DEPARTMENT OF INTERNATIONAL AND EUROPEAN ECONOMIC STUDIES
ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

**Vertical Licensing, Input Pricing, and Entry**

Elpiniki Bakaouka

Chrysovalantou Milliou

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Elpiniki Bakaouka and Chrysovalantou Milliou*

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Abstract

We explore the incentives of a vertically integrated incumbent firm to license the production technology of its core input to an external firm. We find that it opts for licensing even when licensing induces the entry of the licensee in the final goods market. In fact, although the entry of the licensee reduces the licensor's efficiency and the competition that it faces, it reinforces, instead of weakens, the licensing incentives. Vertical licensing is always welfare-enhancing and it is even more welfare-enhancing when it triggers entry.

Keywords: licensing; vertical relations, entry; two-part tariffs; outsourcing

JEL classification: L22; L24; L13; L42; D45

*Bakaouka: Department of International and European Economic Studies, Athens University of Economics and Business, Athens 10434, Greece, e-mail: bakaoukael@aueb.gr; Milliou: Department of International and European Economic Studies, Athens University of Economics and Business, Athens 10434, Greece, e-mail: cmilliou@aueb.gr. We are grateful to Marie-Laure Allain and to Lambros Pechlivanos for their important suggestions. We would like to thank Maria Alipranti, Christos Constantatos, Emmanuel Petrakis, Joel Sandonis, Spyros Vassilakis as well as the conference participants at the Workshop on Competition and Bargaining in Vertical Chains at Toulouse, the CRETE 2016 at Tinos, the EARIE 2016 at Lisbon, and the ASSET 2016 at Thessaloniki for their useful comments. This research has been co-financed by the European Union (European Social Fund - ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) - Research Funding Program: Thalis - Athens University of Economics and Business - "New Methods in the Analysis of Market Competition: Oligopoly, Networks and Regulation". Full responsibility for all shortcomings is ours.
1 Introduction

Original brand manufacturers often license the production technology of their core inputs to external firms.\(^1\) Such licensing activities can trigger the entry of the latter - the licensees - into direct competition with their licensors. An illustration comes from the commercial aircraft market, where, in 2003, Boeing, the US aircraft manufacturer, licensed its wing technology to Mitsubishi Heavy Industries, the Japanese engineering and electrical equipment firm, after agreeing to source the wings of its planes from the latter. Boeing, as well as its main competitor in the market, Airbus, had been already subcontracting the production of various aircraft components to external suppliers. However, never before a commercial aircraft manufacturer had subcontracted its wing production. Boeing’s decision was accepted with wide spread criticism. First, because Boeing’s wing technology has been regarded as its crown jewel, and second, because the transfer of wing manufacturing and assembly expertise effectively gave to Mitsubishi ‘total production competence’ with regard to commercial aircrafts. Mitsubishi is now ready to launch its own passenger jet, which is due to enter service in 2018.\(^2\)

A similar illustration can be found in the electronic appliances market, where, a Chinese producer of electrical equipment for original brand manufacturers at the time, Haier, acquired in 1984 the technology for producing high-quality refrigerators from a German original brand manufacturer, Liebherr. A year after the licensing of Liebherr’s technology, Haier introduced its own first four-star refrigerator into the market. That is, Haier transformed from an original equipment manufacturer to an original brand manufacturer. It soon became the leader refrigerator producer in China and, eventually, it also became a major competitor in the global market.

The above illustrations give rise to a number of important questions regarding vertical licensing, such as: Does a firm have incentives to license its input technology to an external supplier when the latter can turn into its competitor in the final goods market? What is the impact of the licensee’s entry into the licensor’s market on the licensing incentives?

\(^1\)According to empirical evidence, technology licensing is a common practice among firms (e.g., Caves et al., 1983, Nadiri, 1993, Anand and Khanna, 2000). Many well-known firms, such as IBM, Hitachi, Kodak, and Procter & Gamble, have licensed their technology, earning millions of dollars in licensing revenue (e.g., A Market for Ideas, *The Economist* (October 20, 2005)).

\(^2\)Note that without the ability to make wings, Mitsubishi could not become an independent player in the passenger jet industry. For more information regarding this case, e.g., Secret Wounds Of Globalism: Boeing Sells Its Technology – Cheap – To Japan, *Forbes* (January 2014), and Boeing Outsourcing Gives Wing to Concerns, *Chicago Tribune* (December 21, 2003).
How does input pricing affect the licensing incentives? What are the welfare implications of vertical licensing? In this paper, we attempt to address these questions.

To this end, we consider a framework in which two incumbent firms produce two competing final goods using an input that they initially produce in-house. One of the incumbents considers licensing its input technology to an external firm for a fixed licensing fee. Once the licensing agreement is signed, the licensor can source the input from the licensee after bargaining over the terms of a two-part tariff contract. Moreover, once the licensing agreement is signed, the licensee can enter into the final goods market and compete with the two incumbents. We assume that competition in the final goods market takes place in quantities and examine what happens when the licensee does not enter into the final goods market - the ‘no entry case’ as well as when it enters - the ‘entry case’.

We find that independently of whether the licensee enters into the final goods market or not, the incumbent always opts for licensing. The driving force of licensing, however, differs substantially among the entry and the no entry case. In the no entry case, licensing is mainly due to efficiency reasons: when the incumbent licenses its input technology, it becomes more efficient and, thus, enjoys a larger competitive advantage in the final goods market. The higher efficiency of the licensor is not due to a cost advantage of the licensee. It is due, instead, to input pricing. Specifically, in the no entry case, the licensee sets a wholesale price which is below the input’s marginal cost. We refer to this as the input pricing effect of licensing. The licensee has incentives to do so because it can extract part of the resulting higher profits of the licensor through the fixed fee included in the two-part tariff. In fact, due to the higher efficiency under licensing, the incumbent is willing to license its input technology even for free when bargaining power is sufficiently high.

In the entry case, licensing results in an increase in the number of downstream competitors and, thus, in an increase in the intensity of competition. We refer to this as the competition intensity effect of licensing. The licensee now does not subsidize the licensor - it sets a wholesale price that exceeds the input’s marginal cost, decreasing the efficiency of the licensor but alleviating the negative impact of the competition intensity effect. Furthermore, licensing gives rise to two additional effects. First, the market expansion effect, which refers to the fact that licensing, through entry, increases the product variety and, in turn, expands the demand in the final goods market. And second, the business stealing effect: when the licensee enters into the final goods market, it “steals” a part of the sales and market share of the licensor’s rival. These two effects work in favor of licensing. They
outweigh the decreased efficiency and the increased competition intensity and lead to its emergence in equilibrium.

Importantly, the entry of the licensee into the final goods market, although it intensifies the market competition and reduces the efficiency of the licensor, it strengthens, instead of weakens, the licensing incentives. In other words, the incumbent has stronger licensing incentives when the licensee becomes its direct competitor in the final goods market, since the business stealing effect along with the market expansion effect and the inverse input pricing effect augment the licensor’s profits and make entry desirable. Mankiw and Whinston (1986), similar to us, show that a firm through its entry in the final goods market “steals” business from the incumbent firms and increases product variety, leading to higher demand and higher consumers’ and total welfare. Although this is the case, Mankiw and Whinston (1986) show that the firm cannot capture the resulting larger pie. In contrast, we show when firms are involved in vertical licensing, they can use their vertical trading contract as an instrument that allows them to behave in a pro-collusive way and in turn they can take full advantage of the larger pie. Clearly, this works as an additional motive for licensing.

Vertical licensing turns out to be desirable not only for the licensor, but also for the consumers and the economy as a whole. This holds both with and without entry of the licensee in the final goods market. Intuitively, licensing results in lower final prices, and thus, in higher consumers’ surplus, either because it enhances the efficiency of the licensor (in the no entry case) or because it intensifies market competition (in the entry case). In fact, in the entry case, licensing is even more desirable due to the increased product variety and competition.

We also examine the implications of licensing with and without entry in various extensions of the main model. We find that entry reinforces licensing incentives when the licensor and the licensee trade through a wholesale price contract and when the licensor is initially a monopolist in the final goods market. Furthermore, we show that, in case of entry, after the signing of the licensing agreement when the licensor’s rival also sources its input from the licensee or when the licensor has a cost advantage compared to its rival, the licensing incentives are even stronger. Examining what happens when licensing takes place through a per-unit of output royalty, instead of through a fixed licensing fee, licensing incentives occur only for free and only in the no entry case if the bargaining power of the licensor is
sufficiently high.\(^3\)

Our work is related to the vast theoretical literature on technology licensing. This literature has analyzed various aspects of licensing such as, the choice among royalties and licensing fees (e.g., Kamien and Tauman, 1986, Muto, 1993, Wang, 1998), the impact of licensing on innovation (e.g., Gallini and Winter, 1985), the role of information asymmetries (e.g., Gallini and Wright, 1990, Beggs, 1992), and the choice among merger and licensing (e.g., Fauli-Oller and Sandonis, 2003). The majority of this literature has done so assuming that the licensor and the licensee(s) operate in the same one-tier market or that the licensor is not active in the market (it is an outsider). Some recent exceptions include the papers of Mukherjee (2003), Arya and Mittendorf (2006), Mukherjee and Ray (2007), Rey and Salant (2012), which, similar to us, have examined licensing within a vertically related market. The latter papers, however, differ from ours in three important aspects. First, most of them have considered licensing either among downstream or upstream firms, and not among vertically related firms.\(^4\) Second, some of them have analyzed settings in which, after the signing of the licensing agreement, the licensor is not a customer of the licensee. Third, they have examined the possibility that the licensing triggers the entry of a new firm into the market, but not the entry of the licensee into the licensor’s market. In this respect, our paper complements the existing literature on technology licensing and, in contrast to the existing ones, is more appropriate for the analysis of situations such as the ones described above (e.g., Boeing case).

Our paper is also related to the literature on outsourcing. A number of papers within this literature (e.g., Pack and Saggi, 2001, Shy and Stenbacka, 2003, Sappington 2005, Arya et al., 2008a and 2008b, Lim and Tan, 2010) have analyzed a final product manufacturer’s ‘make-or-buy’ decision. That is, its choice among input production in-house and input sourcing from an external firm - outsourcing.\(^5\) Some of these papers have assumed that the input production is outsourced to an already existing vertically integrated rival. Others, instead, have assumed that it is outsourced to an independent upstream firm. In particular,

\(^3\)It should be noted that free licensing is observed quite often. It is observed in the market for open source software as well as in other markets. For instance, in January 2015, Toyota announced that it would make more than 5,600 patents on fuel-cell technologies available for use, free of royalty payments, to a wide array of companies in the transportation sector. For more on this, see e.g. Toyota Offers To License Hydrogen Fuel-Cell Patents, For Free, Green Car Reports (January 5, 2015), and Toyota To Share Hydrogen Fuel Cell Patents, Forbes (January 5, 2015).

\(^4\)An exception in this respect is the paper of Rey and Salant (2012) which considers vertical licensing.

\(^5\)The practice of outsourcing is obviously quite similar to the practice of licensing especially when licensing does not involve fees or royalties.
Pack and Saggi (2001) and Goh (2005) have examined a buyer’s incentives to outsource its technology to a supplier. In the former paper, technology diffusion can result in a new downstream entry while in the latter can result in upstream entry. Both papers have found that the new entry can be beneficial for the two original firms engaged in technology transfer because the input tends to marginal cost. In contrast to these papers, we consider the entry of the upstream firm to which the input production is outsourced and we show that licensing is beneficial even when input price is greater than the marginal cost. Lim and Tan (2010) consider a setting in which the supplier becomes a direct competitor of the buyer after the latter’s outsourcing. However, they focus on the buyer’s rate of learning and brand equity, while we demonstrate the role of the endogenous input pricing on licensing decision.

The remainder of the paper is organized as follows. In Section 2, we describe our main model. In section 3, we examine the licensing incentives in both the no entry case and the entry case, and we characterize the impact of entry on these incentives. In the following section, we evaluate the welfare implications of vertical licensing. In Section 5, we examine the robustness of our main findings when under licensing both incumbent firms source the input from the licensee. In Section 6, we discuss a number of other extensions of our main model. Finally, in Section 7, we conclude. All the proofs are included in Appendix B.

2 The Model

We consider a market consisting initially of two firms, firm 1 and firm 2. Each firm $i$, with $i = 1, 2$, produces a differentiated final good using, in a one-to-one proportion, a core input that it produces in-house at marginal cost $c > 0$.

Both firms hold a patent for their input production technologies. One of them, without loss of generality firm 1, considers licensing its input technology to an external firm, firm $S$, for a fixed licensing fee, $F \geq 0$. After the licensing agreement has been signed, the licensee (firm $S$) is in the position to produce the licensor’s (firm 1’s) patented input. It can produce the input at the same cost as the licensor, i.e., at $c$.\(^6\)

The knowledge that licensing provides regarding the production of the final good’s core input can allow firm $S$ to produce the final good too. Thus, licensing can cause firm $S$’s

\(^6\)In Section 6, we discuss what happens when licensing takes place through a variable royalty instead and show that the licensor prefers using a fixed licensing fee.

\(^7\)We abstract from assuming that firm $S$ is more efficient in input production than firm 1 since the incentives for vertical licensing would be straightforward then.
entry into the final goods market. In what follows, we consider two cases, the ‘entry case’ in which firm S enters into the final goods market and the ‘no entry’ case in which it stays out of it.\(^8\) In both cases, we assume that under licensing, firm 1 stops producing the input in-house and sources it from firm S. The input sourcing terms include the terms of a two-part tariff, i.e., a fixed fee, \(T\), and a wholesale price per unit of input, \(w\), that firm 1 pays to firm S. These terms are determined through Nash bargaining, in which the bargaining power of firm S and firm 1 is given by \(\beta\) and \(1 - \beta\), respectively, with \(0 < \beta < 1\).\(^9\)\(^10\)

The (inverse) demand function for firm \(i\)’s final good is:

\[ p_i(q_i, Q_{-i}) = a - q_i - \gamma Q_{-i}, \quad 0 < \gamma < 1, \quad a > c, \]

where \(p_i\) and \(q_i\) are the price and the quantity of firm \(i\)’s final good, respectively, and \(Q_{-i}\) is the quantity of its rival(s)’ final goods. In particular, \(Q_{-i} = q_j\), with \(i, j = 1, 2\) and \(i \neq j\), in the no entry case, while \(Q_{-i} = q_j + q_k\), with \(i, j, k = 1, 2, S\) and \(i \neq j \neq k\), in the entry case. The parameter \(\gamma\) measures the degree of product differentiation; namely, the higher \(\gamma\) is, the closer substitutes the final goods are.

The timing of moves is as follows. First, firm 1 decides whether to license its input technology to firm S. In case of licensing, it sets the licensing fee \(F\) and, in turn, firm S signs or not the licensing agreement. If the agreement is signed, in the following stage, firm 1 and firm S negotiate over \((w, T)\). In the last stage, firm 1 and firm 2, as well as firm S in the case of licensing and entry, choose their quantities simultaneously and separately.\(^11\)

We solve for the subgame perfect Nash equilibrium of this game.

We make the following assumption throughout the paper in order to guarantee that all

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\(^8\)This could be so because, for instance, the entry costs in the final good’s market are prohibitively high and, thus, entry is blocked or because the ability of producing the core input is not sufficient for producing the final good too.

\(^9\)We should note that in the alternative scenario, in which firm 1 can produce the input in-house in case of disagreement with firm S during the negotiations, the equilibrium \(w\) is the same as in the case in which firm 1 commits to sourcing the input from S. The only thing that changes then is \(T\) which is lower since firm S needs to compensate firm 1 for its outside option. As we will show later on, firm 1 manages to extract \(T\) through the licensing fee. Thus, the change in \(T\) does not affect our main conclusions.

\(^10\)Note that the inclusion of bargaining over the licensing fee too would not affect our main results. For more on this, see Section 6.

\(^11\)We implicitly assume in our analysis that firm 2 (and not only firm 1 and firm S) observes the contract terms \((w, T)\) before it chooses its own quantity. Still, our results would be exactly the same in the alternative scenario with partial observation of firms’ marginal costs. That is, in the scenario in which firm 2 does not observe the contract terms, and thus, the endogenous marginal cost of firm 1, while firms 1 and firm S observe firm 2’s exogenous marginal cost since firm 2 is vertically integrated. The analysis of this scenario is available from the authors upon request. Moreover, in Section 6, we examine what happens when firms compete in prices.
the firms face a non-negative marginal cost in all the cases under consideration:

**Assumption 1:** \( a \leq 5c \)

We start our analysis with the benchmark case in which there is no licensing. In the absence of licensing, firm 1 and firm 2 compete among them in the standard Cournot way. In particular, each firm \( i \) chooses its output in order to maximize its profits: 
\[
\pi_i(q_i, q_j) = (a - q_i - \gamma q_j)q_i - cq_i, \quad \text{with} \quad i, j = 1, 2 \quad \text{and} \quad i \neq j.
\]  
Solving the resulting system of first order conditions, we obtain the Cournot-Nash equilibrium quantities, \( q^B_1 \) and \( q^B_2 \), and the respective equilibrium profits, \( \pi^B_1 \) and \( \pi^B_2 \), included in Table 1 of Appendix A.

### 3 Licensing Incentives

In this section, we analyze the licensing incentives with and without entry of the licensee into the final goods market and examine how they are influenced by entry.

#### 3.1 Licensing and No Entry

We start with the analysis of the case in which the licensing agreement has been signed and firm \( S \) has stayed out of the final goods market.

In the last stage, firm 2 faces the same maximization problem as in the benchmark case. Its competitor, firm 1, chooses \( q_1 \) in order to maximize its own (gross from \( T \) and \( F \)) profits: 
\[
\pi_1(q_1, q_2, w) = (a - q_1 - \gamma q_2)q_1 - wq_1.
\] 
Solving the system of the first order conditions, we derive the equilibrium quantities in terms of \( w \):
\[
q_1(w) = \frac{a(2 - \gamma) + \gamma c - 2w}{4 - \gamma^2} \quad \text{and} \quad q_2(w) = \frac{a(2 - \gamma) - 2c + \gamma w}{4 - \gamma^2}.
\]  
(1)

Obviously, a decrease in the wholesale price results in higher output for firm 1 and lower output for firm 2.

In the following stage, firm \( S \) and firm 1 negotiate over \((w, T)\). In particular, they solve the following generalized Nash bargaining problem:
\[
\max_{w,T} [\pi_S(w) + T]^\beta [\pi_1(w) - T]^{1-\beta},
\]  
(2)

where \( \pi_S(w) = (w - c)q_1(w) \) are firm \( S \)'s profits and \( \pi_1(w) = \pi_1(q_1(w), q_2(w), w) \). Note that the disagreement payoffs of both firms are equal to zero since neither firm has an outside
option. Maximizing (2) with respect to $T$, we find:

$$T = \beta \pi_1(w) - (1 - \beta)\pi_S(w).$$

(3)

Using (3), the gross (from $F$) profits of firm $S$ and firm 1 can be rewritten as:

$$\pi_S(w) + T = \beta(\pi_S(w) + \pi_1(w)) \quad \text{and} \quad \pi_1(w) - T = (1 - \beta)(\pi_S(w) + \pi_1(w)).$$

(4)

Substituting the above into (2), we obtain an expression which is proportional to the joint profits of firm $S$ and firm 1. It follows that $w$ is chosen to maximize these profits:

$$\max_w \pi_S(w) + \pi_1(w) = (a - q_1(w) - \gamma q_2(w))q_1(w) - cq_1(w).$$

(5)

From the first order condition of (5), we obtain the equilibrium wholesale price and, after substituting it into (3), we also obtain the equilibrium fixed fee:

$$w_{LN} = \frac{8c - 2(a + c)\gamma^2 + (a - c)\gamma^3}{4(2 - \gamma^2)} \quad \text{and} \quad T_{LN} = \frac{(a - c)^2(2 - \gamma)^2(2\beta + (1 - \beta)\gamma^2)}{8(2 - \gamma^2)^2}. \quad \text{(6)}$$

One can easily check that $w_{LN} < c$. That is, firm $S$ subsidizes, through the wholesale price, the production of its customer, firm 1.\(^{12}\) As we saw above, by charging a lower wholesale price, firm $S$ increases the aggressiveness of firm 1 in the final goods market and enhances its output at the expense of firm 2’s output. Firm $S$ has incentives to do so because it can use, in turn, the fixed fee $T$ in order to capture part of the resulting higher firm 1’s profits.\(^{13}\) Clearly, the higher is firm $S$’s bargaining power, the larger is the share of firm 1’s profits that it captures through $T$.

The licensing fee is determined in the following way: firm 1 knows that firm $S$ will reject the licensing agreement if and only if its profits without the agreement exceed its profits with the agreement. Since the former profits are equal to 0, it follows that firm 1 will optimally set $F_{LN} = \pi_S(w_{LN}) + T_{LN}$. As a result, firm 1’s net equilibrium profits in the licensing and no entry case are: $\pi_{1LN} = \pi_1(w_{LN}) - T_{LN} + F_{LN} = \pi_1(w_{LN}) + \pi_S(w_{LN})$. Therefore, firm 1 enjoys not only the profits from its own sales in the final goods market but also firm $S$’s profits from the input sales, i.e., it enjoys all of its joint profits with firm

\(^{12}\) Assumption 1 guarantees that $w_{LN} > 0$.

\(^{13}\) A similar rationale exists in the delegation literature (e.g., Vickers, 1985, Fershtman and Judd, 1987, Sklivas, 1987).
Comparing the equilibrium profits of firm 1 in the licensing and no entry case $\pi_1^{LN}$ (included in Table 1 of Appendix A) with its respective profits under no licensing $\pi_1^B$, we reach the following conclusion.

**Proposition 1** When firm $S$ does not enter into the final goods market, firm 1 always has incentives to license its input technology.

As Proposition 1 informs us, firm 1 always licenses its input technology to firm $S$ when the latter does not enter into the final goods market. It is important to note that this holds not only when firm 1 charges a positive fixed fee for the licensing agreement, but also when it offers the licensing agreement for free ($F = 0$), as long as its bargaining power is sufficiently high. This is stated formally in the following Corollary.

**Corollary 1** When firm $S$ does not enter into the final goods market and licensing is for free ($F = 0$), firm 1 has incentives to license its input technology if and only if $\beta$ is sufficiently low.

Why does firm 1 have incentives to transfer its input technology to firm $S$ even for free? Recall that when firm 1 produces the input in-house, it faces marginal cost $c$. When, instead, it licenses its input technology, it faces a lower marginal cost $w^{LN} < c$. We refer to the cost reduction that firm 1 enjoys through the input price as the *input pricing effect* of licensing. It follows that due to the *input pricing effect*, licensing results in an increase in firm 1’s efficiency; hence, in a cost advantage for firm 1 in the final goods market. A straightforward implication of this is that the gross from $T^{LN}$ profits of firm 1 are larger under licensing. When firm $S$’s bargaining power is not too large, firm $S$ obtains only a small share of these profits through $T^{LN}$. As a consequence, firm 1 is willing, then, to license its input technology even for free. Clearly, when a licensing fee is used and firm 1 fully enjoys its own profits from the final goods market, licensing incentives are always present independently of the bargaining power distribution.\(^{14}\)

The above result is in accordance with a result of the literature on vertical separation, according to which vertical separation and, thus, external input sourcing can be preferred to vertical integration for strategic reasons. In particular, a number of papers within this literature (e.g., Vickers, 1985, Jansen, 2003) demonstrate that in settings with downstream

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\(^{14}\)In Section 6, we discuss what happens when firm 1 also bargains with firm $S$ over the licensing fee. We find that still, firm 1 always has incentives for licensing.
quantity competition (strategic substitutability), a vertically separated upstream firm sets the wholesale price below marginal cost in order to increase its downstream customer’s efficiency and profits and extract the latter through the fixed fee. This mechanism obviously coincides with the one that we have identified above. The main difference is that in our setting, licensing allows the downstream, and not the upstream, vertically separated firm to extract the profits at the end.

3.2 Licensing and Entry

We turn now to the examination of the case in which the licensing agreement has been signed and firm $S$ has entered into the final goods market.

In the last stage of the game, firms 1, 2, and $S$ choose their outputs in order to maximize their respective profits:

$$
\pi_1(q_1, q_2, q_S, w) = (a - q_1 - \gamma q_2 - \gamma q_S)q_1 - wq_1; \quad (7)
$$

$$
\pi_2(q_1, q_2, q_S, w) = (a - q_2 - \gamma q_1 - \gamma q_S)q_2 - cq_2; \quad (8)
$$

$$
\pi_S(q_1, q_2, q_S, w) = (a - q_S - \gamma q_1 - \gamma q_2)q_S - c(q_1 + q_S) + wq_1. \quad (9)
$$

Solving the system of the first order conditions, we find:

$$
q_1(w) = \frac{a(2 - \gamma) + 2c\gamma - (2 + \gamma)w}{2(2 - \gamma)(1 + \gamma)}; \quad (10)
$$

$$
q_2(w) = q_S(w) = \frac{2(a - c) - \gamma(a - w)}{4 + 2\gamma - 2\gamma^2}. \quad (11)
$$

In the second stage, firm $S$ and firm 1 solve the following maximization problem:

$$
\max_{w,T} [\pi_S(w) - d_S + T]^\beta [\pi_1(w) - T]^{1-\beta}, \quad (12)
$$

where $\pi_1(w)$ and $\pi_S(w)$ are found after substituting (10) and (11) into (7) and (9), respectively. It is important to note that while the disagreement payoff of firm 1 continues to be null, the same does not longer hold for firm $S$’s disagreement payoff. This is so because firm $S$ now has an outside option in its bargaining with firm 1: in case of disagreement, firm $S$ can still have profits from its own sales in the final goods market. In particular, its disagreement payoff is given by $d_S = (a - q_S^D - \gamma q_2^B)q_S^D - cq_S^D$, where $q_S^D = q_1^B$. 
Maximizing (12) with respect to $T$, we obtain:

$$T = \beta \pi_1(w) - (1 - \beta)\left[\pi_S(w) - d_S\right].$$

(13)

Using the above expression, we find the following:

$$\pi_S(w) - d_S + T = \beta[\pi_S(w) + \pi_1(w) - d_S];$$

(14)

$$\pi_1(w) - T = (1 - \beta)[\pi_S(w) + \pi_1(w) - d_S].$$

(15)

Substituting in turn (14) and (15) into (12), we note that the latter reduces to an expression proportional to the joint profits of firm S and firm 1 minus firm S’s disagreement payoff. The wholesale price that maximizes this expression is:

$$w^{LE} = \frac{a(2 - \gamma)(1 - \gamma)\gamma + c(4 + \gamma(2 - \gamma - 3\gamma^2))}{2(2 + 2\gamma - 2\gamma^2 - \gamma^3)}.$$ (16)

It can be confirmed that $w^{LE} > c$. In other words, in contrast to the no entry case, when firm S enters into the final goods market, it does not subsidize firm 1; there is an inverse input pricing effect in place. Why is that? Initially, one might think that this result is driven by firm S’s incentive to “raise rival’s cost” in order to increase its own market share in the final goods market. However, this is not so here. As we saw above, the wholesale price is chosen in order to maximize the joint profits of firm S and firm 1 and not the profits of firm S alone. Essentially, the two firms manage to behave as a multi-product firm through the setting of the wholesale price. But why their joint profits are higher when the wholesale price exceeds the marginal cost? Firm S’s entry into the final goods market intensifies downstream competition. We refer to this as the competition intensity effect. The two firms alleviate the negative impact of the competition intensity effect on their joint profits through the setting of a higher wholesale price that leads in turn into a lower joint output. In other words, the two firms set the wholesale price in a pro-collusive way.

It remains to determine the presence or absence of licensing incentives in the first stage of the game. We can obtain the equilibrium fixed fee $T^{LE}$ after substituting (16) into (13) and, in turn, substituting the resulting $T^{LE}$ and $w^{LE}$ into (14) and (15), we can also obtain the gross from the licensing fee equilibrium profits of firm 1 and firm S. For the same reasons as the ones explained in subsection 3.1, firm 1 extracts, through the licensing fee, the profits of firm S: $F^{LE} = \pi_S(w^{LE}) + T^{LE}$. Therefore, firm 1’s net equilibrium profits (included in
Table 2 of Appendix A) in the licensing and entry case are: \( \pi_1^{LE} = \pi_1(w^{LE}) + \pi_S(w^{LE}) \). Comparing \( \pi_1^{LE} \) with \( \pi_1^H \), we find that firm 1 has incentives to license its input technology even when licensing reinforces competition.

**Proposition 2** When firm \( S \) enters into the final goods market, firm 1 always has incentives to license its input technology.

The intuition is as follows. As mentioned above, licensing, when it is accompanied by entry, gives rise to the *competition intensity effect* and the inverse *input pricing effect*. These effects clearly have a negative impact on the profits that firm 1 obtains from its own sales in the final goods market. Hence, if licensing was for free, firm 1 would never opt for licensing in the entry case. Besides though the *competition intensity effect* and the inverse *input pricing effect*, when licensing triggers entry, it also brings about a *business stealing effect* and a *market expansion effect*. The former refers to the fact that firm 2’s output falls as the number of firms in the market increases. Thus, when firm \( S \) enters into the final goods market, it “steals” part of the sales and market share of firm 1’s rival. The *market expansion effect* refers instead to the fact that the entry of firm \( S \) corresponds to an increase in the number of differentiated final products and, thus, to an increase in product variety that in turn expands the demand in the final goods market.\(^{15}\) Both of these effects augment the joint profits of firm \( S \) and firm 1 that the licensor enjoys, making licensing profitable even when it intensifies competition.

An important observation is that the incentives for vertical licensing can be stronger than the incentives for horizontal licensing. In particular, consider the case of horizontal licensing in which firm 1 and firm 2 operate in an one-tier market and firm 1 licenses its production technology to an independent firm which enters into this one-tier market. In such a case, licensing would give rise again to the *competition intensity effect*, the *business stealing effect* and the *market expansion effect* that we identify here. This is also shown

\(^{15}\)The *market expansion effect* appears to be present in the licensing case between Boeing and Mitsubishi. The latter is about to introduce the Mitsubishi Regional Jet (MRJ) - a new and differentiated product in the market for short-haul passenger aircrafts known as “regional jets”. In particular, the MRJ will come in versions seating roughly 70-90 passengers, while Boeing and Airbus’ aircrafts seat over 100 passengers. Demand for regional jets has picked up after the MRJ made its maiden flight in November 2015. In fact, Mitsubishi already has 243 orders and 204 options for these jets. According to an article on the press ("Mitsubishi Aims for the Sky After Jet Takes Off", The Wall Street Journal, November 11, 2015) “... estimates that carriers will order 4,360 regional jets through 2034 and it is predicted that Mitsubishi will capture 27% of that market.” For more on this see e.g., "Can Mitsubishi Heavy Industries' MRJ Regional Jet Lift It To A 'Buy'?", Forbes (October 20, 2014), "Japanese Planemaker in Talks for Major Deal", Financial Tribune (July 13, 2016).
by Mankiw and Whinston (1986) where a new entrant in the final goods market “steals” business from the incumbent firms and increases product variety. However, in an one-tier market, the entry does not give rise to the inverse input pricing effect, and thus the entrant cannot capture part of the greater demand and welfare. So, in case of horizontal licensing and entry, the lack of the inverse input pricing effect does not allow the licensor and the licensee to alleviate the negative impact of the competition intensity effect - and thus, to behave in a pro-collusive way. Therefore, the vertical contract - input pricing - can constitute an instrument that allows the firms involved in licensing to behave in a pro-collusive way, strengthening their licensing incentives.

### 3.3 The Impact of Entry

Having explored the incentives for licensing both with and without entry, we are now able to characterize the impact that entry has on them.

**Proposition 3** The entry of firm $S$ into the final goods market reinforces firm 1’s licensing incentives.

Interestingly, the entry of the licensee into direct competition with the licensor has a positive instead of a negative impact on the licensing incentives. Why is that? The entry of the licensee into the final goods market increases the intensity of competition faced by the licensor. This - the competition intensity effect - is clearly a negative effect for the licensor. At the same time though, the entry allows firm 1 to steal away market share from its rival - firm 2 - as well as to enjoy a larger market size. These effects - the business stealing effect and the market expansion effect - augment firm 1’s profits and along with the entry’s inverse input pricing effect which mitigates the negative impact of the competition intensity effect, render entry desirable.

Entry does not only alter the licensing incentives, as Proposition 4 states below, it also alters the impact that product differentiation has on them.

**Proposition 4** An increase in product differentiation has a negative impact on the licensing incentives when firm $S$ does not enter into the final goods market and a positive one when it enters.

In the no entry case, the higher is product differentiation, the higher is the equilibrium wholesale price and, thus, the smaller is the subsidy that firm $S$ offers to firm 1. In other
words, the weaker is the competition in the final goods market, the lower are the incentives of firm $S$ to enhance the competitive position of its customer by decreasing the latter’s variable cost and, thus, the smaller is the efficiency enhancement that firm 1 enjoys. As a result, firm 1 has stronger incentives to license its technology when product differentiation decreases.

In the entry case, instead, a decrease in product differentiation has two negative implications for the licensor: it enhances the *competition intensity effect* and it weakens the *market expansion effect*. In fact, in the entry case, the relationship between the equilibrium wholesale price and product differentiation is U-shaped: $\frac{\partial w^{LE}}{\partial \gamma} < 0$ if and only if $\gamma \geq 0.43792$. Intuitively, an increase in the wholesale price decreases the negative impact of the *competition intensity effect*. When, however, the competition in the final goods market is already too fierce, a (further) increase in the wholesale price is avoided because it can result in the market foreclosure of firm 1.\footnote{For a review of the market foreclosure issues that arise in the presence of vertical integration see Rey and Tirole (2007).} In light of these, it is not surprising that when the licensee enters into the final goods market, licensing incentives get weaker when product differentiation decreases.

Therefore, when the licensee stays out of the licensor’s market, it is more likely to observe licensing in markets with less differentiated products. While, when the licensee enters into the final goods market, licensing is more likely to occur in markets with more differentiated final products.\footnote{In the extreme case in which the final goods tend to be homogeneous, firm 1 is indifferent between entry and no entry. The *market expansion effect* is absent then and the *competition intensity effect* along with the *business stealing effect* cancel out with the *input pricing effect*.} Overall, the impact of entry on the licensing incentives is weaker in markets in which final products are not too differentiated.

### 4 Welfare Implications of Vertical Licensing

We have already seen that vertical licensing is desirable for firm 1. Next, we examine whether vertical licensing is also desirable from a welfare point of view.

**Proposition 5** *Vertical licensing both with and without the entry of firm $S$ into the final goods market always has a positive impact on consumers’ surplus and on total welfare. Its impact is larger with entry than without entry.*
Vertical licensing is beneficial both for the consumers and for the economy as a whole. Intuitively, the positive impact of licensing on consumers’ surplus is due to the increase in firm 1’s efficiency under no entry and to the increase in product variety and competition intensity under entry. The positive impact of licensing on overall welfare is driven by its positive impact on consumers’ surplus and on the licensor’s profits (Propositions 1 and 2) that outweigh its negative impact on firm 2’s profits. In fact, vertical licensing is even more desirable from a welfare viewpoint when it triggers entry into the final goods market than when it does not. Intuitively, under entry, even though firm 1 is less efficient, market competition is fiercer and the market is larger.

Recently, the EU and the US both revised their rules for the assessment of technology licensing agreements under respectively the EU competition law and the US antitrust law. The new regulations continue to reflect the view that licensing, by facilitating the diffusion of technology, is in most cases pro-competitive. Based on this view, a block exemption applies to all the licensing agreements between firms that have limited market shares.\(^\text{18}\) Our analysis points out a novel channel through which licensing can be pro-competitive even in cases in which the licensors have large market shares.\(^\text{19}\) In particular, it points out that vertical licensing, by triggering entry into more than one stages of the vertical chain, it can also generate more intense product market competition.

5 Common Input Supplier

We have assumed throughout our analysis that firm 2 produces its input in-house both with and without licensing. We alter this assumption now and assume that after the licensing agreement has been signed, firm 2 also sources its input from firm S. A consequence of this is that the bargaining game over the contract terms that takes place under licensing in stage two differs from the respective one in our main model. In particular, firm S now bargains with two, instead of one, firms. In modeling the multilateral bargaining game, we

\(^\text{18}\) According to the new EU regulation, Regulation N. 316/2014, in the case of licensing agreements between non-competitors, the block exemption applies when the individual market share of each party does not exceed 30%. Respectively, according to the US Antitrust Guidelines for the Licensing of Intellectual Property (updated on August 12, 2016 by the US Department of Justice and the Federal Trade Commission), the US Agencies will not challenge an agreement if the licensor and the licensee collectively account for no more than 20% of each relevant market significantly affected by a licensing restraint.

\(^\text{19}\) In Section 6, we confirm and reinforce this point by showing that licensing can be welfare-enhancing even when the licensor is initially a monopolist and/or when it has a cost advantage relative to the other incumbent.
invoke the Nash equilibrium of simultaneous generalized Nash bargaining games, in which
the bargaining power of firm S and each firm $i$, with $i = 1, 2$, continues to be given by $\beta$
and $1 - \beta$, respectively. Thus, during the negotiations between firm S and firm $i$, each of
them takes as given the outcome of the simultaneously-run negotiations of firm S and firm
$j$, with $i, j = 1, 2$ and $i \neq j$. A key assumption that underlies this modeling approach is
that firm S bargains with each firm $i$ simultaneously and separately.20 In order to avoid
the multiple equilibria that can arise in such a setting due to the multiplicity of the beliefs
that the downstream firms can form when they receive out-of-equilibrium offers, we impose
pairwise proofness on the equilibrium contracts (e.g., Horn and Wolinsky, 1988, O’Brien
and Shaffer, 1992, Milliou and Petrakis, 2007, Aliprant et al., 2014). That is, we require
that a contract between firm S and firm $i$ is immune to a bilateral deviation of firm S with
firm $j$. Moreover, in order to ensure that all the firms face a non-negative marginal cost in
all the cases under consideration, we assume that $a < 3c$.

In the no entry case, since in the bargaining game over the contract terms firm S bargains
with two firms, it now has an outside option: the profits that it would make when one of the
downstream firms acts as monopolist facing the equilibrium contract terms. The resulting
equilibrium wholesale prices, with $i = 1, 2$, are:21

$$w_{i}^{CN} = c - \frac{\gamma^2(a - c)}{2(2 - \gamma^2)} < w^{LN}$$

We observe that $w_{i}^{CN} < c$. That is, similarly to our main model, we find that, in the no entry
case, firm S subsidizes the final good producers via the wholesale prices. In fact, it subsidizes
the downstream production now more. The reason for the subsidization differs from the one
in our main model. The subsidization here is due to the "commitment problem" faced by
the upstream monopolist, firm S. That is, it is due to the fact that when firm S trades with
firm $i$, it cannot commit that it will not offer better trading terms to firm $j$.22 An important
implication of this is that the gross from the licensing fee profits of firm S are negative when
$\beta < \beta(\gamma) \equiv \frac{\gamma^3}{4 - 2\gamma - 2\gamma^2 + 3\gamma^3}$, with $\frac{\partial \beta(\gamma)}{\partial \gamma} > 0$, $\beta(1) = 1$ and $\beta(0) = 0$, i.e., when either products
are too close substitutes or when products are sufficiently close substitutes and firm S’s

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21 The detailed equilibrium analysis is included in Appendix A.

22 For more on this, see e.g., Milliou and Petrakis (2007), Aliprant et al. (2014).
bargaining power is sufficiently low. This is so because, when products are close substitutes and, thus, downstream competition is fierce, firm S’s incentives to behave opportunistic are strong; hence, its "commitment problem" is severe. Given this, when $\beta < \bar{\beta}(\gamma)$, firm 1 has to offer a negative licensing fee to firm S; it has to pay for the licensing agreement.

Respectively, in the entry case, firm S’s outside option also differs from the one in our main model. Specifically, its outside option now includes not only its profits from its own sales in the final goods market, but also its profits from its input sales, at the equilibrium wholesale price, to firm $i$, with $i = 1, 2$. This results in the following equilibrium wholesale prices:

$$w_{i}^{CE} = \frac{a(2 - \gamma)\gamma + c(2 + \gamma)^2}{4 + 6\gamma} > w^{LE}$$

As in the entry case of the main model, we observe that $w^{CE} > c$. Given though that now, firm 1, firm S and firm 2 can behave, through the setting of the wholesale price, as a multi-product monopolist, the equilibrium wholesale price exceeds the one in our main model in which only firm 1 and firm S could behave as a multi-product firm.

**Proposition 6** When both firm 1 and firm 2 source the input from firm S under licensing,

(i) in the no entry case, firm 1 has incentives to license its input technology if and only if $\gamma < 0.891638$ and $\beta > \beta_1(\gamma)$, with $\frac{\partial \beta_1}{\partial \gamma} > 0$, $\beta_1(0.891638) = 1$ and $\beta_1(0) = 0$, and $\beta_1(\gamma) > \bar{\beta}(\gamma)$ for all values of $\gamma$,

(ii) in the entry case, firm 1 always has incentives to license its input technology,

(iii) entry always reinforces firm 1’s licensing incentives,

(iv) licensing always has a positive impact on consumers’ surplus and on total welfare and its positive impact is higher with entry than without entry if and only if products are sufficiently differentiated.

Proposition 6(i) informs us that when both firms 1 and 2 source the input from firm S in the licensing and no entry case, firm 1 licenses its input technology only when the final products are not too close substitutes and the bargaining power of firm S is sufficiently high. This clearly contrasts with what happens when only firm 1 sources the input from firm S under licensing, where, as we saw in Proposition 1, firm 1 always licenses its input technology. This difference is due to the fact that, as we mentioned above, in the case of the common input supplier, if either products are too close substitutes or products are sufficiently close substitutes and firm S’s bargaining power is sufficiently low, firm 1 has
to pay firm $S$ for the licensing agreement. Moreover, when products are sufficiently close substitutes, both firms 1 and 2 enjoy great subsidization under licensing.

According to Proposition 6(ii), in the entry case, firm 1 always opts for licensing when it shares its input supplier with its rival. This happens because, similarly to the main model, first, the setting of a higher wholesale price decreases the negative impact of competition intensity effect and, second, the entry of firm $S$ into the final goods market gives rise to both the market expansion effect and the business stealing effect.

As Proposition 6(iii) informs us, the entry of firm $S$ into the final goods market has again a positive impact on the licensing incentives. This is again due to the market expansion effect and the business stealing effect which increase firm 1’s profits as well as due to the inverse input pricing effect which mitigates the negative impact of competition intensity effect.

In the common input supplier case too, licensing is welfare-enhancing in the no entry case due to input pricing effect and in the entry case due to market expansion effect. However, now licensing with entry is beneficial both for the consumers and total welfare relative to licensing without entry if and only if product differentiation is sufficiently high. This is so, because in the entry case when product differentiation is low, the market expansion effect is weak and, thus it does not outweigh the lower efficiency of both firm 1 and firm 2 due to the higher wholesale prices under entry.

**Corollary 2** (i) In the no entry case, firm 1’s licensing incentives are stronger when both firm 1 and firm 2 source the input from firm $S$ under licensing than when only firm 1 sources the input from firm $S$ if and only if $\gamma < 0.881239$ and $\beta > \beta_2(\gamma)$, with $\frac{\partial \beta_2}{\partial \gamma} > 0$, $\beta_2(0.881239) = 1$ and $\beta_2(0) = 0$.

(ii) In the entry case, firm 1’s licensing incentives are stronger when both firm 1 and firm 2 source the input from firm $S$ under licensing than when only firm 1 sources the input from firm $S$.

In the no entry case, firm 1 can have stronger licensing incentives when both firm 1 and firm 2 source the input from firm $S$ than when only firm 1 sources the input from firm $S$. This holds when products are not close substitutes or when products are sufficiently close substitutes and firm $S$’s bargaining power is sufficiently high. Intuitively, when both firms source their inputs from firm $S$, not only firm 1 but also firm 2 benefits from the input pricing effect. Firm 1, in turn, extracts a share of the increased profits of firm 2 via the
fixed licensing fee. When, however, the products are close substitutes, firm 1 has to pay firm S for the licensing agreement. This fact along with firm 2’s greater efficiency weaken then firm 1’s licensing incentives.

In the entry case, firm 1 always has stronger licensing incentives when both firms 1 and 2 source the input from firm S than when only firm 1 sources the input from firm S. This happens because when both firms share their input supplier, then all the firms in the market manage to behave as a multi-product monopolist. When instead firm 1 alone sources the input from firm S, only firms 1 and S can behave as a multiproduct firm competing with firm 2. Thus, the competition intensity effect is weaker in the common input supplier case. Moreover, the business stealing effect is greater then: firm 2 loses a greater part of its market share that in turn increases firm S’s market share even more in the common input supplier case. The greater business stealing effect is due to the greater inverse input pricing effect.

Finally, in the no entry case, both the consumers’ welfare and total welfare are greater when both firms 1 and 2 source the input from firm S than when only firm 1 sources the input from firm S. This is due to the greater input pricing effect that occurs in the former case and, thus, to the fact that both firms enjoy higher efficiency. However, in the entry case, when both firm 1 and firm 2 source the input from firm S, the greater inverse input pricing effect along with the weaker market expansion effect, have as a result the consumers’ surplus and the total welfare to be lower than the respective ones when only firm 1 sources the input from firm S.

6 Extensions - Discussion

In this section, we discuss briefly a number of further extensions of our main model to extract some additional insights.

(i) Wholesale Price Contract
In our main model, we have assumed that input trading occurs through a two-part tariff. We consider here what would happen if, instead, it occurred through a wholesale price contract. For simplification reasons, we assume that firm S makes a take-it-or-leave-it offer to firm 1 regarding \( w \); i.e., there is no bargaining. Our main conclusion is included in the following Proposition.

\(^{23}\)As we discuss later on, the inclusion of bargaining would not affect our main results.
Proposition 7 When vertical trading takes place through a wholesale price contract, firm 1 has incentives to license its input technology if and only if firm S enters into the final goods market.

When firms trade through a wholesale price contract, licensing does not arise in equilibrium when it is not accompanied by entry. Intuitively, under trading through a wholesale price contract, firm S is not in position to extract part of firm 1’s profits through the fixed fee and there is double marginalization both with and without entry, \( \tilde{w}^{LN} > c \) and \( \tilde{w}^{LE} > c \). Therefore, under trading through a wholesale price, there is a inverse input pricing effect in the no entry case too. As a result, the licensor is less efficient with licensing. Its lower efficiency translates into lower profits with licensing in the no entry case. The licensing incentives are restored when a licensing fee is optimally charged and entry occurs mainly because of the market expansion effect which is in place under a wholesale price contract too.\(^{24}\) In fact, the market expansion effect results also in a greater consumer and total welfare under licensing with entry than under no licensing.

On the basis of the above, we can draw two conclusions. First, the contract type used in input trading can be crucial for the licensing incentives: when the licensee enters into the final goods market, licensing arises in equilibrium when firms trade through a two-part tariff contract, but not when they trade through a wholesale price contract. And second, when input trading takes place through a wholesale price contract, the licensee’s entry not only does not discourage vertical licensing, but, in fact, it constitutes its driving force.

(ii) Ex-ante Monopoly

We have assumed throughout our analysis that there are initially two vertically integrated incumbents in the market. One might wonder whether this assumption is innocuous. In order to examine this, we consider here the alternative case in which only firm 1 is initially in the market - it is an ex-ante monopolist.

In the no entry case now, the licensee, in contrast to our main analysis, does not subsidize the production of firm 1. This occurs because firm 1 does not face any competition, and thus, firm S has no reason to increase its aggressiveness in the final goods market. An implication of this is that firm 1 and firm S operate as a vertically integrated firm; hence,\(^{24}\)If we included bargaining over \( w \), the equilibrium wholesale prices would be lower than \( \tilde{w}^{LN} \) and \( \tilde{w}^{LE} \). In fact, they would be decreasing with the bargaining power of firm 1. Clearly, this means, that the licensing incentives in the entry case would not only be present but that they would be even stronger then. Moreover, firm 1 again would not have licensing incentives in the no entry case.
they operate as in the benchmark case of no licensing. This means that firm 1 is now indifferent between licensing without entry and no licensing.

In the entry case, there are two important differences relative to our main analysis. The first is that the competition intensity effect is stronger because firm S’s entry into the final goods market transforms the latter from a monopoly to a duopoly. The second is that the business stealing effect is absent since firm 2 does not exist in the final goods market. In light of these, one would expect that firm 1 does not opt for licensing now. This is not so though; firm 1 opts for licensing even when licensing causes the loss of its monopoly status.

**Proposition 8** When firm 1 is initially a monopolist in the market, it has incentives to license its input technology if firm S enters into the final goods market. Otherwise, it is indifferent between licensing and no licensing.

The presence of licensing incentives, which is more striking here than in our main analysis, is driven exclusively by the market expansion effect and, thus, by the positive impact that entry has on the size of the final goods market. Furthermore, the market expansion effect is also the driving force for the positive impact that licensing under entry has on both the consumer and total welfare.

Summing up, the market structure can influence the licensing incentives: when licensing does not trigger entry, licensing is more likely to occur when the licensor is not a monopolist. In addition, the entry of the licensee into the licensor’s market encourages instead of discourages the licensing incentives even though the licensor seizes to be monopolist.

**(iii) Cost Asymmetry**

We have assumed throughout our analysis that the two incumbents are symmetric. One might wonder what would happen if the two firms had asymmetric costs, and in particular, if the licensor had a cost advantage relative to the other incumbent.

In order to examine this, we assume now that the marginal costs that firm 1 and firm 2 face initially are, respectively, $c$ and $c_2$, with $c < c_2$. Under this assumption, we find that the results of our main model are qualitatively similar. Moreover, we find that the licensing incentives, both under entry and no entry, are stronger when the licensor enjoys a cost-advantage than when it does not as well as that the positive impact of licensing on consumer and total welfare is larger in the former case. This occurs because when the licensor enjoys a cost advantage, then both the input pricing effect and the market expansion effect are stronger while the competition intensity effect is weaker.
(iv) Bargaining over the Licensing Fee

In our main model, the licensor makes a take-it-or-leave-it offer over the fixed licensing fee. We consider here what would happen if the licensor bargained with the licensee not only over the input’s trading terms but also over the licensing fee. To do so, we assume that the firms bargain in a Nash bargaining fashion in the first stage of the game over \( F \) and that they bargaining powers are the same as the ones in their negotiations over the input’s terms of trade.

When the bargaining power of firm \( S \) is positive, firm 1 is not in the position to extract all of firm \( S \)’s profits through \( F \). Still, even in this case, firm 1 always has incentives to license its input technology both with and without entry. This holds because during the negotiations over \( F \), firm \( S \) has to compensate firm 1 for the profits that the latter makes without licensing - otherwise, firm 1 would not offer a licensing agreement to firm \( S \) and the latter would make zero profits. Because of this, in the extreme case in which firm 1 has no bargaining power (\( \beta = 1 \)), it is indifferent among licensing and no licensing. In all other cases, it prefers licensing to no licensing and the higher is its bargaining power the more it prefers licensing.

The allocation of the licensing fee does not affect consumer and total welfare. Therefore, the conclusions of our main analysis regarding the impact of licensing on them remain unchanged in the presence of bargaining.

(v) Licensing through Royalty

One might wonder what would happen if licensing took place though a per-unit of output royalty, \( r \geq 0 \), instead of through a fixed licensing fee.

Clearly, when a royalty is used, firm 1 is not able to extract all the potential profits of the licensee. Moreover, firm \( S \)’s marginal cost is \( r + c \) instead of \( c \). The increase in the input supplier’s marginal cost can translate into worse input sourcing terms for firm 1 and, thus, into lower firm 1’s profits from its own sales in the final goods market. Not surprisingly thus, in the no entry case, firm 1 always optimally sets \( r^N = 0 \). Given this, does it have incentives to license its technology? The answer to this question is already provided in our main analysis. More specifically, recall that when \( F = 0 \), firm 1 opts for licensing if and only if its bargaining power is sufficiently high and the licensee does not enter into the final goods market.

In the entry case, when the royalty is imposed only on firm 1’s output, then again
When the latter occurs, then, in contrast to what happens when licensing takes place through a fixed fee, the entry of the licensee into the final goods market discourages licensing incentives - licensing does not arise in equilibrium. This is so mainly for two reasons. First, because the *market expansion effect* is weaker when $r^E > 0$. And second, because since now firm 1 does not manage to extract firm S’s profits, it is not in the position to take full advantage of the *market expansion effect*.

It follows from the above that the form of the licensing contract, whether it is a fixed licensing fee or a variable royalty, can affect the licensing incentives. It also follows that when licensing is through a royalty, entry can discourage instead of encourage licensing. This is in sharp contrast with our main conclusion. Still, we should mention that firm 1 is better off under licensing through a fixed licensing fee than under licensing through a royalty; firm 1 would choose to license its technology through a fixed licensing fee and not through a royalty. Actually, firm 1’s choice would be aligned with the interest of consumers and the economy as a whole since the positive impact of licensing on consumer and total welfare is even larger when licensing takes place through a fixed licensing fee rather than through a royalty. Stated differently, not only from the licensor’s viewpoint but also from a welfare viewpoint, vertical licensing is preferable when it occurs through a fixed licensing fee.

*(vi) Downstream Price Competition*

Next, we discuss what happens if the firms compete in prices in the final products market instead of in quantities.

As it is well known from the literature, prices, in contrast to quantities, are strategic complements. Because of this, under downstream price competition, in the no entry case, firm S does not subsidize firm 1’s final production via the wholesale price; it does not wish its downstream partner to behave aggressively in the final market competition - it charges a wholesale price that exceeds its marginal cost. Still, similarly to our main model, firm 1 always has incentives to license its input technology. In fact, it has again incentives to license its technology even for free if the bargaining power of firm S is low enough. Why is that? As the literature on vertical separation (e.g., Bonanno and Vickers, 1988, Lin, 1988, Gal-Or, 1990, Cyrenne, 1994) has demonstrated, vertical separation (vertical licensing in our case) dampens downstream competition, leading to higher downstream profits that the
upstream firm (the downstream firm in our case) extracts through the fixed fee.

In the entry case now, firm 1 has incentives to license its input technology, but not always. It has incentives to do so if and only if the final products are not too close substitutes (if and only if $\gamma < 0.9793$). Its licensing incentives are driven now by the market expansion effect alone; the business stealing effect is absent under price competition. Moreover, competition in the market is fiercer under price competition than under quantity competition. Hence, the competition intensity effect of licensing is stronger in the former case and dominates its market expansion effect when products tend to be homogeneous since then downstream competition is already quite fierce. Clearly, this implies that in markets in which firms’ products are quite similar, vertical licensing, when it is accompanied by entry, is more likely to be observed when firms compete in quantities than when they compete in prices.

It follows that the impact of entry on the licensing incentives is positive under downstream price competition too, but only as long as the final products are not too close substitutes (in particular, if and only if $\gamma < 0.8801$). Licensing continues to be welfare-enhancing under downstream price competition but only when it triggers entry. In the no entry case, licensing is now welfare detrimental due to the decrease that it causes (through the input price) in the licensor’s efficiency.

7 Concluding Remarks

We have examined the incentives of a vertically integrated incumbent to license its input technology to an external firm. We have done so in a setting in which after the signing of the licensing agreement, the licensor sources the input from the licensee and the latter can enter into the final goods market and compete with the licensor.

We have shown that licensing emerges in equilibrium not only when the licensee does not enter into direct competition with the licensor, but also when it enters. In fact, we have shown that in the latter case, although market competition is more intense, the licensing incentives are stronger. Intuitively, in the absence of entry, vertical licensing is motivated by the low input price at which the licensor sources the input from the licensee. The low input price translates into higher efficiency and, thus, into larger market share and profits for the licensor.

When, instead, the licensee enters into the final goods market, first, the licensor is less efficient - it pays a high input price - and second, it competes with more firms than in the no
licensing case. Still, the licensor opts for licensing because the entry of the licensee results in the expansion of the final goods market and in business stealing from the rival incumbent firm. The licensor takes full advantage of these effects by extracting the resulting profits of the licensee through the licensing fee. So, the entry of the licensee encourages instead of discourages the licensing incentives even when licensing intensifies market competition.

Our welfare analysis has revealed, first, that vertical licensing is always beneficial both for the consumers and for the economy as a whole, and second, that it is even more beneficial in the entry case. In particular, it indicates that vertical licensing, by triggering entry into more than one stages of a vertical chain, it can also generate more intense product market competition.

Extending our main analysis in various directions, we have shown that in many instances the emergence of licensing in equilibrium would be impossible without the entry of the licensee in the final goods market. This is, for instance, the case when the licensor and the licensee trade via a wholesale price contract or when the licensor is initially the only firm in the market. Moreover, the entry of the licensee reinforces licensing incentives even when the rival firm sources its input by the same supplier as well as when downstream competition takes place in prices. However, the entry of the licensee seems to obstruct licensing when the latter takes place through royalties.

Summing up, we have provided an explanation for the commonly observed practice of vertical licensing in markets where licensing can transform the licensee from an input supplier to a direct competitor of the licensor. Our explanation lies on strategic considerations and not on exogenously assumed efficiencies of the licensee. Clearly, if we had assumed that the licensee is either more efficient in input production than the licensor or that the input production is characterized by economies of scale, then vertical licensing and the positive impact of entry on the licensing incentives would have been much less surprising than in our setting.

Still, we should mention that our analysis is just a first step in the direction of understanding licensing in vertically related markets. In future work, we plan to extend our analysis by endogenizing firms’ investments in their input production technology and examining how they could affect the licensing incentives.
8 Appendix A

Table 1: Equilibrium Values in the Benchmark Case and in the Licensing with No Entry

<table>
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<th>Case</th>
<th>Table 1: Equilibrium Values in the Benchmark Case and in the Licensing with No Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_1^B = q_2^B = \frac{a-c}{2+\gamma} ); ( \pi_1^B = \pi_2^B = \frac{(a-c)^2}{(2+\gamma)^3} )</td>
<td>( \frac{\pi_1^B}{q_1^B} - T_{LN} = \frac{1-\beta(a-c)^2}{2(2-\gamma)^2} )</td>
</tr>
<tr>
<td>( q_1^{LN} = \frac{(a-c)(2-\gamma)}{2(2-\gamma^2)} ); ( q_2^{LN} = \frac{(a-c)(4-\gamma)(2+\gamma)}{4(2-\gamma)^2} )</td>
<td>( \frac{\pi_1^{LN}}{\pi_2^{LN}} = \frac{(a-c)(4-\gamma)(2+\gamma)^2}{16(2-\gamma)^2} )</td>
</tr>
<tr>
<td>( \pi_1^{LN} = \frac{(a-c)(4-\gamma)(2+\gamma)^2}{8(2-\gamma)^2} )</td>
<td>( \pi_2^{LN} = \frac{(a-c)(4-\gamma)(2+\gamma)^2}{8(2-\gamma)^2} )</td>
</tr>
</tbody>
</table>

Table 2: Equilibrium Values in the Licensing with Entry Case

<table>
<thead>
<tr>
<th>Case</th>
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</tr>
</thead>
<tbody>
<tr>
<td>( q_1^{LE} = \frac{(a-c)(4-2\gamma-\gamma^2)}{4(2+2\gamma-2\gamma^2-\gamma^3)} )</td>
<td>( \pi_1^{LE} = \frac{(a-c)^2}{16(2+2\gamma-2\gamma^2-\gamma^3)^2} )</td>
</tr>
<tr>
<td>( q_2^{LE} = q_S^{LE} = \frac{(a-c)(4-3\gamma)}{4(2+2\gamma-2\gamma^2-\gamma^3)} )</td>
<td>( F^{LE} = \frac{(a-c)^2[16(1+\gamma)-8\gamma^2(2+\gamma)+\beta(4-2\gamma-\gamma^2)^2]}{8(2+\gamma)^2(2+2\gamma-2\gamma^2-\gamma^3)^2} )</td>
</tr>
<tr>
<td>( \pi_2^{LE} = \frac{(a-c)^2}{16(2+2\gamma-2\gamma^2-\gamma^3)^2} )</td>
<td>( \pi_1^{LE} = \frac{(a-c)^2}{8(2+2\gamma-2\gamma^2-\gamma^3)^2} )</td>
</tr>
</tbody>
</table>

Table 3: Equilibrium Values in the Licensing with Entry and Wholesale Price Contract

<table>
<thead>
<tr>
<th>Case</th>
<th>Table 3: Equilibrium Values in the Licensing with Entry and Wholesale Price Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{q}_1^{LE} = \frac{(a-c)(4-2\gamma-\gamma^2)}{2(8+8\gamma-3\gamma^2-2\gamma^3)} )</td>
<td>( \hat{\pi}_1^{LE} = \frac{(a-c)^2}{4(8+8\gamma-3\gamma^2-2\gamma^3)^2} )</td>
</tr>
<tr>
<td>( \hat{q}_2^{LE} = \hat{q}_S^{LE} = \frac{(a-c)(2-\gamma)(4+3\gamma^2)}{2(8+8\gamma-3\gamma^2-2\gamma^3)} )</td>
<td>( \hat{\pi}_2^{LE} = \frac{(a-c)^2}{4(8+8\gamma-3\gamma^2-2\gamma^3)^2} )</td>
</tr>
<tr>
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</tr>
<tr>
<td>( \hat{\pi}_2^{LE} = \frac{(a-c)^2}{4(8+8\gamma-3\gamma^2-2\gamma^3)^2} )</td>
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</tr>
</tbody>
</table>

8.1 Common Input Supplier

(a) No Entry

After each firm \( i \) chooses its quantity in order to maximize its profits: \( \pi_i(q_i, q_j, w_i) = (a - q_i - \gamma q_j)q_i - w_i q_i \) with \( i, j = 1, 2 \) and \( i \neq j \), we obtain the equilibrium quantities for
given levels of input prices:

\[ q_i(w_i, w_j) = \frac{a(2 - \gamma) - 2w_i + \gamma w_j}{4 - \gamma^2}. \]  

Letting \( w_{j}^{CN} \) denote the equilibrium outcome of the negotiations of firm \( S \) and firm \( j \), \( w_i \) is chosen to maximize the generalized Nash bargaining product:

\[
\max_{w_i,T_i} [\pi_S(w_i, w_{j}^{CN}) + T_i + T_j - d_{S\!i}(w_{j}^{CN}, T_{j}^{CN})]^{\beta} [\pi_i(w_i, w_{j}^{CN}) - T_i]^{1-\beta},
\]

where \( \pi_S(w_i, w_{j}^{CN}) = (w_i - c)q_i(w_i, w_{j}^{CN}) + (w_{j}^{CN} - c)q_j(w_i, w_{j}^{CN}) \) and \( \pi_i(w_i, w_{j}^{CN}) = \pi_i(q_i(w_i, w_{j}^{CN}), q_j(w_i, w_{j}^{CN}), w_i, w_{j}^{CN}) \). Note that firm \( S \)'s profits arise from sales to two instead of one final good producer. We notice here that firm \( S \) has an outside option, which means that its disagreement payoff is no longer null. In particular, if an agreement between firm \( S \) and firm \( i \) is not reached, then firm \( S \)'s disagreement payoff is given by \( d_{S\!i}(w_{j}^{CN}, T_{j}^{CN}) = (w_{j}^{CN} - c)q_{j}^{\text{mon}}(w_{j}^{CN}) + T_{j}^{CN} \), where \( q_{j}^{\text{mon}}(w_{j}^{CN}) = (\alpha - w_{j}^{CN})/2 \) is the quantity expected to be produced by the monopolist final good producer which faces an input price \( w_{j}^{CN} \). In other words, in case of disagreement with one of the final good producers, firm \( S \) is expected to receive from the remaining firm in the final goods market the equilibrium fixed fee \( T_{j}^{CN} \) plus the revenues from input sales at the equilibrium wholesale price \( w_{j}^{CN} \).

From the first order conditions of (18) we obtain the equilibrium wholesale price and the respective net equilibrium profits:

\[ w^{CN} = w_{i}^{CN} = w_{j}^{CN} = c - \frac{\gamma^2(a - c)}{2(2 - \gamma^2)}; \quad (19) \]

\[ \pi_{1}^{CN} = \frac{(a - c)^2(2 - \gamma)((1 + \beta)(4 - 2\gamma - 2\gamma^2) - \gamma^3(1 - \beta))}{8(2 - \gamma^2)^2}; \quad (20) \]

\[ \pi_{2}^{CN} = \frac{(1 - \beta)(a - c)^2(2 - \gamma)^2}{8(2 - \gamma^2)}. \quad (21) \]

(b) Entry

After each firm \( i \) chooses its quantity in order to maximize its profits: \( \pi_i(q_i, q_j, q_S, w_i, w_j) = (a - q_i - \gamma q_j - \gamma q_S)q_i - w_i q_i \), with \( i, j = 1, 2 \) and \( i \neq j \), while firm \( S \) chooses its quantity in order to maximize its profits given by: \( \pi_S(q_i, q_j, q_S, w_i, w_j) = (a - q_S - \gamma q_i - \gamma q_j)q_S - c(q_i + \ldots \).
$q_j + q_S) + w_i q_i + w_j q_j$, we obtain the equilibrium quantities for given levels of input prices:

\[
q_i(w_i, w_j) = \frac{a(2 - \gamma) - (2 + \gamma)w_i + \gamma(c + w_j)}{2(2 - \gamma)(1 + \gamma)}, \tag{22}
\]

\[
q_S(w_i, w_j) = \frac{a(2 - \gamma) - (2 + \gamma)c + \gamma(w_i + w_j)}{2(2 - \gamma)(1 + \gamma)}. \tag{23}
\]

Letting $w_j^{CE}$ denote the equilibrium outcome of the negotiations of firm $S$ and firm $j$, $w_i$ is chosen to maximize the generalized Nash bargaining product:

\[
\max_{w_i, T_i} \left[ \pi_S(w_i, w_j^{CE}) + T_i + T_j - d_{S_2}(w_j^{CE}, T_j^{CE}) \right]^{\beta} \left[ \pi_i(w_i, w_j^{CE}) - T_i \right]^{1-\beta}, \tag{24}
\]

where $\pi_S(w_i, w_j^{CE})$ and $\pi_i(w_i, w_j^{CE})$ are found after substituting $q_S(w_i, w_j)$ and $q_i(w_i, w_j)$ into $\pi_S(q_i, q_j, q_S, w_i, w_j)$ and $\pi_i(q_i, q_j, q_S, w_i, w_j)$. In the entry case firm $S$ has an outside option that differs to the one of the no entry case. In particular, if an agreement between firm $S$ and firm $i$ is not reached, then firm $S$‘s disagreement payoff is given by $d_{S_2}(w_j^{CE}, T_j^{CE}) = (a - q_S^D - \gamma q_j^D(w_j^{CE}) - c)q_S^D + (w_j^{CE} - c)q_j^D(w_j^{CE}) + T_j^{CE}$, where $q_S^D = q_i^D = \frac{a-c}{2+\gamma}$ and $q_j^D(w_j^{CE}) = \frac{a-w_j^{CE}}{2+\gamma}$ are the quantities expected to be produced, respectively, by firm $S$ and firm $j$ which faces an input price $w_j^{CE}$. In other words, in case of disagreement with firm $i$, firm $S$ can still have profits from its own sales in the final goods market, as well as it receives the equilibrium fixed fee $T_j^{CE}$ plus the revenues from input sales at the equilibrium wholesale price $w_j^{CE}$ from the rival remaining firm in the final goods market.

From the first order conditions of (24) we obtain the equilibrium wholesale price and the respective net equilibrium profits:

\[
w^{CE} = w_i^{CE} = w_j^{CE} = \frac{a(2 - \gamma)\gamma + c(2 + \gamma)^2}{4 + 6\gamma}; \tag{25}
\]

\[
\pi_1^{CE} = \frac{(a - c)^2 A}{8(2 - \gamma^2)^2} \text{ and } \pi_2^{CE} = \frac{(1 - \beta)(a - c)^2(16(1 - \gamma - \gamma^2) - 8\gamma^3 + 7\gamma^4 + \gamma^5)}{4(2 + \gamma)^2(2 + 3\gamma)^2} \tag{26}
\]

where $A = 32 + 16\beta + 80\gamma + 16\beta\gamma + 64\gamma^2 - 16\beta\gamma^2 + 8\gamma^3 - 8\beta\gamma^3 - 10\gamma^4 + 7\beta\gamma^4 - \gamma^5 + \beta \gamma^5 > 0$. 

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9 Appendix B

Proof of Proposition 1: Calculating the difference in the profits of firm 1 in the case of licensing and no entry and in the benchmark case, we find:

\[
\pi_1^{LN} - \pi_1^B = \frac{(a - c)^2 \gamma^4}{8(2 + \gamma)^2(2 - \gamma^2)} > 0.
\]

Thus, firm 1 always has incentives to license its input technology under no entry.

Proof of Corollary 1: Calculating the difference in the profits of firm 1 in the case of licensing and no entry without receiving the licensing fee, and in the benchmark case, we find that

\[
\pi_1(q_1^{LN}) - T^{LN} - \pi_1^B = \frac{(a - c)^2(16\beta - 8\beta\gamma^2 - \gamma^4 + \beta\gamma^4)}{8(2 + \gamma)^2(2 - \gamma^2)} > 0 \text{ if and only if } 0 < \beta < \frac{\gamma^4}{16 - 8\gamma^2 + \gamma^4}.
\]

Thus, under no entry firm 1 has incentives to license its input technology for free when its bargaining power is sufficiently high.

Proof of Proposition 2: Calculating the difference in the profits of firm 1 in the case of licensing and entry and in the benchmark case, we find:

\[
\pi_1^{LE} - \pi_1^B = \frac{(a - c)^2(4 - 2\gamma - \gamma^2)^2}{8(2 + \gamma)^2(2 + 2\gamma - 2\gamma^2 - \gamma^3)} > 0.
\]

Therefore, firm 1 always has incentives to license its input technology under entry.

Proof of Proposition 3: Calculating the difference in the profits of firm 1 in the case of licensing with entry and in the case of licensing without entry, we find:

\[
\pi_1^{LE} - \pi_1^{LN} = \frac{(a - c)^2(1 - \gamma)(8 - 8\gamma + 2\gamma^3 - \gamma^4)}{8(2 - \gamma^2)(2 + 2\gamma - 2\gamma^2 - \gamma^3)} > 0.
\]

It follows that firm 1 has stronger incentives to license its input technology with entry than without entry.

Proof of Proposition 4: We differentiate \((\pi_1^{LN} - \pi_1^B)\) and \((\pi_1^{LE} - \pi_1^B)\) in terms of \(\gamma\), respectively:

\[
\frac{\partial (\pi_1^{LN} - \pi_1^B)}{\partial \gamma} = \frac{(a - c)^2 \gamma^3(4 + \gamma - \gamma^2)}{(2 + \gamma)^2(2 - \gamma^2)^2} > 0
\]

\[
\frac{\partial (\pi_1^{LE} - \pi_1^B)}{\partial \gamma} = \frac{(a - c)^2(-4 + 2\gamma + \gamma^2)(48 + 16\gamma - 44\gamma^2 - 14\gamma^3 + 4\gamma^4 + \gamma^5)}{8(2 + \gamma)^2(2 + 2\gamma - 2\gamma^2 - \gamma^3)^2} < 0
\]

It follows that an increase in product differentiation has a negative impact on the licensing incentives under no entry and a positive one under entry.
Proof of Proposition 5: In the benchmark case, the consumers’ surplus is:

$$CS^B = aq_1^B + aq_2^B - \frac{1}{2}[(q_1^B)^2 + (q_2^B)^2 + 2\gamma q_1^B q_2^B] - p_1q_1^B - p_2q_2^B = \frac{(a - c)^2(1 + \gamma)}{(2 + \gamma)^2}.$$  

In the case of licensing with no entry, the consumers’ surplus is:

$$CS^{LN} = aq_1^{LN} + aq_2^{LN} - \frac{1}{2}[(q_1^{LN})^2 + (q_2^{LN})^2 + 2\gamma q_1^{LN} q_2^{LN}] - p_1q_1^{LN} - p_2q_2^{LN} = \frac{(a - c)^232(1 - \gamma^2) + \gamma^3(4 + 5\gamma)}{32(2 - \gamma^2)^2}.$$  

In the case of licensing with entry, the consumers’ surplus is:

$$CS^{LE} = aq_1^{LE} + aq_2^{LE} + aq_s^{LE} - \frac{1}{2}[(q_1^{LE})^2 + (q_2^{LE})^2 + (q_s^{LE})^2] + 2\gamma q_1^{LE} q_2^{LE} + 2\gamma q_1^{LE} q_s^{LE} + 2\gamma q_2^{LE} q_s^{LE}] - p_1q_1^{LE} - p_2q_2^{LE} - psq_s^{LE} = \frac{(a - c)^248 + 80\gamma - 84\gamma^2 - 108\gamma^3 + 43\gamma^4 + 30\gamma^5}{32(2 + 2\gamma - 2\gamma^2 - \gamma^3)}.$$  

Calculating the following differences: $CS^{LN} = CS^B = \frac{(a - c)^2\gamma^2(32 + 16\gamma - 28\gamma^2 - 8\gamma^3 + 5\gamma^4)}{32(2 + \gamma)^2(2 - \gamma)^2}$ and $CS^{LE} = CS^B = \frac{(a - c)^2(4 - 2\gamma - \gamma^2)(16 + 40\gamma - 34\gamma^3 - 7\gamma^4 + 2\gamma^5)}{32(2 + \gamma)^2(2 - \gamma - 2\gamma^2 - \gamma^3)}$, we find that they are always positive.

Total welfare is defined as the sum of consumers’ and producers’ surplus, namely: $W^k = CS^k + \pi_1^k + \pi_2^k + \pi_3^k$, where $k = B$, LN and LE. Calculating the following differences: $W^{LN} - W^B = \frac{(a - c)^2\gamma^2(32 - 16\gamma - 20\gamma^2 + 8\gamma^3 + 3\gamma^4)}{32(2 + \gamma)^2(2 - \gamma)^2}$ and $W^{LE} - W^B = \frac{(a - c)^2(4 - 2\gamma - \gamma^2)(12 + 12\gamma - 11\gamma^2 - 6\gamma^3)}{32(2 + \gamma)^2(2 - \gamma - 2\gamma^2 - \gamma^3)}$, we find that they are always positive. Thus, both the consumers’ surplus and total welfare are greater under licensing than under no licensing.

Moreover, calculating the following differences, $CS^{LE} - CS^{LN} = \frac{(a - c)^2(1 - \gamma)B}{32(2 - \gamma)^2(2 + 2\gamma - 2\gamma^2 - \gamma^3)}$, where $B = 64 + 128\gamma - 144\gamma^2 - 272\gamma^3 + 104\gamma^4 + 200\gamma^5 - 20\gamma^6 - 60\gamma^7 - 7\gamma^8 + 5\gamma^9 > 0$, and $W^{LE} - W^{LN} = \frac{(a - c)^2(1 - \gamma)\Gamma}{32(2 - \gamma)^2(2 + 2\gamma - 2\gamma^2 - \gamma^3)}$, where $\Gamma = 192 - 496\gamma^2 + 80\gamma^3 + 376\gamma^4 - 40\gamma^5 - 124\gamma^6 - 4\gamma^7 + 17\gamma^8 + 3\gamma^9 > 0$, we find that they are always positive and, therefore, both consumers’ surplus and total welfare are greater with entry than without entry.

Proof of Proposition 6:

(i): In the Common Input Supplier case, calculating the difference in the profits of firm 1 in the case of licensing without entry and in the benchmark case, we find:

$$\pi_1^{CN} - \pi_1^B = \frac{(a - c)^2(32\beta - 32\beta\gamma^2 - 16\gamma^3 - 6\gamma^4 + 10\beta\gamma^4 - 4\gamma^5 - \gamma^6 + \beta\gamma^6)}{8(2 + \gamma)^2(-2 + \gamma^2)^2} > 0,$$
if and only if \( \gamma < 0.891638 \) and \( \beta > \beta_1(\gamma) = \frac{-16\gamma^3 - 6\gamma^4 + 4\gamma^5 + \gamma^6}{-32 + 32\gamma^2 - 10\gamma^4 - \gamma^6} \), with \( \frac{\theta_2}{\theta_1} > 0, \beta_1(0.891638) = 1 \) and \( \beta_1(0) = 0 \), and \( \beta_1(\gamma) > \frac{1}{3}\beta(\gamma) \) for all values of \( \gamma \). Namely, firm 1 does not have always incentives for licensing without entry.

(ii): In the Common Input Supplier case, calculating the difference in the profits of firm 1 in the case of licensing with entry and in the benchmark case, we find:

\[
\pi^{CE}_{1} - \pi^{B}_{1} = \frac{(a - c)^2 \Delta}{4(2 + \gamma)^2(2 + 3\gamma)^2} > 0,
\]

where \( \Delta = 16 + 16\beta + 32\gamma + 16\beta\gamma + 28\gamma^2 - 16\beta\gamma^2 + 8\gamma^3 - 8\beta\gamma^3 - 10\gamma^4 + 7\beta\gamma^4 - \gamma^5 + \beta\gamma^5 > 0 \). Therefore, firm 1 always has incentives to license its input technology under entry.

(iii): In the Common Input Supplier case, calculating the difference in the profits of firm 1 in the case of licensing with entry and in the case of licensing without entry, we find:

\[
\pi^{CE}_{1} - \pi^{CN}_{1} = \frac{(a - c)^2 E}{8(2 + \gamma^2)(2 + 3\gamma^2)(-2 + \gamma^2)^2} > 0,
\]

where \( E = 128 + 256\gamma - 256\beta\gamma + 96\gamma^2 - 416\beta\gamma^2 - 128\gamma^3 + 192\beta\gamma^3 - 56\gamma^4 + 464\beta\gamma^4 + 192\gamma^5 - 16\beta\gamma^5 + 138\gamma^6 - 174\beta\gamma^6 - 24\gamma^7 - 12\beta\gamma^7 - 29\gamma^8 + 23\beta\gamma^8 - 2\gamma^9 + 2\beta\gamma^9 > 0 \). It follows that firm 1 has stronger incentives to license its input technology with entry than without entry.

(iv): In the Common Input Supplier case, in the case of licensing with entry and in the case of licensing without entry, the consumers’ surplus are given by:

\[
CS^{CN} = \frac{(a - c)^2 (2 - \gamma)^2 (1 + \gamma)}{4(-2 + \gamma^2)^2} \quad \text{and} \quad CS^{CE} = \frac{(a - c)^2 (12 + \gamma(28 + 9\gamma))}{8(2 + 3\gamma)^2}.
\]

Calculating the differences in consumers’ surplus between licensing and no licensing, namely, \( CS^{CN} - CS^{B} = \frac{(a - c)^2 \gamma^2 (1 + \gamma)(8 - 9\gamma^2)}{4(2 + \gamma)^2(-2 + \gamma^2)^2} \) and \( CS^{CE} - CS^{B} = \frac{(a - c)^2 (16 + 32\gamma - 8\gamma^2 - 8\gamma^3 + 9\gamma^4)}{8(2 + \gamma)^2(2 + 3\gamma)^2} \), we find that they are always positive.

Total welfare is defined as the sum of consumers and producers’ surplus, namely:

\[
W^{k} = CS^{k} + \pi^{k}_{1} + \pi^{k}_{2} + \pi^{k}_{S}, \quad \text{where} \ k = B, \ CN \ \text{and} \ CE.
\]

Calculating the following differences, \( W^{CN} - W^{B} = \frac{(a - c)^2 \gamma^2 (8 - 5\gamma^2 - \gamma^3)}{4(2 + \gamma)^2(-2 + \gamma^2)^2} \) and \( W^{CE} - W^{B} = \frac{(a - c)^2 (48 + 32\gamma - 56\gamma^2 - 8\gamma^3 + 3\gamma^4)}{8(2 + \gamma)^2(2 + 3\gamma)^2} \), we find that they are always positive. Thus, both the consumers’ surplus and total welfare are greater under licensing than under no licensing.

Calculating the following differences, we find \( CS^{CE} - CS^{CN} = \frac{(a - c)^2 Z}{8(2 + 3\gamma)^2(-2 + \gamma^2)^2} > 0 \) where \( Z = 16 + 16\gamma - 60\gamma^2 - 48\gamma^3 + 6\gamma^4 + 10\gamma^5 + 9\gamma^6 > 0 \) if and only if \( \gamma < 0.54958 \)
\[ W^{CE} - W^{CN} = \frac{(a-c)^2 H}{8(2+3\gamma)^2(-2+\gamma^2)} > 0 \] where \( H = 48-16\gamma-116\gamma^2+32\gamma^3+42\gamma^4-2\gamma^5+3\gamma^6 > 0 \) if and only if \( \gamma < 0.714414 \). Thus, both consumers’ surplus and total welfare are higher with entry than without entry if and only if products are sufficiently differentiated.

**Proof of Corollary 2:**

(i): Calculating the difference in the profits of firm 1 in the case of licensing and no entry when firm 1 and firm 2 source the input from firm S with the case when only firm 1 sources the input from firm S, we find that, \( \pi_1^{CN} - \pi_1^{LN} = \frac{(a-c)^2(2-\gamma)}{8(2+\gamma)^2} > 0 \) where \( \Theta = 4\beta-2\beta\gamma-2\beta\gamma^2-2\gamma^3 + \beta\gamma^3 > 0 \) if and only if \( \gamma < 0.881239 \) and \( \beta > \beta_2(\gamma) = \frac{2\gamma^3}{4-2\gamma-2\gamma^2+\gamma^3} \), with \( \frac{6\beta_2}{\pi_1} > 0, \beta_2(0.881239) = 1 \) and \( \beta_2(0) = 0 \). Thus, under no entry firm 1 has stronger licensing incentives when both firm 1 and firm 2 source the input from firm S than when only firm 1 sources the input from firm S if the products are not close substitutes or when products are sufficiently close substitutes and firm S’s bargaining power is sufficiently high.

(ii): Calculating the difference in the profits of firm 1 in the case of licensing and entry when firm 1 and firm 2 source the input from firm S with the case when only firm 1 sources the input from firm S, we find that, \( \pi_1^{CE} - \pi_1^{LE} = \frac{(a-c)^2 I}{8(2+\gamma)^2(2+3\gamma)^2(2+2\gamma-2\gamma^2+\gamma^3)} > 0 \) where \( I = 64\beta + 64\gamma + 128\beta\gamma + 240\gamma^2 - 64\beta\gamma^2 + 160\gamma^3 - 192\beta\gamma^3 - 200\gamma^4 + 28\beta\gamma^4 - 180\gamma^5 + 96\beta\gamma^5 + 11\gamma^6 - 8\beta\gamma^6 + 24\gamma^7 - 18\beta\gamma^7 + 2\gamma^8 - 2\beta\gamma^8 > 0 \). Thus, under entry firm 1 always has stronger licensing incentives when both firm 1 and firm 2 source the input from firm S than when only firm 1 sources the input from firm S.

**Proof of Proposition 7:** Calculating the difference in the profits of firm 1 in the case of licensing with entry and in the case of licensing without entry, we find:

\[ \tilde{\pi}_1^{LE} - \tilde{\pi}_1^{LN} = \frac{(a-c)^2 K}{8(2+\gamma)^2(8+8\gamma-3\gamma^2+2\gamma^3)^2} > 0, \]

where \( K = 512 + 256\gamma - 448\gamma^2 - 32\gamma^3 + 170\gamma^4 - 32\gamma^5 - 35\gamma^6 + 8\gamma^7 + 4\gamma^8 > 0 \). Therefore, firm 1 always has stronger incentives to license its input technology with entry than without entry.

**Proof of Proposition 8:** Calculating the difference in the profits of firm 1 in the case of licensing with entry and in the benchmark case, when firm 1 is initially a monopolist, we find:

\[ \tilde{\pi}_1^{LE} - \tilde{\pi}_1^{B} = \frac{(a-c)^2(1-\gamma)^2}{4-3\gamma^2} > 0. \]

Therefore, firm 1 has always incentives to license its input technology under entry.
Calculating the difference in the profits of firm 1 in the case of licensing without entry and in the benchmark case, we observe that $\pi_1^{LN} - \pi_1^B = 0$, namely, firm 1 is indifferent between licensing without entry and no licensing.

10 References


