of the agent and should depend on both the characteristics of the agents as well as on 'external' characteristics of the economy. For example the average utility level V_{ij} may depend on a number of economic factors, such as the wage difference between the two countries, the difference in the cost of living etc, as well as on a number of qualitative variables, modeled in statistics in terms of categorical variables. Such variables may model structural issues, for example, are there good provisions for the welcome of migrants in the host country, are there migrant networks that may facilitate the settlement of migrants in the host country etc. A particularly convenient class of models are linear models for V_{ij} of the form

$$V_{ij} = \sum_{\ell=1}^{m} \beta_\ell Y_\ell^{ij}$$

where $Y^{ij} = (Y_1^{ij}, \cdots, Y_m^{ij})$ is a vector of m variables (continuous valued or categorical) that models several characteristics of regions i and j that may influence the migrants decision to move from region i to region j. For example Y_1^{ij} may correspond to the ratio or difference of the wage rates in the two regions, Y_2^{ij} may correspond to the ratio or difference of the unemployment rate in the two regions etc. whereas other variables may be categorical, i.e. may take the value 1 is there is a network of migrants from region i operating in region j that facilitates settlement and 0 otherwise, or 1 is there are bilateral agreements between the governments of i and j and 0 otherwise etc. The vector of coefficients $\beta = (\beta_1, \cdots, \beta_m)$ (assumed for simplicity to be common for all utility pairs V_{ij}) provide an idea of the sensitivity of the utility levels on the various possible external factors. This choice leads to the multinomial logit model for the probability of migration between sites,

$$w(i,j) = \frac{\exp(\sum_{\ell=1}^m \beta_\ell Y_\ell^{ij})}{\sum_{k=1}^N \exp(\sum_{\ell=1}^m \beta_\ell Y_\ell^{ik})}$$

This model can be estimated using standard statistical techniques e.g. maximum likelihood methods that will allow us to estimate the vector β, and thus calibrate the model into real life data.

One way to understand the above model is as $w(i,j) = w(i,j \mid Y^{ij})$, that is that the above formula provides the probability of an agent to migrate from i to j is

$$w(i,j) = \frac{\exp(\sum_{\ell=1}^m \beta_\ell y_\ell^{ij})}{\sum_{k=1}^N \exp(\sum_{\ell=1}^m \beta_\ell y_\ell^{ik})}$$

given that the macroeconomic or behavioural or structural variables Y^{ij} take the value $Y^{ij} = y^{ij}$. Clearly, these macroeconomic variables are subject to periodic or stochastic variability, that may be accounted for to shocks that arise in the economy.

5 Including the migration model into an economic growth model

Migration is at the same time affected by economic factors but also affects the economy, since labour (including migrant labour) is a very important factor of production. The question of how migration affects global economic growth (i.e., the economic growth of the various regions in the world economy, modelled here as the graph $\mathscr{G}$) is an interesting one that deserves our attention.

Obviously, this is a very intriguing and deep question, (see for instance Chapter 9 in [4] and the references therein to get an idea of the considerable activity in the field; see also e.g., [33], [11], [12], [35] and references therein for alternative or related models) the answer of which would require a full treatise rather than simply an expository chapter, whose main objective is to outline a fundamental modelling framework. In this vein, we propose a simple economic growth model which when coupled with the population equation (7) can model and predict growth and population patterns on the graph $\mathscr{G}$.

Our starting point in the modelling of economic growth is the celebrated Solow model. Let K_i be the capital stock of region $i \in \mathscr{G}$. This capital stock is subject to temporal change through production and consumption. The production of output Y_i at site $i \in \mathscr{G}$ is modelled through a neoclassical production function $Y_i = f(A_i, K_i, L_i)$ where L_i is the labour population at site i and A_i is a region specific productivity parameter. The temporal change of capital stock at site $i \in \mathscr{G}$ is given by

$$K_i(t+1) = s_i F(A_i, K_i(t), L_i(t)) + (1-\delta) K_i(t)$$

where s_i is the average propensity to save of region i, and δ is the depreciation rate of capital. For simplicity, we may assume that the production function and the depreciation rate of capital are common for all regions. Clearly, this assumption may (and will) be relaxed. Let us now assume full employment and that u_i represents the total population of labour force in region i therefore, $L_i = u_i$. We may therefore express the capital stock change in i by

$$K_i(t+1) = s_i F(A_i, K_i(t), u_i(t)) + (1-\delta) K_i(t), \; i \in \mathscr{G}.$$

A convenient choice for the production function is a Cobb-Douglas type production function of the form

$$F(A,K,L) = K^{\alpha}(AL)^{1-\alpha}, \; \alpha < 1,$$

where A is the index of the technology (Harrod neutral, see, e.g., [4] p. 52). This capital stock update equation is complemented with the population monitoring equation

$$u(t+1) = \mathsf{T} u_t \tag{3}$$

However, one should note that the migration decision depends on the macroeconomic variables, and in the present model this is shown by the dependence of the transition probabilities $w(i,j)$ on macroeconomic variables.

The model may also be expressed in terms of per capita variables, i.e. in terms of $k_i = K_i/(A_i u_i)$. Then noting that $F(K, Au) = Auf(k)$ where $f(k) := F(k,1)$ we see that

$$k_i(t+1) = \frac{A_i(t) u_i(t)}{A_i(t+1) u_i(t+1)} (f(k_i(t)) + (1-\delta) k_i(t)). \tag{4}$$

Note that in a single region setting and under the standard assumption of steady growth of the population, i.e. that $u_i(t+1) = (1+n) u_i(t)$ and constant $A_i(t)$ this equation would reduce to the standard form of the Solow model, involving only the per capita capital stock. An alternative approach is to assume technological change of the form $A_i(t+1) = (1 + g_{i,A}) A_i(t)$, leading essentially to the same model but with a modified parameter on account of the technological change. The effect of the technological change may be included in the local growth rate of labour, modifying the factor f_i. In the following, we assume, that the factor f_i accounts also for the effect of technological change and, without loss of generality, set $A_i = 1$[1]. On the other hand, concerning labour, here we assume that the population evolves according to the population monitoring equation (3) which is in fact a coupled system of n equations and this assumption is not in general valid, except perhaps in an asymptotic sense. Another way to express (4) is to express

$$\frac{u_i(t+1)}{u_i(t)} = f_i + w(i,i) + \sum_{\{j:(i,j)\in\mathscr{E}\},j\neq i} w(j,i)\frac{u_j(t)}{u_i(t)}$$

and then rewrite the per capita capital stock evolution law as

$$k_i(t+1) = \left(f_i + w(i,i) + \sum_{\{j:(i,j)\in\mathscr{E}\},j\neq i} w(j,i)\frac{u_j(t)}{u_i(t)}\right)^{-1} (Af(k_i(t)) + (1-\delta)k_i(t)) \quad (5)$$

The modelling of the transition probabilities can be done using the discrete choice model of Section 4, by proper interpretation of the "decision variables" Y. For example a decision variable Y_1^{ij} may be the difference between the per capita output in each region

$$Y_1^{ij} = \frac{Y_j}{u_j} - \frac{Y_i}{u_i} = f(k_j) - f(k_i).$$

This implies that agents will turn to migrate from region i towards region i with more propensity, the higher the difference between the per capita output in the corresponding regions is. Within the context of the Solow model, under constant returns to scale, this quantity is related to the wage difference between the regions. On the other hand if $Y_1^{ij} < 0$ then an agent originating at region i has no incentive to migrate to region j therefore, $w(i,j) = 0$. Another possible choice for the decision variable can be the difference between the capital stock between the various regions, with the same interpretation as above.

Another factor we can introduce is a qualitative factor (a caterogical variable) modelling the existence or not of migrant networks facilitating the settlement of migrants. This can be modelled by $Y_2^{ij} = \varepsilon_{ij}$ a matrix consisting of 1 and 0, the element ij is 1 if there exists a network of migrants from region i that facilitates the settlement in region j and 0 otherwise. Empirical studies of migration decision imply that such qualitative factors (e.g. networks, herding effects as well as cultural factors) may play a very important role and in some cases even outweight economic factors.

One may introduce other factors, e.g., the effect of the supply and demand of labour in the employment rate etc. Furthermore, we may introduce capital flows and in addition to labour flows, different sectors in the economy and separate labour force into skilled and unskilled (dual markets), time lags in the migration decision process etc. In some sense the sky is the limit when it comes to modelling, however,

[1] Alternatively we simply measure all quantities in efficiency units AL.

one should be careful to keep a model down to its bare essentials in order to keep it functional and useful. Since the main objective of this expository chapter is to present a simple working model as an illustration of the modelling strategy we will limit ourselves to this rather simplistic model.

Collecting all the above, we end up with the following "bioeconomic" type model for the interaction of migration and the macroeconomy

$$\begin{aligned}
K_i(t+1) &= s_i F(K_i(t), A_i u_i(t)) + (1-\delta) K_i(t), \; i \in \mathscr{G}, \\
u_i(t+1) &= (f_i + w(i,i)) u_i + \sum_{\{j:(i,j)\in\mathscr{E}\}, j\neq i} w(j,i) u_j(t) \\
w(i,j) &= \frac{\exp(\sum_{\ell=1}^{2} \beta_\ell y_\ell^{ij})}{\sum_{k=1}^{N} \exp(\sum_{\ell=1}^{2} \beta_\ell y_\ell^{ik})}
\end{aligned}$$

or its scaled form

$$\begin{aligned}
k_i(t+1) &= \left(f_i + w(i,i) + \sum_{\{j:(i,j)\in\mathscr{E}\}, j\neq i} w(j,i) \frac{u_j(t)}{u_i(t)} \right)^{-1} (A f(k_i(t)) + (1-\delta) k_i(t)), \; i \in \mathscr{G}, \\
u_i(t+1) &= (f_i + w(i,i)) u_i + \sum_{\{j:(i,j)\in\mathscr{E}\}, j\neq i} w(j,i) u_j(t) \\
w(i,j) &= \frac{\exp(\sum_{\ell=1}^{2} \beta_\ell y_\ell^{ij})}{\sum_{k=1}^{N} \exp(\sum_{\ell=1}^{2} \beta_\ell y_\ell^{ik})}
\end{aligned}$$

where

$$\begin{aligned}
Y_1^{ij} &= \frac{Y_j}{u_j} - \frac{Y_i}{u_i} = f(k_j) - f(k_i) \\
Y_2^{ij} &= \varepsilon_{ij} = \begin{cases} 1 & \text{network exists} \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}$$

This is a dynamical system, the evolution of which may provide some intuition concerning the transitory and asymptotic spatio-temporal behaviour of the migration process as well as its effects and dependences on the economic variables. As there are no constraints to the movement of migrants from region to region, and as we assume that labour force that has arrived from region j to region i is treated in the same terms as local labour force, our model may be a reasonable model for e.g. the Eurozone[2]. The model may be augmented with such features for as to include sanctions, difference in wages between local labour and migrant labour etc.

The modeling proposed here is very schematic and of course may be improved along various directions. For example, a more elaborate model for economic growth than that of Solow may be used, which may internalize employment rates, care for

[2] Since the Treaty of Rome which aimed at the establishment of a common market the free movement of capital, goods and people has been established. In the begining, the right of free movement refered to workers and later on (the Maastrich, Amsterdam and Shengen treaties) refered to the general free mobility of nationals of the EU member states.

more than one sectors in the economy, consider the effects of dual markets etc. Also the decision to migrate can be modelled in a more elaborate fashion, including more factors, time lags etc. Empirical evidence based on econometric studies of migration seems to show that qualitative effects such as culture, network formation etc. may in some cases outweight economic effects (see e.g. [5], [14] or [30] and references therein). The modelling of migration networks included here is rather schematic, more elaborate models can be investigated using ideas from [1] or [6]. At any rate, even this simple model serves well within the limited scope of this expository chapter to capture the intricate interdependence structure of migration and the economy and designate the salient features of the bioeconomics of migration.

6 Economic, mathematical and simulation challenges of the model

The long term behaviour of the proposed dynamical system is of interest from both the point of view of demographic and migration issues as well as from the point of view of economic dynamics. A good understanding of such issues will be important from the point of view of economic and migration policy as well. Eventhough our model is simple enough, it is still too complicated to deal with analytically, therefore one should resort to extended simulations and scenario based studies. However, there are still some interesting qualitative results we may obtain from this simple model proposed here, which is one of the main reasons for introducing our various simplifying assumptions.

6.1 Long term behaviour

To get an idea of the type of a priori information we can obtain from our model, let us recall one of the fundamental features of the Solow growth model. In the standard (single region) model, assuming a steady growth rate for the population $(1+n_i)$ it is well known (see e.g. [4]) that the economy reaches a steady state in per capita output $k_i^* = A_i^{1/(1-\alpha_i)}(n+\delta)^{1/(\alpha_i-1)}$. This steady state depends on the characteristics of the economy (i.e., the production function) as well as the population growth rate n_i. Of course the coupled nature of the system does not allow us to assume a constant growth rate for each economy. However, it is seen that if we manage to find an upper bound for the quantities $u_i(t+1)/u_i(t)$, $i \in \mathscr{G}$, this will provide an upper bound for the quantity n_i and therefore, a lower bound for k_i^*. If we can show that an upper bound exists for the growth rate of the population and if the capital evolution equation satisfy monotonicity properties that guarantee the validity of a comparison principle, then we can provide a lower bound for the limiting state for the per capita capital k_i^*.

Suppose that for some reason the transition matrix is constant in time and given. This means that somehow the economies have approached an equilibrium state so that the per capita ratios $K_i/u_i = k_i$ that provide the incentives for migration[3] are either constant or are varying but at a very slow rate, so that they may well be approximated as constant. This approximation is also true if we assume that other effects except economic effects play an important role in the decision to migrate, a fact supported sometimes by empirical studies. The population equation is then effectively a constant coefficient difference equation of the form $u(t+1) = \mathsf{T}u(t)$. Assuming that all eigenvalues of the matrix T are real and different ordered as $\sigma(\mathsf{T}) = \{\lambda_0, \lambda_1, \cdots, \lambda_{N-1}\}$ with corresponding eigenvectors $\{v_0, v_1, \cdots, v_{N-1}\}$ the general solution may be expressed as

$$u(t) = \sum_{j=0}^{N-1} C_j v_j \lambda_j^t$$

where C_j are arbitrary constants to be specified by the initial condition and in particular are such that $u(0) = \sum_{j=0}^{N-1} C_j v_j$. This formula will hold for complex valued eigenvalues and has to be modified accordingly in case where there are multiple eigenvalues. This general solution gives us a general idea about the possible growth rates of the various populations. For example if v_0 does not contain any zero elements then for a "generic" initial condition $u(0)$ we expect all elements of the vector $u(t)$ to grow at the rate λ_0^t. At the positions where v_0 has a zero element we expect a slower growth rate which will coincide with one of the other eigenvalues etc. Crude as this information may sound it is probably the most we can say for the behaviour of the population asymptotically.

The non negativity of the matrix T ensures that there is a great deal of structure on the eigenvalues and eigenvectors of T, given by the celebrated Perron-Frobenius theorem. Furthermore, this theorem links the topology and connectivity of the migration graph with the long term behaviour of the system. For the sake of the reader we briefly recall (for a full account of this beautiful theory see e.g. [32]) that according to the Perron-Frobenius theory, If the matrix T is irreducible and primitive then it has a non-negative algebraically simple eigenvalue λ_0 that strictly dominates all the others, and the corresponding left and right eigenvectors related to this eigenvalue are positive.

This, importantly involves the geometric and topological properties of the migration graph. This comes through the condition of irreducibility and primitivity that is needed for the validity of the Perron-Frobenius theorem. Recall, that a positive matrix $\mathsf{T} \geqslant 0$ is called irreducible and primitive if there exists a $m \in \mathbb{N}$ such that $\mathsf{T}^m > 0$. This definition, implies that the irreducibility of the matrix T is related to the connectivity properties of the graph related to the migration process. In particular T is irreducible if the graph $\mathscr{G}$ is strongly connected. That means that there is a path tak-

[3] Recall that in our model the incentive for migration between two regions is the difference in $\frac{Y_i}{u_i}$ which for the Cobb-Douglas function is equal to the difference between $f_i(k_i)$. Note also that the functions f_i are monotone.

ing an agent from any location i to any location j as long as we wait long enough (at least as long as m time units where m is least integer such that $\mathsf{T}^m > 0$). Furthermore the Perron-Frobenius eigenvalue (which in fact coincides with the spectral radius of the matrix T) can be a priori estimated by

$$\min_i \sum_{j=1}^N t_{ij} \leqslant r \leqslant \max_i \sum_{j=1}^N t_{ij}$$

i.e. lies between the minimum and the maximum of the row sums of the matrix T.

The long term behaviour of the system $u(t+1) = \mathsf{T}u(t)$ is given by the Perron-Frobenius theorem. In fact $u(t) = \mathsf{T}^t u(0)$ where $u(0)$ is the initial state of the population, and as a consequence of the Perron-Frobenius theorem

$$\lim_{t\to\infty} \frac{\mathsf{T}^t}{\lambda_0} = vw^{tr}$$

where v and w are the left and right eigenvectors corresponding to λ_0, normalized so that $v^{tr}w = 1$. This result leads us to the approximate asymptotic result that the fastest growth rate of the populations in $\mathscr{G}$ will be related to λ_0. This implies that $u_i(t+1)/u_i(t)$ will tend to λ_0 asymptotically as $t \to \infty$. The action of the matrix vw^{tr} on the initial population distribution (i.e., in effect the structure of the left eigenvector v) will reveal which regions are expected to have a growth rate equal to the Perron-Frobenius eigenvector. When inserted into the Solow equations this provides an idea of the growth rates of these economies, in the sense that in the long run the per capita capital for these regions should follow

$$k_i(t+1) \simeq \frac{1}{\lambda_0}(A_i f(k_i) - (1-\delta)k_i).$$

This may give us a rough idea of the growth rate allowed for these economies. The economies (regions) which correspond to zeroes of the left Perron-Frobenius will necessarily have population growth rates which are lower that λ_0 and that will correspond to a different eigenvalue in the spectrum of T. Knowledge of this eigenvalue (and assuming that it is simple) in conjunction with the Solow equation for this region (where the relevant population growth rate is inserted) will give an estimate of the relevant long run steady state for the per capita capital.

This procedure has a slight catch. The matrix T depends on the steady state of the economies, and so in turn the eigenvalues of T which give the long run behaviour of the population in the various regions depend on this steady state. On the other hand, these eigenvalues characterize the steady state of the economies since the population growth rate enters the Solow equation. This is a nonlinear problem and the procedure we have just described may be turned into a fixed point argument so that we may obtain approximations to the relevant long run behaviour of the system. The implementation of such a fixed point argument will require certain results on the behaviour of the eigenvalues of T as the matrix varies.

There is an alternative way to view the above arguments. Let us assume that we wish to drive the system into a prescribed steady state $\bar{k}^* = (k_1^*, \cdots, k_N^*)$. This is equivalent to prescribing a given capital growth rate for each economy $\bar{g} = (g_1, \cdots, g_N)$, since a simple calculation using the Solow equation and the Cobb-Douglas production function implies that a prescribed growth rate g_i for country i (in terms of the capital stock K_i) specifies the asymptotic behaviour of the per capita capital, in terms of

$$\rho_i := \frac{K_i}{u_i} = k_i = \left(\frac{g_i + \delta}{s_i A_i}\right)^{\frac{1}{1-\alpha_i}}.$$

This ratio determines the migration matrix and through that the matrix T. Therefore, given a growth rate vector $\bar{g}$, the vector $\bar{\rho} = (\rho_1, \cdots, \rho_N)$ is specified and that determines the matrix T which we will denote as $\mathsf{T}(\bar{\rho})$ for the same reason as above. The above asymptotic behaviour in turn prescribes an asymptotic growth rate for the population in each region since by the definition of k_i we have that asymptotically in time

$$u_i(t) \sim \frac{1}{\rho_i} K_i(t) \sim \frac{1}{\rho_i}(1 + g_i)^t.$$

Since the asymptotic behaviour of the population dynamics equation is specified by the spectral decomposition of the matrix $\mathsf{T}(\bar{\rho})$, this provides a link between the vector $\bar{g}$ (or equivalently $\bar{\rho}$) and the eigenvalues of the matrix $\mathsf{T}(\bar{\rho})$. This is a nonlinear, inverse eigenvalue problem that will allow us to specify the allowed values of the vector $\bar{g}$ for which such a prescribed behaviour may be asymptotically supported by the system. These problems can be treated numerically and in general are difficult problems to handle. However, if treated it may provide interesting information on the "spatial" structure of the vector $\bar{g}$ (or equivalently $\bar{k}$) and allow us to infer on question of whether migration of labour may contribute to phenomena like unconditional convergence, conditional convergence or club convergence of the economies (for a definition of the relevant notions in a single country model see [13]; these notions may be extended to our model of N coupled economies and provide important insight on the spatial distribution of growth). The particular case where the problem is solvable for a spatially homogeneous $\bar{g}$ (i.e., in the case where $g_i = g$ for all $i \in \mathscr{G}$) corresponds to the phenomenon of convergence for the economies.

6.2 Non homogeneous and random versions of the model

It is a very naive assumption that the probabilities an agent from location i will migrate to location j, as well as the local growth rates of labour are constant for every time period. These depend on the economic and social conditions and clearly change with time. Therefore, a better model for the population would be a temporally non homogeneous model of the form

$$u(t+1) = \mathsf{T}(t)u(t)$$

therefore, given the state $u(s)$ we may find the state $u(t)$, $s < t$ by the forward iteration scheme,

$$u(t) = \mathsf{T}(t-1)\mathsf{T}(t-2)\cdots\mathsf{T}(s)u(s)$$

or in more compact form in terms of the "propagator"

$$\begin{aligned} &u(t) = U(t,s)u(s), \;\; s < t, \\ &U(t,s) := \mathsf{T}(t-1)\mathsf{T}(t-2)\cdots\mathsf{T}(s). \end{aligned}$$

Each of the matrices $T(k)$, $k = s, \cdots, t-1$ is a positive matrix, therefore, we are now dealing with products of positive matrices. Ergodic theory may provide some help in understanding the long term behaviour of the system.

As also argued in Section 2 the migration decision often depends on contingencies (the general economic and social framework) which can be modelled as random variables using a properly selected probability space. Generalization of the model to $u(t+1) = \mathsf{T}(t,\omega)u(t)$ where in general T is a stochastic process are also clearly feasible. This is interesting since it allows us to introduce into the model random effects depending on uncertainty and fluctuations in the environment, as well as to introduce effects related to the agents decision making, which may be rational or irrational.

A very convenient way to introduce these random effects into our model is by assuming that the macroeconomic variables Y^{ij}, that play the role of indicating variables in the discrete choice model of Section 4 that determine the migration probabilities, depend on a set of hidden random variables, H. These take values on a metric space $\mathbb{X}$, which without loss of generality assume that $\mathbb{X} = \mathbb{R}^d$. The random variables H are in some sense the factors that drive the economy. Then, the transition matrix structure for the system $u(t+1) = \mathsf{T}u(t)$ can be expressed simply by observations of the factors H that drive the economy. If $H(t)$ is the value of the factors at time t then $\mathsf{T}_t(\omega) = \mathsf{T}(H(t))$. A simpler structure can be given to this model if we assume that each of the values that the random variable $H(t)$ can take is obtained by choosing random variables from a probability space $(\Omega, \mathscr{F}, P) := (\mathbb{X}, \mathscr{F}, P)$ in the following way: Consider a transformation $\theta : \Omega \to \Omega$ that preserves the probability measure P, i.e. $P(\theta(A)) = P(A)$ for every $A \subset \Omega$. Let us choose a sequence of matrix valued random variables $\{\mathsf{T}_t\}$. Each of these T_t is assumed to contain the transition probabilities that may describe migration tendencies between the different regions from time t to time $t+1$. We now make the further assumption that all these matrices may be generated through a single (random) matrix variable $\mathsf{T} : \Omega \to \mathbb{R}_+^{N\times N}$ using the transformation θ by

$$\mathsf{T}_t(\omega) = \mathsf{T}(\theta^t \omega). \tag{6}$$

This assumption introduces a stationarity assumption in the migration probabilities.

The state of the population at time t at each region is then understood as a random variable, $u(t, omega)$ which can be obtained by the solution of the random dynamical system

$$u(t+1,\omega) = \mathsf{T}_t(\omega)u(t,\omega). \tag{7}$$

The treatment of this system is greatly simplified by the assumption that $\mathsf{T}_t(\omega)$ is of the special form (6), as this allows the use of the powerful techniques of ergodic theory in the study of the long term dynamics of (7).

This stationarity assumption allows us to state and understand the long term behaviour of the system, using ergodic theory. A random version of the Perron-Frobenius theory (see e.g. [2]) is useful in that. This theory guarantees that, under certain technical assumptions, that there exist a unique positive random unit vector u and a positive random scalar r with $\ln^+ r \in L^1(\Omega, \mathscr{F}, P)$ such that $\mathsf{T}(\omega)u(\omega) = r(\omega)u(\omega)$. This captures the asymptotic long term behaviour of the system in the sense that there exists an invariant splitting of $\mathbb{R}^N$ as $\mathbb{R}^N = W(\omega) \oplus \mathbb{R}\,u(\omega)$, such that $\mathsf{T}(\omega)W(\omega) \subset W(\theta\omega)$, and if we define $\phi_t(\omega) = \mathsf{T}(\theta^{t-1}\omega)\cdots\mathsf{T}(\omega)$, then, if $x \neq W(\omega)$

$$\lim_{t\to\infty} \frac{1}{t} \ln(|\phi_t(\omega)x|) = \lambda = \mathbb{E}[r]$$

where λ is the top Lyapunov exponent (in case $x \in W(\omega)$ then this limit is strictly less that λ). Therefore, even for the random case we have that in some sense the long term temporal behaviour of the system is captured by the random vector u.

When viewed in full coupling with the Solow model, we obtain a fully coupled nonlinear random dynamical system, the long term behaviour of which will provide us with information on the asymptotic distribution of population and capital in the economy. The steady state of the random dynamical system is now not a vector but a vector valued random variable whose law is invariant under the action of the random dynamical system. The calculation of this random fixed point is not a very easy task and in most cases requires detailed numerical work. The question of random effects in growth theory has been studied within the standard Solow model (including one region and without the introduction of migration) in [31] where using the theory of random dynamical systems and a random version of the Banach fixed point theorem the existence of a random fixed point was proved. The multi-region case coupled with the migration dynamics presents a considerably more complicated problem, which is worth of further investigation. The theory of monotone random dynamical systems (see e.g. [8]) is expected to play an important role in this study.

6.3 Simulation

It must be clear from the above discussion that analytic techniques will not take us very far with our model, which albeit simple is still very complex for analytic treatment. One must resort to simulation, which can help us generate multiple scenarios concerning the evolution of the population dynamics and the economy which may assist in policy making. The simulation of the model as such is not too demanding, when the transition probabilities are not assumed to be random. What seems to be more demanding is the calibration of the model to realistic parameters that may fit the real world. For example, the calibration of the discrete choice model that determines the location in which a migrant chooses to migrate to has to be based on questionnaires, or micro-econometric studies (see e.g. [23]). While there are standard techniques for dealing with such problems, the collection of data can be a problem, especially when it comes to undocumented migration.

Country	Output per worker (y)	Capital stock per worker (k)	Workers (u)
Greece GR (1)	17,717 USD	39,423 USD	3,867,047
Portugal PT (2)	16,637 USD	28,973 USD	4,435,469
Germany GDR (3)	30,099 USD	79,049 USD	30,126,905

Table 1 Initial conditions for the simulation (Source: Micro Time Series, available on line at http://econ.worldbank.org/WBSITE/EXTERNAL/EXTDEC/EXTRESEARCH/0,,contentMDK:20701055∼pagePK:64214825∼piPK:64214943∼theSitePK:469382,00.html)

We now present the results of simulations of the model, for 3 regions[4]. As initial conditions for the simulation we have used data for the output per worker (y), capital stock per worker (k) and worker population (u) taken from the Macro Time Series (available online from the World Bank). The initial condition is chosen to be the 1990 data for two reasons. The first is technical, this is the last date where capital per worker data has been published and without this data available the estimation of the production function requires more sophisticated econometric techniques outside the scope of the present paper. The second is that this date is sufficiently far from the present day, so that (a) we may test for the plausibility of our results using our historical experience and (b) be sufficiently remote from the present day Eurozone crisis. For the sake of argument we have used data from Greece, Portugal and Germany and therefore we name the relevant regions after these countries for simplicity. The data are presented in Table 1. Using the available data and assuming the coefficient α_i in the range $0.2-0.4$ we estimate the production function and then use this production function to simulate the model. For the particular run presented here we have chosen $\alpha_1 = \alpha_2 = \alpha_3 = 0.3$. The f_i are taken to be $f_1 = f_2 = 0.015$ and $f_3 = 0.02$, where in this we have also included the rate of technical change. The propensity to save is taken to be $s_1 = s_2 = 0.1$ and $s_3 = 0.2$ which is a reasonable estimate and in accordance with available data.

In figures 1-2 we present the results of a simulation of the model for these initial conditions and these parameters. In Figure 1 we present on the top panel the evolution of the capital stock per worker (k), on the mid panel the evolution of the output per worker (y) and the ratio of workers divided by the number of workers in the

[4] The number of regions is chosen to be so low only for simplicity and economy in the presentation of the results, there is no limitation as to the number of regions.

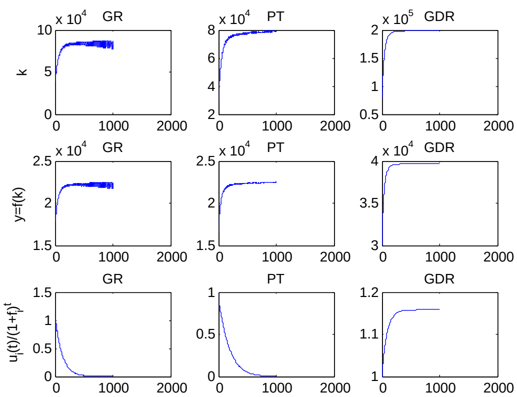

Fig. 1 Simulation of the model for 3 countries. Country number 3 (Germany) has an advantage in production (as shown in the choice of the relevant Cobb-Douglas parameter). The long run macroeconomic variables for the 3 regions and the evolution of the worker population.

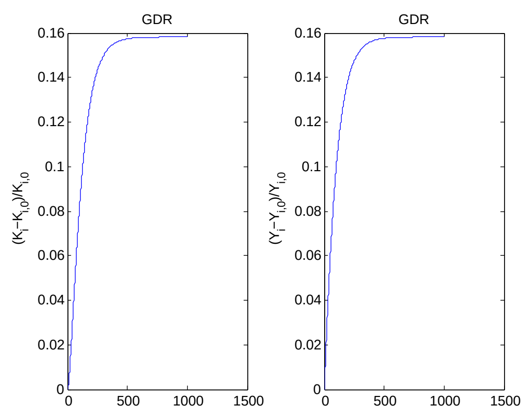

Fig. 2 Effect of migration on the macroeconomic variables of the receiving country.

absence of migration $u_i(t)/(1+f_i)^t$ as a function of time for the three regions. The time scale is deliberately taken to be long in order to show that the model is well behaves globally in time. The results show a migratory movement from regions 1 and 2 to region 3 as expected, and the log run macro economic quantities k and y show a lead for region 3, also as expected. The long run behaviour of the population as predicted by the simulation matches the pattern predicted by the Perron-Frobenius theory. In Figure 2 we present the effects of migration on the capital stock K_i and the overall production Y_i for region 3 (the receiving country). On the first panel we plot the temporal evolution of the quantity $\frac{K_i - K_{i,0}}{K_{i,0}}$ where K_i is the prediction for the capital stock in region i from the model presented here and $K_{i,0}$ is the relevant quantity as predicted by the standard Solow model without migration. On the second panel we plot the quantity $\frac{Y_i - Y_{i,0}}{Y_{i,0}}$ where Y_i is the prediction for the output in region i from the model presented here and $K_{i,0}$ is the relevant quantity as predicted by the standard Solow model without migration. It is seen that region 3 benefits as an effect of the migrant worker influx. A similar calculation for the sending countries shows that regions 1 and 2 suffer a loss as an effect of labour outflow.

It has to be stressed that the simulation presented here is for the sake of illustration of the general behaviour of the model only and as it is cannot be used for quantitative predictive purposes. For that, one has to employ more sophisticated econometric methods for the estimation of the parameters of the growth model than the simple estimation performed here. Furthermore, detailed micro-econometric modelling of the migration probabilities should be made, based on quantitative and qualitative methods. This is important as the migration probabilities play an important role in the model. The proper calibration of the model is clearly beyond the scope of the present work. However, when properly calibrated one may use this model to run a number of different scenarios, including any number of regions, and trying different types of network effects. This may provide a fairly good understanding of the dynamics of this complex system, and the relative importance of the various factors and parameters in its evolution. The inclusion of randomness complicates things a bit further as it requires detailed modelling of the random effects as well as Monte-Carlo simulations in order to assess the effects of randomness in the model.

7 Conclusion

We have attempted a selective review of migration theories with a focus towards a modelling framework for the bioeconomics of migration. We also present a simple model, based on a set of difference equations monitoring the motion of agents between N regions, where the migration probabilities depend on economic as well as on network effects. This system is coupled with a simple multi-region Solow model that monitors economic growth and the migration decision is based partly on the evolution of the economy in these regions. The final model is a nonlinear dynamical system that provides information on the evolution of the economies and the population. Certain qualitative features of the model are addressed and some comments on simulation are made. Extensions are numerous and quite interesting and are expected to provide useful insight for the decision making process in migration policy.

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