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**TRANSBOUNDARY CAPITAL  
AND POLLUTION FLOWS AND THE  
EMERGENCE OF REGIONAL INEQUALITIES**

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# Transboundary Capital and Pollution Flows and the Emergence of Regional Inequalities <sup>\*</sup>

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## Abstract

We seek to explain the emergence of spatial heterogeneity regarding development and pollution on the basis of interactions associated with the movement of capital and polluting activities from one economy to another. We use a simple dynamical model describing capital accumulation along the lines of a fixed-savings-ratio Solow-type model capable of producing endogenous growth and convergence behavior, and pollution accumulation in each country with pollution diffusion between

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<sup>\*</sup>We are pleased to dedicate this to Stephen Cantrell on the occasion of his 60th birthday

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countries or regions. The basic mechanism underlying the movements of capital across space is the quest for locations where the marginal productivity of capital is relatively higher than the productivity at the location of origin. The notion that capital moves to locations of relatively higher productivity but not necessarily from locations of high concentration to locations of low concentration, does not face difficulties associated with the Lucas paradox. We show that, for a wide range of capital and pollution rates of flow, spatial heterogeneity emerges even between two economies with identical fundamental structures. These results can be interpreted as suggesting that the neoclassical convergence hypothesis might not hold under differential rates of flow of capital and polluting activities among countries of the same fundamental structure.

**Keywords:** Transboundary flows, Capital, Pollution, Diffusion, Turing instability, Spatial heterogeneity

**JEL Classification:** O44, R12, Q52, C65

## 1. Introduction

The study of economic growth, when the detrimental effects of environmental pollution that emerge from the joint production of pollutants are fully accounted for, dates from the early 1970s.<sup>1</sup> During this period, various models coupling growth with environmental effects were developed. As a basis for the economic part, these models have used models of descriptive growth (the Solow model), models of optimal growth using

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<sup>1</sup>See for example, Keeler and Zeckhauser (1971), Brock (1973), Becker (1982)

the Ramsey model as the basis, and models of new economic growth with increasing returns.<sup>slev</sup> Although the above literature provides a detailed analysis of the temporal dynamics - and in particular transition dynamics, steady states and convergence properties, as well as a thorough study of policy issues - the spatial dimension of the problem has not been addressed.

In environmental and resource economics, spatial issues have been analyzed mainly in terms of regulation of ambient pollution (Kolstad 1986, Kyriakopoulou and Xepapadeas 2012), land use in urban settings (Henderson 1977; Arnot et al. 2008), location and pollution haven issues (Levinson and Taylor, 2008), or resource management in spatial settings (e.g. Wilen, 2007; Smith et al. 2009, Brock and Xepapadeas 2010). Spatial considerations have not, however, been extended to models of growth and environment.

The study of economic growth in spatial settings is still in the early stages since there are both conceptual and analytical difficulties in extending static models of new economic geographic to a dynamic setup, although there are a few notable exceptions of growth models that incorporate spatiotemporal dynamics (Quah 1996; Bucekine et al. 2013; Brock et al. 2014, Xepapadeas et al. 2014)).

A much researched relationship in the context of growth and environment is the link between growth and pollution and its structure across countries. Empirical evidence (Hettige et al., 1990) suggests that a long-term upward trend in industrial emissions, both relative to GDP and to manufacturing output, is higher among lower-income coun-

tries. This result is consistent with an industrial displacement effect of dirty industries as a result of more stringent environmental regulation in industrialized countries since 1970. This evidence could be interpreted as suggesting that in a country or region that has reached a high development stage and where industrialization has led to accumulation of polluting activities and environmental pollution, certain mechanisms could go into effect that might cause transport of polluting activities to less-developed regions where industrialization is not heavy, environmental regulation is relatively less stringent, and polluting activities and pollution accumulation might be less relative to those of the industrialized/developed region. This transport or relocation of polluting activities will induce a corresponding transport of pollution from the developed region to the less developed region.

Pollution, however, also moves across space due to natural mechanisms affecting regions other than the regions where the pollution was generated. Atmospheric Brown Clouds (ABC) can be regarded as related to this type of air pollution. As stated in a recent UNEP study (Ramanathan et al., 2008), ABC consist of particles (or primary aerosols) and pollutant gases - such as nitrogen oxides (NO<sub>x</sub>), carbon monoxide (CO), sulphur dioxide (SO<sub>2</sub>), ammonia (NH<sub>3</sub>), and hundreds of organic gases and acids. ABC plumes, which result from the combustion of biofuels from indoors, biomass burning outdoors and fossil fuels, are found in all densely inhabited regions and oceanic regions downwind of populated continents. In this case emissions generated in a certain location

move to other locations.<sup>2</sup> Thus pollution can move in two ways across space: first through the relocation of capital stock and the subsequent joint production of output and emissions in the new location, and second through natural mechanisms that transport pollution across locations.

This set-up of capital flows and pollution flows across locations could imply a non-homogeneous spatial pattern for development and pollution, in the sense that in the regional context we might observe spatially heterogeneous development and pollution patterns as a result of the processes described above. The mechanism driving this spatial heterogeneity, and the question of whether this heterogeneity increases or decreases over time as globalization forces tend to work towards closer integration, might be important for understanding regional inequalities with respect to development and environmental quality, and for formulating policies to eliminate them.

In the present paper we seek to explain the emergence of spatial heterogeneity regarding development and pollution on the basis of interactions between economies. These interactions are associated with the movement of capital and polluting activities from one economy to another; they are characterized by negative effects of pollution accumulated through polluting activities in a country on domestic capital accumulation.

Our methodological approach seeks to capture, as a factor explaining spatially heterogeneous patterns of development and environmental quality, current tendencies to

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<sup>2</sup>Five regional ABC hotspots around the world have been identified: i) East Asia, ii) Indo-Gangetic Plain in South Asia, iii) Southeast Asia, iv) Southern Africa; and v) the Amazon Basin.

wards increased integration on a global scale that induce movements among countries or regions of both capital and pollution. We model flows of capital and polluting activities by a simple dynamical model consisting of an equation describing capital accumulation and an equation describing pollution accumulation in each country. The capital accumulation equation is formulated along the lines of a fixed-savings-ratio Solow type model capable of producing endogenous growth and convergence behavior which is augmented to account for capital flows and negative effects from pollution. The pollution accumulation equation describes the accumulation and the diffusion of polluting activities between countries or regions.

In modelling capital flows we assume that the basic mechanism underlying the movements of capital across space is the quest for locations where the marginal productivity of capital is relatively higher than the productivity at the location of origin, without imposing the constraint that capital moves from locations of high concentration to locations of low concentration which is implied by standard models with diminishing returns to capital. The assumption that capital flows towards locations of high returns is a plausible assumption underlying capital flows if rates of return to capital differ across countries (e.g., Acemoglu 2009), with velocity depending on endogenous factors such as the existing stock of capital or the size of profitability. The major advantage of assuming that capital moves towards locations of higher productivity rather than a mechanism where capital moves necessarily from higher to lower concentration locations, is that

the latter behavior seems not to be supported by empirical findings, as pointed out in the context of the Lucas paradox (Lucas, 1990, 2003).<sup>3</sup> Our approach, which is based on the notion that capital moves to locations of relatively higher productivity but not necessarily from locations of high concentration to locations of low concentration, does not face this difficulty.

By using the methodology underlying Turing diffusion-induced instability (Turing 1952), we show that, for a wide range of capital and pollution rates of flow, spatial heterogeneity emerges even between two economies with identical fundamental structures. These results can be interpreted as suggesting that the neoclassical convergence hypothesis might not hold under differential rates of flow of capital and polluting activities among countries of the same fundamental structure. In fact, under differential flow rates, economies starting close to each other might tend to diverge from each other and converge to different steady states. In this respect, observed regional inequalities regarding development and environmental quality might be a permanent rather than a transient phenomenon. On the other hand, policies directed towards reducing the differential flow rates, and in particular towards increasing the flow of capital and reducing the flow of polluting activities, tend to make the economies converge to a common steady state and eliminate regional inequalities.

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<sup>3</sup>For a detailed analysis of this approach and the implications for spatial growth models with fixed saving ratios, see Xepapadeas et al. (2014).



## 2. Capital accumulation and capital diffusion

We consider two similar economies, one located in the north (denoted by  $N$ ) and the other located in the south (denoted by  $S$ ). Let  $K_j(t)$ ,  $j = N, S$  denote the stock of capital at time  $t > 0$  in each economy. To provide a general set up we assume that in each country output is produced according to a production function capable of delivering both endogenous growth and convergence behavior in which the poorer economy grows faster. This following Jones and Manuelli (1990), Barro and Sala-i-Martin (2004) production technology is specified as

$$Y_j = f_j(K_j) = A_j K_j + B_j K_j^\alpha, 0 < \alpha < 1. \quad (2.1)$$

In production function (2.1) the part  $A_j K_j$  will deliver endogenous growth, while the Cobb-Douglas part  $B_j K_j^\alpha$  will deliver convergence. Assuming that the population in each of the two regions is constant (2.1) can be interpreted in per capita terms. We will use per capita interpretation of all variables in this paper.

As explained in the introduction capital flows from one region to the other chasing higher net returns relative to the 'home' region at a speed  $D_K$ . Thus the net flux into region  $N$  is proportional to the rate of return difference. This can be written as

$$D_K (r_N - r_S), \quad (2.2)$$

where  $r_j$  is the net return on capital in each region and the proportionality coefficient is incorporated into  $D_K$ . Since the representative firm in each region is a profit maximizer the net return is the marginal product of capital net of depreciation  $\delta_j$ , or

$$r_j = r_j(K_j) = \frac{\partial f_j}{\partial K_j} - \delta_j, \frac{\partial f_j}{\partial K_j} = A_j + \alpha B_j K_j^{\alpha-1}, j = N, S \quad (2.3)$$

To describe the evolution of capital stock in each region we adopt the “behaviorist” tradition (Solow 1956) that savings-investment is a given fraction  $s_j$  of output. In this context, the evolution of the stock of capital in the two regions is determined by the growth equations:

$$\frac{dK_N(t)}{dt} = sY_N(t) - \delta_N K_N(t) + D_K [r_S(t) - r_N(t)] \quad (2.4)$$

$$\frac{dK_S(t)}{dt} = sY_S(t) - \delta_S K_S(t) + D_K [r_N(t) - r_S(t)] \quad (2.5)$$

where  $\delta$  is the depreciation rate, population is assumed constant, and  $D_K$  is the diffusion coefficient characterizing the movement of capital from one economy to the other.

In a spatially homogenous model where  $D_K = 0$  and  $s$  and  $\delta$  are fixed, the growth equation becomes

$$\dot{K} = s(AK + BK^\alpha) - \delta K$$

### 3. Pollution accumulation and pollution diffusion

In each region polluting activities contribute to aggregate emissions  $e(t)$ , defined as  $e(t) = vY_j(t)$ ,  $j = N, S$  where  $v$  denotes emissions per unit of output,<sup>4</sup> and pollution accumulation is denoted by  $P_j(t)$ . We assume that polluting activities move from the region higher accumulated pollution to the region of lower accumulated pollution. We assume that this transport of polluting activities induces a pollution flow that is proportional to the difference between the accumulated pollution in the two regions, or  $D_P [P_S(t) - P_N(t)]$ . This underlies the assumption that if economies are very similar regarding pollution accumulation, there will be little room for transport of polluting activities from one to the other. Under these assumptions the evolution of the pollution stock in each economy is determined by:

$$\frac{dP_N(t)}{dt} = vY_N(t) - \gamma P_N(t) + D_P [P_S(t) - P_N(t)] \quad (3.1)$$

$$\frac{dP_S(t)}{dt} = vY_S(t) - \gamma P_S(t) + D_P [P_N(t) - P_S(t)] \quad (3.2)$$

where  $D_P$  is the diffusion coefficient characterizing the movement of pollution from one region to the other, and  $\gamma$  is a natural pollution depreciation rate. In our model the economy feeds pollution accumulation through capital accumulation. The pollution

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<sup>4</sup>The emission coefficient in a more sophisticated model could be defined as  $v(K_j)$ , where  $v(K_j)$  is a non-increasing function of  $K_j$ , indicating that as capital accumulates relatively “cleaner” techniques are used.

module of the model is linked to the economy by the assumption that pollution is detrimental to capital defined in a broad sense. This assumption (Gradus and Smulders 1993) underlies the idea that pollution, in the form of air pollution, smog and heavy metals, increases the depreciation rate of human capital due to health effects. In this case the depreciation rate of capital can be written as

$$\delta \equiv \delta(P_j(t)), \quad \frac{\partial \delta}{\partial P_j} > 0, \quad j = N, S$$

and the growth equations for the two regions become

$$\frac{dK_N(t)}{dt} = sY_N(t) - \delta(P_N(t))K_N(t) + D_K[r_S(t) - r_N(t)] \quad (3.3)$$

$$\frac{dK_S(t)}{dt} = sY_S(t) - \delta(P_S(t))K_S(t) + D_K[r_N(t) - r_S(t)] \quad (3.4)$$

Thus the effect of pollution can override the stimulatory effects of capital inflows.

The system of (3.1), (3.2), (3.3) and (3.4) determines the evolution of the capital stock and the pollution stock in each economy.

#### 4. Steady State Equilibrium without Diffusion

If there is no diffusion, that is no transport of capital or polluting activities, then a steady-state equilibrium in north and south is defined as:

$$(\bar{P}_j, \bar{K}_j) : \frac{dP_j}{dt} = \frac{dK_j}{dt} = 0 \text{ or} \quad (4.1)$$

$$0 = v\bar{Y}_j - \gamma\bar{P}_j, \quad 0 = s\bar{Y}_j - \delta(\bar{P}_j)\bar{K}_j, \quad j = N, S \quad (4.2)$$

To examine the stability properties of the spatially independent steady state, we form the linearization matrix around the steady state, which is defined as

$$J_{1j} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} -\gamma & v f'(\bar{K}_j) \\ -\delta'(\bar{P}_j)\bar{K}_j & s f'(\bar{K}_j) - \delta(\bar{P}_j) \end{pmatrix} \quad (4.3)$$

$$\begin{pmatrix} -\gamma & v(A_j + \alpha B_j \bar{K}_j^{\alpha-1}) \\ -\delta'(\bar{P}_j)\bar{K}_j & s(A_j + \alpha B_j \bar{K}_j^{\alpha-1}) - \delta(\bar{P}_j) \end{pmatrix}, j = N, S \quad (4.4)$$

For a positive symmetric steady state we require  $sB\bar{K}^{\alpha-1} = \frac{\delta(\bar{P})}{s} - A > 0$ . Furthermore  $a_{11} < 0$ ,  $a_{12} > 0$ ,  $a_{21} < 0$  by inspection, and at a positive steady state  $\alpha s B \bar{K}^{\alpha-1} < \frac{\delta(\bar{P})}{s} - A$  since  $\alpha < 1$ ; therefore  $a_{22} < 0$ . The positive steady state is stable provided the eigenvalues of  $J_1$  have negative real parts; that is,  $\text{tr}(J_{1j}) = a_{11} + a_{22} < 0$ , which is always true, and  $\det(J_{1j}) = a_{11}a_{22} - a_{12}a_{21} > 0$ . Henceforth we assume that the steady

state is stable.

Thus, without diffusion, both north and south converge to a stable long-run capital stock and pollution stock equilibrium. This steady state will be spatially homogeneous if the economies have the same structure  $A_j = A, B_j = B, j = N, S$ . In this case independent of initial conditions both economies converge to the same steady state. This result can be regarded as an extension of the neoclassical convergence result to the convergence of both capital and pollution to a stable steady state.

It is interesting to note that the inhibitory effect of pollution on capital accumulation and output production prevents sustained growth, which would have been possible in a model with an  $AK$  structure and capital depreciation independent of the pollution stock, or  $\delta(P) \equiv \delta$ .

## **5. Pollution Transportation, Capital Mobility, and Spatial Pattern Formation**

To analyze the effects of capital flows and pollution flows between the two economies, we consider whether small perturbations enhanced by diffusion, that is transport of polluting activities and capital mobility between the two regions, can destabilize the spatially homogeneous steady state. In this we extend the classical arguments of Turing (1952), and standard methods (Murray 1993). We consider therefore the linearization matrix of the system (3.1), (3.2), (3.3) and (3.4), around the spatially homogeneous

steady state  $(\bar{P}_N, \bar{K}_N, \bar{P}_S, \bar{K}_S) = (\bar{P}, \bar{K}, \bar{P}, \bar{K})$ . The linearization matrix is defined as:

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \quad (5.1)$$

where

$$m_{11} = -\gamma - D_p, m_{12} = v f'(\bar{K}_N), m_{13} = D_p, m_{14} = 0 \quad (5.2)$$

$$m_{21} = \delta'(\bar{P}_N)(D_K - \bar{K}_N), m_{22} = s f'(\bar{K}_S) - \delta(\bar{P}_N) - D_K f''(\bar{K}_N) \quad (5.3)$$

$$m_{23} = -D_K \delta'(\bar{P}_S), m_{24} = D_K f''(\bar{K}_S) \quad (5.4)$$

$$m_{31} = D_p, m_{32} = 0, m_{33} = -\gamma - D_p, m_{34} = v f'(\bar{K}_S) \quad (5.5)$$

$$m_{41} = -D_K \delta'(\bar{P}_N), m_{42} = D_K f''(\bar{K}_N) \quad (5.6)$$

$$m_{43} = \delta'(\bar{P}_S)(D_K - \bar{K}_S), m_{44} = s f'(\bar{K}_N) - \delta(\bar{P}_N) - D_K f''(\bar{K}_N) \quad (5.7)$$

To obtain a more tractable representation of matrix  $M$  and its eigenvalues define

$$J_{2j} = \begin{pmatrix} D_p & 0 \\ -D_K \delta'(\bar{P}_j) & D_K f''(\bar{K}_j) \end{pmatrix}, j = N, S \quad (5.8)$$

$$J = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \quad (5.9)$$

Then the linearization matrix  $M$  can be written as

$$M = I \otimes J_{1j} + J \otimes J_{2j} \quad (5.10)$$

where  $\otimes$  denotes tensor product. By standard procedures (Levin 1974) the eigenvalues of  $M$  are the eigenvalues of the various matrices  $J_{1j} + \lambda J_{2j}$ , where  $\lambda$  is an eigenvalue of  $J$ . Since the eigenvalues of  $J$  are 0 and  $-2$  it follows that the eigenvalues of  $M$  are the eigenvalues of the  $J_{1j}$  and the eigenvalues of matrix  $J_3 = J_{1j} - 2J_{2j}$ , with all matrices evaluated at the spatially homogeneous steady state.

Since we have already assumed that the spatially homogeneous steady state is stable, which means that matrix  $J_{1j}$ ,  $j = N, S$  has eigenvalues with negative real parts, capital flows with capital seeking higher returns and pollution diffusion between the two countries can destabilize the spatially homogenous steady state if and only if matrix  $J_3$  has at least one positive eigenvalue. This requires that  $\det J_3 < 0$  when evaluated at



the spatially homogeneous steady state.

Assume that  $D_p = z^2 D_K$ , then

$$\det J_3 = Q(z, D_K) = \tag{5.11}$$

$$\left[ sf'(\bar{K}) - \delta(\bar{P}) - 2D_K f''(\bar{K}) \right] (-\gamma - 2z^2 D_K) - v f'(\bar{K}) \delta'(\bar{P}) (2D_K - \bar{K})$$

It should be noted that for  $D_K = 0$ , the determinant is positive, since  $sf'(\bar{K}) - \delta(\bar{P}) < 0$ , and thus the spatially homogeneous steady state is stable. If  $D_K$  is sufficiently increased to make the bracketed term positive, then for sufficiently large  $z$ , the determinant becomes negative and we have diffusion induced instability of the spatially homogeneous steady state. Thus, for a given  $D_K$ , there is a critical ratio  $z$  that breaks symmetry and induces spatial instability. Alternatively, one could fix  $z$  and then increase  $D_K$  until instability results (i.e.  $Q(z, D_K) < 0$ ) because the  $D_K$  squared term in (5.11) must eventually dominate. This instability can be regarded as a precursor of spatial pattern formation which could result in different capital and pollution accumulation in the two regions in the long run.

The emergence of spatial instability can be made more clear with the help of a numerical example.

### 5.1. A numerical example

We consider two regions (economies) characterized by the following structure common to both regions:

$$\text{Production Function} \quad AK + BK^\alpha \quad A = 1, B = 2, a = 0.4$$

$$\text{Depreciation Function} \quad P^{1+\delta} \quad \delta = 0.1$$

$$\text{Savings Ratio} \quad s \quad s = 0.2$$

$$\text{Emission Coefficient} \quad v \quad v = 0.05$$

$$\text{Pollution Depreciation} \quad \gamma \quad \gamma = 0.05$$

In the absence of capital mobility and transboundary pollution effects, the spatially homogeneous steady state is the same for both economies and is determined by the solution of the system

$$0 = s(AK + BK^\alpha) - KP^{1+\delta}$$

$$0 = v(AK + BK^\alpha) - \gamma P$$

The spatially homogeneous steady state is

$$\bar{K} = \bar{K}_N = \bar{K}_S = 0.497 \tag{5.12}$$

$$\bar{P} = \bar{P}_N = \bar{P}_S = 2.001, \tag{5.13}$$

The linearization matrix  $J_1 = J_{1N} = J_{1S}$  has negative eigenvalues

$$\zeta_1 = -1.670, \zeta_2 = -0.090 \quad (5.14)$$

Thus the spatially homogeneous steady state is stable.

To examine destabilization of this steady state under capital mobility and transportation of pollution between the two regions we study the determinant of matrix  $J_3$  given by (5.11) which for the specific parametrization is:

$$Q(z, D_K) = 0.1504 - 0.4085D_K + 3.4204D_Kz^2 - 5.8806D_K^2z^2 \quad (5.15)$$

The surface corresponding to  $Q(z, D_K)$  is shown in Figure 1

In this figure the set  $In = \{(z, D_K) : Q(z, D_K) < 0\}$  determines the region of spatial instability, while the region of spatial stability determined by the set  $St = \{(z, D_K) : Q(z, D_K) > 0\}$ . The curve AB is the boundary curve separating the two sets. This numerical example confirms our theoretical findings. For small  $D_K$  and  $z$  the spatially homogeneous steady state is stable and the two economies converge to this steady state even when they start from different initial conditions. Thus weak capital and pollution mobility promotes convergence and regional homogeneity. For sufficiently large  $D_K$  and  $z$  the spatially homogeneous steady state is destabilized and patterns emerge. Thus strong capital and pollution mobility could lead to spatially heterogeneous regional

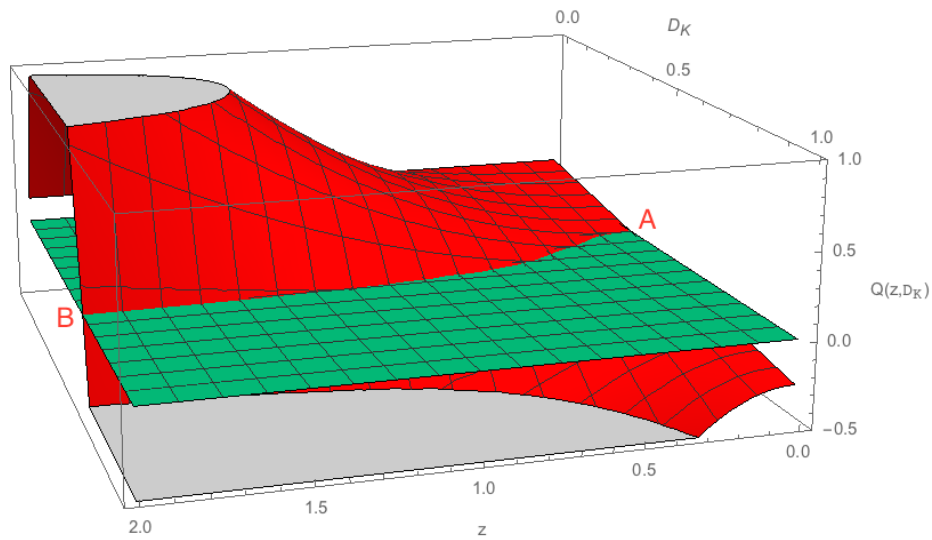


Figure 5.1: Spatial stability and instability

development and pollution patterns.

## 6. Concluding Remarks

Inequality in the distribution of capital across nations has many contributing factors, some tied to environmental resources, and some historical. Population, culture and political systems are obviously important in this context, and there is no single simple explanation. What is perhaps surprising, however, is that such inequality can arise endogenously, even when all else is symmetric, through the magnification of random initial differences, and even in the face of convergence. Basically, productivity begets productivity, in the process creating negative externalities that can serve to increase disparities, and lock the distribution into an asymmetric pattern that resists full convergence.

We illustrate that in this paper by considering two nations (North and South), which are initially identical in their resources, and in their stocks of capital. Following standard approaches, we assume that each nation has a production function incorporating both endogenous growth (represented by an  $AK$  production function) and convergence behavior (represented by a Cobb-Douglas function). Capital flows from nation to nation, not random, but moving according to where the higher net returns can be realized. We then introduce a negative externality associated with polluting activities, which flows from the higher pollution area to the lower according to Fickian diffusion. Pollution generation is assumed to be proportional to production, and to have a restraining effect on its growth.

With these simple assumptions, and following the ideas of Alan Turing in his discussion of pattern formation in embryogenesis, we find that inhomogeneity can arise endogenously, and be reinforced and stabilized in a permanent pattern. There is no suggestion that self-organization is the complete answer to the patterns of inequity on the globe; that would clearly be incorrect. But it does seem clear that once inhomogeneous patterns are established through a combination of exogenous and endogenous factors, the dynamics of capital accumulation and negative externalities can serve to make those patterns resistant to efforts at equalization.

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